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Roughness in Semigroups Based on Soft Sets

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Abstract

In this paper, in order to combination of soft sets and rough sets with semigroups, the notion of soft rough semigroup is introduced according to Feng's idea. Roughness in semigroups with respect to soft rough approximation spaces is studied. In particular, some basic properties of lower and upper soft rough approximations of a subset with respect to two different soft approximation spaces are investigated. Finally, sufficient conditions for the lower and upper soft rough approximations of an arbitrary subset of a semigroup to be subsemigroup and (bi-, interior) ideal are provided.

Keywords: soft set, soft rough set, soft approximation space, soft rough semigroup, lower and upper soft rough approximation

1 Introduction

In 1999, Molodtsov introduced the notion of soft sets [15] as a new approach to tackle the problems of vagueness and uncertainty. In fact, classical methods are not always successful for modeling uncertain data, because the uncertainties appearing in different sciences such as medical science, engineering, economics, environment, social science, etc., may be of various types. In order to solve these problems, many mathematical tools are available for modeling uncertainties such as probability theory, fuzzy set theory [24], rough set theory [17], etc., but all of these theories have their inherent difficulties as pointed out in [15]. In this case, Molodtsov put forward the concept of soft sets as a new mathematical tool for dealing with uncertainties. Soft set theory has many application in areas such as the smoothness of functions, game theory, operations research, Riemann integration and so on, as reported by Molodtsov [15] in his work. Later on many researchers began to focus on the emerging theory and develop it in both theoretical study and practical application. Maji et al. [13] discussed in detail some new operations on soft sets such as subset, union, intersection and complements of soft sets and so on with examples and considered the application of soft set in decision making problem. Based on the analysis of several operations on soft sets introduced in [13], Ali et al. [2] introduced some new notions such as the restricted intersection, the restricted union, the restricted difference and the extended intersection of two soft sets. In recent years, research works on the relationship between soft sets, rough sets and fuzzy sets have been investigated by many authors. In 2010, Feng et al. [5] investigated the problem of combining soft sets with fuzzy sets and rough sets and proposed three different types of hybrid models, which are called rough soft sets, soft rough sets and soft-rough fuzzy sets. The combination of soft sets, fuzzy sets, and rough sets was also discussed by some researchers such as [1, 7, 10, 11, 14, 16, 20, 22, 29, 30].

In recent years, the connection between algebraic structures (especially, semigroups and hemirings) and the theories of uncertainty has been studied by many authors. Shabir and Irfan Ali [18] studied soft semigroups and soft ideals over a semigroup which characterize generalized fuzzy ideals and fuzzy ideals with thresholds of a semigroup. In [23], Yang gave the notations of fuzzy soft semigroups and fuzzy soft ideals, and discussed the α-level set, union and intersection of them. Hamouda [8] introduced the notions of soft left and soft right ideals, soft quasi-ideal and soft bi-ideal in ordered semigroups. Based on the idea in [5], Zhan et al. [26] firstly applied rough soft sets to hemirings and described some characterizations of rough soft hemirings. Zhan and Zhu [28], proposed a novel concept of soft rough fuzzy sets which is called Z-soft rough fuzzy set and introduced the notion of Z-soft rough fuzzy ideals of hemirings. After that, Zhan et al. [25] introduced

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the notion of Z-soft fuzzy rough set as an important generalization of Z-soft rough fuzzy sets and presented the application of this notion in semigroups. The notion of Z-soft fuzzy rough ideals of hemirings was also investigated by Ma et al. [12].

In particular, the notion of soft rough set introduced by Feng et al. [5] (we call it Feng-soft rough set) has a restricted condition, that is, the soft set must be full. Therefore, in order to strengthen this concept, a new approach called modified soft rough set was presented by Shabir et al. [19]. Recently, Zhan et al. [21, 27] investigated soft rough semigroups and soft rough hemirings according to Shabir's idea. It should be noted that the relationship between Feng-soft rough set and algebraic structures is not studied until now, therefore, research on this topic can be interesting. For this reason, we apply Feng-soft rough set to semigroup, as the first work in this area.

The aim of this paper is to make a connection between soft sets, rough sets and semigroups all together according to the idea of Feng et al. [5], which leads to introduce the notion of soft rough semigroups. In fact, we consider a soft set over a semigroup and use it instead of an equivalence relation to obtain lower and upper soft rough approximations of a subset of semigroup. Then we study the characterizations of soft rough sets in semigroups. In particular, we show that the lower and upper soft rough approximations of an arbitrary subset of a semigroup can be subsemigroup and (bi-, interior) ideal under sufficient conditions. Moreover, we consider two different soft approximation spaces over a common universe and investigate some basic properties of lower and upper soft rough approximations of a subset with respect to these soft approximation spaces.

2 Preliminaries and Notations

In this section, we recall some basic notions and definitions related to soft sets, rough sets and semigroups.

A *semigroup* is a system $(S, \cdot)$, where S is a non-empty set and the binary operation "$\cdot$" is associative. In what follows let S denote a semigroup.

A non-empty subset X of S is called a *subsemigroup* of S if $XX \subseteq X$. A nonempty subset X of S is called a *left* (*right*) *ideal* of S if $SX \subseteq X$ ($XS \subseteq X$). By *two-sided ideal* (simply, *ideal*), we mean a subset of S, which is both a left and right ideal of S. A subsemigroup X of S is called a *bi-ideal* of S if $XSX \subseteq X$. A subsemigroup X of S is called an *interior ideal* of S if $SXS \subseteq X$. For more details, please see [4, 9].

Definition 2.1. ([15]) *Let U be the universe set, E be the set of all possible parameters with respect to U and $P(U)$ denotes the set of all subsets of U. A pair $\mathfrak{S} = (F, A)$ is called a soft set over U, where $A \subseteq E$ and F is a set-valued mapping given by $F : A \longrightarrow P(U)$.*

As pointed out in [13], for any parameter $e \in A$, the subset $F(e) \subseteq U$ may be considered as the set of e-approximate elements, or as the set of e-elements in the soft set (F, A).

Definition 2.2. ([5]) *A soft set (F, A) over U is called a full soft set if $\bigcup_{a\in A} f(a) = U$.*

Definition 2.3. ([13]) *Let (F, A) and (G, B) be two soft sets over U. (F, A) is called a soft subset of (G, B), denoted by $(F, A) \subseteq (G, B)$, if $A \subseteq B$ and $F(a) \subseteq G(a)$ for all $a \in A$.*

Definition 2.4. ([2]) *Let (F, A) and (G, B) be two soft sets over U. The extended union of (F, A) and (G, B) is denoted by $(F, A) \cup (G, B) = (F \cup G, C)$, which is soft set over U, where $C = A \cup B$ and the set-valued mapping is given by*

$$(F \cup G)(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

for all $e \in C$.

