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Decision Theory can Explain Why Buying and Selling Prices are Different

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Abstract

According to a naive understanding of economic behavior, for each object, we should have an internal estimate of how much this object is worth for us. If anyone offers us to buy this object at a smaller amount, we should agree, and if anyone offers to buy it from us for a larger amount, we should agree as well. In practice, however, contrary to this understanding, the price for which we are willing to buy and the price at which we are willing to sell are often different. In this paper, we show that this seemingly counterintuitive phenomenon can be explained within decision theory – if we use the standard Hurwicz optimism-pessimism recommendations for decision making under uncertainty.

Keywords: selling and buying prices, decision making under uncertainty, Hurwicz optimism-pessimism criterion

1 Buying and Selling Prices are Different: A Phenomenon and Its Current Quantitative Explanations

Buying and selling prices are different: a phenomenon. According to the naive understanding of economic behavior, we should decide, for ourselves, how much each object is worth to us. Then:

- This worth amount should be the largest amount that we should be willing to pay if we are buying this object.
- This same amount should be the smallest amount for which we should agree to sell this objects.

However, in many experiments, the price participants are willing to pay to buy a certain item is different from the price they are willing to accept to part with this item. For example:

- students are willing to pay at most $3 for a mug but
- these same students require to be paid at least $7 to sell it back.

In other words, people estimate the consequences of losing an object differently than the consequences of gaining the same object; see, e.g., [5, 8] and references therein.

Current explanations of this phenomenon. The current explanation of this phenomenon is based on the fact that people are not clear on the value of each object. Instead of the exact monetary amount, at best, they have a range $[\underline{u}, \overline{u}]$ of possible values of this object's worth; see, e.g., [2, 3].

Need for a more detailed analysis. While [2, 3] provide a qualitative explanation for the loss aversion phenomenon, it is desirable to extend this to a quantitative analysis, an analysis that takes into account known results about rational decision making under interval uncertainty.

This is what we do in this paper.

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2 Qualitative Explanation

Decision making under interval uncertainty: a brief reminder. How can we make a decision if, instead of the exact value of an object, we only know the interval $[\underline{u},\overline{u}]$ of possible values? In other words, what is the value $u(\underline{u},\overline{u})$ that we are willing to pay for this object?

Clearly, since we know that the object is worth at least $\underline{u}$ and at most $\overline{u}$, this means that the price $u(\underline{u},\overline{u})$ that we are willing to pay should also be at least $\underline{u}$ and at most $\overline{u}$:

$$\underline{u} \le u(\underline{u},\overline{u}) \le \overline{u}. \tag{1}$$

This property is known as *boundedness*.

Another reasonable requirement is that if we have two different objects, with values in $[\underline{u},\overline{u}]$ and $[\underline{v},\overline{v}]$, then the price that we are willing to pay to buy both should be equal to the prices that we pay for each of them. Let us describe this second requirement in precise terms.

When we get two objects together, the smallest possible value of our purchase is when both objects are worth their smallest amounts $\underline{u}$ and $\underline{v}$. In this case, the overall worth of both objects is equal to the sum

$$\underline{u}+\underline{v}.$$

Similarly, the largest possible value of our purchase is when both objects are worth their largest amounts $\overline{u}$ and $\overline{v}$. In this case, the overall worth of both objects is equal to the sum

$$\overline{u}+\overline{v}.$$

Thus, for two objects sold together the interval of possible worth values is

$$[\underline{u}+\underline{v},\overline{u}+\overline{v}].$$

So, the second requirement takes the following form:

$$u(\underline{u}+\underline{v},\overline{u}+\overline{v}) = u(\underline{u},\overline{u}) + u(\underline{v},\overline{v}). \tag{2}$$

This property is known as *additivity*.

