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An Inexact Two-Stage Stochastic Dependent-Chance Programming Model for Water Resources Management

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Abstract

In order to optimize the allocation of water resources, an inexact two-stage stochastic dependent-chance programming model, which integrates stochastic dependent-chance programming, two-stage programming and interval programming, is given. Compared with the existing other water resources management models, this model reflects the dynamic characteristics and randomness of the water resource management system, emphasizes the importance of water users, and maximizes the probability of achieving the required economic goals set by the water manager. In order to solve the model with the data of multiply distributed stochastic boundaries, a hybrid algorithm, which incorporates stochastic simulation, back propagation neural network, and genetic algorithm, is proposed. Finally, the model is applied to a case study of Handan City's water resources management in 2024 and 2025, through which the optimized water-allocation in Handan City is realized.

Keywords: water resources management, stochastic dependent-chance programming, two-stage programming, interval programming

1 Introduction

Water resources are crucial to the survival of human beings and the development of the society. With the increasing number of water users, the consumption of water is increasing rapidly. Meanwhile the extreme weather, deterioration of ecological environment and water-consuming economics also lead to the increasing shortage of water resources. The efficient utilization of water resources has become an urgent problem.

For the better use of water resources, the optimal configuration of water resources has attracted a lot of attention [2, 5, 14, 15, 17, 20]. Stochastic programming is one of the important optimization methods to handle the randomness in the water resources management system. For example, Gu et al. [3] gave a water resources management method by incorporating interval programming, multistage programming and joint-probabilistic integer programming. Taking into account the development and distribution processes for water, Li and Zhang [8] balanced the water allocation and system benefits through two-stage interval stochastic programming. Mo et al. [13] gave more discussion of urban water resources allocation problems by putting forward a multistage stochastic integer programming. In order to handle the relationship between water users and regional water exchange, Fu et al. [1] proposed a two-stage interval-parameter stochastic programming model to replace water resources from the areas where water is used inefficiently to the areas where water is used efficiently. Actually, there are multiple events in a complex water resources management system [9]. Sometimes, the water manager wants to maximize the chance functions, such as probability measure, credibility measure, chance measure and so on, of meeting these events [9]. Dependent-chance programming [9] is a method to solve the problem. Up to now, dependent-chance programming has been used to optimize the disposition of water resources. Guo et al. [4] gave an uncertainty theory [10, 11] based dependent-chance goal programming model to manage water resources. Wang et al. [18] gave an optimal scheme of allocating irrigation water by establishing an interval quadratic fuzzy dependent-chance programming model. Peng and Zhou [16] proposed a fuzzy multi-objective dependent-chance programming model to allocate the water resources in Dalian

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City of China. However these models did not discuss the water resources management in the stochastic environment nor consider the significance of different water users.

In order to solve the above problems, an inexact two-stage stochastic dependent-chance programming model is given firstly, which combines stochastic dependent-chance programming, two-stage programming and interval programming. We set weights according to the importance of the different water users, obtain the maximum probability of achieving the aims formulated by the decision maker at different stages with different levels of inflow (such as high inflow, medium inflow and low inflow), and acquire optimized configuration schemes. It is difficult to solve the model by translating the model into deterministic equivalents because there are data with multiply distributed stochastic boundaries. Then a hybrid algorithm incorporating stochastic simulation, back propagation neural network, and genetic algorithm is introduced to solve the model. Next, this model is applied the water resource management system in Handan City. The optimized allocation of water resources can help balance the development of society, economy and environment under the existing system constraints of Handan City. In the end, the solutions of the model provide optimal water allocation schemes to municipal user, ecological user, agricultural user and industrial user in 2024 and 2025.

2 An Inexact Multistage Stochastic Programming Model for Water Resources Management

The maximum expected system benefits can be obtained through formulating the optimal water allocation schemes to different water users, such as municipal, ecological, industrial and agricultural users and so on. A water manager promises to provide a certain amount of water for each user in advance. If the promised amount of water can be satisfied, the users can gain profits or expand production [7]. If the promised amount of water is not available, the users have to reduce production or obtain water in other ways at higher prices, resulting a lower profit [12]. Because the supply of water is subject to the randomness of the amount of water in the future and certain dynamic characteristics exist in the long-term water resource allocation plan, Li et al. [7] proposed the following scenario-based multistage stochastic programming model for water resources management under uncertainties,

$$
\begin{aligned}
\max \quad & f=\sum_{i=1}^{m}\sum_{j=1}^{n} NB_{ij}T_{ij}-\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\left(\sum_{k=1}^{K_j} p_{jk}D_{ijk}\right)\\
s.t. \quad & \sum_{i=1}^{m}\left(T_{ij}-D_{ijk}\right)\le Q_{jk}+\varepsilon_{(j-1)k},\ j=1,2,\ldots,n,k=1,2,\ldots,K_j\\
& \varepsilon_{(j-1)k}=\left[Q_{(j-1)k}-\sum_{i=1}^{m}\left(T_{i(j-1)}-D_{i(j-1)k}\right)\right]+\varepsilon_{(j-2)k},\ j=1,2,\ldots,n,k=1,2,\ldots,K_j\\
& T_{ij\max}\ge T_{ij}\ge D_{ijk}\ge 0, i=1,2,\ldots,m,\ j=1,2,\ldots,n,k=1,2,\ldots,K_j.
\end{aligned}
\tag{1}
$$

