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An Uncertain Bilevel Newsboy Model with a Budget Constraint

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Abstract

In a classical newsboy problem, the demand of market is usually assumed to be a random variable and can be described by a probability distribution. However, the historical data are not reliable or even rare for probabilistic reasoning. In such situations, we have to rely on experts to estimate the demand. As a result, some researchers assumed that the demand is an uncertain variable which can involve in the expert experience and intuition. Then they investigated some uncertain newsboy models under this assumption. In this paper, we propose an uncertain bilevel newsboy model, in which the manufacturer is the leader and the retailers are followers. The leader first decides his wholesale prices of multiple kinds of newspapers, and each follower decides the order quantity to maximize her own expected profit according to the leader's policy, other followers' reactions, and her uncertain demand. Then the manufacturer modifies his wholesale prices with the intention of obtaining his own maximum profit. Moreover, we assume that there are budget constraints for the retailers in the uncertain bilevel newsboy problem. Illustrative examples are given to show the application of the proposed models.

Keywords: bilevel newsboy problem, uncertainty theory, uncertain variable, budget constraints

1 Introduction

Classical newsboy problem refers to a kind of inventory problems, where the products usually become obsolete out of the period like newspaper. Hadley and Whitin [5] rendered the early treatment for the newsboy problem. The inventory strategy of newsboy problem is to determine the optimal order quantity with the aim of maximizing expected profit or minimizing the cost. And the most of classical newsboy problem with budget constraints extensions [7, 14] usually treated the problem only in single context, not in bilevel environment.

But in practice, the manufacturer usually needs the retailer to purchase his kinds of products. In this situation, the manufacturer and the retailer are separate, and each decision-maker intends to maximize its own profit and is affected by the actions of the other one, this brings about the consideration in hierarchical system. Iyer and Bergen [6] considered the quick response in manufacturer-retailer channels. Pasternack [15] developed the optimal pricing and return policies for perishable commodities. Lau [8] proposed some two-echelon style-goods inventory models with asymmetric market information and Lau [9] also considered comparative normative optimal behavior in two-echelon multiple-retailer distribution systems for a single-period product.

However, these extensive models above used randomness to describe the demand of the retailer and almost solved the problem by means of probability. But some non-deterministic events are lack of historical data for some reasons, such as economic influences or some newly promulgated policies, so that it is hard to give the probability distributions to describe them. In order to handle these non-deterministic events which cannot be described by probability theory. Liu [10] has proposed the uncertainty theory in 2007. And it can be found that uncertainty theory is another way to interpret human uncertainty. Uncertainty theory is a branch of mathematics based on normality, duality, subadditivity and product axioms.

Qin and Kar [16] firstly introduced the uncertainty theory to newsboy problem and assumed the demand as an uncertain variable. From then on, many researchers have applied uncertainty theory into newsboy problem and some extension models have been brought forward. Gao et al. [4] obtained the optimal (s, S) policy considering the setup cost and initial stock. Gao and Ding [1] got (σ, S) policy for multi-product newsboy problem. Ding [2] took chance constraint into account and derived an uncertain multi-product

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model. In addition, Ding [3] applied uncertain random variable into newsboy problem and proposed the relationships among uncertain newsboy problem, stochastic newsboy problem and uncertain random newsboy problem.

In this paper, an uncertain bilevel newsboy model with a budget constraint in the lower level for multi-product is firstly proposed. For solving the bilevel model, we first solve the multi-product newsboy problem with a budget constraint in single context, the Lagrangian function method is introduced and when obtaining an expression for the Lagrangian multiplier μ , we have the value of optimal or near optimal order quantity of the retailer and then substitute to the top level of the model, the optimal or near optimal wholesale prices of the manufacturer can be obtained, in special, the single product case is also be considered with K-T conditions method and get the exact solution.

The rest of this paper is organized as follows. Section 2 gives some fundamental concepts of uncertainty theory which are useful to understand the paper. In Section 3, we present an uncertain bilevel newsboy model with a budget constraint for multi-product case and give the solution procedure, and as a remark, single product case is also be researched. Illustrative examples to show the application of the proposed models are given in Section 4. A conclusion of the paper is provided in Section 5.

