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Averaging of a System of Set-Valued Differential Equations with the Hukuhara Derivative

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Abstract

In the introduction of this article, we gave a brief overview of publications on the theory of set-valued equations. Next, we considered slow-fast systems of set-valued differential equations and substantiated the possibility of applying the averaging method for approximate resolution or investigation of the properties of solutions of such systems.

Keywords: set-valued differential equation, Hukuhara, averaging

1 Introduction

Lately the development of calculus in metric spaces became an object of attention of many researchers [16, 39, 43, 63, 81, 95, 98, 106]. Earlier F.S. de Blasi and F. Iervolino [10] begun studying of set-valued differential equations (SDEs) in semilinear metric spaces. Now it transformed into the theory of set-valued equations (SEs) as an independent discipline. The properties of SDEs [1, 2, 3, 11, 9, 14, 107, 108, 109, 17, 18, 19, 20, 27, 28, 35, 38, 39, 40, 41, 42, 43, 45, 47, 48, 50, 44, 57, 70, 76, 81, 82, 83, 86, 88, 89, 96, 97, 104, 106, 113], set-valued differential inclusions [12, 22, 23, 34, 71, 72, 73, 74, 79, 91, 95, 102], the set-valued integral and integro-differential equations and inclusions [8, 30, 32, 33, 36, 65, 80, 84, 85, 87, 89, 90, 91, 103, 105, 114, 99], the fractional SDEs [110, 111], the set-valued stochastic differential equations (SSDEs) [31, 46, 51, 52, 53, 54, 55, 56, 58, 59, 60, 66, 116, 113, 117, 118], the impulse set-valued differential equations (ISDEs) [4, 63, 64, 81, 101, 102], set-valued differential equations on time scales [49, 115] and control set-valued systems [6, 7, 5, 34, 67, 68, 69, 71, 75, 77, 93, 95, 99] were considered. On the other hand, set-valued equations are useful in other areas of mathematics. For example, SDEs are used as an auxiliary tool to prove the existence results for differential inclusions [63, 96, 97, 106]. Also, one can employ SEs in the investigation of fuzzy differential equations, fuzzy integral equations and other [38, 39, 41, 43, 44, 81]. Moreover, SDEs are a natural generalization of usual ordinary differential equations in finite (or infinite) dimensional Banach spaces [44, 106]. On the other hand, in many cases, when modeling real-world phenomena, information about the behavior of a dynamical system is uncertain and one has to consider these uncertainties to gain better understanding of the full models. The set-valued equations can be used to model dynamical systems subjected to uncertainties. We have given only a small part of the literature on the theory of set-valued equations (see also the references therein). We also want to note that we did not consider interval equations. They are a particular case of set-valued equations, but they have their own specifics.

The averaging methods combined with the asymptotic representations began to be applied as the basic constructive tool for solving the complicated problems of analytical dynamics described by the differential equations [13, 15, 95, 100]. Averaging theory for ordinary differential equations has a rich history, dating to back to the work of N.M. Krylov and N.N. Bogoliubov [37]. The possibility of using some averaging schemes for set-valued equations was studied in [22, 23, 26, 29, 32, 33, 34, 62, 63, 64, 78, 79, 81, 92, 93, 94, 95, 102, 103] and the references therein.

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In works [61, 62, 63, 94] the authors considered systems of differential equations with the Hukuhara derivative:

$$\begin{array}{ll} D_H X_1 = F_1(t, X_1, \cdots, X_m), & X_1(0) = X_1^0, \\ D_H X_2 = F_2(t, X_1, \cdots, X_m), & X_2(0) = X_2^0, \\ \vdots & \vdots \\ D_H X_m = F_m(t, X_1, \cdots, X_m), & X_m(0) = X_m^0, \end{array} \tag{1}$$

where $D_H X$ - Hukuhara derivative, $t \in [0, T]$, $X_i \in conv(R^{n_i})$, $F_i : [0, T] \times conv(R^{n_1}) \times \cdots \times comv(R^{n_m}) \to conv(R^{n_i})$, proved the existence theorem for them, and substantiated the possibility of some averaging schemes for such systems with a small parameter

$$\begin{array}{ll} D_H X_1 = \varepsilon F_1(t, X_1, \cdots, X_m), & X_1(0) = X_1^0, \\ D_H X_2 = \varepsilon F_2(t, X_1, \cdots, X_m), & X_2(0) = X_2^0, \\ \vdots & \vdots \\ D_H X_m = \varepsilon F_m(t, X_1, \cdots, X_m), & X_m(0) = X_m^0, \end{array} \tag{2}$$

where $\varepsilon > 0$ is a small parameter.