where f is the net benefit of the water management system (¥); m is the water users' number; i represents the index of water users; n is the number of the period; j represents the index of the periods; K_j is the sum of scenarios in period j; NB_{ij} is the net benefit when per cubic meter of water is allocated to user i in period j (¥/m^3); T_{ij} is the fixed amount of water which the water manager promises to distribute to user i in period j (m^3); Q_{jk} is the water flow in the scenario k occurs in period j (m^3); p_{jk} is the probability of occurrence for scenario k in period j; D_{ijk} is the water shortage amount of user i in the scenario k occurs in period j (m^3); C_{ij} is the loss when per cubic meter of water is not allocated to user i in period j (¥/m^3); $\varepsilon_{(j-1)k}$ is the surplus water inflow in the scenario k occurs in period $j-1$ (m^3); $T_{ij\max}$ is the amount of maximum allowable water allocation for user i in period j (m^3).

Suppose there are four water users such as municipal user, ecological user, the agricultural user and industrial user. The priorities of water users are different. For example, the water supply for the municipal user must be guaranteed first since water for life is crucial to people's survival. Besides that, due to the government's focus on eco-construction, the water supply for the ecological user should also be satisfied preferentially as compared with other users. Therefore, we introduce the priority coefficients ω_i [19] into the model (1) and establish the following model (2) to measure the priority level of water user i,

$$\begin{aligned}
\max \quad & f=\sum_{i=1}^{m}\sum_{j=1}^{n}\omega_i NB_{ij}T_{ij}-\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K_j}p_{jk}\omega_i C_{ij}D_{ijk} \\
s.t \quad & \sum_{i=1}^{m}\left(T_{ij}-D_{ijk}\right)\le Q_{jk}+\varepsilon_{(j-1)k},\ j=1,2,\ldots,n, k=1,2,\ldots,K_j \\
& \varepsilon_{(j-1)k}=\left[Q_{(j-1)k}-\sum_{i=1}^{m}\left(T_{i(j-1)}-D_{i(j-1)k}\right)\right]+\varepsilon_{(j-2)k},\ j=1,2,\ldots,n, k=1,2,\ldots,K_j \\
& T_{ij\max}\ge T_{ij}\ge D_{ijk}\ge 0, i=1,2,\ldots,m,\ j=1,2,\ldots,n, k=1,2,\ldots,K_j.
\end{aligned} \tag{2}$$

Take into account the difficulties of precise data collection, some fixed values, such as T_{ij}, NB_{ij}, C_{ij} and Q_{jk}, cannot be determined exactly. Thus, an inexact multistage stochastic programming model (3), which introduces interval parameters into model (2), is proposed as follows,

$$\begin{aligned}
\max \quad & f^{\pm}=\sum_{i=1}^{m}\sum_{j=1}^{n}\omega_i NB_{ij}^{\pm}T_{ij}^{\pm}-\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K_j}p_{jk}\omega_i C_{ij}^{\pm}D_{ijk}^{\pm} \\
s.t. \quad & \sum_{i=1}^{m}\left(T_{ij}^{\pm}-D_{ijk}^{\pm}\right)\le q_{jk}^{\pm}+\varepsilon_{(j-1)k}^{\pm},\ j=1,2,\ldots,n, k=1,2,\ldots,K_j \\
& \varepsilon_{(j-1)k}^{\pm}=\left[q_{(j-1)k}^{\pm}-\sum_{i=1}^{m}\left(T_{i(j-1)}^{\pm}-D_{i(j-1)k}^{\pm}\right)\right]+\varepsilon_{(j-2)k}^{\pm},\ j=2,3,\ldots,n, k=1,2,\ldots,K_j \\
& T_{ij\max}^{\pm}\ge T_{ij}^{\pm}\ge D_{ijk}^{\pm}\ge 0, i=1,2,\ldots,m,\ j=1,2,\ldots,n, k=1,2,\ldots,K_j.
\end{aligned} \tag{3}$$

3 Stochastic Dependent-Chance Constrained Programming

When the water manager wants to maximize the probability measure of meeting the events in a complex water resources management system, stochastic dependent-chance programming [9] can be employed.

(1) Stochastic environment, stochastic event and chance function

According to the reference [9], stochastic environment, stochastic events and chance function are basic concepts in stochastic dependent-chance programming. Let $\boldsymbol{x}$ be a decision vector and $\boldsymbol{\xi}$ a stochastic vector.

