Journal of Uncertain Systems
Vol.12, No.4, pp.243-255, 2018
Online at: www.jus.org.uk

# A Bi-objective Distributionally Robust Model on Green Supplier Selection of SCN Design with Demand Uncertainty

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Received 20 January 2018; Revised 17 July 2018

**Abstract**

Efficient supply chain configuration and supplier selection are two critical factors for the development of enterprise. In this paper, we combine these two factors to build a bi-objective distributionally robust supply chain network (SCN) design model considering green supplier selection. Particularly, the distributions of uncertain demands are not precisely known and only partial information is available. Furthermore, to cope with this situation, we transfer robust counterparts of service level constraints under Box-Ellipsoid-Budget set to a computationally tractable safe approximations. Finally, a numerical experiment is implemented to evaluate the performance of the proposed model. The computational results reveal the significance and applicability of the developed model and solution method.


**Keywords:** green supplier selection, bi-objective optimization, supply chain network design, distributionally robust optimization

## 1 Introduction

With the enhancing of public environmental protection consciousness and the improving of environmental regulations gradually, an increasing number of enterprises attract their attentions in green supply chain management. For contributing to green supply chain management, Sheu et al. [26] pointed out environmental pollution issues should be addressed together with supply chain management. After that many authors studied the green supply chain by considering "green" factors, such as carbon footprint [18, 21, 27], carbon regulatory policies [4, 19, 22] and green image[6, 15].

In practical SCN problems, there are typically structured with five key areas: external suppliers, production centers, distribution centers, demand zones, and transportation assets [17], where supplier selection can strongly influence the performance of a SCN [16]. Kisomi et al. [16] introduced an integrated closed-loop supply chain model by considering supplier selection problem. However, many firms concern the detrimental impacts of businesses operations on the environment, many researchers and practitioners add "green" to the supplier selection problem. They used different approaches to evaluate and choose the optimal number of suppliers [5, 6, 11, 10, 14]. Among them, Hashemi et al. [11] used an integrated ANP-improved GRA approach to select suitable green supplier and built a comprehensive green supplier selection model by considering both environmental and economic criteria; Fallahpour et al. [5] integrated the Kourosh and Arash method [15] as a robust model of DEA with genetic programming to develop a mathematical GP-based model for green supplier selection; and Hsu et al. [14] utilized the decision-making trial and evaluation laboratory approach to evaluate green suppliers in green supply chain management.

The uncertainty is a critical issue in the decision making process, such as uncertainties in the customers' demands, costs and supply [9]. To cope with these uncertainties, stochastic optimization approach is often used to handle randomness uncertainty originated from historical data. By this approach, Ma and Liu [20] discussed a stochastic mathematical model for the design of closed loop supply chain network with uncertain transportation costs and customers' demands. Ghelichi et al. [7] proposed a two-stage stochastic programming model for the design of an integrated green biodiesel SCN. They obtained an extension of a two-stage scenario-based stochastic programming approach which incorporated min-max relative regret in a soft worst-case

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framework. Numerous previous studies about designing SCNs have existed in the literature [13, 22, 23]. However, if the assumed probability distributions are different from the actual distribution, the optimal solution of a stochastic programming model may perform poorly [3]. Therefore, when decision makers don't know the ture distributions of uncertain parameters but can obtain a known support, scholars often use a robust optimization approach to handle uncertainties of SCN [1, 12, 24]. Pishvaee et al. [25] applied robust optimization approach to formulate a closed-loop supply chain network model by considering uncertain the quantity of returned products, demands and transportation costs. Zokaee et al. [29] presented a robust supply chain network design model, in which demand, supply capacity and major cost data are uncertain parameters.

However, the general robust optimization would be overly and unnecessarily conservative, when the probability distributions of uncertainties are precisely known [8]. To bridge the gap between the conservatism of robust optimization and the specificity of stochastic programming, some researchers utilize the distributionally robust optimization method to study the uncertainty of model parameters. They can obtain the optimal decisions by seeking for the worst-case probability distributions within a family of possible distributions, which is defined by certain properties such as their support and moments [8]. In this paper, we consider that probability distributions of demands are not complete, and derive safe approximations of service level constraints under uncertainty set (i.e., Box-Ellipsoid-Budget). Furthermore, a bi-objective mathematical model is formulated by utilizing the greenness of parts and products for the problem of SCN design and green supplier selection. The objectives contain profit the total profit of the SCN and the greenness of products and parts. In order to cope with bi-objective distributionally robust model, a weighted sum method is used to obtain the set of Pareto optimal solution, and safe tractable approximations are given by dual theory. In the following, we summarize the contributions of this paper,

- We introduce the greenness for dealing with the green supplier selection in a new bi-objective distributionally robust SCN model.
- To handle the proposed model, we use a weighted sum method transformed the bi-objective into a single objective. Further, we employ the Box-Ellipsoid-Budget set as the support to turn the proposed model into its safe approximation, which can be solved by conventional optimization softwares.
- A numerical experiment is addressed to evaluate the performance of the proposed model, and sensitivity analyses and a comparison study are conducted on some important parameters.

This paper is organized as follows. In Section 2, the problem is described in detail and a bi-objective distributionally robust mathematical model is presented in this section. Section 3 discusses the method for solving multi-objective mathematical programming problem and the safe approximations of ambiguous service level constraints. In Section 4 we address a numerical example and show computational results to illustrate the proposed model. Finally, we conclude this research with note for future research directions in Section 5.

