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Soft Simple Graphs over Hemirings

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Abstract

In this paper, we have extended the concept of soft hemiring in graph theory and introduced soft simple graphs over hemirings and soft simple graph isomorphism over hemirings. We also investigated some structural properties of them.

Keywords: soft set, hemiring, soft hemiring, graph, simple graph, soft simple graph, soft simple graph isomorphism

1 Introduction

In our practical life, we often face some troubles with uncertainties in which right judgment is very crucial. But most of our conventional mathematic apparatus cannot solve these troubles. To triumph over these limitations in 1999, Russian researcher Molodtsov [23] initiated the idea of soft set, which can be treated as an inventive mathematical tool for dealing with uncertainties. Also Molodtsov pointed out some applications of soft set in [24]. Recently, soft set theory has been developed speedily and focused by a host of scholars in theory and application. In 2003, Maji et al.[21] defined some basic operations of soft set such as equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set and absolute soft set with examples. In 2009, Ali et al. [5] introduced several new operations of soft sets and developed the idea of complement of soft set. Babitha and Sunil [6] in 2010, introduced the concept of soft set relations as a sub soft set of the Cartesian product of the soft sets and many related concepts such as equivalent soft set relation, partition, composition and function are discussed. The theoretical aspects of soft set were also studied by some researchers [11, 22, 26, 27, 28, 30]. Soft set theory has also probable applications in numerous fields of science and engineering such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory and measurement theory. Mostly it has been applied to soft decision making in [7, 8, 20, 34]. In algebra, the concept of soft set was also successfully applied by some researchers [1, 4, 9, 13-18, 25]. On the other hand the concept of hemiring was first introduced by LaTorre [19] in 1965; as a special kind of semiring. Golan [10] in 1999 redefined hemiring and discussed some special classes of hemirings. Later some researchers studied hemirings such as [2, 31]. Soft hemirings are also studied by Yasin Ali et al. [32, 33]. In graph theory, the concept of soft set first used Thumbakara and George [29]. They introduced soft graphs and investigated some of its properties. Akram and Nawaz [3] introduced vertex-induced soft graphs, edge-induced soft graphs and complement of soft graphs also investigated some properties of soft graphs. In this work, we have extended the concept of soft hemirings in undirected simple graph theory. The paper is organized as follows. In section 2 we have discussed some basic definitions and notations used in this article. In section 3 we have introduced soft simple graph over hemirings. In section 4, soft simple graph isomorphism is described.

2 Preliminaries

2.1 Hemirings

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Definition 2.1.1: [31] A *hemiring* is a nonempty set H on which operations of addition and multiplication have been defined such that the following conditions are satisfied:

- $(H\,;\,+)$ is a commutative monoid with identity element 0;
- $(H\,;\,\cdot)$ is a semigroup;
- Multiplication distributes over addition from either side;
- The element 0 is the absorbing element of the multiplication i.e., $0\cdot r = 0 = r\cdot 0$.

A non-empty subset A of H is called a *subhemiring* of H if it contains zero and is closed with respect to the addition and multiplication of H.

Example 2.1.1 (a): The set $H = \{0, a, b, c, d\}$ is a hemiring with the operations table in Table 1.

Table 1: The operations table of the hemiring H

$+$	0	a	b	c	d
0	0	a	b	c	d
a	a	a	b	c	d
b	b	b	b	c	d
c	c	c	c	c	d
d	d	d	d	d	d

$\cdot$	0	a	b	c	d
0	0	0	0	0	0
a	0	a	a	a	a
b	0	a	b	b	b
c	0	a	b	c	c
d	0	a	b	c	d

2.2 Soft Sets [23]

Suppose that U is an initial universe set and E is a set of parameters, let $P(U)$ denotes the power set of U. A pair (F, E) is called a *soft set* of U where F is a mapping given by $F: E \to P(U)$. Clearly a soft set is a mapping from parameters to $P(U)$ and it is not a set, but a parameterized family of subsets of the universe.

2.3 Soft Hemirings [33]

Let H be a hemiring and A be two non-empty set. We consider an arbitrary binary relation R between an element of A and an element of H, that is, R is a subset of $A \times H$ without otherwise specified. A set-valued function $F: A \to P(H)$ which is defined as

$$F(x) = \{y \in H: (x, y) \in R\} \text{ for all } x \in A.$$

Then the pair (F, A) is called a *soft set* of H, which is derived from the relation R. For a soft set (F, A), the set $Supp(F, A) = \{x \in A: F(x) \neq \emptyset\}$ is called the *support* of the soft set. A soft set is called *null soft set* if the support is empty, and we say that a soft set is *non-null* if $Supp(F, A) \neq \emptyset$.

Definition 2.3.1: Let $(H; +,\cdot)$ be a hemiring and A be a non-empty set. Then a non-null soft set (F, A) is called a *soft hemiring* of H, if $F(x)$ is a subhemiring of H for all $x \in Supp(F, A)$.