Stochastic environment is defined by the stochastic constraints of

$$g_j\left(\boldsymbol{x},\boldsymbol{\xi}\right)\le 0\ ,\quad j=1,2,\ldots,p.$$

Stochastic event is defined by the inequalities of

$$h_k\left(\boldsymbol{x},\boldsymbol{\xi}\right)\le 0, k=1,2,\ldots,q.$$

The chance function of an event characterized by $h_k\left(\boldsymbol{x},\boldsymbol{\xi}\right)\le 0, k=1,2,\cdots,q$ is defined as the probability measure of the event, i.e.,

$$f(x)=\Pr\left\{h_k\left(\boldsymbol{x},\boldsymbol{\xi}\right)\le 0, k=1,2,\ldots,q\right\}$$

subject to the stochastic environment.

(2) Stochastic dependent-chance single-objective programming

In order to maximize the chance function of the event in the stochastic environment, the following stochastic dependent-chance single-objective programming [9] is given,

$$\begin{cases}\max & \Pr\left\{h_k\left(\boldsymbol{x},\boldsymbol{\xi}\right)\le 0, k=1,2,\ldots,q\right\} \\ s.t. & g_j\left(\boldsymbol{x},\boldsymbol{\xi}\right)\le 0,\ j=1,2,\ldots,p,\end{cases} \tag{4}$$

where $h_k(\boldsymbol{x},\boldsymbol{\xi}) \le 0$ ($k = 1,2,\ldots,q$) represent the stochastic event, $g_j(\boldsymbol{x},\boldsymbol{\xi}) \le 0$ ($j = 1,2,\ldots,p$) represent the stochastic environment.

4 An Inexact Two-Stage Stochastic Dependent-Chance Programming Model for Water Resources Management

According to the practical applications, the n is set to 2 in model (3). Then combining the two-stage model with the dependent-chance programming model, an inexact two-stage stochastic dependent-chance programming model (5) for water resources management can be obtained as follows,

$$\begin{aligned}
\max \quad & \Pr\left\{ \sum_{i=1}^{m}\sum_{j=1}^{2} \omega_i NB_{ij}^{\pm}\left(T_{ij}^{-} + \Delta T_{ij} y_{ij}\right) - \sum_{i=1}^{m}\sum_{j=1}^{2}\sum_{k=1}^{K_j} p_{jk}\omega_i C_{ij}^{\pm} D_{ijk}^{\pm} \ge v^{\pm} \right\} \\
s.t. \quad & \sum_{i=1}^{m}\left(T_{ij}^{-} + \Delta T_{ij} y_{ij} - D_{ijk}^{\pm}\right) \le q_{jk}^{\pm} + \varepsilon_{(j-1)k}^{\pm},\ j{=}1,2, k = 1,2,\ldots,K_j \\
& \varepsilon_{(j-1)k}^{\pm} = \left[q_{(j-1)k}^{\pm} - \sum_{i=1}^{m}\left(T_{i(j-1)}^{-} + \Delta T_{i(j-1)} y_{i(j-1)} - D_{i(j-1)}^{\pm}\right)\right] + \varepsilon_{(j-2)k}^{\pm},\ j{=}2, k = 1,2,\ldots,K_j \\
& 0 \le y_{ij} \le 1, i{=}1,2,\ldots,m,\ j{=}1,2 \\
& T_{ij\max}^{\pm} \ge T_{ij}^{-} + \Delta T_{ij} y_{ij} \ge D_{ijk}^{\pm} \ge 0, i{=}1,2,\ldots,m,\ j{=}1,2, k = 1,2,\ldots,K_j
\end{aligned} \tag{5}$$

where $v^{\pm}$ is the given target value. The aim of the programming is to maximize the probability of

$$\sum_{i=1}^{m}\sum_{j=1}^{2} \omega_i NB_{ij}^{\pm}\left(T_{ij}^{-} + \Delta T_{ij} y_{ij}\right) - \sum_{i=1}^{m}\sum_{j=1}^{2}\sum_{k=1}^{K_j} p_{jk}\omega_i C_{ij}^{\pm} D_{ijk}^{\pm} \ge v^{\pm}.$$

5 The Inexact Two-Stage Stochastic Dependent-Chance Programming Model for Water Resources Management in Handan City

Handan City, which has about 1.67 billion cubic meters of water and 191 cubic meters per capita, 9% of the national average level, is short of water resources extremely. Relevant data of Handan City's water resources system can be obtained from Statistics Almanac of Handan City and the evaluation of water resources in Handan City (2008) [6]. Table 1 gives the water allocation targets in 2024 and 2025 of Handan City, Table 2 gives the distribution of water sources, and Table 3 gives relevant economic data in 2024 and 2025. And $v^{\pm}$ is set to be [65, 80]$\times 10^8$ ¥, then we will know how to allocate the water resources so that the probability of the event that the system net benefit is [65, 80] $\times 10^8$ ¥ more than the economic penalty is maximal.