# 2 SCN with Green Supplier Selection

## 2.1 Problem Description and Assumptions

In this section, we introduce a green supplier selection and an SCN problem. The concerned SCN is a multi-part, multi-product, three-echelon network that includes suppliers, assembly centers and customer zones. Figure 1 displays the network in which manufacturers purchase raw materials (parts) with greenness level from a set of potential suppliers.

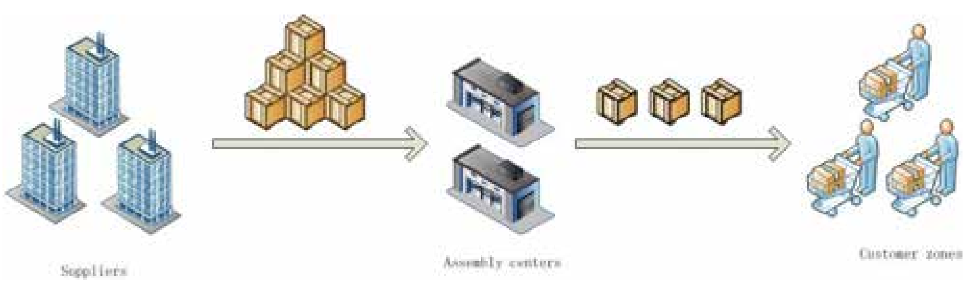


Figure 1: A supply chain network

Green parts are made of highly disposal materials and their prices rely on their greenness levels. In the assembly center, manufacturers assemble multiple products with greenness level, where the greenness level for products is defined according to the level of ability for disassembly/disposal. The ability for disassembly/disposal can be better and the greenness level can be higher. After assembly, manufacturers ship the products from assembly center to customer zones according to the customers' demands. The locations of the customer zone are supposed to be predefined and fixed. Note that the greenness score as the overall value of green parts and products in the network [6]. The following assumptions in the proposed model are described below.

- The location of customer zone is fixed and predefined.
- The greenness indices of parts and products are denoted by greenness level.
- The greenness levels chosen for parts/products are also considered as parameters.
- Demands for products are assumed as random parameters whose distributions belong to a given family.

In our SCN, a bi-objective model is developed to address the problem of supply chain design decisions. The first function is the maximization of the total profit, which includes the total selling prices, fixed costs, operating costs, shipping costs and penalty cost. The second function is the maximization of the greenness score that aggregates the greenness characteristics of the materials and products in the SCN. Considering the above description and assumptions, the notations will be introduced in next subsection.

## 2.2 Notations

In order to formulate the model, the notations are described as follows:

**Sets**

| | |
|---|---|
| $I$ | Set of potential suppliers $i \in I$ |
| $J$ | Set of fixed assembly centers $j \in J$ |
| $K$ | Set of customer zones $k \in K$ |
| $P$ | Set of products $p \in P$ |
| $R$ | Set of parts $r \in R$ |
| $G$ | Set of greenness levels of material used to manufacture parts $g \in G$ |
| $H$ | Set of products greenness levels $h \in H$ |

**Parameters**

| | |
|---|---|
| $d_{kp}$ | Demand product $p$ for customer zone $k$ |
| $cs_i$ | Fixed cost of selecting supplier $i$ |
| $ca_j$ | Fixed cost of assembly center $j$ |
| $cm_{igr}$ | Purchasing cost of part $r$ with greenness level $g$ by supplier $i$ |
| $cm'_{jhp}$ | Assembly cost per unit of product $p$ with greenness level $h$ at assembly center $j$ |
| $cp_{ijr}$ | Unit shipping cost of part $r$ from supplier $i$ to assembly center $j$ |
| $cp'_{jkp}$ | Unit shipping cost of product $p$ from assembly center $j$ to customer zone $k$ |
| $ss_{igr}$ | Supplier $i$ capacity for part $r$ with greenness level $g$ |
| $sa_{jhp}$ | Assembly center $j$ capacity for product $p$ with greenness level $h$ |
| $\delta_{kp}$ | Penalty cost per unit of non-satisfied demand of product $p$ for customer $k$ |
| $\pi_{gpr}$ | Quantity of part $r$ with greenness level $g$ in one unit of product $p$ |
| $\rho_{kp}$ | Selling price of new product $p$ in customer zone $l$ |

**Decision variables**

| | |
|---|---|
| $x_{ijgr}$ | Quantity of part $r$ with greenness level $g$ bought from supplier $i$ to assembly center $j$ |
| $y_{jhp}$ | Quantity of product $p$ with greenness level $h$ assembled at assembly center $j$ |
| $\omega_{kp}$ | Quantity of non-satisfied demand of product $p$ for customer $k$ |
| $z_{jkhp}$ | Quantity of product $p$ with greenness level $h$ sent from assembly center $j$ to customer zone $k$ |
| $u_i$ | 1 if a supplier $i$ is selected, 0 otherwise |
| $v_j$ | 1 if a assembly center $j$ is opened, 0 otherwise. |

### 2.3 Formulation of Problem

The first objective function aims at maximizing the total profit of the network which as follows:

$$\begin{aligned}Z_1 =&\sum_{j\in J}\sum_{k\in K}\sum_{h\in H}\sum_{p\in P}\rho_{kp}z_{jkhp} - \sum_{i\in I} cs_i u_i - \sum_{j\in J} ca_j v_j - \sum_{i\in I}\sum_{j\in J}\sum_{g\in G}\sum_{r\in R} cm_{igr}x_{ijgr}\\ &-\sum_{j\in J}\sum_{h\in H}\sum_{p\in P} cm^{'}_{jhp}y_{jhp} - \sum_{i\in I}\sum_{j\in J}\sum_{g\in G}\sum_{r\in R} cp_{ijr}x_{ijgr} - \sum_{j\in J}\sum_{k\in K}\sum_{h\in H}\sum_{p\in P} cp^{'}_{jkp}z_{jkhp}\\ &-\sum_{k\in K}\sum_{p\in P}\delta_{kp}\omega_{kp}.\end{aligned} \tag{1}$$

The first term is the total selling price of the whole network. The second and third terms are the fixed costs of opening supplier and assembly center, respectively. The fourth term shows the supply cost for purchasing the parts from suppliers, and assembly cost for products from assembly centers is represented by the fifth term. The sixth and seventh terms indicate the shipping costs between two facilities.