Definition 2.5. ([2]) *Let (F, A) and (G, B) be two soft sets over U. The extended intersection of (F, A) and (G, B) is denoted by $(F, A) \cap (G, B) = (F \cap G, C)$, which is soft set over U, where $C = A \cup B$ and the set-valued mapping is given by*

$$(F \cap G)(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cap G(e) & \text{if } e \in A \cap B \end{cases}$$

for all $e \in C$.

Definition 2.6. ([5]) *A soft set (F, A) over U is called an intersection complete soft set if for any $a_1, a_2 \in A$, there exists $a_3 \in A$ such that $F(a_3) = F(a_1) \cap F(a_2)$ whenever $F(a_1) \cap F(a_2) \neq \emptyset$.*

Definition 2.7. ([10]) *Let (F, A) be a soft set over U. If for any $a, b \in A$, there exists $c \in A$ such that $f(c) = f(a) \cup f(b)$, then (F, A) is called an union complete soft set (UCS-set) over U.*

Definition 2.8. ([17]) *Let U be a non-empty finite set and R be an equivalence relation on U. A pair (U, R) is called an approximation space. For any subset X of U, the operations $\underline{R}(X)$ and $\overline{R}(X)$ are called the R-lower and the R-upper approximation of X, with respect to R, respectively. If $\underline{R}(X) = \overline{R}(X)$, then X is called definible with respect to R; otherwise, X is said to be rough with respect to R.*

Now, we present the notions of soft semigroups and soft (bi-, interior) ideals of semigroups.

Definition 2.9. ([18]) *Let (F, A) be a soft set over S. Then (F, A) is said to be a soft subsemigroup over S if $F(a)$ is a subsemigroup of S for all $a \in A$ with $F(a) \neq \emptyset$. Note that a soft subsemigroup also is called a soft semigroup.*

Definition 2.10. ([18]) *A soft set (F, A) over S is called a soft ideal (resp. soft left ideal, soft right ideal) over S, if $F(a)$ is an ideal (resp. left ideal, right ideal) of S for all $a \in A$ with $F(a) \neq \emptyset$.*

Definition 2.11. ([3]) *A soft subsemigroup (F, A) over S is called a soft bi-ideal over S, if $F(a)$ is a bi-ideal of S for all $a \in A$ with $F(a) \neq \emptyset$.*

Definition 2.12. *A soft semigroup (F, A) over S is called a soft interior ideal over S, if $F(a)$ is an interior ideal of S for all $a \in A$ with $F(a) \neq \emptyset$.*

3 Soft Rough Sets and Some of Their Properties

In this section, we concentrate on some basic properties of soft rough sets and investigate lower and upper soft rough approximations of a subset with respect to two different soft approximation spaces. We also introduce the notion of product complete soft sets to discuss the fundamental properties of lower and upper soft rough approximations of a subset of a semigroup. First we recall the notion of soft rough sets in [5].

Definition 3.1. ([5]) *Let $\mathfrak{S} = (F, A)$ be a soft set over U. The pair $P = (U, \mathfrak{S})$ is called a soft approximation space. Based on P, the following two operations are defined:*

$$\underline{apr}_P(X) = \{u \in U \mid \exists a \in A\,[u \in f(a) \subseteq X]\},$$

$$\overline{apr}_P(X) = \{u \in U \mid \exists a \in A\,[u \in f(a), f(a) \cap X \neq \emptyset]\},$$

for any subset X of U. Two sets $\underline{apr}_P(X)$ and $\overline{apr}_P(X)$ are called the lower and upper soft rough approximations of X with respect to P, respectively. If $\underline{apr}_P(X) \neq \overline{apr}_P(X)$, X is said to be soft P-rough set, otherwise X is called a soft P-definable.

Clearly, $\underline{apr}_P(X) \subseteq X$ and $\underline{apr}_P(X) \subseteq \overline{apr}_P(X)$ for all $X \subseteq U$ but $X \subseteq \overline{apr}_P(X)$ may not hold in general as shown in the following example.

Example 3.2. *Suppose that $U = \{u_1, u_2, \ldots, u_8\}$ be a universe and $A = \{e_1, e_2, \ldots, e_6\}$ be subset of parameter set. Let $\mathfrak{S} = (F, A)$ be a soft set over U given by Table 1 and $P = (U, \mathfrak{S})$ be the soft approximation space.*

Table 1: Tabular representation of the soft set $\mathfrak{S}$

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
e_1	0	0	1	0	1	0	0	0
e_2	1	0	0	0	1	0	0	0
e_3	1	1	0	0	0	0	1	1
e_4	0	0	1	0	1	0	0	1
e_5	0	1	1	0	0	0	0	0
e_6	0	0	0	0	0	1	0	1

For $X = \{u_2, u_3, u_4, u_5\} \subseteq U$, *we have* $\underline{apr}_P(X) = \{u_2, u_3, u_5\}$ *and* $\overline{apr}_P(X) = \{u_1, u_2, u_3, u_5, u_7, u_8\}$. *So,* $\underline{apr}_P(X) \neq \overline{apr}_P(X)$ *and* X *is soft P-rough set. It is easy to see that* $\underline{apr}_P(X) \subseteq X$ *and* $\underline{apr}_P(X) \subseteq \overline{apr}_P(X)$ *but* $X \nsubseteq \overline{apr}_P(X)$.

Theorem 3.3. ([6]) *Let* $\mathfrak{S} = (F, A)$ *be a full soft set over* U *and* $P = (U, \mathfrak{S})$ *be a soft approximation space. Then* $X \subseteq \overline{apr}_P(X)$, *for all* $X \subseteq U$.

From the definition of soft rough set, we have the following neat result.

