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Modeling Stochastic Multi-Period Multi-Product Closed-Loop Supply Chain Network by Joint Service Level Constraints

Hui-Li Pei*, Hong-Liang Li
College of Mathematics and Information Science, Hebei University, Baoding 071002, Hebei, China

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Abstract

The high importance of economic benefits and environmental impacts of using scrapped products have caused most industries to move to design closed-loop supply chain network. This paper studies a multi-period, multi-product, multi-layer, capacitated closed-loop supply chain network, in which the uncertain customer demands are characterized by a known joint probability distribution. The considered supply chain network problem is built as a chance-constrained programming (CCP) model. For general probability distribution, it is usually impossible to convert the probabilistic constraints to their deterministic equivalent forms. To avoid this difficulty, we consider a special case that uncertain demand follows a discrete distribution with finite scenarios. Furthermore, by introducing auxiliary binary variables, the proposed chance-constrained programming model is reformulated as an equivalent mixed-integer programming model, which can be solved by conventional optimization software like CPLEX. Finally, some numerical experiments are performed to demonstrate the effectiveness of our proposed optimization method.

Keywords: closed-loop supply chain network, uncertain demand, CCP, joint service level, mixed-integer programming

1 Introduction

A closed-loop supply chain network is an integrated system including both the forward and the reverse supply chains. The closed-loop supply chain network design includes decision making on the numbers, locations, capacities of facilities and the flows between them in order to optimize the entire supply chain operations [1]. With the increased concentrations of environmental issues, the depletion of our natural resources and the restrictive environmental regulations, the problem of closed-loop supply chain design has received growing attention in the literature [5, 17, 19]. Some researchers are interested in the performing of the closed-loop supply chain network design activities such as recycling and disposing operations [4]. Since the integrated design of forward and reverse supply chain is a critical factor in reducing the costs, improving service level and responding to environmental issues, some interesting work [3, 6, 9, 12, 18, 21] have focused on the closed-loop supply chain network design problem.

In a practical decision-making system, the supply chain network design problem is subject to subjective and objective uncertainties [22, 23, 24, 26, 27]. Consideration of uncertainties will lead to a more realistic supply chain. A comprehensive list of the sources of uncertainty are identified by Simangunsong [20]. In forward supply chain, customers' demands are quite uncertain. Reverse supply chain is very complex and tend to be high degree of uncertainty. The variety of returns and the quality and quantity of returned products are highly uncertain even in a short period of time. The significance of uncertainty has prompted many researchers to develop methodologies to tackle uncertainty in supply chain network design. The methodologies include stochastic programming, robust optimization and fuzzy programming. For example, Giri et al. [8] addressed a single-manufacturer single-retailer closed-loop supply chain with stochastic product returns considering worker experience under learning and forgetting in production and inspection of returned items at the manufacturer. Ma and Liu [14] proposed a stochastic chance-constrained CLSC network design model with value-at-risk objective, in which both transportation cost and customer's demand are stochastic parameters with known

*Corresponding author.
Email: peihuili81@126.com (H. Pei).

joint distributions. Kisomi [11] studied supply chain configuration and proposed an integrated mathematical programming model based on robust optimization. On the basis of credibility measure theory [13], Bai and Liu [2], and Yang and Liu [25] studied the supply chain network design problem under fuzzy uncertainty, where the possibility distributions of customer demands and transportation costs are partially available. Mohammed [15] formulated a multi-period multi-product MILP model with scenario-based stochastic customer demand and end-of-life product return, in which demand and return are modeled independently. For the recent developments of supply chain network design problems, the interested readers may refer to literatures [7, 10, 16, 28].

Motivated by the work mentioned above, this paper is to develop a multi-period, multi-product, multi-layer, capacitated closed-loop supply chain network problem, in which the customers' demands are stochastic parameters with known joint probability distribution. To the best of our knowledge, there is no research work about this issue. Our problem is to minimize the total cost of the closed-loop supply chain network including the opening cost of facilities, production cost, inventory cost, collection cost, recycling cost, disposal cost and transportation cost with joint service level constraints. The key difficulty in solving our stochastic supply chain network design problem is to compute the joint service level effectively. In practical applications, discrete distributions about uncertain demand are frequently used, and it may be available through experience distribution or approximating continuous probability distribution. In this paper, we assume that the stochastic parameters follow a discrete joint probability distributions, and reformulate the service level constraints as their equivalent deterministic forms. As a result, the original uncertain supply chain network design problem is equivalent to a deterministic mixed-integer programming problem, which can be solved by optimization solvers like CPLEX.

The rest of the paper is organized as follows. In section 2, we develop a multi-period, multi-product, capacitated closed-loop supply chain network. Section 3 derives the equivalent mixed-integer programming model under stochastic customer demand. In Section 4, some numerical experiments are performed to demonstrate the the effectiveness of the proposed optimization method. Section 5 presents the conclusions of this paper.

