

## Reviews of Papers Related to Uncertain Systems

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### Abstract

In this section, we provide brief reviews of papers related to uncertain systems.

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Ariyan, S.P., and H. Kluger (eds.), *The Melanoma Handbook*, Demos Medical/Springer, New York, 2017.

Zager, J.S., Sondak, V.K., and R. Kudchadkar (eds.) *Melanoma*, Oxford University Press, New York, 2016.

One of the main criteria that makes a mole suspicious for cancer is irregular shape. But why do faster-growing (cancerous) moles have more irregular shapes than benign (slow-growing) ones? A possible general-systems explanation is as follows. We need to compare the shape of the two moles which are both, at present, of size  $r$ . Let  $v_1$  and  $v_2$  denote the average growth of each mole during a certain time period  $\tau$ .

The growths in different directions are, in general, different. This difference can be described by the standard deviations  $\sigma_i$  of the growth rate. It is reasonable to assume that this standard deviation  $\sigma_i$  constitutes approximately the same proportion of the growth, i.e., in precise terms, that it is proportional to the average growth rate:  $\sigma_i = c \cdot v_i$ , for some  $c > 0$ . It is also reasonable to assume that the deviations at different time periods are independent.

For each  $i$ , to attain the current size  $r$ , the  $i$ -th mole had to go through  $r/v_i$  growth cycles. Since the deviations at different cycles are independent, the resulting variance  $\sigma^2$  – describing variation of the size in different directions – is equal to the sum of the corresponding variances, i.e., to

$$\sigma^2 = \frac{r}{v_i} \cdot \sigma_i^2 = \frac{r}{v_i} \cdot (c \cdot v_i)^2 = C \cdot v_i,$$

where  $C \stackrel{\text{def}}{=} r \cdot c^2$ . Thus, indeed, the larger the mole's growth rate  $v_i$ , the larger the standard deviation  $\sigma$  that describes the irregularity of the mole's shape.

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Baginski, P., and S.T. Chapman, Factorizations of algebraic integers, block monoids, and additive number theory, *American Mathematical Monthly*, vol.118, no.10, pp.901–920, 2011.

Jenssen, M.O., Montealegre, D., and V. Ponomarenko, Irreducible factorization lengths and the elasticity problem within  $N$ , *American Mathematical Monthly*, vol.120, no.4, pp.322–328, 2013.

Schwab, E.D., and G. Schwab, Non-unique factorization in a class of non-commutative monoids, *Abstracts of the 19th UTEP/NMSU Workshop on Mathematics, Computer Science, and Computational Science*, El Paso, Texas, November 5, 2016.

The main objective of control is to transform the system from its original state to a desired state. For example, we want to make sure that a train goes from its origin to its destination; in chemical engineering, we want to make sure that oil is transformed into gasoline, etc.

In the beginning, we would like to come up with a control that achieves the desired objective: e.g., we want to make sure that the spaceship lands on the Moon. Once this objective is achieved, the next step is to

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come up with the *optimal* control – e.g., control that consists of the smallest possible number of elementary steps.

Let  $a_1, a_2, \text{etc.}$ , be elementary control actions. We can apply these actions one after another; let us denote the result of applying  $x$  after  $x'$  by  $xx'$ . From this viewpoint,  $x(x'x'')$  means the same as  $(xx')x''$ , so this combination operation is associative. Also, in most cases, there is a do-nothing action  $e$  for which  $xe = ex = x$  for all  $x$ . Sets with an associative operation with such a unit element  $e$  are known as *monoids*.

Once we have found a sequence of actions that leads to the desired transformation, i.e., once we have found a way to represent the desired transformation  $d$  as a superposition of elementary actions  $d = a_{i_1}a_{i_2} \dots a_{i_k}$ , a natural question is how to come up with an optimal control, i.e., with a representation which has the smallest possible number of elements. How much we can decrease the number of elementary actions is described by the notion of *elasticity*: for each element  $d$ , its elasticity  $\rho(d)$  is the ratio of its longest representation to its shortest one. So, for example, if we know that for a given element  $d$ , we have  $\rho(d) \leq 2$ , this means that no matter what sequence we start with, at best, we can improve the number of actions by a factor of two.