Proposition 3.4. ([5, 6]) *Let* $\mathfrak{S} = (F, A)$ *be a soft set over* U *and* $P = (U, \mathfrak{S})$ *be a soft approximation space. Then*

$$\underline{apr}_P(X) = \bigcup_{f(a) \subseteq X} f(a) \quad \text{and} \quad \overline{apr}_P(X) = \bigcup_{f(a) \cap X \neq \emptyset} f(a).$$

Theorem 3.5. ([5, 6]) *Let* $\mathfrak{S} = (F, A)$ *be a soft set over* U *and* $P = (U, \mathfrak{S})$ *be a soft approximation space. Then for all* $X, Y \subseteq U$, *we have*

1. $\underline{apr}_P(\emptyset) = \overline{apr}_P(\emptyset) = \emptyset$;

2. $\underline{apr}_P(U) = \overline{apr}_P(U) = \bigcup_{a \in A} f(a)$;

3. $X \subseteq Y \Longrightarrow \underline{apr}_P(X) \subseteq \underline{apr}_P(Y)$ *and* $\overline{apr}_P(X) \subseteq \overline{apr}_P(Y)$;

4. $\underline{apr}_P(X \cap Y) \subseteq \underline{apr}_P(X) \cap \underline{apr}_P(Y)$;

5. $\underline{apr}_P(X \cup Y) \supseteq \underline{apr}_P(X) \cup \underline{apr}_P(Y)$;

6. $\overline{apr}_P(X \cup Y) = \overline{apr}_P(X) \cup \overline{apr}_P(Y)$;

7. $\overline{apr}_P(X \cap Y) \subseteq \overline{apr}_P(X) \cap \overline{apr}_P(Y)$;

8. $\underline{apr}_P(\overline{apr}_P(X)) = \overline{apr}_P(X)$;

9. $\overline{apr}_P(\underline{apr}_P(X)) \supseteq \underline{apr}_P(X)$;

10. $\underline{apr}_P(\underline{apr}_P(X)) = \underline{apr}_P(X)$;

11. $\overline{apr}_P(\overline{apr}_P(X)) \supseteq \overline{apr}_P(X)$.

Theorem 3.6. ([5, 6])*Let* $\mathfrak{S} = (F, A)$ *be an intersection complete soft set over* U *and* $P = (U, \mathfrak{S})$ *be a soft approximation space. Then*

$$\underline{apr}_P(X \cap Y) = \underline{apr}_P(X) \cap \underline{apr}_P(Y).$$

As we know, Pawlak's rough set is based on equivalence relations. In Feng-soft rough set, equivalence relation is replaced by a soft set. Based on this idea, soft rough approximations and soft rough sets are introduced. In the following, we consider two different soft approximation spaces based on a pair of soft sets over a common universe and study some basic properties of lower and upper soft rough approximations of a subset X of a universe with respect to these soft approximation spaces. Firstly, we introduce the notion of soft approximation subspaces. After that, we define the intersection and union of two soft approximation spaces. Moreover, in order to understand the obtained results, we give a practical example.

Definition 3.7. *Let* $\mathfrak{S} = (F, A)$ *and* $\mathfrak{T} = (G, B)$ *be two soft sets over* U *and* $P = (U, \mathfrak{S})$, $Q = (U, \mathfrak{T})$ *be two soft approximation spaces. Then* P *is called a soft approximation subspace of* Q, *if* $\mathfrak{S} \subseteq \mathfrak{T}$. *We write* $P \sqsubseteq Q$.

Theorem 3.8. *Let* $\mathfrak{S} = (F, A)$ *and* $\mathfrak{T} = (G, B)$ *be two soft sets over* U *and* $P = (U, \mathfrak{S})$, $Q = (U, \mathfrak{T})$ *be two soft approximation spaces such that* $P \sqsubseteq Q$. *Then for any non-empty subset* X *of* U, $\overline{apr}_P(X) \subseteq \overline{apr}_Q(X)$.

Proof. Let $u \in \overline{apr}_P(X)$, then there exist $a \in A$ such that $u \in f(a)$, $f(a) \cap X \neq \emptyset$. Since $\mathfrak{S} \subseteq \mathfrak{T}$, $a \in B$ and $f(a) \subseteq G(a)$ and so $u \in G(a)$. Clearly, $G(a) \cap X \neq \emptyset$. Therefore, $u \in \overline{apr}_Q(X)$. □

It's worth noting that the inclusion in Theorem 3.8 may be strict, as shown in the following example.

Example 3.9. *Suppose that* $U = \{u_1, u_2, \ldots, u_6\}$ *is a universe and* $A = \{e_1, e_4\}$ *and* $B = \{e_1, e_2, e_3, e_4\}$ *are subsets of parameter set. Let* $\mathfrak{S} = (F, A)$ *and* $\mathfrak{T} = (G, B)$ *be two soft sets over* U *given by Tables 2, 3, respectively and* $P = (U, \mathfrak{S})$, $Q = (U, \mathfrak{T})$ *be two soft approximation spaces. Clearly* $\mathfrak{S} \subseteq \mathfrak{T}$. *For* $X = \{u_1, u_4, u_5, u_6\} \subseteq U$, *we have* $\overline{apr}_P(X) = \{u_1, u_2, u_5, u_6\}$ *and* $\overline{apr}_Q(X) = \{u_1, u_2, u_3, u_5, u_6\}$. *Therefore* $\overline{apr}_P(X) \subset \overline{apr}_Q(X)$.

Table 2: Tabular representation of the soft set $\mathfrak{S}$

	u_1	u_2	u_3	u_4	u_5	u_6
e_1	1	0	0	0	0	1
e_4	1	1	0	0	1	0

Table 3: Tabular representation of the soft set $\mathfrak{T}$

	u_1	u_2	u_3	u_4	u_5	u_6
e_1	1	0	1	0	0	1
e_2	0	0	0	0	1	0
e_3	0	0	0	0	0	0
e_4	1	1	1	0	1	0

Remark 3.10. *In Example 3.9, we have* $\underline{apr}_P(X) = \{u_1, u_6\}$ *and* $\underline{apr}_Q(X) = \{u_5\}$. *Thus, in the case of* $P \sqsubseteq Q$, *there is no relationship between* $\underline{apr}_P(X)$ *and* $\underline{apr}_Q(X)$.

Definition 3.11. *Let* $\mathfrak{S} = (F, A)$ *and* $\mathfrak{T} = (G, B)$ *be two soft sets over* U *and* $P = (U, \mathfrak{S})$, $Q = (U, \mathfrak{T})$ *be two soft approximation spaces. The intersection of* P *and* Q *is the soft approximation space* $(U, \mathfrak{I})$, *where* $\mathfrak{I} = \mathfrak{S} \cap \mathfrak{T}$. *We write* $P \sqcap Q = (U, \mathfrak{I})$.