Example 2.3.1(a): Let $(H = \{0, 1, 2, 3, 4, 5\}; +_6,\ \times_6)$ be a hemiring and $A = \{0,1,2,3,4,5\}$. We consider a set valued function $F: A \to P(H)$ which is defined as

$$F(x) = \{y \in H: xRy \Leftrightarrow x \times_6 y = 0\} \text{ for all } x \in A.$$

Then $F(0) = H, F(1) = \{0\},\ F(2) = \{0, 3\},\ F(3) = \{0, 2, 4\},\ F(4) = \{0, 3\},\ F(5) = \{0\}$.

As we observe that all of these sets are subhemirings of H. So (F, A) is a soft hemiring of H.

2.4 Simple Undirected Graphs [12]

A *graph* $G^* = (V, E)$ consists of a non-empty set of objects V, called *vertices* and a set E of two element subset of V called *edges*. A graph $G_1^* = (V_1, E_1)$ is said to be a *subgraph* of $G^* = (V, E)$ if $V_1 \subseteq V$ and $E_1 \subseteq E$. A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices is called a *simple graph*. The simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ are *isomorphic* if there is a one-to-one and onto mapping from V_1 to V_2 with the property that a and b are adjacent in G_1^* if and only if $f(a)$ and $f(b)$ are adjacent in G_2^*, for all

a and b in V_1. Such a mapping f is called an *isomorphism*. The *degree* of a vertex in an undirected simple graph is the number of edges incident with it.

3 Soft Simple Graphs over Hemirings

Let $G^* = (V, E)$ be a simple undirected graph where, V is a hemiring and E is the relation on V. The elements of V are vertices of G^* and the elements of E are the edges of G^*. Let A be a non-empty set and R be an arbitrary binary relation from A to G^*. A mapping from A to $P(V)$ written as, $F: A \to P(V)$ can be defined as $F(x) = \{y \in V: (x, y) \in R\}$ for all $x \in A$ and a mapping from A to $P(E)$ written as $K: A \to P(E)$ can be defined as $K(x) = \{uv \in E: \{u, v\} \subseteq F(x)\}$ for all $x \in A$. The pair (F, A) is a soft set of V and the pair (K, A) is a soft set of E.

Definition 3.1: The pair $((F, A), (K, A))$ is a called *soft simple graph* over a hemiring V if (F, A) is a soft hemiring of V and $(F(a), K(a))$ is a connected subgraph of G^* for all $a \in A$. It is denoted by G, i.e., $G = ((F, A), (K, A))$ where $(F(a), K(a))$ is denoted by $J(a)$.

Example 3.1 (a): Consider the simple undirected graph $G^* = (V, E)$, where $V = \{0, a, b, c, d\}$ is a hemiring with the operations table in Table 1, and the graph is given as,

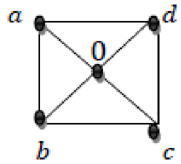


Figure 1: Simple undirected graph G^*

Let $A = \{a, b, c\}$ be a non-empty set and $F: A \to P(V)$ is a set valued function defined by

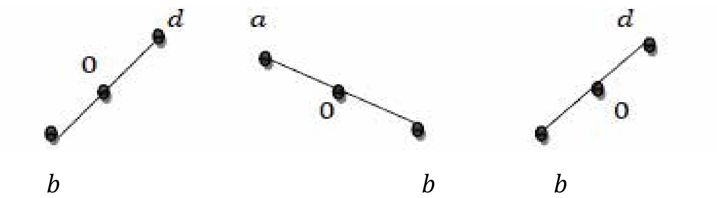
$$F(x) = \{y \in V: xRy \Leftrightarrow d(x, y) = 1\} \text{ for all } x \in A.$$

That is, $F(a) = \{0, b, d\}, F(b) = \{0, a, c\}$ and$F(c) = \{0, b, d\}$. Then clearly (F, A) be a soft hemiring of V.

Let (K, A) be a soft set of E where $K: A \to P(E)$ is a set valued function defined by

$$K(x) = \{uv \in E: \{u, v\} \subseteq F(x)\} \text{ for all } x \in A.$$

That is, $K(a) = \{0b, 0d\}, K(b) = \{0a, 0c\}$ and $K(c) = \{0b, 0d\}$. Then we obtain that $J(a) = \big(F(a), K(a)\big), J(b) = \big(F(b), K(b)\big)$ and $J(c) = \big(F(c), K(c)\big)$ are connected subgraphs of G^* as shown in Figure 2. Hence $G = ((F, A), (K, A))$ is a soft simple graph of G^* over the hemiring V.