The largest of the element's elasticities is known as the elasticity  $\rho(M)$  of the monoid  $M$ . So, if the monoid's elasticity is 3, this means that for each desired action  $d$ , once we found some control sequence of length  $L$  that leads to  $d$ , then the optimal sequence cannot be shorter than  $L/3$ .

Elasticity of different monoids is what the reviewed papers study. The study of elasticity of different monoids has just started. Most of the results are currently about the commutative case, when order of every two actions can be changes without changing the result – the only exceptions are the results by the Schwabs.

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Hatch, S., *Snowball in a Blizzard: A Physician's Notes on Uncertainty in Medicine*, Basic Books, Philadelphia, 2016.

Medical doctors give us patients a lot of advice. Often, we do not follow all this advice – to the detriment of our own health. One of the reasons for this is that we know that some of this advice about diet, etc. may not be solid, since a few years ago, the medical community gave an opposite advice: for example, in the past, eggs were considered bad for cholesterol, now they are good, etc. So, since it is difficult to follow all the doctor's recommendations, we follow only some of them. The problem is that some of the advice that we do not follow is quite solid, based on good evidence – and not following this advice is bad for our health.

To avoid such situations, the author recommends that with each recommendation, doctors supply the patients with their degree of certainty to which this recommendation is valid; then, if we are unable to follow all their recommendations, we would at least follow the most critical ones. In many cases, doctors already have such scales, what is needed is to convey the corresponding degrees to the patients.

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Fosnot, C.T. (ed.), *Constructivism: Theory, Perspectives And Practice*, Teachers College Press, New York, 2005.

According to the constructivism approach to learning, we are not supposed to provide students with ready formulas and algorithms: we should instead provide students with guiding examples, based on which the students themselves will come up with the resulting formula or algorithm.

At first glance, this sounds like a waste of time: if the formula is already known, why not teach this formula to the students? It is usually much faster to provide the formula than to wait until students themselves come up with this formula. However, empirical analysis shows that this additional time is not wasted: in this approach, students retain the knowledge much longer and use it much more efficiently.

How can we explain this? One possible explanation is that people do forget. When a student is simply taught the formula, eventually he or she may forget this formula, and therefore, the student will not be able to use this formula anymore.

On the other hand, if the student is first taught numerous examples, so many examples that the student himself can derive the desired formula from them, then at the end of the class, the student has been exposed to both the examples and the formula. Students forget, so he or she will forget some part of their knowledge. If they forget one of the examples, they still remember the formula. If they forget the formula, they still have, in their memory, sufficiently many examples that help them reconstruct the formula. Thus, constructivism bring redundancy that increases the reliability of the students' memory.

In the long run, many examples will be forgotten. With this in mind, it is important to provide students with more examples than needed to derive the formula – so that even after a significant number of these examples are forgotten, the student will still have enough examples to be able to reconstruct the desired formula.

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Pink, D.H., *Drive: The Surprising Truth about What Motivates Us*, Riverhead Books, New York, 2009.

Research shows that humans (and even apes) like to altruistically solve creative problems – and that, somewhat surprisingly, their productivity in solving these problems drastically decreases when they are first compensated for these solutions and then this compensation is taken away. The author encourages us to use these effects as win-win situation in real life – if we make the problems more creative, people are happier and less compensation is needed.

The author claims that the above phenomenon contradicts to the natural ideas of self-interest and (for apes) biological necessity. In our opinion, however, such seemingly altruistic behavior does not necessarily mean that people are irrational. Indeed, in real life, we sometimes encounter complex situations that require creative solutions. Such situations are rare. So, unless we repeatedly practice solving such complex problems, we may lose this complex-problem-solving skills by the time such a complex situation is encountered. Thus, when we sometimes encounter a situation when we can practice solving complex problems – even when solving the corresponding problem is not crucial for us at this time – it is natural to use this as an opportunity to practice our complex-problem-solving skills (so as not to lose them by the time when a vitally important complex situation emerges).

As for the productivity decrease when the original compensation is withdrawn, there is also an easy explanation: the original compensation shows that someone is willing to pay us for solving problems of this type; so why should we solve these problems for free if we can, in addition to the fun of solving, also get some compensation?

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Raynor, M.E., *The Strategy Paradox: Why Committing to Success Leads to Failure (and What to Do about It)*, Doubleday, New York, 2007.