Definition 3.12. *Let* $\mathfrak{S} = (F, A)$ *and* $\mathfrak{T} = (G, B)$ *be two soft sets over* U *and* $P = (U, \mathfrak{S})$, $Q = (U, \mathfrak{T})$ *be two soft approximation spaces. The union of* P *and* Q *is the soft approximation space* $(U, \mathfrak{U})$, *where* $\mathfrak{U} = \mathfrak{S} \cup \mathfrak{T}$. *We write* $P \sqcup Q = (U, \mathfrak{U})$.

Theorem 3.13. *Let* $\mathfrak{S} = (F, A)$ *and* $\mathfrak{T} = (G, B)$ *be two soft sets over* U, $P = (U, \mathfrak{S})$ *and* $Q = (U, \mathfrak{T})$ *be two soft approximation spaces and* $X \subseteq U$. *Then*

$$\overline{apr}_P(X) \cup \overline{apr}_Q(X) \subseteq \overline{apr}_{P \sqcup Q}(X).$$

Proof. Note that $P \sqsubseteq P \sqcup Q$, so by Theorem 3.8, $\overline{apr}_P(X) \subseteq \overline{apr}_{P \sqcup Q}(X)$. Similarly, $\overline{apr}_Q(X) \subseteq \overline{apr}_{P \sqcup Q}(X)$. Therefore, $\overline{apr}_P(X) \cup \overline{apr}_Q(X) \subseteq \overline{apr}_{P \sqcup Q}(X)$. □

Theorem 3.14. *Let* $\mathfrak{S} = (F, A)$ *and* $\mathfrak{T} = (G, B)$ *be two soft sets over* U, $P = (U, \mathfrak{S})$ *and* $Q = (U, \mathfrak{T})$ *be two soft approximation spaces and* $X \subseteq U$. *Then*

$$\underline{apr}_{P \sqcup Q}(X) \subseteq \underline{apr}_P(X) \cup \underline{apr}_Q(X).$$

Proof. Let $u \in \underline{apr}_{P \sqcup Q}(X)$, then there exists $c \in A \cup B$ such that $u \in (F \cup G)(c) \subseteq X$. By Definition 2.4, if $c \in A - B$, then $c \in A$ and $u \in (F \cup G)(c) = F(c) \subseteq X$, so $u \in \underline{apr}_P(X) \subseteq \underline{apr}_P(X) \cup \underline{apr}_Q(X)$. Similarly, for $c \in B - A$ we have $u \in \underline{apr}_Q(X) \subseteq \underline{apr}_P(X) \cup \underline{apr}_Q(X)$. Finally, if $c \in A \cap B$, then $u \in (F(c) \cup G(c)) \subseteq X$. Thus, $u \in F(c) \subseteq X$ or $u \in G(c) \subseteq X$ and so $u \in \underline{apr}_P(X) \cup \underline{apr}_Q(X)$. Therefore, $\underline{apr}_{P \sqcup Q}(X) \subseteq \underline{apr}_P(X) \cup \underline{apr}_Q(X)$. □

The following example shows that the inclusion in Theorems 3.13 and 3.14 may be strict.

Example 3.15. *Let* $U = \{u_1, u_2, \ldots, u_8\}$ *be the universal set and* $A = \{e_1, e_2, e_3\}$, $B = \{e_2, e_3, e_4\}$ *subsets of parameter set. Let* $\mathfrak{S} = (F, A)$ *and* $\mathfrak{T} = (G, B)$ *be two soft sets over* U *given by Tables* 4, 5 *and* $P = (U, \mathfrak{S})$, $Q = (U, \mathfrak{T})$ *be two soft approximation spaces. One can easy to check that* $(\mathfrak{S} \cup \mathfrak{T})(e_1) = \{u_3, u_8\}$, $(\mathfrak{S} \cup \mathfrak{T})(e_2) = \{u_1, u_2, u_3, u_5, u_6\}$, $(\mathfrak{S} \cup \mathfrak{T})(e_3) = \{u_1, u_2, u_4, u_7, u_8\}$ *and* $(\mathfrak{S} \cup \mathfrak{T})(e_4) = \{u_5, u_7\}$.

(1) *If we take* $X = \{u_4, u_7\}$*, then* $\overline{apr}_{P\sqcup Q}(X) = \{u_1, u_2, u_4, u_5, u_7, u_8\}$*. Since* $\overline{apr}_P(X) = \{u_2, u_4, u_7, u_8\}$ *and* $\overline{apr}_Q(X) = \{u_5, u_7\}$*, then* $\overline{apr}_P(X) \cup \overline{apr}_Q(X) = \{u_2, u_4, u_5, u_7, u_8\}$*. Therefore*

$$\overline{apr}_P(X) \cup \overline{apr}_Q(X) \subset \overline{apr}_{P\sqcup Q}(X).$$

(2) *If we take* $X = \{u_1, u_2, u_3, u_5, u_7\}$*, then* $\underline{apr}_{P\sqcup Q}(X) = \{u_5, u_7\}$*. Also we have* $\underline{apr}_P(X) = \{u_1, u_2, u_5\}$ *and* $\underline{apr}_Q(X) = \{u_5, u_7\}$*, so* $\underline{apr}_P(X) \cup \underline{apr}_Q(X) = \{u_1, u_2, u_5, u_7\}$*. Therefore*

$$\underline{apr}_{P\sqcup Q}(X) \subset \underline{apr}_P(X) \cup \underline{apr}_Q(X).$$

Table 4: Tabular representation of the soft set $\mathfrak{S}$

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
e_1	0	0	1	0	0	0	0	1
e_2	1	1	0	0	1	0	0	0
e_3	0	1	0	1	0	0	1	1

Table 5: Tabular representation of the soft set $\mathfrak{T}$

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
e_2	0	1	1	0	0	1	0	0
e_3	1	0	0	0	0	0	0	1
e_4	0	0	0	0	1	0	1	0