Table 1: Water allocation targets in 2024 and 2025 ($10^8 m^3$)

Water allocation targets	Time periods	
	$j = 1$	$j = 2$
Municipality $(i = 1)$	[2.21,3.50]	[1.99,3.15]
Ecology $(i = 2)$	[0.95,1.53]	[1.04,1.67]
Agriculture $(i = 3)$	[12.40,14.96]	[11.16,13.46]
Industry $(i = 4)$	[2.25,3.31]	[2.02,2.98]

Table 2: The amount of water resources in 2024 and 2025 ($10^8 m^3$)

The flow level	L $(k=1)$	M $(k=2)$	H $(k=3)$
j=1	[3.43,3.58]+ a_{11}	[3.61,3.76] + a_{12}	[6.68,6.79] + a_{13}
j=2	[3.43,3.58] + a_{21}	[3.61,3.76] + a_{22}	[6.68,6.79] + a_{23}
Probability	25.42%	47.46%	27.12%

where $U(a,b)$ represents a uniform distributed random variable and $a_{11} \sim U(12.73,13.43)$, $a_{12} \sim U(12.85,13.57)$, $a_{13} \sim U(10.62,11.29)$, $a_{21} \sim U(11.31,12.73)$, $a_{22} \sim U(11.42,12.85)$, $a_{23} \sim U(9.50,10.62)$.

Table 3: Net benefits and penalties in 2024 and 2025 (¥/m^3)

	Time period	
	$j=1$	$j=2$
Net benefit when water demand is satisfied		
Municipality	$[77.35, 81.14]$	$[72.01, 88.44]$
Ecology	$\left[N(11.97, 0.5^2), N(16.92, 0.5^2)\right]$	$\left[N(12.23, 0.5^2), N(17.22, 0.5^2)\right]$
Agriculture	$\left[N(25.22, 0.5^2), N(30.46, 0.5^2)\right]$	$\left[N(34.2, 0.5^2), N(38.8, 0.5^2)\right]$
Industry	5.82	5.64
Penalty when water is not delivered		
Municipality	$[122.46, 130.90]$	$[156.74, 167.54]$
Ecology	$[78.53, 90.93]$	$[112.48, 128.75]$
Agriculture	$[39.51, 40.12]$	$[48.82, 50.61]$
Industry	188.84	270.64

where $N(\mu, \sigma^2)$ represents a normally distributed random variable.

Due to the survival needs and the eco-priority principle, the priorities of water users are municipal user ($i=1$), ecological user ($i=2$), agricultural user ($i=3$), and industrial user ($i=4$), respectively. ω_i $(i=1,2,3,4)$ represents the priority of the four users which can be set to 0.4, 0.3, 0.2 and 0.1 [19]. Considering the actual situations, the remaining water amount in the previous period is included into the water amount in the latter period, then $\varepsilon_{(j-1)k}$ is no longer introduced separately. According to Tables 1-3, the model is proposed as follows:

$$
\begin{aligned}
\max \quad & \Pr\left\{ \sum_{i=1}^{4}\sum_{j=1}^{2} \omega_i NB_{ij}^{\pm}\left(T_{ij}^{-} + \Delta T_{ij} y_{ij}\right) - \sum_{i=1}^{4}\sum_{k=1}^{3} p_{1k}\omega_i C_{i1}^{\pm} D_{i1k}^{\pm} - \sum_{i=1}^{4}\sum_{k=1}^{9} p_{2k}\omega_i C_{i2}^{\pm} D_{i2k}^{\pm} \geq v^{\pm} \right\} \\
s.t. \quad & \sum_{i=1}^{4}\left(T_{i1}^{-} + \Delta T_{i1} y_{i1} - D_{i1k}^{\pm}\right) \leq q_{1k}^{\pm}, k=1,2,3 \\
& \sum_{i=1}^{4}\left(T_{i2}^{-} + \Delta T_{i2} y_{i2} - D_{i2k}^{\pm}\right) \leq q_{2k}^{\pm}, k=1,2,\ldots,9 \\
& 0 \leq y_{ij} \leq 1, i=1,2,3,4, j=1,2,3 \\
& T_{i1\max}^{\pm} \geq T_{i1}^{-} + \Delta T_{i1} y_{i1} \geq D_{i1k}^{\pm} \geq 0, i=1,2,3,4, k=1,2,3 \\
& T_{i2\max}^{\pm} \geq T_{i2}^{-} + \Delta T_{i2} y_{i2} \geq D_{i2k}^{\pm} \geq 0, i=1,2,3,4, k=1,2,\cdots,9.
\end{aligned}
\tag{6}
$$

The chance function of model (6) is

$$f = \Pr\left\{\begin{array}{l} \sum_{i=1}^{4}\sum_{j=1}^{2}\omega_i NB_{ij}^{\pm}\left(T_{ij}^{-}+\Delta T_{ij}y_{ij}\right)-\sum_{i=1}^{4}\sum_{k=1}^{3}p_{1k}\omega_i C_{i1}^{\pm}D_{i1k}^{\pm}-\sum_{i=1}^{4}\sum_{k=1}^{9}p_{2k}\omega_i C_{i2}^{\pm}D_{i2k}^{\pm}\geq v^{\pm} \\ \sum_{i=1}^{4}\left(T_{i1}^{-}+\Delta T_{i1}y_{i1}-D_{i1k}^{\pm}\right)\leq q_{1k}^{\pm}, k=1,2,3 \\ \sum_{i=1}^{4}\left(T_{i2}^{-}+\Delta T_{i2}y_{i2}-D_{i2k}^{\pm}\right)\leq q_{2k}^{\pm}, k=1,2,\ldots,9 \\ 0\leq y_{ij}\leq 1, i=1,2,3,4, j=1,2,3 \\ T_{i1\max}^{\pm}\geq T_{i1}^{-}+\Delta T_{i1}y_{i1}\geq D_{i1k}^{\pm}\geq 0, i=1,2,3,4, k=1,2,3 \\ T_{i2\max}^{\pm}\geq T_{i2}^{-}+\Delta T_{i2}y_{i2}\geq D_{i2k}^{\pm}\geq 0, i=1,2,3,4, k=1,2,\ldots,9 \end{array}\right\}. \quad (7)$$

