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Why A is Usually 90, B is 80, etc.: A Possible Explanation

Francisco Zapata[1], Olga Kosheleva[2], Vladik Kreinovich[3,*]

[1] *Department of Industrial, Manufacturing, and Systems Engineering*
University of Texas at El Paso, 500 W. University, El Paso, Texas 79968, USA

[2] *Department of Teacher Education, University of Texas at El Paso*
500 W. University, El Paso, Texas 79968, USA

[3] *Department of Computer Science, University of Texas at El Paso*
500 W. University, El Paso, Texas 79968, USA

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Abstract

In the US education system, instructors typically use 90/100, 80/100, 70/100, and 60/100 thresholds to gauge students' knowledge: students who get 90 or more points out of 100 get the highest grade of A, students whose grades are in between 80 and 90 get a B, followed by C, D, and F (fail). In this paper, we show that these seemingly arbitrary threshold have a natural explanation: A means that a student can solve almost all problems; C means that two students with this level of knowledge can solve almost all problems when working together; D means that we need at least three such students; and B means that, working jointly with a D student, they can solve almost all problems.

Keywords: US grading system, seven plus minus two law

1 Formulation of the Problem

Letter grades: the usual practice. In the US, we usually gauge the student's knowledge in a course by a number from 0 to 100 – the number of points that students accumulate through different assignments and exams.

- Theoretically, this numerical grade could be used to evaluate the student's readiness for next classes or for graduate school, or for a company interested in hiring to evaluate how well the student is prepared for the corresponding job.
- However, the numerical grade carries too much information, and in most decision making situations, we do not have time to process such a detailed information.

As a result, what goes into a student's transcript is a *letter grade* that compresses all possible numerical values into five categories:

- A (excellent),
- B (good),
- C (satisfactory),
- D (borderline), and
- F (fail).

In principle, different schools (and even different instructors in the same school) use different thresholds for these grades. However, mostly, the following thresholds are being followed:

*Corresponding author.
Emails: fazg74@gmail.com (F. Zapata), olgak@utep.edu (O. Kosheleva), vladik@utep.edu (V. Kreinovich).

- 90 and above is A,
- 80 and above (but lower than 90) is B,
- 70 and above is C,
- 60 and above is D, and
- below 60 is F.

Why? The fact that in spite of all the experimentation, most instructors continue using the standard thresholds, means that this 5-letter system with these thresholds is probably the most empirically efficient. A natural question is: Why?

- Why 5 letters and not 4 or not 6?
- Why these thresholds?

In this paper, we provide a possible explanation for this.

2 Seven Plus Minus Two Law: A Brief Reminder

Reminder. There is a known phenomenon in psychology called a 7 ± 2 law (see, e.g., [1, 2]), according to which each person can directly keep in mind only a certain number of classes. Depending on the person, this number ranges from $7 - 2 = 5$ to $7 + 2 = 9$ elements.

This explains why there are 5 letter grades. We want a classification which everyone can easily handle.

Since some people can only handle 5 different classes, this means that we can easily have at most 5 different letter grades.

On the other hand, we want the classification into letter grades to be as informative as possible, i.e., have as many different classes as possible. This means that we should select the largest possible number of different letter grades – which is 5.

But why 90, 80, etc.? The above simple explanation is natural and probably not new. Interestingly, it turns out that the same seven plus minus two law can also explain the standard 90, 80, etc., thresholds – we just need to have a somewhat deeper analysis than what we had in this section.

3 Why 90, 80, 70, etc.: An Explanation

What do grades mean. In general, a grade of 90 means that the student correctly solved 90% of the problems. This means that when this student graduates and start working, he or she will correctly solve the problems in 90% of the cases.

Why 90: a possible explanation. We want a student who, after graduation, would be able to solve the vast majority of the corresponding problems. In the first approximation, we expect this student to solve all the problems – not literally all, of course, more like "all" in phrases like "all birds fly".

In other word, it means that while there may be cases when the student will not be able to solve this or that problem, such cases would not be much noticed in the big picture.

In the first approximation, due to the 7 ± 2 law, any proportion which is smaller than $1/n$, where n is from 5 to 9, will not be noticed.