2 Preliminaries

In this section, some basic concepts and theorems are proposed to interpret the uncertainty theory.

Let Γ be a nonempty set, let $\mathcal{L}$ be a σ-algebra over Γ. Each element $\Lambda \in \mathcal{L}$ is assigned a number $\mathcal{M}\{\Lambda\} \in [0,1]$. In order to ensure the $\mathcal{M}\{\Lambda\}$ be an uncertain measure, four axioms: (1) normality axiom, (2) duality axiom, (3) subadditivity axiom, and (4) product axiom are proposed.

Definition 2.1 ([10]) *Let Γ be a nonempty set, and let $\mathcal{L}$ be a σ-algebra over Γ, and let $\mathcal{M}$ be an uncertain measure. Then the triple (Γ,$\mathcal{L}$,$\mathcal{M}$) is called an uncertainty space.*

Definition 2.2 ([10]) *An uncertain variable is a measurable function ξ from an uncertainty space (Γ,$\mathcal{L}$,$\mathcal{M}$) to the set of real numbers, i.e., for any set $\mathbf{B}$ of real numbers, the set*

$$\{\xi \in \mathbf{B}\} = \{\gamma \in \Gamma | \xi(\gamma) \in \mathbf{B}\}$$

is an event.

Definition 2.3 ([10]) *The uncertainty distribution Φ of an uncertain variable ξ is defined by*

$$\Phi(x) = \mathcal{M}\{\xi \leq x\}$$

for any real number x.

Example 2.1 *The linear uncertain variable $\xi \sim \mathcal{L}(a,b)$ has an uncertainty distribution*

$$\Phi(x) = \begin{cases} 0, & x < a \\ (x-a)/(b-a), & a \leq x \leq b \\ 1, & b < x. \end{cases}$$

Example 2.2 *The normal uncertain variable $\xi \sim \mathcal{N}(e,\sigma)$ has an uncertainty distribution*

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi \ (e-x)}{\sqrt{3}\sigma}\right)\right)^{-1}.$$

Definition 2.4 ([11]) *The uncertain variables $\xi_1, \xi_2, ..., \xi_n$ are said to be independent if*

$$\mathcal{M}\left\{\bigcap_{i=1}^{n}(\xi_i \in \mathbf{B}_i)\right\} = \bigwedge_{i=1}^{n} \mathcal{M}\{\xi_i \in \mathbf{B}_i\}$$

for any Borel sets $\mathbf{B}_1, \mathbf{B}_2, ..., \mathbf{B}_n$ of real numbers.

Definition 2.5 ([12]) *An uncertainty distribution Φ is said to be regular if its inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0,1)$.*

Definition 2.6 ([12]) *Let ξ be an uncertain variable with regular uncertainty distribution $\Phi(x)$. Then the inverse function $\Phi^{-1}(\alpha)$ is called inverse uncertainty distribution of ξ.*

Example 2.3 *The inverse uncertainty distribution of linear uncertain variable $\mathcal{L}(a,b)$ is*

$$\Phi^{-1}(\alpha) = (1-\alpha)a + \alpha b.$$

Example 2.4 *The inverse uncertainty distribution of normal uncertain variable $\mathcal{N}(e,\sigma)$ is*

$$\Phi^{-1}(\alpha) = e + \frac{\sigma\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}.$$

Definition 2.7 ([10]) *Let ξ be an uncertain variable. Then the expected value of ξ is defined by*

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \geq x\}\mathrm{d}x - \int_{-\infty}^0 \mathcal{M}\{\xi \leq x\}\mathrm{d}x$$

provided that at least one of the two integrals is finite.

The expected value of ξ can also be obtained by the formula

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha)\mathrm{d}\alpha.$$

3 Model Description

We consider an uncertain bilevel newsboy model with a budget constraint for multi-product. The demand for each product is assumed as an independent uncertain variable $\xi_i, (i = 1,2,...,n)$ with regular uncertainty distribution $\Phi_i(x)$, and $\xi_1, \xi_2, ..., \xi_n$ are independent, the goal is to establish an uncertain bilevel newsboy model with a budget constraint for multi-product to gain the maximum profit both for the manufacturer and the retailer. The mathematical formulations of the model is developed with the following notations for product i, $i = 1,2,...,n$ as follows.