$J(a)$ corresponding to 'a' $J(b)$ corresponding to 'b' $J(c)$ corresponding to 'c'

Figure 2: Connected subgraphs $J(a), J(b), J(c)$

Also we can represent this soft simple graph $G = ((F, A), (K, A))$ over the hemiring V by the Table 2.

Table 2: Tabular representation of the soft simple graph

F	0	a	b	c	d
a	1	0	1	0	1
b	1	1	0	1	0
c	1	0	1	0	1

K	$0a$	$0b$	$0c$	$0d$	ab	bc	cd	da
a	0	1	0	1	0	0	0	0
b	1	0	1	0	0	0	0	0
c	0	0	0	1	0	0	0	0

Definition 3.2: Let $G_1 = ((F_1, A), (K_1, A))$ and $G_2 = ((F_2, B), (K_2, B))$ be two soft simple graphs of G^* over a hemiring V. Then G_2 is a *soft simple subgraph* of G_1if

(i) $B \subseteq A$;
(ii) $F_2(x)$ is a subhemiring of $F_1(x)$ for all $x \in B$;
(iii) $J_2(x)$ is a connected subgraph of $J_1(x)$ for all $x \in B$.

Example 3.2 (a): Consider the simple undirected graph G^* in the Example 3.1(a).

Let $A = \{a, b, c\}$ and $B = \{a, b\}$ be two non-empty sets. Let $F_1: A \to P(V)$ be a set valued function defined by

$$F_1(x) = \{y \in V: xRy \Leftrightarrow d(x, y) \leq 1\} \text{ for all } x \in A.$$

That is, $F_1(a) = \{0, a, b, d\}$, $F_1(b) = \{0, a, b, c\}$ and $F_1(c) = \{0, b, c, d\}$. Then obviously (F_1, A) be a soft hemiring of V.

Let (K_1, A) be a soft set of E where $K_1: A \to P(E)$ is a set valued function defined by

$$K_1(x) = \{uv \in E: \{u, v\} \subseteq F_1(x)\} \text{ for all } x \in A.$$

That is, $K_1(a) = \{0a, 0b, 0d, ab, ad\}, K_1(b) = \{0a, 0b, 0c, ab, bc\}$ and $K_1(c) = \{0b, 0c, 0d, bc, cd\}$. Thus $J_1(a) = (F_1(a), K_1(a)), J_1(b) = (F_1(b), K_1(b))$ and $J_1(c) = (F_1(c), K_1(c))$ are connected subgraphs of G^*as shown in figure 3. Hence $G_1 = ((F_1, A), (K_1, A))$ is a soft simple graph of G^* over the hemiring V.

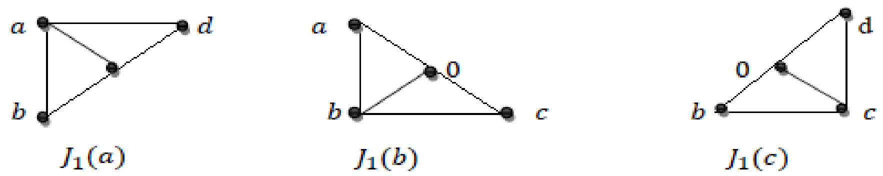


Figure 3: Connected subgraphs $J_1(a), J_1(b), J_1(c)$

Again let $F_2: B \to P(V)$ is a set valued function defined by

$$F_2(x) = \{y \in V: xRy \Leftrightarrow d(x, y) = 1\} \text{ for all } x \in B.$$

That is, $F_2(a) = \{0, b, d\}$ and $F_2(b) = \{0, a, c\}$. Then evidently (F_2, B) be a soft hemiring of V.

Let (K_2, B) be a soft set of E where $K_2: B \to P(E)$ is a set valued function defined by

$$K_2(x) = \{uv \in E: \{u, v\} \subseteq F_2(x)\} \text{ for all } x \in B.$$

That is, $K_2(a) = \{0b, 0d\}$ and $K_2(b) = \{0a, 0c\}$. Thus $J_2(a) = (F_2(a), K_2(a))$, $J_2(b) = (F_2(b), K_2(b))$ are connected subgraphs of G^*as shown in figure 4. Hence $G_2 = ((F_2, B), (K_2, B))$ is a soft simple graph of G^* over the hemiring V.

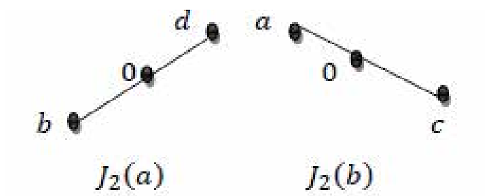


Figure 4: Connected subgraphs $J_2(a)$, $J_2(b)$

Now, we observe that $B \subseteq A$, $F_2(x)$ is a subhemiring of $F_1(x)$ and $J_2(x)$ is a connected subgraph of $J_1(x)$ for all $x \in B$. Hence G_2 is a soft simple subgraph of G_1.