In this article, we consider a slow-fast system of two differential equations with the Hukuhara derivative:

$$\begin{array}{ll} D_H X = \varepsilon F(t, X, Y, \varepsilon), & X(0, \varepsilon) = X_0, \\ D_H Y = \Phi(t, X, Y, \varepsilon), & Y(0, \varepsilon) = Y_0, \end{array} \tag{3}$$

where $\varepsilon \in [0, \varepsilon_0]$ is a small parameter; $t \in [0, T]$; $X \in conv(R^n)$ is a fast variable; $Y \in conv(R^m)$ is a slow variable; $F : R_+ \times conv(R^n) \times conv(R^m) \times [0, \varepsilon_0] \to conv(R^n)$, $\Phi : R_+ \times conv(R^n) \times conv(R^m) \times [0, \varepsilon_0] \to conv(R^m)$ are given set-valued mappings.

This paper is organized as follows. In Section 2, we recall some basic concepts and notations about set-valued analysis and set-valued differential equations. In Section 3, we justify the possibility of applying one averaging scheme for system (3).

2 Preliminaries

Let $conv(R^n)$ be a space of all nonempty convex compact subsets of R^n with the Hausdorff metric

$$h(A, B) = \min_{r \geq 0} \{B \subset S_r(A), \;\; A \subset S_r(B)\}$$

where $A, B \in conv(R^n)$, $S_r(A)$ be a r-neighborhood of the set A.

The usual set operations, i.e., well-known as Minkowski addition and scalar multiplication, are defined as follows

$$A + B = \{a + b \,:\, a \in A,\, b \in B\}$$

and

$$\lambda A = \{\lambda a \,:\, a \in A,\, \lambda \in R\}.$$

Lemma 2.1. ([81, 98]) *The following properties hold:*
1. $(conv(R^n), h)$ *is a complete metric space,*
2. $h(A + C, B + C) = h(A, B)$,
3. $h(\lambda A, \lambda B) = |\lambda| h(A, B)$ *for all* $A, B, C \in conv(R^n)$ *and* $\lambda \in R$.

For any $A \in conv(R^n)$, it can be seen $A + (-1)A \neq \{0\}$ in general, thus the opposite of A is not the inverse of A with respect to the Minkowski addition unless $A = \{a\}$ is a singleton. To partially overcome this situation, the Hukuhara difference has been introduced [21].

Definition 2.1. ([21]) *Let* $X, Y \in conv(R^n)$. *A set* $Z \in conv(R^n)$ *such that* $X = Y + Z$ *is called a Hukuhara difference of the sets* X *and* Y *and is denoted by* $X \overset{h}{-} Y$.

An important property of Hukuhara difference is that $A \overset{h}{-} A = \{0\}$, for all $A \in conv(R^n)$ and $(A + B) \overset{h}{-} B = A$, for all $A, B \in conv(R^n)$; Hukuhara difference is unique, but a necessary condition for $A \overset{h}{-} B$ to exist is that A contains a translate $\{c\} + B$ of B.

Let $X : [0,T] \to conv(R^n)$ be a set-valued mapping; $(t_0-\Delta, t_0+\Delta) \subset [0,T]$ be a Δ- neighborhood of a point $t_0 \in [0,T]; \Delta > 0$.

For any $t \in (t_0-\Delta, t_0+\Delta)$ consider the following Hukuhara differences $X(t) \overset{h}{-} X(t_0)$, $t \geq t_0$, and $X(t_0) \overset{h}{-} X(t)$, $t \geq t_0$ if these differences exist.

Definition 2.2. ([21]) *We say that the mapping $X : [0,T] \to conv(R^n)$ has a Hukuhara derivative $D_H X(t_0)$ at a point $t_0 \in [0,T]$, if there exists an element $D_H X(t_0) \in conv(R^n)$ such that the limits*

$$\lim_{t \downarrow t_0} \frac{1}{t-t_0}(X(t) \overset{h}{-} X(t_0)) \quad \textit{and} \quad \lim_{t \uparrow t_0} \frac{1}{t_0-t}(X(t_0) \overset{h}{-} X(t))$$

exist in the topology of $conv(R^n)$ and are equal to $D_H X(t_0)$.