It turns out (see, e.g., [6]) that the only functions that satisfy both requirements (1) and (2) are functions of the type

$$u(\underline{u},\overline{u}) = \alpha\cdot\overline{u} + (1-\alpha)\cdot\underline{u}, \tag{3}$$

for some $\alpha\in[0,1]$. This fact easily follows from the fact that all bounded additive functions are linear (see, e.g., [1]), so

$$u(\underline{u},\overline{u}) = \alpha\cdot\overline{u} + \beta\cdot\underline{u}.$$

For the case when

$$\underline{u}=\overline{u},$$

the requirement (1) implies that

$$u(\underline{u},\overline{u}) = \underline{u}.$$

Thus, for the above general linear formula, we get

$$\alpha\cdot\underline{u}+\beta\cdot\underline{u}=\underline{u}.$$

For $\underline{u}\ne 0$, this implies that

$$\alpha+\beta=1,$$

i.e., that

$$\beta=1-\alpha.$$

Thus, that we have the formula (3). All that remains to prove is that $\alpha\in[0,1]$.

Indeed, for $\underline{u}=0<\overline{u}$, the formula (3) leads to

$$u(0,\overline{u})=\alpha\cdot\overline{u}.$$

Thus, the requirement (1) implies that

$$0 \le \alpha \cdot \overline{u} \le \overline{u}.$$

Dividing all three sides of this inequality by $\overline{u} > 0$, we conclude that

$$0 \le \alpha \le 1.$$

The formula (3) was first proposed by a future Nobelist Leo Hurwicz and is thus known as Hurwicz optimism-pessimism criterion [4, 7]. The relation with optimism and pessimism is straightforward:

- when $\alpha = 1$, the person pays the highest possible price $\overline{u}$ – which makes sense if this person believes that the best possible scenario will take place; this is exactly what we usually mean by extreme optimism;

- on the other hand, when $\alpha = 0$, the person is only willing to pay the lowest possible price $\underline{u}$ – which makes sense if this person does not believe in the possibility of higher worth values; this is exactly what we usually mean by extreme pessimism.

Hurwicz criterion explains the difference between buy and sell prices. When we buy an object whose worth is between $\underline{u}$ and $\overline{u}$, the best possible gain is $\overline{u}$ and the worst possible gain is $\underline{u}$. Thus, according to the Hurwicz criterion, we should be willing to pay the amount u_b (b for *buy*) which is equal to

$$u_b = \alpha \cdot \overline{u} + (1 - \alpha) \cdot \underline{u}. \tag{4}$$

On the other hand, if we already own this object and we sell it, then our loss is between $-\overline{u}$ and $-\underline{u}$. The most optimistic estimate for our resulting state is $-\underline{u}$ and the most pessimistic estimate is $-\overline{u}$. In this case, according to the Hurwicz criterion, this is equivalent to the value of

$$\alpha \cdot (-\underline{u}) + (1 - \alpha) \cdot (-\overline{u}). \tag{5}$$

Thus, to compensate for this loss, we need to get the amount u_s (s for *sell*) that, when added to the value (5), will result in 0, i.e., the value

$$u_s = \alpha \cdot \underline{u} + (1 - \alpha) \cdot \overline{u}. \tag{6}$$

We can see that, in general, the expressions for the buy u_b and sell u_s prices are different. Indeed, the only time when the prices are equal, i.e., when $u_b = u_s$, is when

$$\alpha \cdot \underline{u} + (1 - \alpha) \cdot \overline{u} = \alpha \cdot \overline{u} + (1 - \alpha) \cdot \underline{u}.$$

Moving all the terms to the left-hand side and adding resulting coefficients at $\overline{u}$ and $\underline{u}$, we conclude that

$$(2\alpha - 1) \cdot \underline{u} - (2\alpha - 1) \cdot \overline{u} = 0,$$

i.e., $(2\alpha - 1) \cdot (\underline{u} - \overline{u}) = 0$. Since we consider the case when we have uncertainty, i.e., when $\underline{u} \ne \overline{u}$, we thus conclude that $2\alpha - 1 = 0$, i.e., that $\alpha = 0.5$.

So, only people with $\alpha = 0.5$ buy and sell at exactly the same price. For everyone else – who is even slightly more optimistic or even slightly less optimistic than $\alpha = 0.5$ – the buy and sell prices are different, and this is exactly what we observe.

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