Definition 3.3: Let $G_1 = ((F_1, A), (K_1, A))$ and $G_2 = ((F_2, B), (K_2, B))$ be two soft simple graphs of G^* over a hemiring V. Then the ***intersection*** of G_1 and G_2 is defined by $G = G_1 \cap G_2 = ((F, C), (K, C))$ where $C = A \cap B$ and for all $x \in C$, $F(x) = F_1(x) \cap F_2(x)$ and $K(x) = K_1(x) \cap K_2(x)$

Example 3.3 (a): Consider the simple undirected graph G^* in the Example 3.1(a).

Let $A = \{b, c\}$ and $B = \{c, d\}$ be two non-empty sets. Let $F_1: A \to P(V)$ is a set valued function defined by

$$F_1(x) = \{y \in V: xRy \Leftrightarrow d(x, y) = 1\} \text{ for all } x \in A.$$

That is, $F_1(b) = \{0, a, c\}$ and $F_1(c) = \{0, b, d\}$, and therefore (F_1, A) be a soft hemiring of V.

Let (K_1, A) be a soft set of E where $K_1: A \to P(E)$ is a set valued function defined by

$$K_1(x) = \{uv \in E: \{u, v\} \subseteq F_1(x)\} \text{ for all } x \in A.$$

That is, $K_1(b) = \{0a, 0c\}$ and $K_1(c) = \{0b, 0d\}$. Thus $J_1(b) = \big(F_1(b), K_1(b)\big), J_1(c) = \big(F_1(c), K_1(c)\big)$ are connected subgraphs of G^*as shown in figure 5. Hence $G_1 = ((F_1, A), (K_1, A))$ is a soft simple graph of G^* over the hemiring V.

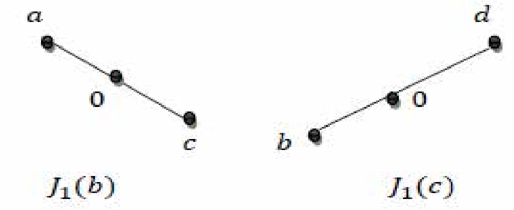


Figure 5: Connected subgraphs $J_1(b), J_1(c)$

Again let $F_2: B \to P(V)$ is a set valued function defined by

$$F_2(x) = \{y \in V: xRy \Leftrightarrow d(x, y) \leq 1\} \text{ for all } x \in B.$$

That is $F_2(c) = \{0, b, c, d\}$ and $F_2(d) = \{0, a, c, d\}$. So by routine calculations give that (F_2, B) is a soft hemiring of V.

Let (K_2, B) be a soft set of E where $K_2: B \to P(E)$ is a set valued function defined by

$$K_2(x) = \{uv \in E: \{u, v\} \subseteq F_2(x)\} \text{ for all } x \in B.$$

That is, $K_2(c) = \{0b, 0c, 0d, bc, cd\}$ and $K_2(d) = \{0a, 0c, 0d, cd, da\}$. Consequently

$$J_2(c) = \big(F_2(c), K_2(c)\big), J_2(d) = \big(F_2(d), K_2(d)\big)$$

are connected subgraphs of G^*as shown in Figure 6. Hence $G_2 = ((F_2, A), (K_2, A))$ is a soft simple graph of G^* over the hemiring V.

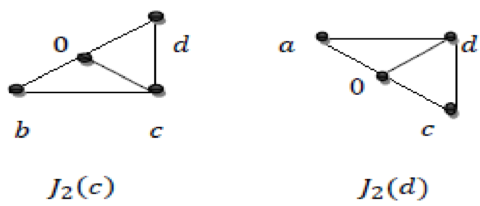


Figure 6: Connected subgraphs $J_2(c), J_2(d)$

Here $C = A \cap B = \{c\}$. Then

$$F(c) = F_1(c) \cap F_2(c) = \{0, b, d\} \text{ and } K(c) = K_1(c) \cap K_2(c) = \{0b, od\}.$$

The subgraph $J(c) = \big(F(c), K(c)\big)$ of G^* is shown Figure 7.

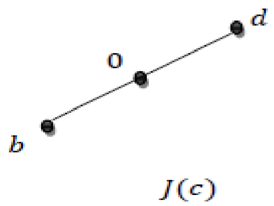


Figure 7: Subgraph $J(c)$

Theorem 3.4: Let $G_1 = ((F_1, A), (K_1, A))$ and $G_2 = ((F_2, B), (K_2, B))$ be two soft simple graphs of G^* over a hemiring V. If $A \cap B \neq \emptyset$ and $F(x) = F_1(x) \cap F_2(x) \neq \emptyset$. Then $G_1 \cap G_2$ is an also soft simple graph of G^* over the hemiring V.