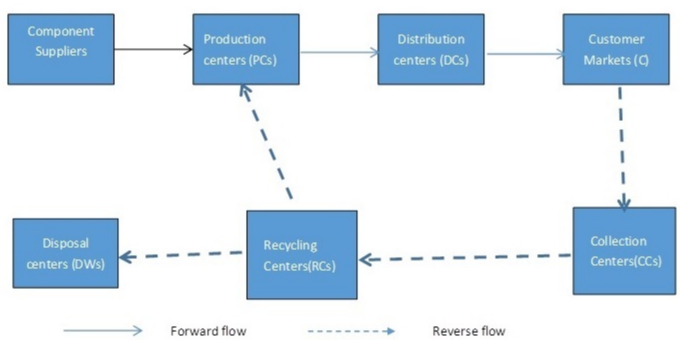

Figure 1: The concerned closed-loop supply chain network

2 Problem Description and Model Formulation

2.1 Problem Description and Model Assumptions

2.1.1 Problem description

In this paper, the concerned closed-loop supply chain is a multi-period, multi-product, capacitated network that includes suppliers, production centers (PCs), distribution centers (DCs) and markets in the forward supply chain, and contains collection centers (CCs), recycling centers (RCs) and disposal centers(DWs) in the reverse supply chain. Figure 1 displays our supply chain network, in which PCs get new components through suppliers and recycled components from RCs. Each PC could produce multiple product types using technologies that may differ from other producers. Each technology has its operation and production costs. The finished products are shipped to the markets through the DCs. The kinds of transportation modes are available for shipping products among facilities at different costs. In the reverse supply chain, the end-of-life products are collected by the CCs. Collected products are shipped to the RCs. At the RCs, the products are disassembled into components, inspected and sorted into recyclable and non-recyclable components. Non-recyclable components are shipped to DWs for disposal.

2.1.2 Model Assumptions

(A1) The number, capacity and locations of PCs, DCs, CCs, RCs and DWs are known;
(A2) The number and locations of the markets are known;
(A3) Customers' demand is characterized by a joint probability distribution with finite support;
(A4) The recycled components are as good as new components.

2.2 Model Notations

In order to formulate our CCP model, some notations are first described as follows:
Sets
$\mathcal{P}$ set of potential location for production centers PCs;
$\mathcal{D}$ set of potential location for distribution centers DCs;
$\mathcal{C}$ set of markets;
$\mathcal{J}$ set of potential location for collection centers CCs;
$\mathcal{R}$ set of potential location for recycling centers RCs;
$\mathcal{W}$ set of potential location for disposal centers DWs;
$\mathcal{I}$ set of products;
$\mathcal{N}$ set of components;
$\mathcal{M}$ set of transportation modes;
$\mathcal{H}$ set of production technologies;
$\mathcal{T}$ set of periods;
$\mathcal{S}$ set of scenarios.
Parameters
D^t_{ci} demand for product i by market c in time period t;
R^t_{ci} end-of-life returns of product i from market c in time period t;
μ^f_i proportion of end-of-life product i returned after f years of service, $f = 0$ means in the same year, $\sum\limits_{f=0}^{F_i} \mu^f_i \leq 1$, F_i maximum life of product i;
φ_{in} number of units of component n in a unit of product i;
δ_n fraction of component n that could be recycled;
tp_{ih} time to produce a unit of product i using technology h;
tr_n time to recycle a component n;
vs_i space required for storing a unit of product i;
vd_n space required for disposing a unit of component n;
Pr_s the occurrence probability of scenario s;
M a large enough scalar;
f^P_{ph} fixed cost of constructing a PC in location p with technology h;
f^D_d fixed cost of constructing a DC in location d;
f^J_j fixed cost of constructing a CC in location j;
f^R_r fixed cost of constructing a RC in location r;
f^W_w fixed cost of constructing a DW in location w;
cp^t_{np} unit cost of a new component n purchased for the PC in location p in time period t;
cm^t_{phi} unit production cost of product i at PC in location p using technology h in time period t;
ch^t_{np} unit holding cost of component n at the PC in location p in time period t;
chc^t_{di} unit holding cost of product i at the DC in location d in time period t;
cc^t_{ji} unit collection cost of end-of-life product i at the CC in location j in time period t;
cr^t_{rn} unit recycling cost of component n at the RC in location r in time period t;
cd^t_{wn} unit disposal cost of scrapped component n at the DW in location w in time period t;
spd^t_{pdim} cost of shipping a unit of product i from the PC in location p to the DC in location d using transportation mode m in time period t;
sdc^t_{dcim} cost of shipping a unit of product i from the DC in location d to market c using transportation

mode m in time period t;
scj^t_{cjim} cost of shipping a unit of returned product i from market c to the CC in location j using transportation mode m in time period t;
sjr^t_{jrim} cost of shipping a unit of returned product i from the CC in location j to the RC in location r using transportation mode m in time period t;
srp^t_{rpnm} cost of shipping a unit of recycled component n from the RC in location r to the PC in location p using transportation mode m in time period t;
srw^t_{rwnm} cost of shipping a unit of recycled component n from the RC in location r to the DW in location w using transportation mode m in time period t.