3 The Method of Averaging

Consider the slow-fast systems of differential equations with the Hukuhara derivative (3). We take $\varepsilon = 0$, then we obtain the following system of differential equations with Hukuhara derivative

$$\begin{array}{ll} D_H X = \{0\}, & X(0,0) = X_0, \\ D_H Y = \Phi(t,X,Y,0), & Y(0,0) = Y_0. \end{array} \tag{4}$$

It's obvious that the solution $X(t,0)$ of the first equation of system (4) such that $X(t,0) \equiv X_0$ for every $t \geq 0$.

Let $Y(t,0)$ is solution of the second differential equation of system (4), i.e.

$$D_H Y = \Phi(t, X_0, Y, 0), \;\; Y(0,0) = Y_0.$$

Let there exists $\overline{F}(X) \in conv(R^n)$ such that a limit

$$h\left(\lim_{T\to\infty} \frac{1}{T}\int_0^T F(t,X,Y(t,0),0)dt, \overline{F}(X)\right) = 0 \tag{5}$$

exists and the integral is understood in the sense of [21].

Now consider following problem with the small parameters

$$D_H Z = \varepsilon \overline{F}(Z), \;\; Z(0,\varepsilon) = X_0. \tag{6}$$

Let $Q = \{t \geq 0,\ X \subset P \in conv(R^n),\ Y \subset G \in conv(R^m),\ \varepsilon \in [0,\varepsilon^0]\}$.

Theorem 3.1. *Suppose the following conditions hold:*

1) mappings $F(\cdot,X,Y,\varepsilon)$, $\Phi(\cdot,X,Y,\varepsilon)$ are continuous, for all $(X,Y,\varepsilon) \in Q$;

2) there exists $\lambda > 0$ such that

$$h(F(t,X',Y',\varepsilon), F(t,X'',Y'',0)) \leq \lambda(h(X',X'') + h(Y',Y'') + \varphi_1(\varepsilon)),$$

$$h(\Phi(t,X',Y',\varepsilon), \Phi(t,X'',Y'',0)) \leq \lambda(h(X',X'') + h(Y',Y'') + \varphi_2(\varepsilon)),$$

for all $(t,X',Y',\varepsilon), (t,X'',Y'',0) \in Q$, and $\lim_{\varepsilon\downarrow 0} \varphi_i(\varepsilon) = 0,\ i = 1,2$;

3) there exists $\gamma > 0$ such that $h(F(t,X,Y,\varepsilon), \{0\}) \leq \gamma$ for every $(t,X,Y,\varepsilon) \in Q$;

4) there exists $\alpha > 0$ such that

$$h(\overline{F}(Z'), \overline{F}(Z'')) \leq \alpha h(Z',Z'')$$

for all $Z', Z'' \in P$;

5) limit (5) exists in every $(X_0,Y_0) \subset P \times G$;

6) the solution of the problem (6) together with a $\rho-$neighborhood belong to the domain P for $t \in [0, L\varepsilon^{-1}]$.

Then for any $\eta \in (0,\rho]$ and $L > 0$ there exists $\varepsilon_0(\eta,L) \in (0,\varepsilon^0]$ such that for all $\varepsilon \in (0,\varepsilon_0]$ and $t \in [0,L\varepsilon^{-1}]$ the following inequality holds

$$h(X(t,\varepsilon), Z(t,\varepsilon)) < \eta. \tag{7}$$

Proof. Let $X(\cdot,\varepsilon),\ Y(\cdot,\varepsilon)$ are solutions of the system of differential equations (3), and $Z(\cdot,\varepsilon)$ is solution of the differential equation (6). Since

$$X(t,\varepsilon)=X_0+\varepsilon\int\limits_0^t F(s,X(s,\varepsilon),Y(s,\varepsilon),\varepsilon)ds,$$
$$Y(t,\varepsilon)=Y_0+\int\limits_0^t \Phi(s,X(s,\varepsilon),Y(s,\varepsilon),\varepsilon)ds,$$
$$Z(t,\varepsilon)=X_0+\varepsilon\int\limits_0^t \overline{F}(Z(s,\varepsilon))ds,$$