The aim of solving the model is to obtain $\max(f)$.

In the following, the process of solving the model is given.

(1) Solve the model (8) as follows:

$$\begin{array}{ll} \max & \Pr\left\{\sum_{i=1}^{4}\sum_{j=1}^{2}\omega_i NB_{ij}^{+}\left(T_{ij}^{-}+\Delta T_{ij}y_{ij}\right)-\sum_{i=1}^{4}\sum_{k=1}^{3}p_{1k}\omega_i C_{i1}^{-}D_{i1k}^{-}-\sum_{i=1}^{4}\sum_{k=1}^{9}p_{2k}\omega_i C_{i2}^{-}D_{i2k}^{-}\geq v^{+}\right\} \\ s.t. & \sum_{i=1}^{4}\left(T_{i1}^{-}+\Delta T_{i1}y_{i1}-D_{i1k}^{-}\right)\leq q_{1k}^{+}, k=1,2,3 \\ & \sum_{i=1}^{4}\left(T_{i2}^{-}+\Delta T_{i2}y_{i2}-D_{i2k}^{-}\right)\leq q_{2k}^{+}, k=1,2,\ldots,9 \\ & 0\leq y_{ij}\leq 1, i=1,2,3,4, j=1,2,3 \\ & T_{i1\max}^{\pm}\geq T_{i1}^{-}+\Delta T_{i1}y_{i1}\geq D_{i1k}^{-}\geq 0, i=1,2,3,4, k=1,2,3 \\ & T_{i2\max}^{\pm}\geq T_{i2}^{-}+\Delta T_{i2}y_{i2}\geq D_{i2k}^{-}\geq 0, i=1,2,3,4, k=1,2,\ldots,9 \end{array} \quad (8)$$

where D_{ijk}^{-} and y_{ij} are decision variables. The chance function of model (8) is

$$f_1 = \Pr\left\{\begin{array}{l} \sum_{i=1}^{4}\sum_{j=1}^{2}\omega_i NB_{ij}^{+}\left(T_{ij}^{-}+\Delta T_{ij}y_{ij}\right)-\sum_{i=1}^{4}\sum_{k=1}^{3}p_{1k}\omega_i C_{i1}^{-}D_{i1k}^{-}-\sum_{i=1}^{4}\sum_{k=1}^{9}p_{2k}\omega_i C_{i2}^{-}D_{i2k}^{-}\geq v^{+} \\ \sum_{i=1}^{4}\left(T_{i1}^{-}+\Delta T_{i1}y_{i1}-D_{i1k}^{-}\right)\leq q_{1k}^{+}, k=1,2,3 \\ \sum_{i=1}^{4}\left(T_{i2}^{-}+\Delta T_{i2}y_{i2}-D_{i2k}^{-}\right)\leq q_{2k}^{+}, k=1,2,\ldots,9 \\ 0\leq y_{ij}\leq 1, i=1,2,3,4, j=1,2,3 \\ T_{i1\max}^{\pm}\geq T_{i1}^{-}+\Delta T_{i1}y_{i1}\geq D_{i1k}^{-}\geq 0, i=1,2,3,4, k=1,2,3 \\ T_{i2\max}^{\pm}\geq T_{i2}^{-}+\Delta T_{i2}y_{i2}\geq D_{i2k}^{-}\geq 0, i=1,2,3,4, k=1,2,\ldots,9 \end{array}\right\},$$

and solving model (8) is equivalent to obtaining $\max f_1$.

In order to solve the model (8), a hybrid algorithm is proposed, which incorporates back propagation neural network, stochastic simulation and genetic algorithm.

Step 1: We use stochastic simulation to generate input-output data for $U_1:(y_{ij}, D^-_{i1k}, D^-_{i2k}) \to f_1$.

Step 2: BP neural network is used to simulate objective function according to the generated input-output data.

Step 3: GA is used to obtain the optimal solution. Firstly, a certain number of chromosomes are initialized according to the distribution function. Then, the chromosomes are selected by running a standard scheme of the roulette wheel and updated by crossover and mutation operations. In this process, the trained BP neural network is used to calculate the values of the objective function as fitness value. Finally, the best chromosome is obtained as the optimal solution and the optimal value is also achieved after a given number of cycles.