Proof: By Definition 3.3, we can write

$$G_1 \cap G_2 = \big((F_1, A), (K_1, A)\big) \cap \big((F_2, B), (K_2, B)\big) = ((F, C), (K, C)),$$

where $C = A \cap B, F(x) = F_1(x) \cap F_2(x)$ and $K(x) = K_1(x) \cap K_2(x)$ for all $x \in C$.

By hypothesis for all $x \in C = A \cap B \neq \emptyset$, we have $F(x) = F_1(x) \cap F_2(x) \neq \emptyset$. Thus $F_1(x)$ and $F_2(x)$ are both subhemirings of V. Hence $F(x)$ is a subhemiring of V and consequently$(F(x), K(x))$ is a connected subgraph of G^* for all $x \in C$ and so $\big((F, C), (K, C)\big) = \big((F_1, A), (K_1, A)\big) \cap \big((F_2, B), (K_2, B)\big)$ is a soft simple graph of G^* over a hemiring V.

Corollary 3.5: Let $G_1 = ((F_1, A), (K_1, A))$ and $G_2 = ((F_2, A), (K_2, A))$ be two soft simple graphs of G^* over a hemiring V. If $F(x) = F_1(x) \cap F_2(x) \neq \emptyset$. Then we have the following

(i) The intersection $G_1 \cap G_2$ is a soft simple graph of G^* over the hemiring V.
(ii) $G_1 \cap G_2$ is a soft connected subgraph of G_1 and G_2 over the hemiring V.

Proof: Trivial.

Definition 3.6: Let $G_1 = ((F_1, A), (K_1, A))$ and $G_2 = ((F_2, B), (K_2, B))$ be two soft simple graphs of G^* over a hemiring V. Then the *union* of G_1 and G_2 is defined by $G = G_1 \cup G_2 = ((F, C), (K, C))$ where $C = A \cup B$ and for all $x \in C$,

$$F(x) = \begin{cases} F_1(x) & if\ x \in A - B; \\ F_2(x) & if\ x \in B - A; \\ F_1(x) \cup F_2(x) & if x \in A \cap B. \end{cases}$$

And

$$K(x) = \begin{cases} K_1(x) & if\ x \in A - B; \\ K_2(x) & if\ x \in B - A; \\ K_1(x) \cup K_2(x) & if x \in A \cap B. \end{cases}$$

Example 3.6 (a): Consider the soft simple graphs $G_1 = ((F_1, A), (K_1, A))$ and $G_2 = ((F_2, B), (K_2, B))$ in the example 3.3(a).

Here $C = A \cup B = \{b, c, d\}$. Then

$F(b) = F_1(b) = \{0, a, c\}$ and $K(b) = \{0a, 0c\}$;
$F(c) = F_1(c) \cup F_2(c) = \{0, b, c, d\}$ and $K(c) = \{0b, 0c, 0d, bc, cd\}$;
$F(d) = F_2(d) = \{0, a, c, d\}$ and $K(d) = \{0a, 0c, 0d, cd, da\}$.

Subgraphs of G^* are

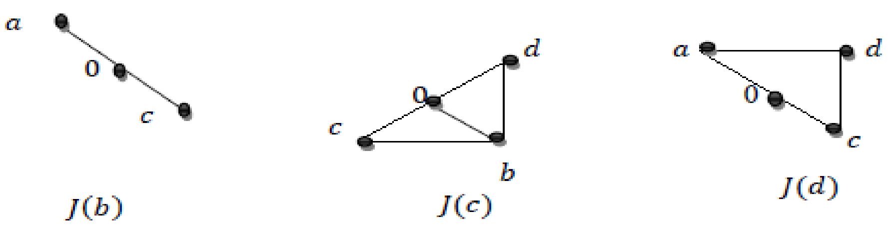


Figure 8: Subgraphs $J(b), J(c), J(d)$

Theorem 3.7: Let $G_1 = ((F_1, A), (K_1, A))$ and $G_2 = ((F_2, B), (K_2, B))$ be two soft simple graphs of G^* over a hemiring V. If $A \cap B = \emptyset$, then $G_1 \cup G_2$ is an also soft simple graph of G^* over a hemiring V.

Proof: We have from the Definition 3.6, $G = G_1 \cup G_2 = ((F, C), (K, C))$ where $C = A \cup B$ and for all $x \in C$,

$$F(x) = \begin{cases} F_1(x) & if\ x \in A - B; \\ F_2(x) & if\ x \in B - A; \\ F_1(x) \cup F_2(x) & if x \in A \cap B. \end{cases}$$

And

$$K(x) = \begin{cases} K_1(x) & if\ x \in A - B; \\ K_2(x) & if\ x \in B - A; \\ K_1(x) \cup K_2(x) & if x \in A \cap B. \end{cases}$$

Since $A \cap B = \emptyset$, it follows that $x \in A - B$ or $x \in B - A$ for all $x \in C$.