Remark 3.16. *In Example 3.15, we have* $(\mathfrak{S} \cap \mathfrak{T})(e_1) = \{u_3, u_8\}$*,* $(\mathfrak{S} \cap \mathfrak{T})(e_2) = \{u_2\}$*,* $(\mathfrak{S} \cap \mathfrak{T})(e_3) = \{u_8\}$ *and* $(\mathfrak{S} \cap \mathfrak{T})(e_4) = \{u_5, u_7\}$*. If we take* $X = U - \{u_3\}$*, then* $\underline{apr}_{P\sqcap Q}(X) = \{u_2, u_5, u_7, u_8\}$*. Since* $\underline{apr}_P(X) = \{u_1, u_2, u_4, u_5, u_7, u_8\}$ *and* $\underline{apr}_Q(X) = \{u_1, u_5, u_7, u_8\}$*,* $\underline{apr}_P(X) \cap \underline{apr}_Q(X) = \{u_1, u_5, u_7, u_8\}$*. Therefore, there is no relationship between* $\underline{apr}_{P\sqcap Q}(X)$ *and* $\underline{apr}_P(X) \cap \underline{apr}_Q(X)$*, in general.*

Remark 3.17. *Let* $U = \{u_1, u_2, \ldots, u_8\}$ *be the universal set and* $A = \{e_1, e_2, e_3\}$*,* $B = \{e_2, e_3, e_4\}$ *be subsets of parameter set. Let* $\mathfrak{S} = (F, A)$ *and* $\mathfrak{T} = (G, B)$ *be two soft sets over* U *given by Tables 4, 6 and* $P = (U, \mathfrak{S})$*,* $Q = (U, \mathfrak{T})$ *be two soft approximation spaces. Then* $(\mathfrak{S} \cap \mathfrak{T})(e_1) = \{u_3, u_8\}$*,* $(\mathfrak{S} \cap \mathfrak{T})(e_2) = \{u_2\}$*,* $(\mathfrak{S} \cap \mathfrak{T})(e_3) = \{u_8\}$ *and* $(\mathfrak{S} \cap \mathfrak{T})(e_4) = \{u_5, u_7\}$*. If we take* $X = \{u_1, u_2, u_3, u_5, u_7\}$*, then* $\overline{apr}_{P\sqcap Q}(X) = \{u_2, u_3, u_5, u_7, u_8\}$*. Also we have* $\overline{apr}_P(X) = U - \{u_6\}$ *and* $\overline{apr}_Q(X) = \{u_1, u_2, u_5, u_6, u_7, u_8\}$*, so* $\overline{apr}_P(X) \cap \overline{apr}_Q(X) = \{u_1, u_2, u_5, u_7, u_8\}$*. Therefore, there is no relationship between* $\overline{apr}_{P\sqcap Q}(X)$ *and* $\overline{apr}_P(X) \cap \overline{apr}_Q(X)$*, in general.*

Table 6: Tabular representation of the soft set $\mathfrak{T}$

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
e_2	0	1	0	0	0	1	0	0
e_3	1	0	0	0	0	0	0	1
e_4	0	0	0	0	1	0	1	0

In order to illustrate the results presented above, consider the following example.

Example 3.18. *Let* $H = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7\}$ *be the set of seven houses as the universe and* $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ *be the set of possible parameters. For* $i = 1, 2, ..., 8$*, the parameters* x_i *stand for "cheap", "beautiful", "close to subway", "comfortable", "modern", "wooden", "in good repair" and "in the green surroundings", respectively. Let "Dm1" and "Dm2" are two decision makers and they consider the set of parameters* $A = \{e_1, e_2, e_4, e_7, e_8\} \subset E$ *and* $B = \{e_1, e_2, e_3, e_5, e_7\} \subset E$*, respectively, to evaluate the houses. After evaluation, "Dm1" and "Dm2" construct the two soft sets* $\mathfrak{S} = (F, A)$ *and* $\mathfrak{T} = (G, B)$*, respectively, which are representing in Tables 7, 8. Now, Assume that Mr. X wants to buy a house from the set* H *and he may think the set of houses* $X = \{h_1, h_3, h_5, h_6\}$ *are suitable for purchase. Let us choose* $P = (H, \mathfrak{S})$ *and* $Q = (H, \mathfrak{T})$ *as the two soft approximation spaces. By definition we have* $\underline{apr}_P(X) = \{h_1, h_3\}$*,* $\overline{apr}_P(X) =$

$\{h_1, h_2, h_3, h_5\}$, $\underline{apr}_Q(X) = \{h_3, h_5\}$, $\overline{apr}_Q(X) = \{h_2, h_3, h_5, h_6, h_7\}$. We also have $\underline{apr}_{P\sqcap Q}(X) = \{h_3, h_5\}$, $\overline{apr}_{P\sqcap Q}(X) = \{h_2, h_3, h_5, h_6\}$, $\underline{apr}_{P\sqcup Q}(X) = \{h_3, h_5\}$, $\overline{apr}_{P\sqcup Q}(X) = H - \{h_4\}$. With respect to Theorems 3.13 and 3.14, it can be seen that the houses h_3 and h_5 are more suitable than h_1 for Mr. X to purchase and the houses h_2, h_6 and h_7 may be suitable.

Table 7: Tabular representation of the soft set $\mathfrak{S}$

	h_1	h_2	h_3	h_4	h_5	h_6	h_7
e_1	0	0	1	0	0	0	0
e_2	1	0	1	0	0	0	0
e_4	0	1	0	0	1	0	0
e_7	0	0	0	0	0	0	0
e_8	0	1	1	0	0	0	0

Table 8: Tabular representation of the soft set $\mathfrak{T}$

	h_1	h_2	h_3	h_4	h_5	h_6	h_7
e_1	0	0	1	0	1	0	0
e_2	0	1	0	0	1	0	0
e_3	0	1	0	0	0	1	0
e_5	0	0	0	0	1	0	0
e_7	0	0	1	0	0	0	1

Definition 3.19. *Let $\mathfrak{S} = (F, A)$ be a soft set over S. If for any $a, b \in A$, there exists $c \in A$ such that $f(c) = f(a)f(b)$, then $\mathfrak{S}$ is called a product complete soft set (PCS-set) over S.*

Example 3.20. *Let $(S, \cdot)$ be a semigroup where $S = \{a, b, c, d, e\}$ and "$\cdot$" is defined with the following Cayley table:*

$\cdot$	a	b	c	d	e
a	a	a	a	a	a
b	a	a	a	a	a
c	a	a	c	c	e
d	a	a	d	d	e
e	a	a	c	c	e

Let $\mathfrak{S} = (F, A)$ be a soft set over S, where $A = \{e_1, e_2, e_3, e_4\}$ and $F(e_1) = \{a\}$, $F(e_2) = \{c, d\}$, $F(e_3) = \{e\}$ and $F(e_4) = \{c\}$. Let $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. By simple calculations, we can see that (F, A) is a PCS-set over S.