Step 4: The model (8) is solved. The optimal solution are D^-_{ijkopt} and y_{ijopt}, and the optimal value $\max f_1$ is obtained.

(2) Solve the following model (9):

$$\begin{aligned}
\max\quad & \Pr\left\{\sum_{i=1}^{4}\sum_{j=1}^{2}\omega_i NB^-_{ij}\left(T^-_{ij}+\Delta T_{ij}y_{ijopt}\right)-\sum_{i=1}^{4}\sum_{k=1}^{3}p_{1k}\omega_i C^+_{i1}D^+_{i1k}-\sum_{i=1}^{4}\sum_{k=1}^{9}p_{2k}\omega_i C^+_{i2}D^+_{i2k}\ge v^-\right\}\\
s.t.\quad & \sum_{i=1}^{4}\left(T^-_{i1}+\Delta T_{i1}y_{i1opt}-D^+_{i1k}\right)\le q^-_{1k},\quad k=1,2,3\\
& \sum_{i=1}^{4}\left(T^-_{i2}+\Delta T_{i2}y_{i2opt}-D^+_{i2k}\right)\le q^-_{2k},k=1,2,\ldots,9\\
& T^\pm_{i1\max}\ge T^-_{i1}+\Delta T_{i1}y_{i1opt}\ge D^-_{i1k}\ge 0, i=1,2,3,4, k=1,2,3\\
& T^\pm_{i2\max}\ge T^-_{i2}+\Delta T_{i2}y_{i2opt}\ge D^-_{i2k}\ge 0, i=1,2,3,4, k=1,2,\ldots,9
\end{aligned}\tag{9}$$

where D^+_{ijk} is the decision variable. The chance function of model (9) is

$$f_2=\Pr\left\{\begin{array}{l}
\sum_{i=1}^{4}\sum_{j=1}^{2}\omega_i NB^-_{ij}\left(T^-_{ij}+\Delta T_{ij}y_{ijopt}\right)-\sum_{i=1}^{4}\sum_{k=1}^{3}p_{1k}\omega_i C^+_{i1}D^+_{i1k}-\sum_{i=1}^{4}\sum_{k=1}^{9}p_{2k}\omega_i C^+_{i2}D^+_{i2k}\ge v^-\\
\sum_{i=1}^{4}\left(T^-_{i1}+\Delta T_{i1}y_{i1opt}-D^+_{i1k}\right)\le q^-_{1k},k=1,2,3\\
\sum_{i=1}^{4}\left(T^-_{i2}+\Delta T_{i2}y_{i2opt}-D^+_{i2k}\right)\le q^-_{2k},k=1,2,\ldots,9\\
T^\pm_{i1\max}\ge T^-_{i1}+\Delta T_{i1}y_{i1opt}\ge D^-_{i1k}\ge 0, i=1,2,3,4, k=1,2,3\\
T^\pm_{i2\max}\ge T^-_{i2}+\Delta T_{i2}y_{i2opt}\ge D^-_{i2k}\ge 0, i=1,2,3,4, k=1,2,\ldots,9
\end{array}\right\},$$

and solving model (9) is equivalent to obtaining $\max f_2$.

In order to solve the model (9), the hybrid algorithm is also used.

Firstly, we use stochastic simulation to generate input-output data for $U_2:(D^+_{i1k}, D^+_{i2k}) \to f_2$.

Then, BP neural network is used to simulate objective function, GA is used to obtain the optimal solution. Finally, the model (9) is solved. The optimal solution is D^+_{ijkopt} and the optimal value $\max f_2$ is obtained.

(3) The real water amount allocating to user i when the scenario k occurs in period j can be obtained by $A^\pm_{ijkopt}=T^-_{ij}+\Delta T_{ij}y_{ijopt}-D^\pm_{ijkopt}$.

(4) The optimized interval solution is $D^\pm_{ijkopt}=\left[D^-_{ijkopt}, D^+_{ijkopt}\right]$, and the real optimized interval amount of allocating water is $A^\pm_{ijkopt}=\left[A^-_{ijkopt}, A^+_{ijkopt}\right]$. We can also obtain the optimal value.

6 Results and discussions

Table 4 and Table 5 indicate the amounts of the real allocation targets, the water shortages and optimized allocations in 2024 and 2025, respectively. Due to $y_{11opt}=0.899$, $y_{21opt}=0.447$, $y_{31opt}=0.697$, $y_{41opt}=1$, $y_{12opt}=0.578$, $y_{22opt}=0.9998$, $y_{32opt}=0.555$, $y_{42opt}=0.429$, the optimal allocation schemes are $T^{\pm}_{11opt}=3.46$, $T^{\pm}_{21opt}=3.18$, $T^{\pm}_{31opt}=12.40$, $T^{\pm}_{41opt}=1.10$, $T^{\pm}_{12opt}=2.12$, $T^{\pm}_{22opt}=2.35$, $T^{\pm}_{32opt}=11.16$, $T^{\pm}_{42opt}=1.20$ ($10^8 m^3$), respectively.