Capacities

CP_p production capacity of the PC in location p, in hours;
CD_d storage capacity of the DC in location d, in m^3;
CJ_j storage capacity of the CC in location j, in m^3;
CR_r recycling capacity of the RC in location r, in hours;
CW_w storage capacity of the DW in location w, in m^3;
cpd_{pdm} Minimum capacity of transportation mode m between PC in location p and DC in d, in tons;
Cpd_{pdm} Maximum capacity of transportation mode m between PC in location p and DC in d, in tons;
cdc_{dcm} Minimum capacity of transportation mode m between DC in location d and market c, in tons;
Cdc_{dcm} Maximum capacity of transportation mode m between DC in location d and market c, in tons;
ccj_{cjm} Minimum capacity of transportation mode m between market c and CC in location j, in tons;
Ccj_{cjm} Maximum capacity of transportation mode m between market c and CC in location j, in tons;
cjr_{jrm} Minimum capacity of transportation mode m between CC in location j and RC in r, in tons;
Cjr_{jrm} Maximum capacity of transportation mode m between CC in location j and RC in r, in tons;
crp_{rpm} Minimum capacity of transportation mode m between RC in location r and PC in p, in tons;
Crp_{rpm} Maximum capacity of transportation mode m between RC in location r and PC in p, in tons;
crw_{rwm} Minimum capacity of transportation mode m between RC in location r and DW in w, in tons;
Crw_{rwm} Maximum capacity of transportation mode m between RC in location r and DW in w, in tons.

Decision variables

Zp_{ph} 1 if a PC is constructed in potential location p using technology h, 0 otherwise;
Zd_d 1 if a DC is constructed in potential location d, 0 otherwise;
Zj_j 1 if a CC is constructed in potential location j, 0 otherwise;
Zr_r 1 if a RC is constructed in potential location r, 0 otherwise;
Zw_w 1 if a DW is constructed in potential location w, 0 otherwise;
Upd^t_{pdm} 1 if transportation mode m is used between the PC in location p and the DC in location d in time period t, 0 otherwise;
Udc^t_{dcm} 1 if transportation mode m is used between the DC in location d and the market in location c in time period t, 0 otherwise;
Ucj^t_{cjm} 1 if transportation mode m is used between the market in location c and the CC in location j in time period t, 0 otherwise;
Ujr^t_{jrm} 1 if transportation mode m is used between the CC in location j and the RC in location r in time period t, 0 otherwise;
Urp^t_{rpm} 1 if transportation mode m is used between the RC in location r and the PC in location p in time period t, 0 otherwise;
Urw^t_{rwm} 1 if transportation mode m is used between the RC in location r and the DW in location w in time period t, 0 otherwise;
XE^t_{np} quantity of new component n purchased by the PC in time period t;
XP^t_{phi} quantity of product i produced in the PC in location p using technology h in time period t;
XPD^t_{pdim} quantity of product i shipped from the PC in location p to the DC in location d using transportation mode m in time period t;
XDC^t_{dcim} quantity of product i shipped from the DC in location d to the market in location c using transportation mode m in time period t;
XCJ^t_{cjim} quantity of returned product i shipped from the market in location c to the CC in location j using transportation mode m in time period t;
XJR^t_{jrim} quantity of returned product i shipped from the CC in location j to the RC in location r using transportation mode m in time period t;

XRP^t_{rpnm} quantity of component n shipped from the RC in location r to the PC in location p using transportation mode m in time period t;
XRW^t_{rwnm} quantity of component n shipped from the RC in location r to the DW in location w using transportation mode m in time period t;
IP^t_{pn} inventory of component n at the PC in location p in time period t;
ID^t_{di} inventory of product i at the DC in location d in time period t.

2.3 The Constraints

2.3.1 Balance Constraints

The left side of (1) gives the quantity of component n generated by all RCs, the number of new component purchased from suppliers and previous period inventory at the PC in location p in time period t. The right side gives the quantity of component n needed for manufacturing product i that is produced in the PC in location p in time period t in addition to end of period inventory.

$$\sum_{r\in\mathcal{R}}\sum_{m\in\mathcal{M}} XPD^t_{rpnm} + XE^t_{np} + IP^{t-1}_{np} = \sum_{h\in\mathcal{H}}\sum_{i\in\mathcal{I}} \varphi_{in} XP^t_{phi} + IP^t_{np}, IP^0_{np} = 0,\ p\in\mathcal{P}, n\in\mathcal{N}, t\in\mathcal{T}. \quad (1)$$

The left of (2) gives the quantity of product i produced by the PC in location p in time period t. The right gives the sum of the shipments of the same product and the same PC to the DCs in time period t.

$$\sum_{h\in\mathcal{H}} XP^t_{phi} = \sum_{d\in\mathcal{D}}\sum_{m\in\mathcal{M}} XPD^t_{pdim}, \qquad p\in\mathcal{P}, i\in\mathcal{I}, t\in\mathcal{T}. \quad (2)$$

The left side of (3) gives the quantity of product i that the DC in location d receives from all PCs in addition to the last period ending inventory. The right side gives the quantity of the same product that has been shipped to the markets in the same time period.

$$\sum_{p\in\mathcal{P}}\sum_{m\in\mathcal{M}} XPD^t_{pdim} + ID^{t-1}_{di} = \sum_{c\in\mathcal{C}}\sum_{m\in\mathcal{M}} XDC^t_{dcim} + ID^t_{di},\quad ID^0_{di} = 0 \quad d\in\mathcal{D}, i\in\mathcal{I}, t\in\mathcal{T}. \quad (3)$$