In many engineering situations, we know the future (at least in the short term): in an automobile plant, we need to produce cars; in petroleum engineering, we need to extract and process oil, etc. In such situations, it is desirable to find a strategy  $x$  that optimizes the corresponding objective function  $F(x)$  (usually, the company's profit), i.e., to find the strategy  $x$  for which the value  $F(x)$  is the largest.

In many other business endeavors – and in long-term engineering situations – the future is uncertain, there are many possible options  $e$ . The traditional approach was to select the most probable outcome  $e_0$  and to optimize the strategy  $x$  under the assumption that the future will follow this most-probable option. In other words, we select the strategy  $x$  for which the corresponding value  $F(x, e_0)$  of the objective function is the largest possible.

The actual future environment  $e$  may differ from  $e_0$ . As a result, a strategy  $x$  which is optimal for  $e_0$  may turn out to be disastrous for  $e \neq e_0$ ; the author provides many examples of such disasters.

A usual engineering approach to this problem is to look for *robust* strategies, i.e., strategies that remain reasonable even when  $e$  deviates from  $e_0$ , i.e., when  $e$  is in an appropriate neighborhood  $N(e_0)$  of the most probably environment  $e_0$ . The usual formalization of this requirement is to find the strategy  $x$  for which the value  $v(x) \stackrel{\text{def}}{=} \min_{e \in N(e_0)} F(x, e)$  is the largest possible. Such a robust strategy enables us to avoid disaster, but the resulting performance is often suboptimal (mediocre).

An ideal solution would be to come up with a family of strategies  $x(e)$  so that for every possible future environment  $e$ , we would apply the strategy which is optimal for this particular  $e$ , i.e., for which  $F(x(e), e) = \max_x F(x, e)$ . However, this is often not practically possible: computing even one optimal strategy usually requires a large amount of effort, and thus, computing a large number of such strategies is not feasible.

The author proposes, instead of come up with a finite “portfolio” of solutions  $x_1, \dots, x_n$  so that for each possible future environment  $e$ , we will be able to select the most appropriate of these prepared strategies, i.e., a strategy  $x_i$  for which  $F(x_i, e) = \max_j F(x_j, e)$ . Similarly to the robust case, these strategies can be selected

based on the requirement that the worst-case outcome is the largest possible, i.e., strategies that maximize the value  $v(x) = \min_{e \in N(e_0)} \max_j F(x_j, e)$ .

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Scerri, E., *A Tale of Seven Scientists and a New Philosophy of Science*, Oxford University Press, New York, 2016.

Since Thomas Kuhn's idea of scientific revolutions, the prevailing view is that a large part of progress of science is achieved by a sequence of drastic revolutionary changes. This is indeed the impression we get if we only look at the main Nobel-prize winning results.

However, if we look deeper, in many cases, we can see that each ground-breaking result actually appeared gradually, with numerous not-so-well-known researchers who provided small steps, steps whose combination eventually resulted in the Nobel-prize-worthy discovery. This book convincingly shows the gradual character of science on an important example of attempts to understand the physical meaning of chemical properties of atoms via their electron configurations.

But why is the progress gradual? One possible explanation may be that no matter how discrete the results of our thoughts can be, inside the brain, all the quantities are continuous. So, in order to change from one state of the brain to another, we have to go gradually, via continuously changing intermediate states. Sometimes, these intermediate states occur in the brain of the same scientist – and so, we see a scientific revolution. However, frequently, this transition happens in the brains of several scientists, many of whom publish their preliminary ideas and results – thus creating the gradual evolution so convincingly illustrated in the book.

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Urban, H., *Life's Greatest Lessons: 20 Things That Matter*, Simon & Schuster, New York, 2003.

One of the advices from the book is to maintain correct attitude: think with an open mind, think for yourself, and think constructively. This can be naturally interpreted in terms of *fuzzy logic*, where each statement is characterized by a *degree of confidence*: degree 1 means that we are absolutely sure that this statement is true, while smaller degrees indicate that we are not absolutely sure.

- Open mind means not to be absolutely confident in any statements that you think are true, i.e., to assign to all such statements degrees of confidence less than 1.
- Think for yourself means not to be absolutely confident in statements by outside experts, to assign to all such statements degree of confidence less than 1.

Thinking constructively means thinking about how to change things, not just about what is the state of the world.