Theorem 3.21. *Let $\mathfrak{S} = (F, A)$ be a PCS-set over S, $P = (S, \mathfrak{S})$ be the corresponding soft approximation space and X, Y any two non-empty subsets of S. Then*
1) $\overline{apr}_P(X)\overline{apr}_P(Y) \subseteq \overline{apr}_P(XY)$,
2) $\underline{apr}_P(X)\underline{apr}_P(Y) \subseteq \underline{apr}_P(XY)$.

Proof. 1) Let $m \in \overline{apr}_P(X)\overline{apr}_P(Y)$, so $m = xy$, where $x \in \overline{apr}_P(X)$ and $y \in \overline{apr}_P(Y)$. Then there exist $a, b \in A$ such that $x \in f(a)$, $f(a) \cap X \neq \emptyset$ and $y \in f(b)$, $f(b) \cap Y \neq \emptyset$. Since $\mathfrak{S}$ is a PCS-set, there exist $c \in A$ such that $f(c) = f(a)f(b)$ and so $xy \in f(c)$. Clearly, $f(c) \cap XY \neq \emptyset$. Therefore, $xy \in \overline{apr}_P(XY)$.

2) Let $m \in \underline{apr}_P(X)\underline{apr}_P(Y)$, so $m = xy$, where $x \in \underline{apr}_P(X)$ and $y \in \underline{apr}_P(Y)$. Then there exist $a, b \in A$ such that $x \in f(a) \subseteq X$ and $y \in f(b) \subseteq Y$ and so $xy \in f(a)f(b) \subseteq XY$. Since $\mathfrak{S}$ is a PCS-set, there exist $c \in A$ such that $f(c) = f(a)f(b)$ and so $xy \in f(c) \subseteq XY$. Therefore, $xy \in \underline{apr}_P(XY)$. □

The following example shows that the inclusion in Theorem 3.21 may be strict.

Example 3.22. *Consider the soft set $\mathfrak{S} = (F, A)$ in Example 3.20.*
(1) *If we take $X = \{a, c, e\}$ and $Y = \{a, b, d\}$, then $\underline{apr}_P(X) = \{c, e\}$, $\underline{apr}_P(Y) = \{a\}$ and so $\underline{apr}_P(X)\underline{apr}_P(Y) = \{a\}$. Also we have $XY = \{a, c\}$ and so $\underline{apr}_P(XY) = \{a, c\}$. Therefore,*

$$\underline{apr}_P(X)\underline{apr}_P(Y) \subset \underline{apr}_P(XY).$$

(2) *If we take $X = \{b, d\}$ and $Y = \{b, c, e\}$, then $\overline{apr}_P(X) = \{c, d\}$ and $\overline{apr}_P(Y) = \{c, d, e\}$. Thus $\overline{apr}_P(X)\overline{apr}_P(Y) = \{c, d, e\}$. Now, $XY = \{a, d, e\}$ and so $\overline{apr}_P(XY) = \{a, c, d, e\}$. Therefore,*

$$\overline{apr}_P(X)\overline{apr}_P(Y) \subset \overline{apr}_P(XY).$$

4 Characterizations of Soft Rough Semigroups

In this section, we characterize the lower and upper soft rough approximations of a subset of a semigroup. First, we introduce the notion of soft rough semigroups.

Definition 4.1. *Let $\mathfrak{S} = (F, A)$ be a soft set over S and $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. Let X be a soft P-rough subset of S. Then X is called lower (upper) soft rough semigroup, if $\underline{apr}_P(X)$ ($\overline{apr}_P(X)$) is a subsemigroup of S. Moreover, X is called soft rough semigroup, if $\underline{apr}_P(X)$ and $\overline{apr}_P(X)$ are subsemigroup of S.*

Example 4.2. *Consider the semigroup $(\mathbb{Z}_{12}, \odot)$. Let $\mathfrak{S} = (F, A)$ be a soft set over $\mathbb{Z}_{12}$, where $A = \{e_1, e_2, e_3, e_4, e_5\}$ and $F(e_1) = \{\overline{2}, \overline{4}, \overline{6}\}$, $F(e_2) = \{\overline{0}, \overline{8}\}$, $F(e_3) = \{\overline{3}, \overline{7}, \overline{9}\}$, $F(e_4) = \{\overline{4}\}$ and $F(e_5) = \{\overline{0}, \overline{2}, \overline{8}, \overline{10}\}$. Let $P = (\mathbb{Z}_{12}, \mathfrak{S})$ be the corresponding soft approximation space. If we take $X = \{\overline{0}, \overline{1}, \overline{4}, \overline{5}, \overline{8}, \overline{11}\}$, then $\underline{apr}_P(X) = \{\overline{0}, \overline{4}, \overline{8}\}$ and $\overline{apr}_P(X) = \{\overline{0}, \overline{2}, \overline{4}, \overline{6}, \overline{8}, \overline{10}\}$, which are subsemigroup of $\mathbb{Z}_{12}$. Therefore, X is a soft rough semigroup over $\mathbb{Z}_{12}$.*

Theorem 4.3. *Let $\mathfrak{S} = (F, A)$ be an intersection complete soft set over S and $P = (S, \mathfrak{S})$ the corresponding soft approximation space. Let X and Y be lower soft rough semigroups of S. Then $X \cap Y$ is a lower soft rough semigroup of S.*

Proof. Let $x, y \in \underline{apr}_P(X \cap Y)$. By Theorem 3.5 (3), $x, y \in \underline{apr}_P(X)$ and so $xy \in \underline{apr}_P(X)$. Similarly, $xy \in \underline{apr}_P(Y)$. Thus $xy \in \underline{apr}_P(X) \cap \underline{apr}_P(Y)$. By Theorem 3.6, we deduce that $xy \in \underline{apr}_P(X \cap Y)$. Therefore, $X \cap Y$ is a lower soft rough semigroup of S. □

The following example shows that if X and Y are upper soft rough semigroup of S, then $X \cap Y$ is not an upper soft rough semigroup of S in general.