Constraint (4) shows inventory balance of returned products at each CC in time period t.

$$\sum_{c\in\mathcal{C}}\sum_{m\in\mathcal{M}} XCJ^t_{cjim} = \sum_{r\in\mathcal{R}}\sum_{m\in\mathcal{M}} XJR^t_{jrim}, \qquad j\in\mathcal{J}, i\in\mathcal{I}, t\in\mathcal{T}. \quad (4)$$

Constraint (5) shows inventory balance of recycled components at each RC in each time period.

$$\sum_{p\in\mathcal{P}}\sum_{m\in\mathcal{M}} XRP^t_{rpnm} = \sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}} \delta_n \varphi_{in} XJR^t_{jrim}, \qquad r\in\mathcal{R}, n\in\mathcal{N}, t\in\mathcal{T}. \quad (5)$$

Constraint (6) shows inventory balance of disposable components at each RC in time period t.

$$\sum_{w\in\mathcal{W}}\sum_{m\in\mathcal{M}} QRW^t_{rwnm} = \sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}} (1-\delta_n)\varphi_{in} XJR^t_{jrim}, \qquad r\in\mathcal{R}, n\in\mathcal{N}, t\in\mathcal{T}. \quad (6)$$

2.3.2 Capacity Constraints

Constraint (7) ensures the total production time of the PC in location p of all products in time period t using technology h does not exceed the capacity of the same PC using the same technology.

$$\sum_{i\in\mathcal{I}} tp_{ih} XP^t_{phi} \le CP_p Zp_{ph}, \qquad p\in\mathcal{P}, h\in\mathcal{H}, t\in\mathcal{T}. \quad (7)$$

Constraint (8) ensures that the volume of carryover inventory of the previous period and volume of products shipped from the PCs to each DC does not exceed the storage capacity of the same DC in the same time period.

$$\sum_{i\in\mathcal{I}} vs_i ID^{t-1}_{di} + \sum_{p\in\mathcal{P}}\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}} vs_i XPD^t_{pdim} \le CD_d Zd_d, ID^0_{di} = 0, \qquad d\in\mathcal{D}, t\in\mathcal{T}. \quad (8)$$

Constraint (9) ensures that the volume of end-of-life products shipped from all markets does not exceed the capacity of CC in location j in time period t.

$$\sum_{c\in\mathcal{C}}\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}} vs_i XCJ^t_{cjim} \le CJ_j Zj_j, \qquad j\in\mathcal{J}, t\in\mathcal{T}. \tag{9}$$

The left of (10) gives the number of all types processing at the RC in location r in time period t.

$$\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{M}} tr_n\varphi_{in} XJR^t_{jrim} \le CR_r Zr_r, \qquad r\in\mathcal{R}, t\in\mathcal{T}. \tag{10}$$

The left side of constraint (11) gives the volume of all scrapped components at the DW in location w in time period t.

$$\sum_{j\in\mathcal{J}}\sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{M}} vd_n\varphi_{in} XRW^t_{rwnm} \le CW_w Zw_w, \qquad w\in\mathcal{W}, t\in\mathcal{T}. \tag{11}$$

Constraint (12) ensures that if location p is used for constructing a PC, then, only one technology is adopted and this will be done at most once.

$$\sum_{h\in\mathcal{H}}\sum_{t\in\mathcal{T}} Zp_{ph} \le 1, \qquad p\in\mathcal{P}. \tag{12}$$

2.3.3 Constraints on the Flow of Items In and Out of a Facility

Constraints (13)-(26) ensure that if a flow takes place in time period t, then the facility should be constructed at t or prior to it.

$$\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} XPD^t_{pdim} \le M^*\sum_{h\in\mathcal{H}} Zp_{ph}, \qquad p\in\mathcal{P}. \tag{13}$$

$$\sum_{n\in\mathcal{N}} XE^t_{np} \le M^*\sum_{h\in\mathcal{H}} Zp_{ph}, \qquad p\in\mathcal{P}. \tag{14}$$

$$\sum_{h\in\mathcal{H}}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} XP^t_{phi} \le M^*\sum_{h\in\mathcal{H}} Zp_{ph}, \qquad p\in\mathcal{P}. \tag{15}$$

$$\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} IP^t_{pn} \le M^*\sum_{h\in\mathcal{H}} Zp_{ph}, \qquad p\in\mathcal{P}. \tag{16}$$

$$\sum_{r\in\mathcal{R}}\sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} XRP^t_{rpnm} \le M^*\sum_{h\in\mathcal{H}} Zp_{ph}, \qquad p\in\mathcal{P}. \tag{17}$$

$$\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} ID^t_{di} \le M^* Zd_d, \qquad d\in\mathcal{D}. \tag{18}$$

$$\sum_{p\in\mathcal{P}}\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} XPD^t_{pdim} \le M^* Zd_d, \qquad d\in\mathcal{D}. \tag{19}$$

$$\sum_{c\in\mathcal{C}}\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} XDC^t_{dcim} \le M^* Zd_d, \qquad d\in\mathcal{D}. \tag{20}$$