If $x \in A - B$, then $F(x) = F_1(x)$, is a subhemiring of V, since (F, A) is a soft hemiring of V and consequently, $(F(x), K(x))$ is a connected subgraph of G^* over the hemiring V. If $x \in B - A$, then $F(x) = F_2(x)$ is a subhemiring of V, since (F_2, B) is a soft hemiring of V and therefore, $(F(x), K(x))$ is a connected subgraph of G^* over a hemiring V. Hence $G_1 \cup G_2 = ((F, C), (K, C))$ is a soft simple graph of G^* over the hemiring V.

Definition 3.8: Let $G_1 = ((F_1, A), (K_1, A))$ and $G_2 = ((F_2, B), (K_2, B))$ be two soft simple graphs of G^* over a hemiring V.

Then $G_1\ AND\ G_2$ denoted by $G_1 \wedge G_2 = \big((F_1, A), (K_1, A)\big) \wedge \big((F_2, B), (K_2, B)\big)$is defined by $\big((F_1, A), (K_1, A)\big) \wedge \big((F_2, B), (K_2, B)\big) = ((F, C), (K, C))$ where $C = A \times B$ and $F(x, y) = F_1(x) \cap F_2(y)$ and $K(x, y) = K_1(x) \cap K_2(y)$ for all $(x, y) \in C$.

Example 3.8 (a): Consider the simple undirected graph G^* in the example 3.1(a).

Let $A = \{a, c\}$ and $B = \{b\}$ be two parameter sets. Let $F_1: A \to P(V)$ is a set valued function defined by

$$F_1(x) = \{y \in V: xRy \Leftrightarrow d(x, y) \leq 1\} \text{ for all } x \in A.$$

That is, $F_1(a) = \{0, a, b, d\}$ and $F_1(c) = \{0, b, c, d\}$. We notice that (F_1, A) be a soft hemiring of V.

Let (K_1, A) be a soft set of E where $K_1: A \to P(E)$ is a set valued function defined by

$$K_1(x) = \{uv \in E: \{u, v\} \subseteq F_1(x)\} \text{ for all } x \in A.$$

That is, $K_1(a) = \{0a, 0b, 0d, ad, ab\}$ and $K_1(c) = \{0b, 0c, 0d, bc\ dc\}$. Therefore $J_1(a) = \big(F_1(a), K_1(a)\big), J_1(c) = \big(F_1(c), K_1(c)\big)$ are connected subgraphs of G^*as shown in figure 9. Thus $G_1 = ((F_1, A), (K_1, A))$ is a soft simple graph of G^* over the hemiring V.

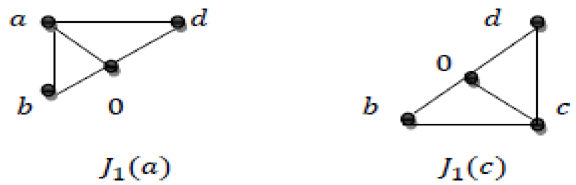


Figure 9: Connected subgraphs $J_1(a), J_1(c)$

Again let $F_2: B \to P(V)$ is a set valued function defined by

$$F_2(x) = \{y \in V: xRy \Leftrightarrow d(x, y) \leq 1\} \text{ for all } x \in B.$$

That is, $F_2(b) = \{0, a, b, c\}$, so therefore (F_2, B) be a soft hemiring of V.

Let (K_2, B) be a soft set of E where $K_2: B \to P(E)$ is a set valued function defined by

$$K_2(x) = \{uv \in E: \{u, v\} \subseteq F_2(x)\} \text{ for all } x \in B.$$

That is, $K_2(b) = \{0a, 0b, 0c, ab, bc\}$. Thus $J_2(b) = \big(F_2(b), K_2(b)\big)$ is a connected subgraph of G^*as shown in figure 10.

Consequently $G_2 = ((F_2, A), (K_2, A))$ is a soft simple graph of G^* over a hemiring V.

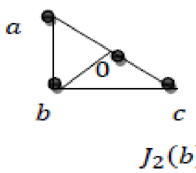


Figure 10: Connected subgraph $J_2(b)$

Here $C = A \times B = \{(a,b)(c,b)\}$. Then

$$F(a,b) = F_1(a) \cap F_2(b) = \{0,a,b\} \text{ and } K(a,b) = K_1(a) \cap K_2(b) = \{0a, 0b, ab\};$$

$$F(c,b) = F_1(c) \cap F_2(b) = \{0,b,c\} \text{ and } K(c,b) = K_1(c) \cap K_2(b) = \{0b, 0c, bc\}.$$

Subgraphs of G^* are

Figure 11: Subgraphs $J(a,b), J(c,b)$

Theorem 3.9: Let $G_1 = ((F_1,A),(K_1,A))$ and $G_2 = ((F_2,A),(K_2,A))$ be two soft simple graphs of G^* over a hemiring V. If $F(x) = F_1(x) \cap F_2(x) \neq \emptyset$. Then $G_1 \wedge G_2$ is an also soft simple graph of G^* over the hemiring V.