Table 4 shows the optimized solutions for four kinds of users under 3 scenarios in 2024. For example, $D^{\pm}_{211opt}=[1.142,1.644]$, $D^{\pm}_{212opt}=[2.689,2.720]$ and $D^{\pm}_{213opt}=[0.564,1.616]$ ($10^8 m^3$) are water shortages for the ecological user ($i=2$) when the level of the water flow is low (the probability is 25.42%), medium (the probability is 47.76%), and high (the probability is 27.12%), respectively. Accordingly, the water allocations are $A^{\pm}_{211opt}=[1.082,1.583]$, $A^{\pm}_{212opt}=[0.005,0.036]$ and $A^{\pm}_{213opt}=[1.109,2.161]$ ($10^8 m^3$). In this case,

$$\max \Pr\left\{\sum_{i=1}^{4}\sum_{j=1}^{2}\omega_i NB^{+}_{ij}\left(T^{-}_{ij}+\Delta T_{ij}y_{ij}\right)-\sum_{i=1}^{4}\sum_{k=1}^{3}p_{1k}\omega_i C^{-}_{i1}D^{-}_{i1k}-\sum_{i=1}^{4}\sum_{k=1}^{9}p_{2k}\omega_i C^{-}_{i2}D^{-}_{i2k}>80\right\}$$

is 98.33%. In this sense, if we allot the water as the scheme in Table 4, the maximal probability that the system net benefit exceeds 80×10^8 ¥ as compared with the economic penalty is 98.33%.

Table 5 gives the optimized solutions for four kinds of users under 3 scenarios in 2025. For example, $D^{\pm}_{127opt}=[0.336,1.590]$, $D^{\pm}_{128opt}=[0.507,1.439]$ and $D^{\pm}_{129opt}=[1.504,2.279]$ ($10^8 m^3$) are water shortages for the municipality user ($i=1$) when the level of the water flow is low, medium and high, respectively, and the corresponding probability is 6.89%, 12.95% and 7.35%, respectively. Accordingly, the water allocations are $A^{\pm}_{127opt}=[1.070,2.324]$, $A^{\pm}_{128opt}=[1.221,2.154]$ and $A^{\pm}_{129opt}=[0.382,1.157]$ ($10^8 m^3$). In this case,

$$\max \Pr\left\{\sum_{i=1}^{4}\sum_{j=1}^{2}\omega_i NB^{-}_{ij}\left(T^{-}_{ij}+\Delta T_{ij}y_{ij}\right)-\sum_{i=1}^{4}\sum_{k=1}^{3}p_{1k}\omega_i C^{+}_{i1}D^{+}_{i1k}-\sum_{i=1}^{4}\sum_{k=1}^{9}p_{2k}\omega_i C^{+}_{i2}D^{+}_{i2k}>65\right\}$$

is 100%. In this sense, if we allot the water as the scheme in Table 5, the maximal probability that the system net benefit exceeds 65×10^8 ¥ as compared with the economic penalty is 100%.

The optimal values represent that the maximal probability is [98.33%, 100%] of the event which the system net benefit is $[65,\ 80]\times10^8$ ¥ more than the economic penalty subject to the constraints, which provides two extreme values. When the value of every parameter fluctuate between its lower and upper bounds, the probabilities would change correspondingly, which simultaneously reflects the balance between the system profit and the constraints.

Table 4: Solutions in 2024

Scenario (*ijk*)	User	The level of water flow	Probability (%)	Targets ($10^8 m^3$)	Shortage ($10^8 m^3$)	Allocation ($10^8 m^3$)
111	Municipal	L	25.42	3.37	[1.458,2.356]	[1.013,1.911]
211	Ecological	L	25.42	2.725	[1.142,1.644]	[1.082,1.583]
311	Agricultural	L	25.42	1.35	[1.276,1.348]	[0.002,0.074]
411	Industrial	L	25.42	14.956	[12.808,14.916]	[0.041,2.148]
112	Municipal	M	47.76	3.37	[0.064,1.913]	[1.456,3.306]
212	Ecological	M	47.76	2.725	[2.689,2.720]	[0.005,0.036]
312	Agricultural	M	47.76	1.35	[1.188,1.193]	[0.157,0.162]
412	Industrial	M	47.76	14.956	[4.128,11.812]	[3.145,10.828]
113	Municipal	H	27.12	3.37	[3.369,3.370]	[0.000,0.001]
213	Ecological	H	27.12	2.725	[0.564,1.616]	[1.109,2.161]
313	Agricultural	H	27.12	1.35	[0.143,0.989]	[0.361,1.208]
413	Industrial	H	27.12	14.956	[8.418,13.356]	[1.601,6.538]

$y_{11opt}=0.899$, $y_{21opt}=0.447$, $y_{31opt}=0.697$, $y_{41opt}=1$.