$$\sum_{c\in\mathcal{C}}\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}XCJ^t_{cjim} \le M^* Zj_j, \qquad j\in\mathcal{J}. \tag{21}$$

$$\sum_{r\in\mathcal{R}}\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}XJR^t_{jrim} \le M^* Zj_j, \qquad j\in\mathcal{J}. \tag{22}$$

$$\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}XJR^t_{jrim} \le M^* Zr_r, \qquad r\in\mathcal{R}. \tag{23}$$

$$\sum_{p\in\mathcal{P}}\sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}XRP^t_{rpnm} \le M^* Zr_r, \qquad r\in\mathcal{R}. \tag{24}$$

$$\sum_{w\in\mathcal{W}}\sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}XRW^t_{rwnm} \le M^* Zr_r, \qquad r\in\mathcal{R}. \tag{25}$$

$$\sum_{r\in\mathcal{R}}\sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}XRW^t_{rwnm} \le M^* Zw_w, \qquad w\in\mathcal{W}. \tag{26}$$

The value of M^* should be large enough to guarantee the right side be not less than the left side. In this papeer, we use $M^* = \sum\limits_{i\in\mathcal{I}}\sum\limits_{c\in\mathcal{C}} maxD^t_{ci}$.

2.3.4 Constraints about the Transportation between Facilities.

Constraints (27)-(36) allow the transportation between a pair of facility at time t if these facilities are part of the network at t or prior to it.

$$Upd^t_{pdm} \le \sum_{h\in\mathcal{H}} Zp_{ph}, \qquad p\in\mathcal{P}, d\in\mathcal{D}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{27}$$

$$Urp^t_{rpm} \le \sum_{h\in\mathcal{H}} Zp_{ph}, \qquad p\in\mathcal{P}, r\in\mathcal{R}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{28}$$

$$Upd^t_{pdm} \le Zd_d, \qquad p\in\mathcal{P}, d\in\mathcal{D}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{29}$$

$$Udc^t_{dcm} \le Zd_d, \qquad c\in\mathcal{C}, d\in\mathcal{D}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{30}$$

$$Ucj^t_{cjm} \le Zj_j, \qquad c\in\mathcal{C}, j\in\mathcal{J}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{31}$$

$$Ujr^t_{jrm} \le Zj_j, \qquad r\in\mathcal{R}, j\in\mathcal{J}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{32}$$

$$Ujr^t_{jrm} \le Zr_r, \qquad r\in\mathcal{R}, j\in\mathcal{J}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{33}$$

$$Urp^t_{rpm} \le Zr_r, \qquad r\in\mathcal{R}, p\in\mathcal{P}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{34}$$

$$Urw^t_{rwm} \le Zr_r, \qquad r\in\mathcal{R}, w\in\mathcal{W}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{35}$$

$$Urw^t_{rwm} \le Zw_w, \qquad r\in\mathcal{R}, w\in\mathcal{W}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{36}$$

2.3.5 Transportation Mode Capacity Constraints

Constraints (37)-(48) guarantee that if a transportation mode is used then the shipment must be between the minimum and maximum capacity of this mode.

$$\sum_{i\in\mathcal{I}} XPD^t_{pdim} \geq cpd_{pdm}Upd^t_{pdm}, \qquad p\in\mathcal{P}, d\in\mathcal{D}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{37}$$

$$\sum_{i\in\mathcal{I}} XPD^t_{pdim} \leq Cpd_{pdm}Upd^t_{pdm}, \qquad p\in\mathcal{P}, d\in\mathcal{D}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{38}$$

$$\sum_{i\in\mathcal{I}} XDC^t_{dcim} \geq cdc_{dcm}Udc^t_{dcm}, \qquad c\in\mathcal{C}, d\in\mathcal{D}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{39}$$

$$\sum_{i\in\mathcal{I}} XDC^t_{dcim} \leq Cdc_{dcm}Udc^t_{dcm}, \qquad c\in\mathcal{C}, d\in\mathcal{D}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{40}$$

$$\sum_{i\in\mathcal{I}} XCJ^t_{cjim} \geq ccj_{cjm}Ucj^t_{cjm}, \qquad c\in\mathcal{C}, j\in\mathcal{J}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{41}$$

$$\sum_{i\in\mathcal{I}} XCJ^t_{cjim} \leq Ccj_{cjm}Ucj^t_{cjm}, \qquad c\in\mathcal{C}, j\in\mathcal{J}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{42}$$

$$\sum_{i\in\mathcal{I}} XJR^t_{jrim} \geq cjr_{jrm}Ujr^t_{jrm}, \qquad r\in\mathcal{R}, j\in\mathcal{J}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{43}$$

$$\sum_{i\in\mathcal{I}} XJR^t_{jrim} \leq Cjr_{jrm}Ujr^t_{jrm}, \qquad r\in\mathcal{R}, j\in\mathcal{J}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{44}$$

$$\sum_{n\in\mathcal{N}} XRP^t_{rpnm} \geq crp_{rpm}Urp^t_{rpm}, \qquad r\in\mathcal{R}, p\in\mathcal{P}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{45}$$

$$\sum_{n\in\mathcal{N}} XRP^t_{rpnm} \leq Crp_{rpm}Urp^t_{rpm}, \qquad r\in\mathcal{R}, p\in\mathcal{P}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{46}$$

$$\sum_{n\in\mathcal{N}} XRW^t_{rwnm} \geq crw_{rwm}Urw^t_{rwm}, \qquad r\in\mathcal{R}, w\in\mathcal{W}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{47}$$

$$\sum_{n\in\mathcal{N}} XRW^t_{rwnm} \leq Crw_{rwm}Urw^t_{rwm}, \qquad r\in\mathcal{R}, w\in\mathcal{W}, m\in\mathcal{M}, t\in\mathcal{T}. \tag{48}$$