$\Phi_i(x)$	the distribution of demand;
p_i	unit selling price;
h_i	unit salvage value;
w	the permitted budget of the retailer
$f_i(q_i, y_i, \xi_i)$	the profit of the retailer;
$F(\boldsymbol{q}, \boldsymbol{y}, \boldsymbol{\xi})$	the profit of the manufacturer;
$\hat{\boldsymbol{y}} = (\hat{y}_1, \hat{y}_2, \cdots, \hat{y}_n)$	the optimal order quantity array without any constraint;
$\boldsymbol{y_i^*} = (y_1^*, y_2^*, \cdots, y_n^*)$	the optimal order quantity array with a budget constraint;
$\boldsymbol{q} = (q_1, q_2, \cdots, q_n)$	the wholesale price array of manufacturer;
$\boldsymbol{d} = (d_1, d_2, \cdots, d_n)$	the manufacturing cost array;
$\boldsymbol{\xi} = (\xi_1, \xi_2, \cdots, \xi_n)$	the demand array of retailer;
$\boldsymbol{y} = (y_1, y_2, \cdots, y_n)$	the order quantity array of retailer.

Obviously, we have $p_i > q_i > h_i > 0$. Then the profit functions of the retailer can be expressed as

$$f_i(q_i, y_i, \xi_i) = \begin{cases} (p_i - q_i)y_i, & y_i \leq \xi_i \\ (h_i - q_i)y_i + (p_i - h_i)\xi_i, & y_i > \xi_i \end{cases}$$

for $i = 1,2,...,n$, respectively. In single context, the manufacturer and the retailer can be treated as a whole. As Qin and Kar (2013) proved that

$$E\left[f_i(q_i, y_i, \xi_i)\right] = (p_i - q_i)y_i - (p_i - h_i)\int_0^{y_i} \Phi_i(x)\mathrm{d}x$$

where $\Phi_i(x)$ is the uncertainty distribution of the $\xi_i (i = 1, 2, ..., n)$.

We consider there is only one retailer and the retailer purchases n kinds of products, in the bilevel decision system, the manufacturer first decides his wholesale price array $\boldsymbol{q} = (q_1, q_2, \cdots, q_n)$ and then the retailer adjusts his order quantity array $\boldsymbol{y} = (y_1, y_2, \cdots, y_n)$ in view of the decision of the manufacturer. In order to maximize the expected profit both for the manufacturer and the retailer, the uncertain bilevel newsboy model with a budget constraint can be expressed as

$$\begin{cases} \max\limits_{\boldsymbol{q}} \quad F(\boldsymbol{q}, \boldsymbol{y^*}, \boldsymbol{\xi}) \\ \text{subject to} : \boldsymbol{q} > 0 \\ \qquad\qquad\quad \text{where } \boldsymbol{y^*} \text{ solves problems } (i = 1, 2, ..., n) \\ \qquad\qquad\quad \begin{cases} \max\limits_{\boldsymbol{y}} \quad E[f(\boldsymbol{q}, \boldsymbol{y}, \boldsymbol{\xi})] \\ \text{subject to} : \sum\limits_{i=1}^{n} q_i y_i \leq w, \end{cases} \end{cases} \tag{1}$$

where $F(\boldsymbol{q}, \boldsymbol{y^*}, \boldsymbol{\xi}) = \sum_{i=1}^{n}(q_i - d_i)y_i^*$, $f(\boldsymbol{q}, \boldsymbol{y}, \boldsymbol{\xi}) = \sum_{i=1}^{n} f_i(q_i, y_i, \xi_i), i = 1, 2, ..., n.$

To solve the uncertain bilevel newsboy problem with a budget constraint, we consider to solve the model forms in single context as follows.

$$\begin{cases} \max\limits_{y} \quad E\left[f(q, y, \xi)\right] = (p - q)y - (p - h) \displaystyle\int_0^y \Phi(x)\mathrm{d}x \\ \text{subject to} : qy - w \leq 0 \\ \qquad\qquad\quad y \geq 0. \end{cases} \tag{2}$$

