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Common Sense Addition Explained by Hurwicz Optimism-Pessimism Criterion

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Abstract

If we place a can of coke that weighs 0.35 kg into a car that weighs 1 ton = 1000 kg, what will be the resulting weight of the car? Mathematics says 1000.35 kg, but common sense says 1 ton. In this paper, we show that this common sense answer can be explained by the Hurwicz optimism-pessimism criterion of decision making under interval uncertainty.

Keywords: commonsense reasoning, Hurwicz optimism-pessimism criterion, decision making under uncertainty

1 Common Sense Addition

Suppose that we have two factors that affect the accuracy of a measuring instrument. One factor leads to errors ±10% – meaning that the resulting error component can take any value from −10% to +10%. The second factor leads to errors of ±0.1%. What is the overall error?

From the purely mathematical viewpoint, the largest possible error is 10.1%. However, from the common sense viewpoint, an engineer would say: 10%.

A similar common sense addition occurs in other situations as well. For example, if we have a car that weights 1 ton = 1000 kg, and we place a coke can that weighs 0.35 kg in the car, what will be now the weight of the car? Mathematics says 1000.35 kg, but common sense clearly says: still 1 ton.

How can we explain this common sense addition?

2 Towards Precise Formulation of the Problem

We know that the overall measurement error Δx is equal to $\Delta x_1 + \Delta x_2$, where:

- the value Δx_1 can take all possible values from the interval $[-\Delta_1, \Delta_1]$, and
- the value Δx_2 can take all possible values from the interval $[-\Delta_2, \Delta_2]$.

What can we say about the largest possible value Δ of the absolute value $|\Delta|$ of the sum

$$\Delta x = \Delta x_1 + \Delta x_2?$$

Let us describe this problem in precise terms. For every pair (x_1, x_2):

- let $\pi_1(x_1, x_2)$ denote x_1 and
- let $\pi_2(x_1, x_2)$ stand for x_2.

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Let $\Delta_1 > 0$ and $\Delta_2 > 0$ be two numbers. Without losing generality, we can assume that

$$\Delta_1 \geq \Delta_2.$$

By $\mathcal{S}$, let us denote the class of all possible sets

$$S \subseteq [-\Delta_1, \Delta_1] \times [-\Delta_2, \Delta_2]$$

for which

$$\pi_1(S) = [-\Delta_1, \Delta_1] \text{ and } \pi_2(S) = [-\Delta_2, \Delta_2].$$

We are interested in the value

$$\Delta(S) = \max\{|\Delta x_1 + \Delta x_2| : (\Delta x_1, \Delta_2) \in S\}$$

corresponding to the actual (unknown) set S.

We do not know what is the actual set S, we only know that $S \in \mathcal{S}$. For different sets $S \in \mathcal{S}$, we may get different values $\Delta(S)$. The only thing we know about $\Delta(S)$ is that it belongs to the interval $[\underline{\Delta}, \overline{\Delta}]$ formed by the smallest and the largest possible values of $\Delta(S)$ when $S \in \mathcal{S}$:

$$\underline{\Delta} = \min_{S \in \mathcal{S}} \Delta(S), \quad \overline{\Delta} = \max_{S \in \mathcal{S}} \Delta(S).$$

Which value Δ from this interval should we choose?

3 Hurwicz Optimism-Pessimism Criterion: Reminder

Situations when we do not know the value of a quantity, we only know the interval of its possible values, are ubiquitous. In such situations, decision theory recommends using *Hurwicz optimism-pessimism criterion*: selecting the value

$$\alpha \cdot \underline{\Delta} + (1 - \alpha) \cdot \overline{\Delta}$$

for some $\alpha \in [0, 1]$. A usual recommendation is to use $\alpha = 0.5$; see, e.g., [2, 3, 4].

Let us see what will be the result of applying this criterion to our problem.