2.3.6 Joint Service Level Constraints

Constraint (49) shows that for each time period, the probability of the event that the shipments from the DCs more than or equal to the demand for each market is at least α.

$$\Pr\{\sum_{d\in\mathcal{D}}\sum_{m\in\mathcal{M}} XDC^t_{dcim} \geq D^t_{ci}, \quad c\in\mathcal{C}, i\in\mathcal{I}, t\in\mathcal{T}\} \geq \alpha, \tag{49}$$

where α is a prescribed service level.

Constraint (50) shows that for each time period, the probability of the event that the shipments of the returns to the CCs less than or equal to the end-of-life returns of each product is at least β.

$$\Pr\{\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}} XCJ^t_{cjim} \leq R^t_{ci}, \quad c\in\mathcal{C}, i\in\mathcal{I}, t\in\mathcal{T}\} \geq \beta, \tag{50}$$

where $R^t_{ci} = \sum_{f=0}^{F_i} \mu^f_i D^{t-f}_{ci}, t \geq F_i \quad c\in\mathcal{C}, i\in\mathcal{I}, t\in\mathcal{T}$, β is a prescribed service level and D^t_{ci} is random, $D^g_{ci} = 0$ when $g \leq 0$.

2.4 The Objective Function

Fixed cost (FC)= $\sum_{p\in\mathcal{P}}\sum_{h\in\mathcal{H}} f^P_{ph} Zp_{ph} + \sum_{d\in\mathcal{D}} f^D_d Zd_d + \sum_{j\in\mathcal{J}} f^J_j Zj_j + \sum_{r\in\mathcal{R}} f^R_r Zr_r + \sum_{w\in\mathcal{W}} f^W_w Zw_w$.

Material and production costs (MPC)= $\sum_{n\in\mathcal{N}}\sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}} cp^t_{np} XE^t_{np} + \sum_{p\in\mathcal{P}}\sum_{h\in\mathcal{H}}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} cm^t_{phi} XP^t_{phi}$.

Inventory holding cost (IHC)= $\sum_{n\in\mathcal{N}}\sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}} ch^t_{np} IP^t_{np} + \sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} chc^t_{di} ID^t_{di}$.

Collection cost (CC)= $\sum_{c\in\mathcal{C}}\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} cc^t_{ji} XCJ^t_{cjim}$.

Recycling cost (RC)= $\sum_{r\in\mathcal{R}}\sum_{p\in\mathcal{P}}\sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} cr^t_{rn} XRP^t_{rpnm}$.

Disposal cost (DC)= $\sum_{r\in\mathcal{R}}\sum_{w\in\mathcal{W}}\sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} cd^t_{wn} XRW^t_{rwnm}$.

Transportation cost(TPC)

$$\begin{aligned} TPC =& \sum_{p\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} spd^t_{pdim} XPD^t_{pdim} + \sum_{d\in\mathcal{D}}\sum_{c\in\mathcal{C}}\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} sdc^t_{dcim} XDC^t_{dcim} \\ &+ \sum_{c\in\mathcal{C}}\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} scj^t_{cjim} XCJ^t_{cjim} + \sum_{j\in\mathcal{J}}\sum_{r\in\mathcal{R}}\sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} sjr^t_{jrim} XJR^t_{jrim} \\ &+ \sum_{r\in\mathcal{R}}\sum_{p\in\mathcal{P}}\sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} srp^t_{rpnm} XRP^t_{rpnm} + \sum_{r\in\mathcal{R}}\sum_{w\in\mathcal{W}}\sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} srw^t_{rwnm} XRW^t_{rwnm}. \end{aligned}$$

The total cost of the CLSC includes the opening cost of facilities, production cost, inventory cost, collection cost, recycling cost, disposal cost and transportation cost. As a result, the total cost (TC) objective to be minimized is represented as follows:

$$\min TC = FC + MPC + IHC + CC + RC + DC + TPC. \tag{51}$$

Based on the notations description above, our multi-period multi-product closed-loop supply chain design problem is modeled as the following CCP model:

$$\begin{aligned} \min \quad & TC = FC + MPC + IHC + CC + RC + DC + TPC \\ \text{s.t.} \quad & \text{constraints} \quad (1)-(50). \end{aligned} \tag{52}$$

Model (52) is a joint probabilistic constraint programming problem. The traditional solution methods require to convert the joint probabilistic constraints (49) and (50) to their respective deterministic equivalents. However, this conversion is usually difficult to perform and only successful for special case. In the next section, we will discuss the equivalent formulation of the CCP model (52) when the random parameters are characterized by discrete probability distribution with finite support.