we get

$$\begin{aligned}
&h(X(t,\varepsilon),Z(t,\varepsilon))\\
&=\varepsilon h\left(\int\limits_0^t F(s,X(s,\varepsilon),Y(s,\varepsilon),\varepsilon)ds,\int\limits_0^t \overline{F}(Z(s,\varepsilon))ds\right)\\
&\le \varepsilon h\left(\int\limits_0^t F(s,X(s,\varepsilon),Y(s,\varepsilon),\varepsilon)ds,\int\limits_0^t \overline{F}(X(s,\varepsilon))ds\right)+\varepsilon h\left(\int\limits_0^t \overline{F}(Z(s,\varepsilon))ds,\int\limits_0^t \overline{F}(X(s,\varepsilon))ds\right)\\
&\le \varepsilon h\left(\int\limits_0^t F(s,X(s,\varepsilon),Y(s,\varepsilon),\varepsilon)ds,\int\limits_0^t \overline{F}(X(s,\varepsilon))ds\right)+\varepsilon\int\limits_0^t h\left(\overline{F}(Z(s,\varepsilon)),\overline{F}(X(s,\varepsilon))\right)ds\\
&\le \varepsilon h\left(\int\limits_0^t F(s,X(s,\varepsilon),Y(s,\varepsilon),\varepsilon)ds,\int\limits_0^t \overline{F}(X(s,\varepsilon))ds\right)+\varepsilon\alpha\int\limits_0^t h(X(s,\varepsilon),Z(s,\varepsilon))ds.
\end{aligned}\tag{8}$$

Using condition 3) of the theorem, we obtain the following estimate

$$h(X(t,\varepsilon),X(t,0))\le 2\varepsilon\gamma t.\tag{9}$$

By (9) and condition 2) of the theorem, we get

$$\begin{aligned}
&h(Y(t,\varepsilon),Y(t,0))\\
&=h\left(\int\limits_0^t \Phi(s,X(s,\varepsilon),Y(s,\varepsilon),\varepsilon)ds,\int\limits_0^t \Phi(s,X(s,0),Y(s,0),0)ds\right)\\
&\le\int\limits_0^t h\left(\Phi(s,X(s,\varepsilon),Y(s,\varepsilon),\varepsilon),\Phi(s,X(s,0),Y(s,0),0)\right)ds\\
&\le\lambda\int\limits_0^t [h(X(s,\varepsilon),X(s,0))+h(Y(s,\varepsilon),Y(s,0))+\varphi_2(\varepsilon)]ds\\
&\le lambda\int\limits_0^t [2\varepsilon\gamma s+\varphi_2(\varepsilon)]ds+\lambda\int\limits_0^t h(Y(s,\varepsilon),Y(s,0))ds\\
&=\lambda\varepsilon\gamma t^2+\lambda\varphi_2(\varepsilon)t+\lambda\int\limits_0^t h(Y(s,\varepsilon),Y(s,0))ds.
\end{aligned}$$

Using Gronwall-Bellmans inequality, we obtain

$$h(Y(t,\varepsilon),Y(t,0))\le(\lambda\varepsilon\gamma t^2+\lambda\varphi_2(\varepsilon)t)e^{\lambda t}.$$

Let $\beta(t,\varepsilon)=(\lambda\varepsilon\gamma t^2+\lambda\varphi_2(\varepsilon)t)e^{\lambda t}$. Clearly, the function $\beta(t,\varepsilon)$ is nondecreasing in t and such that $\lim\limits_{\varepsilon\downarrow 0}\beta(t,\varepsilon)=0$ for every $t\ge 0$.

Now we take an arbitrary number $\xi>0$. Let

$$t^*(\varepsilon,\xi)=\begin{cases} t^*, & \beta(t*,\varepsilon)=\xi,\\ +\infty, & \beta(t,\varepsilon)\ne\xi \text{ for all } t\ge 0\end{cases}$$

and

$$\Delta(\varepsilon,\xi)=\min\{\varepsilon^{-1/2},t^*(\varepsilon,\xi)\}.$$

It's obviously, for every $\xi>0$

$$\lim_{\varepsilon\downarrow 0}\Delta(\varepsilon,\xi)=+\infty.\tag{10}$$

Divide the interval $[0, L\varepsilon^{-1}]$ into partial intervals by the points $t_j = j\Delta$, $j = \overline{0, m}$, where $m\Delta \geq L\varepsilon^{-1}$. We take any $t \in [0, L\varepsilon^{-1}]$. It's obviously, there exists constant such that $t \in [t_k, t_{k+1})$ and $k < m$. Therefore,

$$k\varepsilon \leq \frac{1}{\Delta}. \tag{11}$$

Since $h(\overline{F}(Z), \{0\}) \leq \gamma$ and $t - t_k < \Delta$, it follows that

$$\begin{aligned} & h(X(t,\varepsilon), Z(t,\varepsilon) \\ & \leq h(X(t_k,\varepsilon), Z(t_k,\varepsilon) + h(X(t,\varepsilon), X(t_k,\varepsilon) + h(Z(t,\varepsilon), Z(t_k,\varepsilon) \\ & \leq h(X(t_k,\varepsilon), Z(t_k,\varepsilon) + 2\varepsilon\gamma\Delta. \end{aligned} \tag{12}$$