The second objective function seeks to maximize the total greenness which includes the greenness of products and parts.

$$Z_2 = \sum_{j\in J}\sum_{k\in K}\sum_{h\in H}\sum_{p\in P} h\rho_{kp}z_{jkhp} + \sum_{i\in I}\sum_{j\in J}\sum_{g\in G}\sum_{r\in R} gcm_{igr}x_{ijgr}. \tag{2}$$

The first term is the product greenness score and the second term is the part greenness score.

Next, we illustrate the constraint aspect of the model. First, the flow balance constraint in assembly centers is

$$\sum_{i\in I} x_{ijgr} = \sum_{h\in H}\sum_{p\in P}\pi_{gpr}y_{jhp},\ \forall\ j,g,r. \tag{3}$$

To ensure that the assembled products are assigned to all customer zones, constraint (4) is given as follows:

$$y_{jhp} = \sum_{k\in K} z_{jkhp},\ \forall\ j,h,p. \tag{4}$$

Generally, decision makers always hope that the demands of all customer zones are satisfied. However, due to the impacts of Policy, culture and natural environment, customers' demands are often uncertain. Thus, suppose that the total amount of product $p$ from assembly center $j$ to customer zone $k$ should meet the demands with service levels $\alpha_{kp}\in(0,1)$, which is represented as

$$\Pr\left\{\sum_{j\in J}\sum_{h\in H} z_{jkhp} + \omega_{kp} \geq d_{kp}\right\} \geq \alpha_{kp},\ \forall\ k,p.$$

However, we often only know partial information of demands' distributions, and the distribution vector $P$ belongs to a given family $\mathcal{P}$ of distributions. Then the ambiguous service level constraints are expressed as follows:

$$\Pr_{P\in\mathcal{P}}\left\{\sum_{j\in J}\sum_{h\in H} z_{jkhp} + \omega_{kp} \geq d_{kp}\right\} \geq \alpha_{kp},\ \forall\ k,p. \tag{5}$$

Since the suppliers and assembly centers have their capacities during production process, decision makers must make sure the quantities of parts and products are less than the provided capacities of the suppliers and assembly centers, respectively. This capacity limitation is indicated in the following constraints,

$$\sum_{j\in J} x_{ijgr} \leq ss_{igr}u_i,\ \forall\ i,g,r, \tag{6}$$

$$y_{jhp} \leq sa_{jhp}v_j,\ \forall\ j,h,p. \tag{7}$$

Finally, considering the reality of the problem, the decision variables must satisfy the following constraints,

$$x_{ijgr}, y_{jhp}, z_{jkhp} \geq 0,\ \forall\ i,j,k,g,h,r,p, \tag{8}$$

$$u_i, v_j \in \{0,1\}, \ \forall\, i,j. \tag{9}$$

Combining Eqs. (1) and (2) as the objective function, we can formulate the following bi-objective mathematical model for SCN design problem,

$$\begin{array}{rl} \max & Z_1 \\ \max & Z_2 \\ \text{s.t.} & \text{constraints (3)-(9).} \end{array} \tag{10}$$

The model (10) is a bi-objective mixed integer programming subject to ambiguous constraints. It is difficult to solve problem (10) by commercial solver. In the next section, to solve this hard optimization, we will turn the problem (10) into a computationally tractable problem which can be solved directly by commercial solver.

# 3 Analysis of SCN Model

In this section, we first solve the bi-objective optimization problem used a weighted sum method. With this method, the bi-objective optimization problem is transformed into a single optimization problem with balancing the total profit objective function (1) and the total greenness objective function (2). Combining Eqs. (1) and (2), the original objection functions are turned into the following single objective function with the weight $\eta \in (0,1)$:

$$\max \quad \eta Z_1 + (1-\eta) Z_2, \tag{11}$$

where $\eta$ is determined by decision makers. The greater $\eta$ means that decision makers pay more attention to economic returns.

Next, we will discuss how service level constraints (5) can be turned into their computationally tractable forms. First, we introduce the Box-Ellipsoid-Budget set below:

$$U = \left\{ \mathbf{d} | \mathbf{d} = \mathbf{d}^0 + \sum_{l=1}^{L} \zeta_l \mathbf{d}^l, \max_{1\le l\le L} |\zeta_l| \le 1, \sqrt{\sum_{l=1}^{L} (\frac{\zeta_l}{\sigma_l})^2} \le \Omega,\ \sum_{l=1}^{L} |\frac{\zeta_l}{\sigma_l}| \le \gamma \right\}, \tag{12}$$

where $\mathbf{d} = (d_{11}, \cdots, d_{kp})^T$ is a vector of uncertain parameters, $\mathbf{d}^0 = (d^0_{11}, \cdots, d^0_{kp})^T$ is the normal vector and $\mathbf{d}^l = (d^l_{11}, \cdots, d^l_{kp})^T, l = 1,2,...,L$ are basic shifts. In addition, $\zeta_l, l = 1,2,...,L$ are random perturbations and $\Omega, \gamma$ are the adjustable parameters controlling the size of the uncertainty set.