3 Equivalent Mixed-integer Programming Model

In this section, we will transform the CCP model (52) to its equivalent deterministic model. For this purpose, we assume that the discrete demand distribution D is characterized by

$$D = (D_{111}, \cdots, D_{cit}) \sim \begin{pmatrix} \hat{D}^1 & \hat{D}^2 & \cdots & \hat{D}^s \\ Pr_1 & Pr_2 & \cdots & Pr_s \end{pmatrix},$$

where $\hat{D}^s = (D_{111}, \cdots, D_{cit})$ is the sth realization of demand. $Pr_s > 0$ for all $s \in \mathcal{S}$ and $\sum_{s=1}^{S} Pr_s = 1$.

We define binary vectors z and w whose components $z^s, w^s, s \in \mathcal{S}$ take 1 if the corresponding set of constraints has to be satisfied and 0 otherwise. In particular, we introduce a large enough number M, then the joint chance constraints (49) and (50) are equivalent to (53) and (54), respectively.

$$\begin{aligned} & \sum_{d\in\mathcal{D}}\sum_{m\in\mathcal{M}} Q^t_{dcim} \geq D^t_{cis} - (1-z^s)M, \quad c\in\mathcal{C}, i\in\mathcal{I}, t\in\mathcal{T}, s\in\mathcal{S}, \\ & \sum_{s\in\mathcal{S}} z^s Pr_s \geq \alpha \quad s\in\mathcal{S}. \end{aligned} \tag{53}$$

$$\begin{array}{l} \sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}} Q_{cjim}^t - (1-w^s)M \le R_{cis}^t, \quad c\in\mathcal{C}, i\in\mathcal{I}, t\in\mathcal{T}, s\in\mathcal{S}, \\ \sum_{s\in\mathcal{S}} w^s Pr_s \ge \beta \quad s\in\mathcal{S}, \end{array} \tag{54}$$

where $R_{cis}^t = \sum_{f=0}^{F_i} \mu_i^f D_{cis}^{t-f}, t \ge F_i \quad c\in\mathcal{C}, i\in\mathcal{I}, t\in\mathcal{T}, s\in\mathcal{S}$, $D_{cis}^g = 0$ for $g \le 0$.

As a consequence, the original CCP model (52) is equivalent to following mixed-integer programming model:

$$\begin{array}{ll} \min & TC = FC + MPC + IHC + CC + RC + DC + TPC \\ \text{s.t.} & \sum_{d\in\mathcal{D}}\sum_{m\in\mathcal{M}} Q_{dcim}^t \ge D_{cis}^t - (1-z^s)M, \quad c\in\mathcal{C}, i\in\mathcal{I}, t\in\mathcal{T}, s\in\mathcal{S} \\ & \sum_{s\in\mathcal{S}} z^s Pr_s \ge \alpha, \quad s\in\mathcal{S} \\ & \sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}} Q_{cjim}^t - (1-w^s)M \le R_{cis}^t, \quad c\in\mathcal{C}, i\in\mathcal{I}, t\in\mathcal{T}, s\in\mathcal{S} \\ & \sum_{s\in\mathcal{S}} w^s Pr_s \ge \beta, \quad s\in\mathcal{S} \\ & z^s, w^s \in \{0,1\}, s\in\mathcal{S} \\ & \text{constraints} \quad (1)-(48). \end{array} \tag{55}$$

4 Numerical Experiments

To demonstrate the effectiveness of the proposed CCP model, this section presents an application example about the closed-loop supply chain network design problem and performs the sensitivity analysis about service levels α and β.

All the numerical experiments are executed on a personal computer (Intel core i5 with 2.40 GHz processor and 4.0 GB of RAM), using the Microsoft Windows 7 operating system. All mixed-integer programs were solved to optimality using ILOG CPLEX 12.6 MIP solver.

4.1 Description of the Numerical Example

We consider a hypothetical firm that manufactures a wide range of products that are used in the automotive industry. The firm has two PCs ($p = 2$) to produce four different kinds of products ($i = 4$), using two technology options at each PC ($h = 2$). The production is used to satisfy customers that are located at five locations ($c = 5$) through the DCs ($d = 3$). In reverse supply chain, the returned products are collected at five CCs ($j = 5$). After inspection carried out at the CCs, recyclable products and scrapped products are separated. Scrapped products are sent to two disposal centers ($w = 2$) and recyclable products are sent to the RCs ($r = 3$). Finally, recycled components are sent to the PCs for manufacturing new products. Each product consists of six components ($n = 6$). For logistic activities between the facilities, three alternative transportation modes are available ($m = 3$). We consider a planning horizon of three periods ($t = 3$).