Proof: Trivial.

Definition 3.10: Let $G_1 = ((F_1,A),(K_1,A))$ and $G_2 = ((F_2,B),(K_2,B))$ be two soft simple graphs of G^* over a hemiring V. Then G_1 OR G_2 denoted by $G_1 \vee G_2 = \big((F_1,A),(K_1,A)\big) \vee \big((F_2,B),(K_2,B)\big)$ is defined by

$$\big((F_1,A),(K_1,A)\big) \vee \big((F_2,B),(K_2,B)\big) = ((F,C),(K,C))$$

where $C = A \times B$ and $F(x,y) = F_1(x) \cup F_2(y)$ and $K(x,y) = K_1(x) \cup K_2(y)$ for all $(x,y) \in C$.

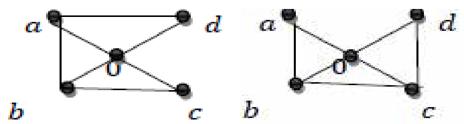
Example 3.10 (a): Consider the soft simple graphs $G_1 = ((F_1,A),(K_1,A))$ and $G_2 = ((F_2,A),(K_2,A))$ in the example 3.8(a).

Here $C = A \times B = \{(a,b)(c,b)\}$. Then

$$F(a,b) = F_1(a) \cup F_2(b) = \{0,a,b,c,d\} \text{ and } K(a,b) = K_1(a) \cup K_2(b) = \{0a, 0b, ab, 0d, ad, 0c, bc\};$$

$$F(c,b) = F_1(c) \cup F_2(b) = \{0,a,b,c,d\} \text{ and } K(c,b) = K_1(c) \cup K_2(b) = \{0a, 0b, 0c, bc, ab, 0d, dc\}.$$

Subgraphs of G^* are



$J(a,b)$ $J(c,b)$

Figure 12: Subgraphs $J(a,b), J(b,c)$

Theorem 3.11: Let $G_1 = ((F_1,A),(K_1,A))$ and $G_2 = ((F_2,A),(K_2,A))$ be two soft simple graphs of G^* over a hemiring V. If $F(x) = F_1(x) \cap F_2(x) \neq \emptyset$. Then $G_1 \vee G_2$ is an also soft simple graph of G^* over the hemiring V.

Proof: Trivial.

4 Soft Simple Graph Isomorphism

Definition 4.1: Let $G_1 = ((F_1,A),(K_1,A))$ and $G_2 = ((F_2,B),(K_2,B))$ be two soft simple graphs of the graphs $G_1^* = (V_1,E_1)$ and $G_2^* = (V_2,E_2)$ respectively over hemirings V_1 and V_2 respectively. Let $f: G_1^* \to G_2^*$ and $g: A \to B$. Then (f,g) is said to be a *soft simple graph homomorphism* if,

(i) f is a graph homomorphism from G_1 to G_2;
(ii) g is a mapping from A to B;
(iii) $f(F_1(x)) = F_2(g(x))$ for all $x \in A$.

Then G_1 is said to be a soft simple graph homomorphic to G_2.

Example 4.1 (a): Consider the graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ as shown in figure 13, where $V_1 = \{0, a, b, c\}$ and $V_2 = \{u(= 0), v\}$ are hemirings under the operations tables, Table 3 and Table 4 respectively.

Table 3: The operations table of the hemiring V_1

$+$	0	a	b	c
0	0	a	b	c
a	a	a	b	c
b	b	b	b	c
c	c	c	c	c

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	a	b	b
c	0	a	b	c

Table 4: The operations table of the hemiring V_2

$+$	u	v
u	0	v
v	v	v

$\cdot$	u	v
u	u	u
v	u	v

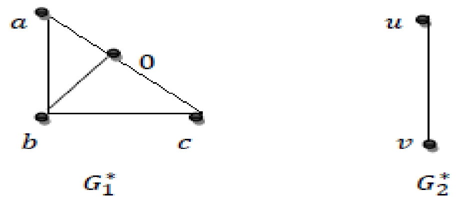


Figure 13: Simple undirected graphs G_1^* and G_2^*

Let $A = \{0, a, b, c\}$ and $B = \{u, v\}$ be non-empty sets. Let (F_1, A) be a soft hemiring of V_1 where $F_1: A \to P(V_1)$ is a set valued function defined by

$$F_1(x) = \{y \in V_1 : xRy \Leftrightarrow d(x, y) \leq 1\} \text{ for all } x \in A.$$

Let (K_1, A) be a soft set of E_1 where $K_1: A \to P(E_1)$ is a set valued function defined by

$$K_1(x) = \{uv \in E_1 : \{u, v\} \subseteq F_1(x)\} \text{ for all } x \in A.$$

Then $G_1 = ((F_1, A), (K_1, A))$ is a soft simple graph of G_1^* over the hemiring V_1.