Then, we present the safe approximations of service level constraints (5) under the Box-Ellipsoid-Budget set.

**Theorem 1.** *Let the random perturbations $\zeta_l, l = 1,2,...,L$ satisfy:*

**(P1)** *$\zeta_l, l = 1,2,...,L$ are independent random variables.*

**(P2)** *When $\mu_l^+ = \mu_l^- = 0$ and $\sigma_l \ge 0$ are known constants, distributions $P_l$ of components $\zeta_l$ are such that*

$$\begin{aligned} \int \exp\{ts\} dP_l(s) &\le \exp\{\max[\mu_l^+ t, \mu_l^- t] + \frac{1}{2}\sigma_l^2 t^2\}, \quad \forall t \in R \\ &= \exp\{\frac{1}{2}\sigma_l^2 t^2\}, \quad \forall t \in R. \end{aligned}$$

*Then the safe approximations of service level constraints (5) under the Box-Ellipsoid-Budget set are*

$$\begin{aligned} &\sum_{l=1}^{L} |\varphi_l| + \Omega \sqrt{\sum_{l=1}^{L} \sigma_l^2 \psi_l^2} + \gamma \max_l |\sigma_l \phi_l| \le \sum_{j\in J} \sum_{h\in H} z_{jkhp} + \omega_{kp} - d^0_{kp}, \\ &\varphi_l + \psi_l + \max_l \phi_l \ge d^l_{kp}, \forall k,p,l, \end{aligned}$$

*where $\alpha_{kp} = 1 - \exp\{-s_{kp}^2/2\}$ and $s_{kp} = \min\{\Omega, \gamma/\sqrt{L}\}, \forall k,p.$*

*Proof.* First, we give the conic representations of random perturbation vector $\boldsymbol{\zeta} = (\zeta_1, \zeta_2, ..., \zeta_L) \in \mathbf{R}^L$ in uncertainty set as follow:

$$\begin{cases} P_1\boldsymbol{\zeta} + p_1 \in \mathbf{K}^1 \\ P_2\boldsymbol{\zeta} + p_2 \in \mathbf{K}^2 \\ P_3\boldsymbol{\zeta} + p_3 \in \mathbf{K}^3, \end{cases}$$

where

•$P_1\boldsymbol{\zeta} = [\boldsymbol{\zeta}; 0], p_1 = [0_{L\times 1}; 1]$ and $\mathbf{K}^1 = \{[\xi; t] \in R^L \times R : t \geq ||\xi||_\infty\}$, whence its dual conic $\mathbf{K}^1_* = \{[\xi; t] \in R^L \times R : t \geq ||\xi||_1\}$;

•$P_2\boldsymbol{\zeta} = [\sum^{-1}\boldsymbol{\zeta}; 0]$ with $\sum = \mathrm{Diag}\{\sigma_1, ..., \sigma_\mathrm{L}\}$, $p_2 = [0_{L\times 1}; \Omega]$ and $\mathbf{K}^2 = \{[\xi; t] \in R^L \times R : t \geq ||\xi||_2\}$, whence its dual conic $\mathbf{K}^2_* = \mathbf{K}^2$;

•$P_3\boldsymbol{\zeta} = [\sum^{-1}\boldsymbol{\zeta}; 0]$ with $\sum = \mathrm{Diag}\{\sigma_1, ..., \sigma_\mathrm{L}\}$, $p_3 = [0_{L\times 1}; \gamma]$ and $\mathbf{K}^3 = \{[\xi; t] \in R^L \times R : t \geq ||\xi||_1\}$, whence its dual conic $\mathbf{K}^3_* = \mathbf{K}^1$.

Setting $y^1 = [\eta_1; \tau_1]$, $y^2 = [\eta_2; \tau_2]$ and $y^3 = [\eta_3; \tau_3]$ with $L$-dimensional $\eta_1$, $\eta_2$, $\eta_3$ and one-dimensional $\tau_1$, $\tau_2$, $\tau_3$, according to [28], the constraints

$$\sum_{j\in J}\sum_{h\in H} z_{jkhp} + \omega_{kp} \geq d_{kp},\ \forall\ d_{kp} \in U$$

can be represented as

$$\begin{aligned} &\tau_1 + \Omega\tau_2 + \gamma\tau_3 \leq \sum_{j\in J}\sum_{h\in H} z_{jkhp} + \omega_{kp} - d^0_{kp}, \\ &(\eta_1 + \tilde{\eta}_2 + \tilde{\eta}_3)_l = d^l_{kp},\ \forall k, p, l, \\ &||\eta_1||_1 \leq \tau_1, \quad [\Leftrightarrow [\eta_1; \tau_1] \in \mathbf{K}^1_*], \\ &||\eta_2||_2 \leq \tau_2, \quad [\Leftrightarrow [\eta_2; \tau_2] \in \mathbf{K}^2_*], \\ &||\eta_3||_\infty \leq \tau_3, \quad [\Leftrightarrow [\eta_3; \tau_3] \in \mathbf{K}^3_*], \end{aligned} \tag{13}$$

where $\tilde{\eta}_2 = \sum^{-1}\eta_2$ and $\tilde{\eta}_3 = \sum^{-1}\eta_3$.