The fixed parameters are set as follows. f_{ph}^P is chosen randomly from the interval (30000, 60000). f_d^D is chosen randomly from the interval (10000, 12000). f_j^J is chosen randomly from the interval (2500, 5000). f_r^R is chosen randomly from the interval (20000, 30000). f_w^W is chosen randomly from the interval (4000, 5000). CP_p is chosen randomly from the interval (40000, 48000). CD_d is chosen randomly from the interval (50000, 60000). CJ_j is chosen randomly from the interval (18000, 24000). CR_r is chosen randomly from the interval (400000, 600000). CW_w is chosen randomly from the interval (500000, 600000). cp_{np}^t is chosen randomly from the interval (11, 13). cm_{phi}^t is chosen randomly from the interval (21, 24). ch_{np}^t is chosen randomly from the interval (2, 4). chc_{di}^t is chosen randomly from the interval (2, 5). cc_{ji}^t is chosen randomly from the interval (6, 9). cr_{rn}^t is chosen randomly from the interval (7, 9). cd_{wn}^t is chosen randomly from the interval (2, 4). tr_n is chosen randomly from the interval (1, 5). tp_{ih} is chosen randomly from the interval (8, 12). vs_i is chosen randomly from the interval (12, 16). vd_n is chosen randomly from the interval (1, 5). $\varphi_{in} = 6$, $\delta_n = 0.6$. The cost of transportation modes are as follow: the cost of heavy duty truck is 0.125 $\$/ton-km$, the cost of Mid-size truck is 0.118 $\$/ton-km$, the cost of light truck is 0.110 $\$/ton-km$. The minimum capacity of the transportation modes is 100 tons, the maximum capacity of the transportation modes is 14000 tons.

We assume the uncertain demand follows the discrete distribution that takes its values on the interval [200, 300] with the same probability $1/S$, where S is the number of scenario.

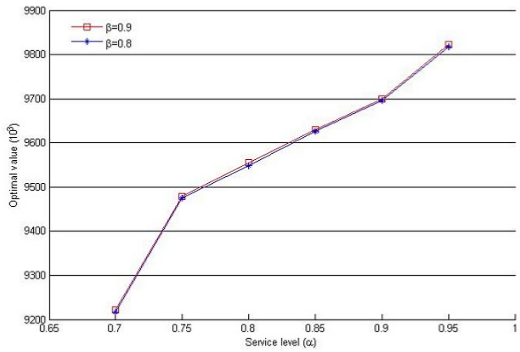

Figure 2: Optimal values under different service level α

4.2 Sensitivity Analysis

In the following, we perform our numerical experiments according to two cases divided by service levels α and β.

Case 1: Computational Results under Different Service Level α

In this case, the demand follows the discrete uniform distribution on the interval [200, 300]. Table 1 and 2 collect the computational results about the proposed CCP model under different service level $\alpha, \alpha \in (0, 1)$, from which we conclude that the increase in α leads to an increase in the total cost of the entire supply chain.

Table 1: Computational results under different service level α

service level	service level α					
β	0.7	0.75	0.8	0.85	0.9	0.95
0.8	9216432.7640	9474715.6738	9548162.7780	9625672.9821	9695119.3989	9818139.0656
0.9	9220948.6013	9479433.7058	9554602.6100	9630508.7142	9699955.1308	9822974.7976

Case 2: Computational Results under Different Service Level β

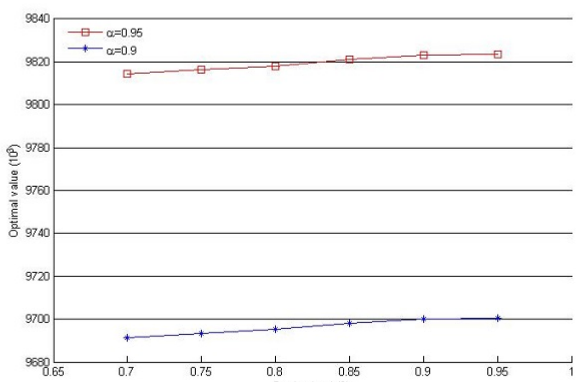

Figure 3: Optimal values under different service level β

In this case, the demand also follows the discrete uniform distribution on the interval [200, 300]. Table 2 and Figure 3 report the computational results about the service level β under various values in the interval $(0, 1)$. From the computational results, we find that the increase in β leads to an increase in the cost of the entire supply chain.

Table 2: Computational results under different service level β

service level	service level β					
α	0.7	0.75	0.8	0.85	0.9	0.95
0.95	9814515.1614	9816419.7174	9818139.0654	9821170.2735	9822974.7974	9823425.5415
0.9	9691495.4948	9693400.0508	9695119.3989	9698150.6068	9699955.1308	9700405.8748

From the above computational results in the two cases, we conclude that when the stochastic demand follows discrete uniform distribution, the total cost of the entire supply chain depends heavily on the service levels α and β.

5 Conclusions

In this paper, we have studied a multi-period, multi-product, multi-layer, capacitated closed-loop supply chain network problem in a stochastic environment. The major results are summarized as follows:

A new CCP model is proposed for our closed-loop supply chain network problem. When demand follows a known discrete probability distribution, by introducing auxiliary binary variables, the proposed CCP model can be transformed to its equivalent mixed-integer programming model.

To demonstrate the effectiveness of the proposed CCP model, we provided an application example about the closed-loop supply chain network design problem and conducted the sensitivity analysis about the service levels α and β with respect to solution quality. A comparison study about the proposed stochastic model with deterministic one is also presented.

In our future research, we will address supply chain network problems under carbon footprint consideration. When the distribution of demand is partially available, it is also a challenge issue to find a new modelling approach to our closed-loop supply chain network problem.

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