4 Analysis of the Problem and the Resulting Explanation of Common Sense Addition

Computing $\overline{\Delta}$. For every set $S \in \mathcal{S}$, from $|\Delta x_1| \leq \Delta_1$ and $|\Delta x_2| \leq \Delta_2$, we conclude that

$$|\Delta x_1 + \Delta x_1| \leq \Delta_1 + \Delta_2.$$

Thus always

$$\Delta(S) \leq \Delta_1 + \Delta_2$$

and hence,

$$\overline{\Delta} = \max \Delta(S) \leq \Delta_1 + \Delta_2.$$

On the other hand, for the set

$$S_0 = \{v, (\Delta_2/\Delta_1) \cdot v) : v \in [-\Delta_1, \Delta_1]\} \in \mathcal{S},$$

we have

$$\Delta x_1 + \Delta x_2 = \Delta x_1 \cdot (1 + \Delta_2/\Delta_1).$$

Thus in this case, the largest possible value $\Delta(S_0)$ of $\Delta x_1 + \Delta x_2$ is equal to

$$\Delta(S_0) = \Delta_1 \cdot (1 + \Delta_2/\Delta_1) = \Delta_1 + \Delta_2.$$

So,

$$\overline{\Delta} = \max \Delta(S) \geq \Delta(S_0) = \Delta_1 + \Delta_2.$$

Hence,

$$\overline{\Delta} = \Delta_1 + \Delta_2.$$

Computing $\underline{\Delta}$. For every $S \in \mathcal{S}$, since

$$\pi_1(S) = [-\Delta_1, \Delta_1],$$

we have

$$\Delta_1 \in \pi_1(S).$$

Thus, there exists a pair

$$(\Delta_1, \Delta x_2) \in S$$

corresponding to

$$\Delta x_1 = \Delta_1.$$

For this pair, we have

$$|\Delta x_1 + \Delta x_2| \geq |\Delta x_1| - |\Delta x_2| = \Delta_1 - |\Delta x_2|.$$

Here, $|\Delta x_2| \leq \Delta_2$, so

$$|\Delta x_1 + \Delta x_2| \geq \Delta_1 - \Delta_2.$$

Thus, for each set $S \in \mathcal{S}$, the largest possible value $\Delta(S)$ of the expression

$$|\Delta x_1 + \Delta x_2|$$

cannot be smaller than $\Delta_1 - \Delta_2$:

$$\Delta(S) \geq \Delta_1 - \Delta_2.$$

Hence,

$$\underline{\Delta} = \min_{S \in \mathcal{S}} \Delta(S) \geq \Delta_1 - \Delta_2.$$

On the other hand, for the set

$$S_0 = \{v, -(\Delta_2/\Delta_1) \cdot v) : v \in [-\Delta_1, \Delta_1]\} \in \mathcal{S},$$

we have

$$\Delta x_1 + \Delta x_2 = \Delta x_1 \cdot (1 - \Delta_2/\Delta_1).$$

Thus in this case, the largest possible value $\Delta(S_0)$ of $\Delta x_1 + \Delta x_2$ is equal to

$$\Delta(S_0) = \Delta_1 \cdot (1 - \Delta_2/\Delta_1) = \Delta_1 - \Delta_2.$$

So,

$$\underline{\Delta} = \min_{S \in \mathcal{S}} \Delta(S) \leq \Delta(S_0) = \Delta_1 - \Delta_2.$$

Thus,

$$\underline{\Delta} \leq \Delta_1 - \Delta_2.$$

Hence,

$$\underline{\Delta} = \Delta_1 - \Delta_2.$$

Let us apply Hurwicz optimism-pessimism criterion. So, if we apply Hurwicz optimism-pessimism criterion with $\alpha = 0.5$ to the interval

$$[\underline{\Delta}, \overline{\Delta}] = [\Delta_1 - \Delta_2, \Delta_1 + \Delta_2],$$

we end up with the value

$$\Delta = 0.5 \cdot \underline{\Delta} + 0.5 \cdot \overline{\Delta} = \Delta_1.$$

For example, for $\Delta_1 = 10\%$ and $\Delta_2 = 0.1\%$, we get $\Delta = 10\%$ – in full accordance with common sense. In other words, *Hurwicz criterion explains the above-described common-sense addition.*

Comment. This result was previously announced in [1].

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