Again let (F_2, B) be a soft hemiring of V_2 where $F_2: B \to P(V_2)$ is a set valued function defined by

$$F_2(x) = \{y \in V_2 : xRy \Leftrightarrow d(x, y) \leq 1\} \text{ for all } x \in B.$$

Let (K_2, B) be a soft set of E_2 and $K_2: B \to P(E_2)$ is a set valued function defined by

$$K_2(x) = \{uv \in E_2 : \{u, v\} \subseteq F_2(x)\} \text{ for all } x \in B.$$

Then $G_2 = ((F_2, B), (K_2, B))$ is a soft simple graph of G_2^* over the hemiring V_2.

Define

$$f: G_1^* \to G_2^* \text{ by } f(0) = u, f(a) = v, f(b) = u, f(c) = v.$$

Then f is a graph homomorphism from G_1^* to G_2^*.

Define

$$g: A \to B \text{ by } g(0) = u, g(a) = v, g(b) = u, g(c) = v.$$

Then g is a mapping from A to B.

Also $f(F_1(x)) = F_2(g(x))$ for all $x \in A$.

Hence G_1 is soft simple graph homomorphic to G_2.

Definition 4.2: Let $G_1 = ((F_1, A), (K_1, A))$ and $G_2 = ((F_2, B), (K_2, B))$ be two soft simple graphs of the graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively over hemirings V_1 and V_2 respectively. Let $f: G_1^* \to G_2^*$ and $g: A \to B$. Then (f, g) is said to be a *soft simple graph isomorphism* if

(i) f is a graph isomorphism from G_1 to G_2;
(ii) g is a bijective mapping from A to B;
(iii) $f(F_1(x)) = F_2(g(x))$ for all $x \in A$.

Then G_1 is said to be a soft simple graph isomorphic to G_2.

Example 4.2 (a): Consider the graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ as shown in figure 14, where $V_1 = \{0, a, b\}$ and $V_2 = \{u(= 0), v, w\}$ are hemirings under the operations tables, Table 5 and Table 6 respectively.

Table 5: The operations table of the hemiring V_1

$+$	0	a	b
0	0	a	b
a	a	a	b
b	b	b	b

$\cdot$	0	a	b
0	0	0	0
a	0	a	a
b	0	a	b

Table 6: The operations table of the hemiring V_2

$+$	u	v	w
u	u	v	w
v	v	v	w
w	w	w	w

$\cdot$	u	v	w
u	u	u	u
v	u	v	v
w	u	v	w

Figure 14: Simple undirected graphs G_1^* and G_2^*

Let $A = \{0, a, b\}$ and $B = \{u, v, w\}$ be non-empty sets. Let (F_1, A) be a soft hemiring of V_1 where $F_1: A \to P(V_1)$ is a set valued function defined by

$$F_1(x) = \{y \in V_1: xRy \Leftrightarrow d(x, y) \leq 1\} \text{ for all } x \in A.$$

Let (K_1, A) be a soft set of E_1 where $K_1: A \to P(E_1)$ is a set valued function defined by

$$K_1(x) = \{uv \in E_1: \{u, v\} \subseteq F_1(x)\} \text{ for all } x \in A.$$

Then $G_1 = ((F_1, A), (K_1, A))$ is a soft simple graph of G_1^* over the hemiring V_1.

Again let (F_2, B) be a soft hemiring of V_2 where $F_2: B \to P(V_2)$ is a set valued function defined by

$$F_2(x) = \{y \in V_2: xRy \Leftrightarrow d(x, y) \leq 1\} \text{ for all } x \in B.$$

Let (K_2, B) be a soft set of E_2 where $K_2: B \to P(E_2)$ is a set valued function defined by

$$K_2(x) = \{uv \in E_2: \{u, v\} \subseteq F_2(x)\} \text{ for all } x \in B.$$

Then $G_2 = ((F_2, B), (K_2, B))$ is a soft simple graph of G_2^* over the hemiring V_2.

Define $f: G_1^* \to G_2^*$ by $f(0) = u, f(a) = v, f(b) = w$. Then f is a graph isomorphism from G_1^* to G_2^*.

Define $g: A \to B$ by $g(0) = u, g(a) = v, g(b) = w$. Then g is a bijective mapping from A to B.

Also $f(F_1(x)) = F_2(g(x))$ for all $x \in A$.

Hence G_1 is soft simple graph isomorphic to G_2.

5 Conclusion

After the introduction of soft set by Molodtsov, a host of researchers used this concept in many branches of mathematics and other disciplines. In this work, we have used the concept of soft hemirings and introduced soft simple graphs and discussed some properties of them.

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