Example 4.4. *Consider the semigroup $(S, \cdot)$ in Example 3.20. Let $A = \{e_1, e_2, e_3, e_4\}$ be a subset of parameter set. We define the soft set $\mathfrak{S} = (F, A)$ over S such that $F(e_1) = \{c, d\}$, $F(e_2) = \{a\}$, $F(e_3) = \{d, e\}$ and $F(e_4) = \{c\}$. Let $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. If we take $X = \{a, b, d, e\}$ and $Y = \{b, c, e\}$, then $\overline{apr}_P(X) = \{a, c, d, e\}$ and $\overline{apr}_P(Y) = \{c, d, e\}$, which are subsemigroups of S. Thus X and Y are upper soft rough semigroups of S. But $\overline{apr}_P(X \cap Y) = \overline{apr}_P(\{b, e\}) = \{d, e\}$ is not a subsemigroup of S.*

Moreover, $X \cup Y$ is neither lower nor upper soft rough semigroup of S, if X and Y are soft rough semigroup of S. Consider the following example.

Example 4.5. *Consider the semigroup $(S, \cdot)$, where $S = \{a, b, c, d, e, f\}$ and "$\cdot$" in defined by the following Cayley table:*

$\cdot$	a	b	c	d	e	f
a	a	a	a	d	a	a
b	a	b	b	d	b	b
c	a	b	c	d	e	e
d	a	a	d	d	d	d
e	a	b	c	d	e	e
f	a	b	c	d	e	f

Let $\mathfrak{S} = (F, A)$ be a soft set over S, where $A = \{e_1, e_2, \ldots, e_6\}$ and $F(e_1) = \{c\}$, $F(e_2) = \{c, d\}$, $F(e_3) = \{a\}$, $F(e_4) = \{b, e\}$, $F(e_5) = \{f\}$ and $F(e_6) = \{f, d\}$. Let $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. If we take $X = \{c\}$ and $Y = \{a, f\}$, then $\underline{apr}_P(X) = \{c\}$, $\overline{apr}_P(X) = \{c, d\}$, $\underline{apr}_P(Y) = \{a, f\}$ and $\overline{apr}_P(X) = \{a, d, f\}$, which are subsemigroups of S. Therefore, X and Y are soft rough semigroups of S. But $\underline{apr}_P(X \cup Y) = \{a, c, f\}$ and $\overline{apr}_P(X \cup Y) = \{a, c, d, f\}$, which are not subsemigroups of S. Therefore, $X \cup Y$ is not lower (upper) soft rough semigroup of S.

Theorem 4.6. *Let $\mathfrak{S} = (F, A)$ be an UCS-semigroup over S and $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. If X is any non-empty subset of S, then $\underline{apr}_P(X)$ is a subsemigroup of S.*

Proof. Let $x, y \in \underline{apr}_P(X)$, then there exist $a, b \in A$ such that $x \in f(a) \subseteq X$ and $y \in f(b) \subseteq X$ and so $x, y \in f(a) \cup f(b) \subseteq X$. Since $\mathfrak{S}$ is a UCS-set, there exists $c \in A$ such that $f(c) = f(a) \cup f(b)$ and so $x, y \in f(c) \subseteq X$. Since $\mathfrak{S}$ is a soft semigroup over S, $xy \in f(c) \subseteq X$. This means that $xy \in \underline{apr}_P(X)$. Therefore, $\underline{apr}_P(X)$ is a subsemigroup of S. $\square$

Theorem 4.7. *Let $\mathfrak{S} = (F, A)$ be an UCS-semigroup over S and $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. If X is any non-empty subset of S, then $\overline{apr}_P(X)$ is a subsemigroup of S.*

Proof. Let $x, y \in \overline{apr}_P(X)$, then there exist $a, b \in A$ such that $x \in f(a)$, $f(a) \cap X \neq \emptyset$ and $y \in f(b)$, $f(b) \cap X \neq \emptyset$. Since $\mathfrak{S}$ is a UCS-set, there exists $c \in A$ such that $f(c) = f(a) \cup f(b)$ and so $x, y \in f(c)$. Since $\mathfrak{S}$ is a soft semigroup over S, $xy \in f(c)$. Clearly, $f(c) \cap X \neq \emptyset$. This means that $xy \in \overline{apr}_P(X)$. Therefore, $\overline{apr}_P(X)$ is a subsemigroup of S. $\square$

Note that in Theorems 4.6 and 4.7, the requirement of (F, A) be an UCS-semigroup over S is not a necessary condition as shown in the following example.

Example 4.8. *Suppose that $S = \{a, b, c, d, e\}$ is a semigroup with the following Cayley table.*

$\cdot$	a	b	c	d	e
a	a	a	a	a	a
b	a	a	a	b	c
c	a	b	c	a	a
d	a	a	a	d	e
e	a	d	e	a	a

Let (F, A) be a soft set over S such that $A = \{e_1, e_2, e_3, e_4\}$ and $F(e_1) = \{a\}$, $F(e_2) = \{a, b, d\}$, $F(e_3) = \{a, d\}$ and $F(e_4) = \{b\}$. It is obvious that (F, A) is neither union complete nor soft semigroup. Let $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. If we take $X = \{a, b, e\}$ then $\underline{apr}_P(X) = \{a, b\}$ and $\overline{apr}_P(X) = \{a, b, d\}$, which are subsemigroup of S.

Theorem 4.9. *Let $\mathfrak{S} = (F, A)$ be an UCS-ideal over S and $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. If X is any non-empty subset of S, then $\underline{apr}_P(X)$ is an ideal of S.*

Proof. According to Theorem 4.6, we have $\underline{apr}_P(X)$ is a subsemigroup of S. Now, let $x \in \underline{apr}_P(X)$ and $s \in S$. Then there exists $a \in A$ such that $x \in f(a) \subseteq X$. Since $\mathfrak{S}$ is a soft ideal over S, $xs \in f(a) \subseteq X$ and $sx \in f(a) \subseteq X$. This means that $xs, sx \in \underline{apr}_P(X)$. Therefore, $\underline{apr}_P(X)$ is an ideal of S. $\square$

Theorem 4.10. *Let $\mathfrak{S} = (F, A)$ be an UCS-ideal over S and $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. If X is any non-empty subset of S, then $\overline{apr}_P(X)$ is an ideal of S.*

Proof. According to Theorem 4.7, we have $\overline{apr}_P(X)$ is a subsemigroup of S. Now, let $x \in \overline{apr}_P(X)$ and $s \in S$. Then there exists $a \in A$ such that $x \in f(a)$, $f(a) \cap X \neq \emptyset$. Since $\mathfrak{S}$ is a soft ideal over S, $xs, sx \in f(a)$ and $f(a) \cap X \neq \emptyset$. This means that $xs, sx \in \overline{apr}_P(X)$. Therefore, $\overline{apr}_P(X)$ is an ideal of S. $\square$

The following example shows that, in Theorems 4.9 and 4.10, the requirement of (F, A) be an UCS-ideal over S is not a necessary condition.