If the constraint is initially relaxed, using Leibniz's rule, one can obtain

$$\Phi_i(\hat{y}_i) = \frac{p_i - q_i}{p_i - h_i}. \tag{3}$$

Consequently, substituting the value of $\hat{y}_i$ in the constraint, if it satisfys the constraint, then $\hat{y}_i$ is the optimal value and the problem is solved. Otherwise, the constraint is set to equality and the Lagrangian function is introduced as in the following (note that μ is the Lagrangian multiplier)

$$L = \sum_{i=1}^{n} \left[(q_i - p_i)y_i + (p_i - h_i) \int_0^{y_i} \Phi_i(x)\mathrm{d}x \right] + \mu \left[\sum_{i=1}^{n} (q_i y_i) - w \right] = 0. \tag{4}$$

Again, using Leibniz's rule and differentiating, we obtain

$$\Phi_i({y_i}^*) = \frac{p_i - q_i(1 + \mu)}{p_i - h_i}. \tag{5}$$

Substituting the values of $\hat{y}_i$ $(i = 1, 2, ..., n)$ in the budget constraint, if it satisfies the budget constraint, then the $y_i^* = \hat{y}_i$. Otherwise define

$$\Delta B = \sum_{i=1}^{n} (q_i \hat{y}_i) - w,$$

and

$$w = \sum_{i=1}^{n} (q_i y_i^*).$$

Assume $\Phi_i(\hat{y}_i)$ is differentiable and denote its derivative by $\phi_i(\hat{y}_i)$, using the Taylor expansion at $\hat{y}_i$ for $\Phi_i(y_i^*)$, that is

$$\Phi_i(y_i^*) = \Phi_i(\hat{y}_i) + \phi_i(\hat{y}_i)(y_i^* - \hat{y}_i). \tag{6}$$

Substituting Eq.(5) in Eq.(6), we can obtain that

$$y_i^* = \hat{y}_i + \frac{1}{\phi_i(\hat{y}_i)} \left[\frac{p_i - q_i(1 + \mu)}{p_i - h_i} - \Phi_i(\hat{y}_i) \right], \tag{7}$$

then, we can get

$$\sum_{i=1}^{n} q_i(\hat{y}_i - y_i^*) = \sum_{i=1}^{n} \frac{q_i\Phi_i(\hat{y}_i)}{\phi_i(\hat{y}_i)} + \sum_{i=1}^{n} \frac{q_i^2}{(p_i - h_i)\phi_i(\hat{y}_i)} + \sum_{i=1}^{n} \frac{q_i^2}{(p_i - h_i)\phi_i(\hat{y}_i)} + \sum_{i=1}^{n} \frac{\mu q_i^2}{(p_i - h_i)\phi_i(\hat{y}_i)}.$$

Then letting

$$A = \sum_{i=1}^{n} \frac{q_i^2}{(p_i - h_i)\phi_i(\hat{y}_i)},$$

$$B = \sum_{i=1}^{n} \left[\frac{q_i\Phi_i(\hat{y}_i)}{\phi_i(\hat{y}_i)} - \frac{p_i q_i}{(p_i - h_i)\phi_i(\hat{y}_i)} \right],$$

that is,

$$\Delta B = A + B + \mu A.$$

Hence,

$$\mu = \frac{\Delta B - (A + B)}{A}.$$

Note that $A + B = 0$, we can get

$$\mu = \frac{\Delta B}{A}.$$

Then the expression of μ can be obtained. As we can see, the value of μ is dependent on the type of demand's distribution. Obtaining an explicit expression for μ, we can have the optimal or near optimal of the order quantity from Eq.(7). Hence, an uncertain newsboy problem with a budget constraint in single context can be solved, then substituting it in the top level of model (1), using SLP Direction algorithm, the optimal wholesale price array of manufacturer can be obtained with the aim of gaining his own maximum profit.

Definition 3.1 *Suppose that $\boldsymbol{q}^*$ is a feasible optimal wholesale price array of the manufacturer and $\boldsymbol{y}^*$ is a optimal order quantity array of retailer with respect to $\boldsymbol{q}$. We call the equilibrium $(\boldsymbol{q}^*, \boldsymbol{y}^*)$ a Stackelberg equilibrium to the uncertain bilevel newboy model* (1) *if*

$$E[F(\boldsymbol{q}, \boldsymbol{y}, \boldsymbol{\xi})] \leq E[F(\boldsymbol{q}^*, \boldsymbol{y}^*, \boldsymbol{\xi})]$$

for any feasible wholesale price array $\boldsymbol{q}$ of the manufacturer and any feasible order quantity array of the retailer $\boldsymbol{y}$ with respect to $\boldsymbol{q}$.