Further, we denote $\tau_1 \geq \bar{\tau_1} \equiv ||\eta_1||_1, \tau_2 \geq \bar{\tau_2} \equiv ||\eta_2||_2, \tau_3 \geq \bar{\tau_3} \equiv ||\eta_3||_\infty$, and $\varphi = \eta_1, \psi = \tilde{\eta}_2, \phi = \tilde{\eta}_3$. Then, Eq. (13) is equivalent to the following form:

$$\begin{aligned} &\sum_{l=1}^{L} |\varphi_l| + \Omega\sqrt{\sum_{l=1}^{L} \sigma_l^2\psi_l^2} + \gamma \max_l |\sigma_l\phi_l| \leq \sum_{j\in J}\sum_{h\in H} z_{jkhp} + \omega_{kp} - d^0_{kp}, \\ &\varphi_l + \psi_l + \max_l \phi_l \geq d^l_{kp}, \forall k, p, l. \end{aligned} \tag{14}$$

Now let us prove that Eq. (14) are the safe approximations of service level constraints. We find that Eq. (5) can be rewritten as

$$\mathrm{Pr}_{P\in\mathcal{P}}\left\{\sum_{j\in J}\sum_{h\in H} z_{jkhp} + \omega_{kp} < d_{kp}\right\} \leq 1 - \alpha_{kp},\ \forall\ k, p.$$

When $|\zeta_l| \leq 1$, we have

$$d^0_{kp} + \sum_{l=1}^{L} \zeta_l d^l_{kp} > \sum_{j\in J}\sum_{h\in H} z_{jkhp} + \omega_{kp}$$

$$\Rightarrow \sum_{l=1}^{L} (\varphi_l + \psi_l + \phi_l)\zeta_l > \sum_{j\in J}\sum_{h\in H} z_{jkhp} + \omega_{kp} - d^0_{kp}$$

$$\Rightarrow \sum_{l=1}^{L} \varphi_l\zeta_l + \sum_{l=1}^{L} \psi_l\zeta_l + \sum_{l=1}^{L} \phi_l\zeta_l > \sum_{l=1}^{L} |\varphi_l| + \Omega\sqrt{\sum_{l=1}^{L} \sigma_l^2\psi_l^2} + \gamma \max_l |\sigma_l\phi_l|$$

$$\Rightarrow \sum_{l=1}^{L}|\varphi_l| + \sum_{l=1}^{L}\psi_l\zeta_l + \sum_{l=1}^{L}\phi_l\zeta_l > \sum_{l=1}^{L}|\varphi_l| + \Omega\sqrt{\sum_{l=1}^{L}\sigma_l^2\psi_l^2} + \gamma\max_l|\sigma_l\phi_l|.$$

Let $(z,\omega,\psi,\phi)$ be feasible for (14), we have

$$||\phi||_2^2 = \sum_{l=1}^{L}\phi_l^2 \le \sum_{l=1}^{L}|\phi_l|||\phi||_\infty \le ||\phi||_\infty\sum_{l=1}^{L}|\phi_l| \le ||\phi||_\infty\sqrt{L}||\phi||_2.$$

Thus,

$$||\phi||_2 \le \sqrt{L}||\phi||_\infty,$$

$$\sum_{l=1}^{L}|\varphi_l| + \sum_{l=1}^{L}\psi_l\zeta_l + \sum_{l=1}^{L}\phi_l\zeta_l > \sum_{l=1}^{L}|\varphi_l| + \Omega\sqrt{\sum_{l=1}^{L}\sigma_l^2\psi_l^2} + \gamma\max_l|\sigma_l\phi_l|$$

$$\Rightarrow \sum_{l=1}^{L}|\varphi_l| + \sum_{l=1}^{L}\psi_l\zeta_l + \sum_{l=1}^{L}\phi_l\zeta_l > \sum_{l=1}^{L}|\varphi_l| + \Omega\sqrt{\sum_{l=1}^{L}\sigma_l^2\psi_l^2} + \frac{\gamma}{\sqrt{L}}\sqrt{\sum_{l=1}^{L}\sigma_l^2\phi_l^2}$$

$$\Rightarrow \sum_{l=1}^{L}(\psi_l+\phi_l)\zeta_l > s_{kp}\sqrt{\sum_{l=1}^{L}\sigma_l^2(\psi_l+\phi_l)^2},$$

where $s_{kp} = \min\{\Omega, \gamma/\sqrt{L}\}, \forall\ k,p$, for convenience. Therefore for $\zeta_l, l = 1,2,...,L$ are independent random variables and every distribution $P_l$ of $\zeta_l$ compatible with Eq. (1), we have

$$\begin{aligned}
&\Pr_{\boldsymbol{\zeta}\sim P}\left\{\sum_{j\in J}\sum_{h\in H}z_{jkhp} + \omega_{kp} < d_{kp}^0 + \sum_{l=1}^{L}\zeta_l d_{kp}^l\right\} \\
&\le \Pr_{\boldsymbol{\zeta}\sim P}\left\{\sum_{l=1}^{L}(\psi_l+\phi_l)\zeta_l > s_{kp}\sqrt{\sum_{l=1}^{L}\sigma_l^2(\psi_l+\phi_l)^2}\right\} \\
&\le \exp\{-\frac{{s_{kp}}^2}{2}\},
\end{aligned}$$

where the last inequality is obtained by [2] and $\alpha_{kp} = 1 - \exp\{-{s_{kp}}^2/2\}, \forall\ k,p$. □

Based on the aforementioned statements, model (10) can be rewritten as

$$\begin{aligned}
\max\quad & \eta Z_1 + (1-\eta)Z_2 \\
\text{s.t.}\quad & \sum_{l=1}^{L}|\varphi_l| + \Omega\sqrt{\sum_{l=1}^{L}\sigma_l^2\psi_l^2} + \gamma\max_l|\sigma_l\phi_l| \le \sum_{j\in J}\sum_{h\in H}z_{jkhp} + \omega_{kp} - d_{kp}^0 \\
& \varphi_l + \psi_l + \max_l\phi_l \ge d_{kp}^l, \forall k,p,l \\
& \text{constraints (3)-(4), (6)-(9).}
\end{aligned} \tag{15}$$

Model (15) is a deterministic linear mixed-integer programming, so it can be solved by commercial solver such as CPLEX. In next section, we will introduce a numerical example to demonstrate the applicability of the proposed methodology, and validate the results obtained.