Example 4.11. *Suppose that $S = \{x, y, z, t\}$ be a semigroup with the following Cayley table.*

$\cdot$	x	y	z	t
x	x	x	x	x
y	x	y	z	x
z	x	x	x	x
t	x	t	x	x

Let (F, A) be a soft set over S, where $A = \{e_1, e_2, e_3, e_4\}$ and $F(e_1) = \{x\}$, $F(e_2) = \{x, t\}$, $F(e_3) = \{y\}$ and $F(e_4) = \{z\}$. Then (F, A) is not an UCS-ideal of S. Let $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. If we take $X = \{x, z\}$, then $\underline{apr}_P(X) = \{x, z\}$ and $\overline{apr}_P(X) = \{x, z, t\}$, which are ideals of S.

Theorem 4.12. *Let $\mathfrak{S} = (F, A)$ be an UCS-bi-ideal over S and $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. If X is any non-empty subset of S, then $\underline{apr}_P(X)$ is a bi-ideal of S.*

Proof. According to Theorem 4.6, we have $\underline{apr}_P(X)$ is a subsemigroup of S. Now, let $x, y \in \underline{apr}_P(X)$ and $s \in S$, then there exist $a, b \in A$ such that $x \in f(a) \subseteq X$ and $y \in f(b) \subseteq X$ and so $x, y \in f(a) \cup f(b) \subseteq X$. Since $\mathfrak{S}$ is a UCS-set, there exists $c \in A$ such that $f(c) = f(a) \cup f(b)$ and so $x, y \in f(c) \subseteq X$. Since $\mathfrak{S}$ is a soft bi-ideal over S, $xsy \in f(c) \subseteq X$. This means that $xsy \in \underline{apr}_P(X)$. Therefore, $\underline{apr}_P(X)$ is a bi-ideal of S. □

Theorem 4.13. *Let $\mathfrak{S} = (F, A)$ be an UCS-bi-ideal over S and $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. If X is any non-empty subset of S, then $\overline{apr}_P(X)$ is a bi-ideal of S.*

Proof. According to Theorem 4.7, we have $\overline{apr}_P(X)$ is a subsemigroup of S. Now, let $x, y \in \overline{apr}_P(X)$ and $s \in S$, then there exist $a, b \in A$ such that $x \in f(a)$, $f(a) \cap X \neq \emptyset$ and $y \in f(b)$, $f(b) \cap X \neq \emptyset$. Since $\mathfrak{S}$ is a UCS-set, there exists $c \in A$ such that $f(c) = f(a) \cup f(b)$ and so $x, y \in f(c)$. Since $\mathfrak{S}$ is a soft bi-ideal over S, $xsy \in f(c)$. Clearly, $f(c) \cap X \neq \emptyset$. This means that $xsy \in \overline{apr}_P(X)$. Therefore, $\overline{apr}_P(X)$ is a bi-ideal of S. □

It is noteworthy that in Theorems 4.12 and 4.13, the condition of being (F, A) an UCS-bi-ideal over S is not a necessary condition. In order to illustrate this fact, we consider the following example.

Example 4.14. *Consider the semigroup $(S, \cdot)$ in Example 3.20. Let $A = \{e_1, e_2, e_3, e_4\}$ be a subset of parameter set. We define a soft set (F, A) over S such that $f(e_1) = \{a\}$, $F(e_2) = \{c, e\}$, $F(e_3) = \{b, c\}$ and $F(e_4) = \{e\}$. Then (F, A) is not an UCS-bi-ideal over S. Let $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. If we take $X = \{a, b, d, e\}$, then $\underline{apr}_P(X) = \{a, e\}$ and $\overline{apr}_P(X) = \{a, b, c, e\}$, which are bi-ideals of S.*

Theorem 4.15. *Let $\mathfrak{S} = (F, A)$ be an UCS-interior ideal over S and $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. If X is any non-empty subset of S, then $\underline{apr}_P(X)$ is an interior ideal of S.*

Proof. According to Theorem 4.6, we have $\underline{apr}_P(X)$ is a subsemigroup of S. Now, let $x \in \underline{apr}_P(X)$ and $s, t \in S$. Then there exists $a \in A$ such that $x \in f(a) \subseteq X$. Since $\mathfrak{S}$ is a soft interior ideal over S, $sxt \in f(a) \subseteq X$. This means that $sxt \in \underline{apr}_P(X)$. Therefore, $\underline{apr}_P(X)$ is an interior ideal of S. □

Theorem 4.16. *Let $\mathfrak{S} = (F, A)$ be an UCS-interior ideal over S and $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. If X is any non-empty subset of S, then $\overline{apr}_P(X)$ is an interior ideal of S.*

Proof. According to Theorem 4.7, we have $\overline{apr}_P(X)$ is a subsemigroup of S. Now, let $x \in \overline{apr}_P(X)$ and $s, t \in S$. Then there exists $a \in A$ such that $x \in f(a)$, $f(a) \cap X \neq \emptyset$. Since $\mathfrak{S}$ is a soft interior ideal over S, $sxt \in f(a)$ and $f(a) \cap X \neq \emptyset$. This means that $sxt \in \overline{apr}_P(X)$. Therefore, $\overline{apr}_P(X)$ is an interior ideal of S. □

The following example shows that in Theorems 4.15 and 4.16, the requirement of being (F, A) an UCS-interior ideal over S is not a necessary condition.

Example 4.17. *Let $S = \{a, b, c, d, e\}$ be a semigroup with the following Cayley table and $A = \{e_1, e_2, e_3, e_4\}$ be a subset of parameter set.*

$\cdot$	a	b	c	d	e
a	a	b	c	d	e
b	b	b	b	b	b
c	c	b	c	c	b
d	d	b	d	d	b
e	e	e	e	e	e

Let (F, A) be a soft set over S, where $F(e_1) = \{b\}$, $F(e_2) = \{c, d, e\}$, $F(e_3) = \{b, c\}$ and $F(e_4) = \{e\}$. Then (F, A) is not an UCS-interior ideal over S. Let $P = (S, \mathfrak{S})$ be the corresponding soft approximation space. If take $X = \{a, b, d, e\}$, we have $\underline{apr}_P(X) = \{b, e\}$ and $\overline{apr}_P(X) = \{b, c, d, e\}$. Thus $\underline{apr}_P(X)$ and $\overline{apr}_P(X)$ are interior ideals of S.

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