Therefore,

$$\begin{aligned} & \varepsilon h\left(\int\limits_0^{t_k} F(s, X(s,\varepsilon), Y(s,\varepsilon), \varepsilon)ds, \int\limits_0^{t_k} \overline{F}(X(s,\varepsilon))ds\right) \\ & \leq \varepsilon \sum_{j=0}^{k-1} h\left(\int\limits_{t_j}^{t_{j+1}} F(s, X(s,0,R_j), Y(s,0,R_j), 0)ds, \int\limits_{t_j}^{t_{j+1}} F(X(t_j,\varepsilon))ds\right) \\ & \quad + \varepsilon \sum_{j=0}^{k-1} h\left(\int\limits_{t_j}^{t_{j+1}} F(s, X(s,0,R_j), Y(s,0,R_j), 0)ds, \int\limits_{t_j}^{t_{j+1}} F(s, X(s,\varepsilon), Y(s,\varepsilon), \varepsilon)ds\right) \\ & \quad + \varepsilon \sum_{j=0}^{k-1} h\left(\int\limits_{t_j}^{t_{j+1}} \overline{F}(X(t_j,\varepsilon))ds, \int\limits_{t_j}^{t_{j+1}} \overline{F}(X(s,\varepsilon))ds\right), \end{aligned} \tag{13}$$

where $R_j = (X(t_j,\varepsilon), Y(t_j,\varepsilon))$.

We take any $0 < \eta < \rho$. Then there exists $\xi > 0$ such that the following estimate is true

$$\lambda\xi \leq \frac{\eta}{4} e^{-\alpha L}. \tag{14}$$

Further we estimate each of the terms in (13). By (10) and condition 3) of the theorem, we can choose numbers $T(\eta) > 0$ and $\varepsilon_1 > 0$ such that

1) $\Delta(\varepsilon, \xi) \geq T(\eta)$;

2)

$$\varepsilon \sum_{j=0}^{k-1} h\left(\int\limits_{t_j}^{t_{j+1}} F(s, X(s,0,R_j), Y(s,0,R_j), 0)ds, \int\limits_{t_j}^{t_{j+1}} F(X(t_j,\varepsilon))ds\right) \leq \varepsilon\gamma\Delta \frac{\eta}{4} e^{-\alpha L} \leq \frac{\eta}{4} e^{-\alpha L} \tag{15}$$

for all $0 < \varepsilon \leq \varepsilon_1$.

Now if we recall condition 2) of the theorem and (11), we obtain

$$\begin{aligned} & \varepsilon \sum_{j=0}^{k-1} h\left(\int\limits_{t_j}^{t_{j+1}} F(s, X(s,0,R_j), Y(s,0,R_j), 0)ds, \int\limits_{t_j}^{t_{j+1}} F(s, X(s,\varepsilon), Y(s,\varepsilon), \varepsilon)ds\right) \\ & \leq \varepsilon \sum_{j=0}^{k-1} \int\limits_{t_j}^{t_{j+1}} h\left(F(s, X(s,0,R_j), Y(s,0,R_j), 0), F(s, X(s,\varepsilon), Y(s,\varepsilon), \varepsilon)\right) ds \\ & \leq \lambda[\varepsilon\gamma\Delta + \xi + \varphi_1(\varepsilon)]. \end{aligned} \tag{16}$$

Similarly, we get

$$\begin{aligned} & \varepsilon \sum_{j=0}^{k-1} h\left(\int\limits_{t_j}^{t_{j+1}} \overline{F}(X(t_j,\varepsilon))ds, \int\limits_{t_j}^{t_{j+1}} \overline{F}(X(s,\varepsilon))ds\right) \\ & \leq \varepsilon \sum_{j=0}^{k-1} \int\limits_{t_j}^{t_{j+1}} h\left(\overline{F}(X(t_j,\varepsilon)), \overline{F}(X(s,\varepsilon))\right) ds \leq \varepsilon\Delta \tfrac{\alpha\gamma}{2}. \end{aligned} \tag{17}$$