# 4 Numerical Experiment

## 4.1 Problem Statement

To illustrate the proposed SCN model with green supplier selection, an example is presented in this section. In this network, there is a set of five suppliers that provide two types of parts with two greenness levels. Two types of products being assembled have two greenness levels and they are shipped to customer zones to satisfy the demands of four customers zones.

Next, we introduce some relevant parameters of the model. The fixed cost of green suppliers is chosen randomly in the interval [40000,60000], and the fixed cost for opening assembly centers is chosen randomly in the interval [70000,90000]. Unit purchasing cost of new parts from supplier is chosen randomly in the interval [50,120]. Besides, the assembly cost per unit of product from assembly centers is chosen randomly in the interval [20,300], and the transportation costs between two facilities are chosen in randomly in the interval [3,15]. For capacities of facilities, we assume that the capacity of supplier is the same and equal to ¥50000 and the capacity of assembly center is the same and equal to ¥10000. Finally, the penalty cost is chosen randomly in the interval [150,190] and the selling price is chosen randomly in the interval [300,600]. Note that all prices are in RMB.

In this numerical experiment, to the demand of customer zone $z$ for product $p$, we built that the structure of uncertain demands is $d_{kp} = d^0_{kp} + \sum_{l=1}^{L} \zeta_l d^l_{kp}, l = 1, ..., L$, where $d^0_{kp}$ are the nominal values and $d^l_{kp}$ are basic shifts. We give the nominal values of uncertain demands in Table 1 and the basic shifts are estimated to be 10 % of the nominal demands.

In addition, for simplicity, we suppose that when $\alpha_{kp}$ is known, $\alpha_{kp} = 1 - \exp\{-s^2_{kp}/2\}$ and $s_{kp} = \min\{\Omega, \gamma/\sqrt{L}\}, \forall k, p$. According to Theorem 1, we set $\Omega = \sqrt{2\ln(1/(1-\alpha_{kp}))}$, $\gamma = \Omega\sqrt{L}$ and $\sigma_l = 2, \forall l$.

Table 1: The nominal value of uncertain demand

| $d_{kp}$ | Customer zone 1 | Customer zone 2 | Customer zone 3 | Customer zone 4 |
|---|---|---|---|---|
| Product 1 | 30000 | 28000 | 32000 | 29000 |
| Product 2 | 28000 | 28000 | 32000 | 29000 |

## 4.2 Results Analysis of Distributionally Robust Model

In this subsection, we take various values of model parameters to examine the influence of the model parameters on the optimal decisions, and solve our optimization problems by Lingo 11.0. All experiments are performed on an Inter(R) Core i5-7200 (can speed up to 2.50 GHz) personal computer with 8.0 GB RAM operating under Windows 10. In the following, we will analyze the impacts of the weight parameter $\eta$ and the service level $\alpha_{kp}$ on the optimal decision.

**Case 1: The influence of the weight parameter**

In this case, we take a sensitivity analysis to examine the influence of the weight parameter $\eta$ on the optimal decision. In reality, firms often prefer the high profit, and then consider the greenness of supply chain. Hence, we just consider the weight parameter of 0.5,0.6,0.7,0.8,0.9 in our numerical experiment. The computational results are shown in Table 2. From Table 2, we conclude that the optimal supplier selection, maximal profit and maximal greenness depend heavily on the parameter $\eta$. For example, given $\eta = 0.5$, the decision makers select the suppliers 2, 3, 4, 5. On the other hand, when $\eta = 0.6$, the optimal suppliers include 2, 4, 5. In addition, when the weight increases, we can obtain higher profit, meanwhile, the greenness of parts and products comes down on trend. Therefore, decision makers can adjust the parameters $\eta$ to balance the relationship between the profit and the greenness, which improve the efficiency of enterprises.

Table 2: The computational results under different $\eta$

| $\eta$ | Optimal supplier selection | Maximal profit ($Z_1$) | Maximal greenness ($Z_2$) |
|---|---|---|---|
| 0.50 | [0 1 1 1 1] | 23729670.00 | 263829900.00 |
| 0.60 | [0 1 0 1 1] | 25513290.00 | 263200000.00 |
| 0.70 | [1 1 1 1 0] | 33188420.00 | 244830400.00 |
| 0.80 | [1 1 0 1 0] | 33828270.00 | 245800000.00 |
| 0.90 | [1 1 1 0 1] | 34491430.00 | 244400000.00 |

**Case 2: The influence of the service level**

To validate the impact of the service level $\alpha_{kp}$ on the optimal decision, we do the following experiments. Figure 2 and Figure 3 are plotted to show the impacts of the service level on the maximal profit and the maximal greenness. In Figure 2, we observe that the maximal profit increases as parameter $\alpha_{kp}$ increases from 0.91 to 0.99. That is, the maximal profit is monotonic increasing with respect to $\alpha_{kp}$. On the other hand, as the service level increases, the change of the greenness is not significant, i.e., the service level has little effect on the maximal greenness. Figure 3 depicts this issue clearly. Therefore, if decision makers could guarantee a high service level, they will obtain a higher profit under a high greenness score.