For an uncertain bilevel newsboy problem with a budget constraint, when the retailer plans to introduce some new kinds of product, lacking historical data, if the relevant experts give the distribution of the demand for each product, we can get a optimal or near optimal order quantity of retailer for each product, then substituting it in the top level of model (1), a Stackelberg equilibrium to the uncertain bilevel newboy model (1) can be obtained.

Remark 3.1 *If we use the above Lagrange function method for single product, it's possible to get a near Stackelberg equilibrium to the uncertain bilevel newboy model with a budget constraint, a Stackelberg equilibrium can be obtained if applying the following K-T condition method.*

Similar to the multi-product case, we firstly get a optimal order quantity array of the retailer and then get a Stackelberg equilibrium to the uncertain bilevel newboy model, the difference is that how to obtain the optimal optimal order quantity array of the retailer.

For single product case, the uncertain newsboy problem with a budget constraint mathematical model in single context can be expressed as

$$\begin{cases} \max\limits_{y} \quad E\left[f(q, y, \xi)\right] = (p - q)y - (p - h)\displaystyle\int_0^y \Phi(x)\mathrm{d}x \\ \text{subject to} : qy - w \leq 0 \\ \qquad\qquad\quad y \geq 0. \end{cases} \tag{8}$$

And the model (8) is equivalent to the following deterministic programming model

$$\begin{cases} \min\limits_{y} \quad -E\left[f(q,y,\xi)\right] = (q-p)y + (p-h)\displaystyle\int_0^y \Phi(x)\mathrm{d}x \\ \text{subject to} : qy - w \leq 0 \\ \qquad\qquad\quad y \geq 0. \end{cases} \tag{9}$$

Theorem 3.1 *If uncertainty distribution $\Phi(x)$ of uncertain variable ξ is regular, then $E\left[f(q,y,\xi)\right]$ is a concave function.*

Proof. Since $E\left[f(q,y,\xi)\right] = (p-q)y - (p-h)\int_0^y \Phi(x)\mathrm{d}x$, the optimal value of y can be obtained by using Leibniz's rule. Thus

$$\Phi(\hat{y}) = \frac{p-q}{p-h}.$$

Then

$$\hat{y} = \Phi^{-1}(\frac{p-q}{p-h}).$$

Since $\Phi(y)$ is an uncertainty distribution, it is monotone increasing function expect $\Phi(y) \equiv 0$ and $\Phi(y) \equiv 1$. In addition, since

$$\frac{\partial E^2[f(q,y,\xi)]}{\partial y^2} < 0.$$

Hence, $E\left[f(q,y,\xi)\right]$ is a strictly concave function. The theorem is proved. □

Since $E\left[f(q,y,\xi)\right]$ is a strictly concave function, then $-E\left[f(q,y,\xi)\right]$ is a strictly convex function. Also, it is obvious that the restriction of model (9) is a convex set, the model (9) is a convex programming problem essentially. Then we can obtain the necessary and sufficient conditions for the existence of optimal order quantity, that is, K-T conditions are

$$\begin{cases} (q-p) + (p-h)\Phi(y) + \lambda q = 0 \\ \lambda(qy - w) = 0 \\ \lambda \geq 0. \end{cases} \tag{10}$$

Theorem 3.2 *If the K-T conditions that is Eq.(10) are satisfied, then we can obtain the unique optimal order quantity as follows*

$$\begin{cases} y^* = \hat{y}, & qy < w \\ y^* = \dfrac{w}{q}, & qy \geq w. \end{cases}$$