Table 5: Solutions in 2025

Scenario symbol (ijk)	User	Water flow level	Probability (%)	Associated water flow	Associated probability (%)	Water target ($10^8 m^3$)	Water shortage ($10^8 m^3$)	Water allocation ($10^8 m^3$)
121	Municipal	L	25.42	L-L	6.46	2.661	[0.803,2.083]	[0.578,1.857]
221	Ecological	L	25.42	L-L	6.46	2.983	[0.806,1.784]	[1.199,2.177]
321	Agricultural	L	25.42	L-L	6.46	1.394	[0.734,0.758]	[0.636,0.660]
421	Industrial	L	25.42	L-L	6.46	12.15	[4.819,9.043]	[3.107,7.331]
122	Municipal	M	47.76	L-M	12.14	2.661	[0.843,1.064]	[1.597,1.818]
222	Ecological	M	47.76	L-M	12.14	2.983	[2.365,2.432]	[0.551,0.618]
322	Agricultural	M	47.76	L-M	12.14	1.394	[1.364,1.394]	[0.000,0.030]
422	Industrial	M	47.76	L-M	12.14	12.15	[1.877,8.932]	[3.218,10.273]
123	Municipal	H	27.12	L-H	6.89	2.661	[0.709,1.075]	[1.585,1.952]
223	Ecological	H	27.12	L-H	6.89	2.983	[0.910,1.927]	[1.056,2.072]
323	Agricultural	H	27.12	L-H	6.89	1.394	[1.146,1.192]	[0.202,0.248]
423	Industrial	H	27.12	L-H	6.89	12.15	[0.701,2.385]	[9.765,11.449]
124	Municipal	L	25.42	M-L	12.14	2.661	[0.326,2.510]	[0.151,2.335]
224	Ecological	L	25.42	M-L	12.14	2.983	[0.803,2.029]	[0.954,2.180]
324	Agricultural	L	25.42	M-L	12.14	1.394	[0.876,1.149]	[0.245,0.518]
424	Industrial	L	25.42	M-L	12.14	12.15	[8.894,12.039]	[0.112,3.257]
125	Municipal	M	47.76	M-M	22.81	2.661	[1.632,2.270]	[0.391,1.028]
225	Ecological	M	47.76	M-M	22.81	2.983	[0.853,2.675]	[0.308,2.130]
325	Agricultural	M	47.76	M-M	22.81	1.394	[0.873,1.386]	[0.009,0.521]
425	Industrial	M	47.76	M-M	22.81	12.15	[10.717,12.041]	[0.110,1.433]
126	Municipal	H	27.12	M-H	12.95	2.661	[0.400,0.477]	[2.183,2.261]
226	Ecological	H	27.12	M-H	12.95	2.983	[1.939,2.922]	[0.061,1.044]
326	Agricultural	H	27.12	M-H	12.95	1.394	[0.260,1.308]	[0.087,1.134]
426	Industrial	H	27.12	M-H	12.95	12.15	[10.898,11.567]	[0.584,1.252]
127	Municipal	L	25.42	H-L	6.89	2.661	[0.336,1.590]	[1.070,2.324]
227	Ecological	L	25.42	H-L	6.89	2.983	[1.510,2.291]	[0.692,1.473]
327	Agricultural	L	25.42	H-L	6.89	1.394	[1.372,1.389]	[0.005,0.022]
427	Industrial	L	25.42	H-L	6.89	12.15	[0.744,10.085]	[2.065,11.406]
128	Municipal	M	47.76	H-M	12.95	2.661	[0.507,1.439]	[1.221,2.154]
228	Ecological	M	47.76	H-M	12.95	2.983	[0.001,2.592]	[0.391,2.982]
328	Agricultural	M	47.76	H-M	12.95	1.394	[1.189,1.392]	[0.002,0.205]
428	Industrial	M	47.76	H-M	12.95	12.15	[5.102,9.259]	[2.891,7.048]
129	Municipal	H	27.12	H-H	7.35	2.661	[1.504,2.279]	[0.382,1.157]
229	Ecological	H	27.12	H-H	7.35	2.983	[0.640,2.185]	[0.798,2.343]
329	Agricultural	H	27.12	H-H	7.35	1.394	[0.006,0.892]	[0.502,1.388]
429	Industrial	H	27.12	H-H	7.35	12.15	[8.813,9.693]	[2.457,3.337]
$y_{12opt} = 0.578$, $y_{22opt} = 0.9998$, $y_{32opt} = 0.555$, $y_{42opt} = 0.429$.								

7 Conclusions

In this study, an inexact two-stage stochastic dependent-chance programming model for water resources management is put forward by incorporating stochastic dependent-chance programming, two-stage stochastic programming and interval programming within an optimization framework. Compared with the existing other dependent-chance programming for water resources management, this model is able to deal with the inexact data with multiply distributed stochastic boundaries, take into account the priority level of water users, and give a certain weight to each water user to make it more realistic. More complex models, such as nonlinear model, can also be solved by the proposed hybrid algorithm. Finally the model is applied to water resource management system in Handan City. The results provide the optimal allocation schemes in Handan City and maximize the probability of achieving the given target value proposed by the decision maker.

Further studies can resort to introducing stochastic dependent-chance multiple-objective programming, fuzzy programming or uncertain programming into the optimization framework. Meanwhile more kinds of water resources, such as reclaimed water, the water from south-to-north water diversion project, and so on, may be taken into consideration.

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