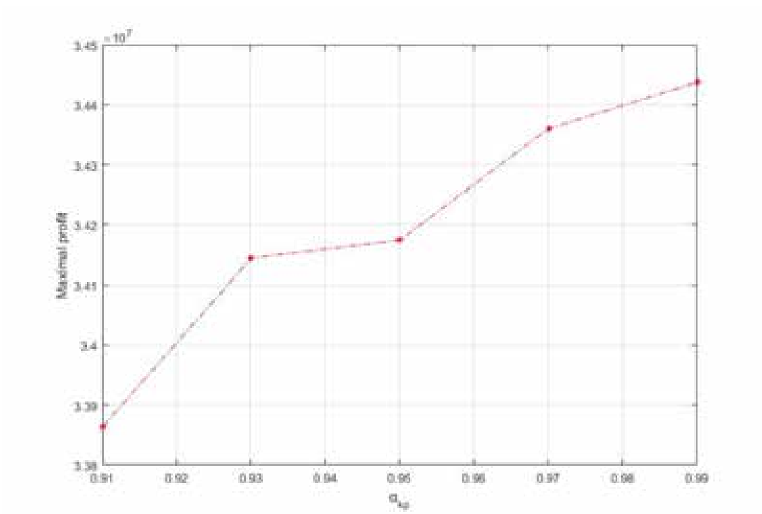


Figure 2: The maximal profit under different $\alpha_{kp}$

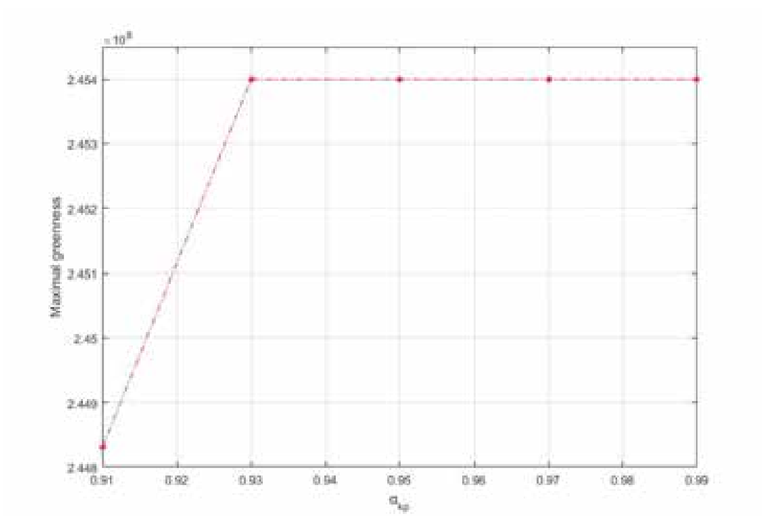


Figure 3: The maximal greenness under different $\alpha_{kp}$

## 4.3 A Comparison Study with Nominal Stochastic Model

In this subsection, we conduct a comparative study with nominal stochastic model to evaluate the advantage our proposed model. Combine (P1) and (P2), $\zeta_l, l = 1, ..., L$ are independent variables which are characterized by Gaussian distribution with $\mu_l = 0$ and $\sigma_l = 2$. To analyze the difference of the nominal stochastic model and the distributionally model, Figures 4–7 show the comparison results under different $\eta$ and $\alpha_{kp}$ on profit and greenness.

The comparison results under different $\eta$ on profit and greenness are expressed in Figures 4 and 5, respectively. From Figure 4, it can be found that the maximal profit of nominal stochastic model is evidently more than those of the proposed distributionally robust model. Analogously, the maximal greenness of nominal stochastic model is also greater than the proposed model in Figure 5. This phenomenon is perfectly consistent with the theoretical facts. This is principally because of the existence of uncertain distribution. Although the distributionally robust model incur lower profit and greenness, they can resist the uncertainty better than nominal stochastic model.

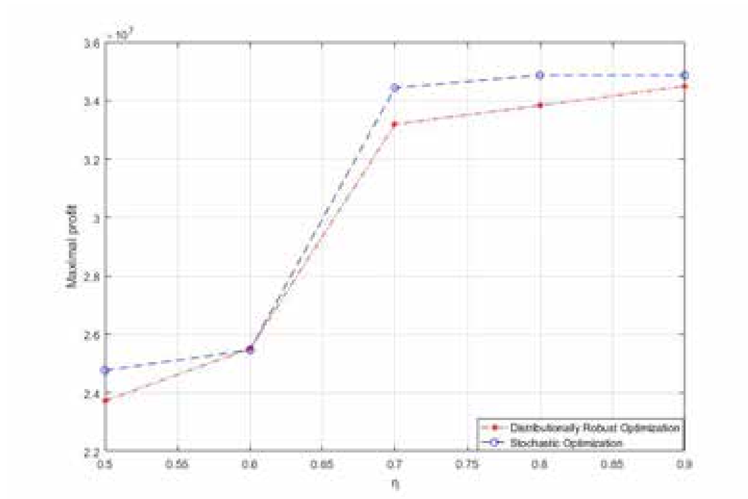


Figure 4: The distributionally robust profit versus stochastic profit, $\alpha_{kp} = 0.99$