- Some people will not notice if this proportion is smaller than 1/5.
- However, the slightly-smaller-than 1/5 proportion will be noticed by someone for whom this number n is greater.

The only way to guarantee that the proportion will not be noticed is when this proportion will be smaller than 1/9. This proportion is $\approx 11\%$. This means that the student would solve more than 89% of the problem, which translated into having more than 89 points.

This means that this top rating of students should correspond to 90 or more points – which is exactly the usual A threshold.

Why 70. If a person's grade is less than 90, then we cannot rely on this person to solve almost all problems. So:

- if we want almost all problems have to be solved,
- we need to hire at least two such persons.

What level of knowledge is needed for a person so that between themselves, they will be able to guarantee that almost all problems are solved?

Let p denote the probability that a student makes a mistake. The problem is solved if at least one of the students comes up with a correct solution. The only time when a problem is not solved is when both make a mistake. When two persons are independently trying to solve the same problem, then, due to independence, the probability that both make the same mistake is equal to the product of the corresponding two probabilities, i.e., to p^2.

To make sure that between these two, they can almost all problems, we need to make sure than p^2 is smaller than the above-described 1/9. The inequality $p^2 < 1/9$ is equivalent to $p < 1/3$. This means that the probability of solving the problem correctly should be greater than $1 - 1/3 = 2/3 \approx 67\%$. This means that the person should get more than 67 points on the test, i.e., at least 68 points. This is very close to the usual 70 points threshold for the C grade.

So, the C grade starts making sense, as the smallest value for which two people will this grade will be able to solve almost all problems.

Why 60: kind of. If a person's grade is smaller than the C level of 70, then even having two people at this level of knowledge working together does not guarantee that almost all problems will be solved. This means that if we only have graduates at this level, we need to hire at least three of them.

What is the smallest grade that guarantees that between 3 of them, they can solve almost all problems? If p is the probability that one of them makes a mistake, then the probability that all three of them make a mistake (and will thus be unable to solve the problem) will be p^3. We want this proportion to be smaller than 1/9. The condition $p^3 < 1/9$ is equivalent to $p < 0.48$. This means that the student's probability of solving the problem should be larger than $1 - 0.48 = 0.52$. This means that this student will gain more than 52 points, i.e., 53 or more. This is loosely related to the usual 60 threshold corresponding to the minimal passing D grade.

So, D means that if we hire three people at this level of knowledge, then together they will be able to solve almost all problems.

Comment. What if the grade is even smaller than that? Should not we be able to hire 4 people for solving problems?

Not really, since when we require that $p^4 < 1/9$, the corresponding probability p is larger than 1/2, so each of them will be wrong in more than a half of the cases: $1 - p > p$. So, the probability that all four will come up with the same wrong answer – which is equal to $(1-p)^4$, will be higher than the probability p^4 of all four of them coming up with the correct answer. Thus, even if all four come up with the same conclusion, it is more probable that this joint solution is wrong.

In other words, such very un-knowledgable students are kind of useless.

Why 80. What if we have one D student. Clearly, he or she cannot solve almost all problems on his/her own, so in principle, if we have two more D students, together they can solve almost all problems. But what if we have only one D student. Who should we supplement this student with, so that their team of two will be able to solve almost all problems?

For the D student, the probability to make a mistake is 0.48. Thus, we need to supplement this student with another one whose probability is p so that the product $0.48 \cdot p$ should be smaller than 1/9. The inequality $0.48 \cdot p < 1/9$ is equivalent to $p < (1/9)/0.48 \approx 0.23$. Thus, the student needs to be able to succeed in more than $100 - 23 = 77$ percent of the cases. In other words, the student needs to get more than 77 points. The resulting threshold of 78 is very close to the usual 80 threshold for B.

So, we get a natural interpretation of B: it is a grade that allows a person teaming up with a D student to solve almost all problems.

This way, all the usual letter grade thresholds are indeed interpreted.

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References

[1] Miller, G.A., The magical number seven plus or minus two: some limits on our capacity for processing information, *Psychological Review*, vol.63, no.2, pp.81–97, 1956.

[2] Reed, S.K., *Cognition: Theories and Application*, Wadsworth Cengage Learning, Belmont, California, 2010.