Proof. If $qy < w$, let $\lambda = 0$, then substituting λ in the first equation of Eq.(10), thus, we have

$$\Phi(y) = \frac{p-q}{p-h}.$$

That is to say

$$\Phi(y^*) = \Phi(\hat{y}),$$

i.e.,

$$y^* = \hat{y}.$$

If $qy \geq w$, in the similar way, we have

$$y^* = \frac{w}{q} \leq y.$$

Divided by $(p-h)$ at the both end the first equation of Eq.(10), then we obtain

$$\frac{q-p}{p-h} + \Phi(y) + \frac{\lambda q}{p-h} = 0,$$

i.e.,

$$-\Phi(\hat{y}) + \Phi(y^*) + \frac{\lambda q}{p-h} = 0.$$

Since $y^* \leq \hat{y}$ and $p > h$, we can obtain that $\lambda > 0$ and it satisfies the K-T conditions, Hence, the

$$y^* = \frac{w}{q}$$

is the optimal order quantity. The theorem is proved. □

Hence, we get the optimal order quantity of retailer, then substituting it in the top level of model (1), a Stackelberg equilibrium can be obtained. In order to show the application of the uncertain bilevel newsboy model with a budget constraint for single product and multi-product, illustrative examples are given in the following part.

4 Illustrative Examples

In this section, illustrative examples for the uncertain bilevel newsboy model with a budget constraint for single product and multi-product case are given. In single product case, we consider the demand of the product obeys normal uncertainty distribution, and show a comparison between the results obtained by K-T conditions method and that obtained by Lagrange function method. In multi-product case, linear distribution and normal uncertainty distribution are considered respectively.

Example 5.1 Consider an uncertain bilevel newsboy problem where the demand $\xi \sim \mathcal{N}(20, 1)$. Single product and a budget of \$100 are considered. Table 1 shows the parameter values and the results.

Table 1: Example 5.1's parameter values and the results for normal uncertainty distribution

method	p	h	d	y^*	q^*	Stackelberg equilibrium	objective value
K-T conditions	10	5	3	10	10	$(10, 10)$	70
Lagrange function method	10	5	3	10.48	9.54	$(9.54, 10.48)$	68.54

Table 1 shows the compared results of K-T conditions method and Lagrange function method, and we can get that the objective value obtained by Lagrange function method has an error of 2.08%.

Example 5.2 For the multi-product uncertain bilevel newsboy problem with a budget constraint, assume that the demand of the product obeys linear uncertainty distribution and normal uncertainty distribution respectively. First, assuming that $\xi_i (i = 1, 2, 3)$ are linear uncertain variables, and a budget of \$800 is considered. Table 2 shows the parameter values and the results. Then, assuming that $\xi_i (i = 1, 2, 3)$ are normal uncertain variables, and a budget of \$200 is considered. Table 3 shows the parameter values and the results.

Table 2: Example 5.2's parameter values and the results for linear uncertainty distribution

item	ξ_i	p_i	h_i	d_i	y_i^*	q_i^*	Stackelberg equilibrium	objective value
1	$\mathcal{L}(200, 400)$	10	4	2	39.89	8.45	$(8.45, 39.89)$	
2	$\mathcal{L}(300, 500)$	12	6	3	9.84	11.81	$(11.81, 9.84)$	
3	$\mathcal{L}(400, 600)$	15	9	4	23.10	15.00	$(15.00, 23.10)$	
								598.27

5 Conclusion

We investigated an uncertain bilevel newsboy model with a budget constraint in the lower level for multi-product case in this paper. For solving the bilevel model, we first solved an uncertain model with a budget constraint for the multi-product newsboy problem in single context, then, the Lagrangian function method was introduced to get the value of optimal or near optimal order quantity. On the basis of the model in single

Table 3: Example 5.2's parameter values and the results for normal uncertainty distribution

item	ξ_i	p_i	h_i	d_i	y_i^*	q_i^*	Stackelberg equilibrium	objective value
1	$\mathcal{L}(20,1)$	10	4	3	19.55	8.17	$(8.17, 19.55)$	
2	$\mathcal{L}(10,1)$	12	6	2	3.36	12.00	$(12.00, 3.36)$	
3	$\mathcal{L}(20,3)$	20	12	4	0.00	20.00	$(20.00, 0.00)$	
								134.64

context, an uncertain bilevel newsboy model with an uncertain demand can be solved. In special, the single product case was also be researched with K-T conditions method and get the exact solution and we also gave the error analysis in illustrative examples section.

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