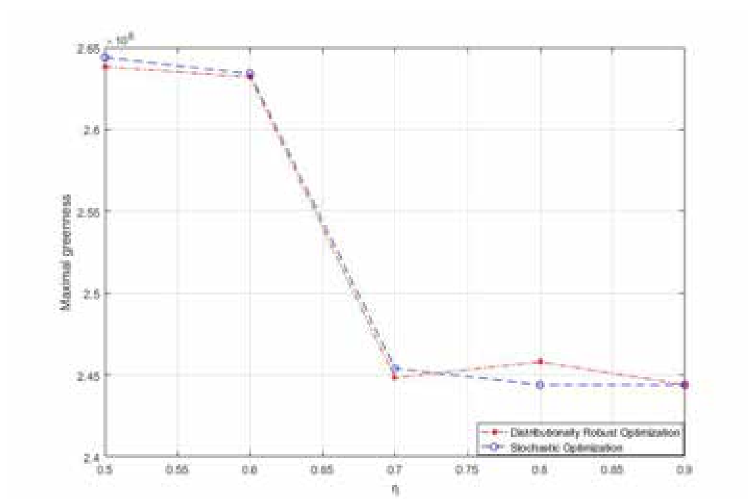


Figure 5: The distributionally robust greenness versus stochastic profit, $\alpha_{kp} = 0.99$

In addition, the comparison results under different $\alpha_{kp}$ on profit and greenness are shown in Figures 6 and 7, respectively. As shown in Figure 6, by increasing the value of the service level $\alpha_{kp}$, the profit for the stochastic model not change, but the profit for the distributionally robust model significantly increase. In other words, the difference between the distributionally robust and nominal stochastic models (robustness price) in profit is significant. Furthermore, we found that the difference between the distributionally robust and nominal stochastic model in greenness is not significant. Therefore, the proposed model are more sensitive compared with the nominal stochastic model.

# 5 Conclusions

This paper integrated an SCN and a supplier selection problem to formulate a bi-objective distributionally robust SCN model by considering the greenness of parts and products. In the proposed SCN, the distributions

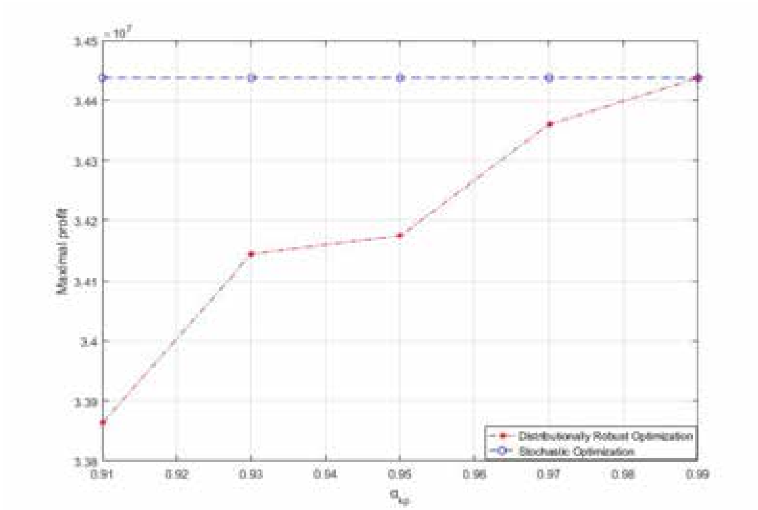


Figure 6: The distributionally robust greenness versus stochastic profit, $\eta = 0.7$

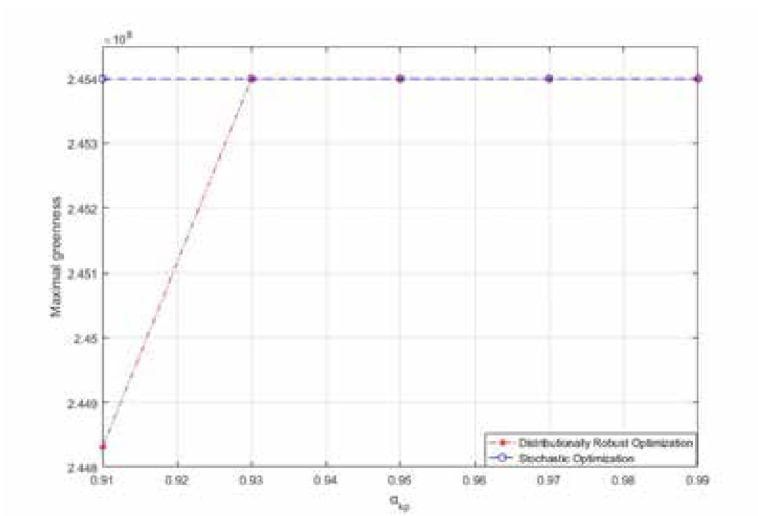


Figure 7: The distributionally robust greenness versus stochastic profit, $\eta = 0.7$

of uncertain demands are partially known and belong to an ambiguity set. Further, a weighted sum method was used to turn the bi-objective functions into a single objective function. Subsequently, we derived approximate expressions of ambiguous service level constraints under Box-Ellipsoid-Budget set, and transformed the proposed model into an approximate mixed-integer programming model, which can be solved by conventional optimization softwares. Finally, we addressed a numerical experiment and presented sensitivity analyses and a comparison study of the computational results to validate and verify of the proposed model.

Anyway, our modeling effort and analysis come with limitations. In the model of this paper, we only consider that the customers' demands are uncertain, but there are other inherent uncertain parameters in SCN, such as the purchasing costs which are not considered. Besides, it is difficult to compute large-scale SCN problems by conventional optimization softwares. In the further studies, the SCN problem should be solved by some efficient heuristic solution methods.

## Acknowledgments

The authors would like to cordially put on record their sincere thankfulness for the valuable time and constructive comments of the respected reviewers. This work was supported by the Post-graduate's Innovation

Fund Project of Hebei University (hbu2018ss44) and the High-Level Innovative Talent Foundation of Hebei University (No. 801260201209).

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