Combining (13)-(17), we obtain

$$\begin{aligned} & \varepsilon h\left(\int\limits_0^{t_k} F(s, X(s,\varepsilon), Y(s,\varepsilon), \varepsilon)ds, \int\limits_0^{t_k} \overline{F}(X(s,\varepsilon))ds\right) \\ & \leq \tfrac{\eta}{2} e^{-\alpha L} + \lambda[\varepsilon\gamma\Delta + \varphi_1(\varepsilon)] + \varepsilon\Delta \tfrac{\alpha\gamma}{2}. \end{aligned} \tag{18}$$

Since $\varepsilon\Delta \leq \sqrt{\varepsilon}$, then we can choose number $\varepsilon_2 > 0$ such that inequality

$$\lambda[\varepsilon\gamma\Delta + \varphi_2(\varepsilon)] + \varepsilon\Delta\frac{\alpha\gamma}{2} \leq \frac{\eta}{2}e^{-\alpha L} \tag{19}$$

is true for all $\varepsilon \in (0, \varepsilon_2]$.

By (18) and (19), we have

$$\varepsilon h\left(\int_0^{t_k} F(s, X(s,\varepsilon), Y(s,\varepsilon), \varepsilon)ds, \int_0^{t_k} \overline{F}(X(s,\varepsilon))ds\right) \leq \frac{3\eta}{4}e^{-\alpha L}. \tag{20}$$

Using (8), (20) and Gronwall-Bellmans inequality, we get

$$h(X(t_k,\varepsilon), Z(t_k,\varepsilon)) \leq \frac{3\eta}{4}. \tag{21}$$

We take $\varepsilon_3 > 0$ such that

$$2\gamma\varepsilon_3\Delta < \frac{\eta}{4}. \tag{22}$$

Summing (12), (21) and (22), we get

$$h(X(t,\varepsilon), Z(t,\varepsilon)) < \eta$$

for all $t \in [0, L\varepsilon^{-1}]$, where $\varepsilon \in (0, \varepsilon_0)$, $\varepsilon_0 = \min\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$. This completes the proof of theorem 3.1.

Remark 3.1. In case, if $X \in R^n$, $Y \in R^m$, $F : R_+ \times R^n \times R^m \times [0, \varepsilon_0] \to R^n$, $\Phi : R_+ \times R^n \times R^m \times [0, \varepsilon_0] \to R^m$, then we have a slow-fast system of ordinary differential equations

$$\begin{array}{ll} \dot{X} = \varepsilon F(t, X, Y, \varepsilon), & X(0,\varepsilon) = X_0, \\ \dot{Y} = \Phi(t, X, Y, \varepsilon), & Y(0,\varepsilon) = Y_0, \end{array} \tag{23}$$

where $\dot{X} = \frac{dX}{dt}$ is ordinary derivative. In this case Theorem 3.1 was proved in the papers [24, 25, 112].

Remark 3.2. Clearly, the theorem 3.1 is also true for a system of interval-valued differential equations, i.e. if

$$\begin{array}{ll} D_H X = \varepsilon F(t, X, Y, \varepsilon), & X(0,\varepsilon) = X_0, \\ D_H Y = \Phi(t, X, Y, \varepsilon), & Y(0,\varepsilon) = Y_0, \end{array} \tag{24}$$

where $\varepsilon \in [0, \varepsilon_0]$ is a small parameter; $t \in [0, T]$, $X \in conv(R)$; $Y \in conv(R)$; $F : R_+ \times conv(R) \times conv(R) \times [0, \varepsilon_0] \to conv(R)$, $\Phi : R_+ \times conv(R) \times conv(R) \times [0, \varepsilon_0] \to conv(R)$ are interval-valued mappings.

Remark 3.3. The same theorem 3.1 can also be proved for a system of fuzzy differential equations

$$\begin{array}{ll} D_H X = \varepsilon F(t, X, Y, \varepsilon), & X(0,\varepsilon) = X_0, \\ D_H Y = \Phi(t, X, Y, \varepsilon), & Y(0,\varepsilon) = Y_0, \end{array} \tag{25}$$

where $D_H X$ - fuzzy Hukuhara derivative; $\varepsilon \in [0, \varepsilon_0]$ is a small parameter; $E^n(E^m)$ is metric space of fuzzy sets [43]; $t \in [0, T]$; $X \in E^n$; $Y \in E^m$; $F : R_+ \times E^n \times E^m \times [0, \varepsilon_0] \to E^n$, $\Phi : R_+ \times E^n \times E^m \times [0, \varepsilon_0] \to E^m$ are fuzzy mappings.

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