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Distributionally Robust Multi-Product Newsvendor Problem under Carbon Policies

Qiang-Wei Liu[1], Zhao-Zhuang Guo[2], Yan-Ju Chen[1,*]

[1] *College of Mathematics and Information Science, Hebei University, Baoding 071002, Hebei, China*

[2] *Fundamental Science Department, North China Institute of Aerospace Engineering Langfang 065000, Hebei, China*

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Abstract

The uncertain market demand and carbon policies influence heavily the order decisions of the retailer for different products. This paper studies uncertain multi-product newsvendor problem under the cap and cap-and-trade policies, respectively. First the probability distribution of market demand is characterized by a box uncertainty set, and the carbon emission is characterized by the intersection of box and budget and the intersection of box and ball uncertainty sets, respectively. We propose two distributionally robust multi-product newsvendor optimization models. Then we transform the proposed models into linear programming and second order cone programming under three types of uncertainty sets. Finally, based on numerical experiments, we compare the nominal solution and the robust solution. The comparison study demonstrates the effectiveness of our proposed models.

Keywords: multi-product newsvendor problem, distributionally robust optimization, uncertainty sets, carbon cap, cap-and-trade

1 Introduction

Multi-product newsvendor problem is one of the important problems in inventory management. Decision maker needs to make order decisions before the random demand is realized. He seeks maximum profit or minimum cost with budget constraint or other constraints. Hadley and Whitin [7] firstly proposed the multi-product newsvendor problem under the budget constraint. Since then, many scholars extended the study of newsvendor problem [8, 13, 14, 16, 21].

In literatures, the classical multi-product newsvendor problem assumes that demand follows a specific distribution with the known parameters. Due to the limited capacity, Zhang and Du [25] studied two outsourcing strategies with independent stochastic demand. Under the loss and budget constraints, Zhou et al. [28] studied the decision maker's loss aversion characteristics. For a multi-constraint multi-product newsvendor model, Zhang [24] developed a multi-tier binary solution method, and gave the optimal solution of the model. On the basis of credibility theory [11], Li and Liu [10] studied a multi-item inventory management problem with fuzzy demand. Shao and Ji [20] investigated the multi-product newsvendor problem with fuzzy demand in the condition of budget constraint.

When the exact demand distribution is unavailable, robust optimization method is often used to deal with the corresponding distribution uncertainty in newsvendor problem. Many scholars adopted distributionally robust optimization method to model these problems. Considering that only the mean and variance of demand distribution were known, Scarf [19] firstly studied the newsvendor problem by distributionally robust method. He maximized profit under the worst possible distribution. Lee and Hsu [9] investigated the effect of advertising on newsvendor problem. Olivares-Nadal et al. [17] combined temporal dependence and tractable robust optimization method to study the single-product newsvendor problem. When only partial information is available, Moon et al. [15] considered multiple discount and multiple upgrading measures to handle overstocks,

*Corresponding author.
Email: chen-yanju@163.com (Y.-J. Chen).

and verified the robustness of distribution-free method. Qiu et al. [18] dealt with the uncertainty of demand distribution, and considered the box and ellipsoid uncertainty sets. Ben-Tal et al. [3] applied robust linear optimization to a numerical multi-item newsvendor example with uncertain probabilities. Xiao and Chen [23] built three robust mean-CVaR models for a newsvendor problem when the probability distribution of market demand was uncertain.

The newsvendor problem with the carbon emission constraint is also an active research area. The carbon emission constraint influences heavily the optimal order quantities for different products. Zhang and Xu [26] proposed a profit-maximization model under the carbon cap-and-trade mechanism and analyzed the optimal production strategy and carbon trading decisions. Chen and Wang [5] and Zheng et al. [27] offered the optimal order quantity and how to choose traffic modes under the carbon policies. When only the mean and the variance of the demand distribution were known, Bai and Chen [2] proposed a robust newsvendor model under the carbon tax and cap-and-trade policies. The literatures [1, 12, 22] also considered the newsvendor problem under the carbon emission constraint. Guo et al. [6] used credibility optimization method to study the inventory management problem with fuzzy carbon emission. However, few researchers have studied the newsvendor problem in which carbon emission is uncertain.

This paper studies multi-product newsvendor problem by robust optimization method, where the probability distribution of market demand is partially available. The uncertain probability and uncertain carbon emission are characterized by uncertainty sets. The expected profit is maximized under two different carbon policies, respectively. We assume that the uncertain probability distribution of demand belongs to a box uncertainty set and the uncertain carbon emission is in a box+budget and a box+ball sets.

The structure of this paper is as follows. In Section 2, we describe the problem and propose the uncertain cap model. We derive the robust counterpart of the uncertain cap model in Section 3. Section 4 extends the uncertain cap model and derives its robust counterpart. In Section 5, numerical experiments are carried out to show the effectiveness of the proposed models. In Section 6, we conclude this paper.

2 Formulation of Uncertain Cap Model

We consider a two-echelon supply chain consisting of multi-supplier and a retailer. Suppliers provide products with short shelf-life for retailer, and retailer needs to order products in advance. Due to carbon emission during transportation, the government will give carbon emission quota to retailer. The retailer can buy additional quota or sell it to the carbon trading market. The retailer orders product i at the price w_i. The unit retail price is p_i and unit transportation cost is c_i. After the sales season, the unsold product i will be cleared at the salvage value s_i. We assume $p_i \geq w_i + c_i \geq s_i$. In order to maximize expected profit, the retailer needs to determine the order quantities $\mathbf{q} = (q_1, \cdots, q_n)$ and carbon emission trading quantity E before the demand is realized.

Before modeling the multi-product newsvendor problem, we first give some notations in Table 1.

2.1 The Objective Function

In our newsvendor problem, we consider that the market demand d_i of the product i is discrete. We have $d_i = d_{i1}, \ldots, d_{ij}, \ldots, d_{im}$, where d_{ij} denotes the possible market demand quantity for the product i. For the product i, the retailer's profit is as follows.

$$\pi(q_i, d_i) = p_i \min(q_i, d_i) + s_i(q_i - d_i)^+ - h_i(d_i - q_i)^+ - c_i q_i - w_i q_i. \tag{1}$$

Let $(\cdot)^+ = \max\{\cdot, 0\}, \min(q_i, d_i) = q_i - (q_i - d_i)^+$, the objective function can be rewritten as:

$$\pi(q_i, d_i) = (p_i - c_i - w_i)q_i - p_i(q_i - d_i)^+ + s_i(q_i - d_i)^+ - h_i(d_i - q_i)^+. \tag{2}$$

The probability of possible market demand d_{ij} is p_{ij}, i.e., $\Pr\{d_i = d_{ij}\} = p_{ij}$, such that $\sum_{j=1}^m p_{ij} = 1$, $p_{ij} \geq 0, i = 1, 2, \ldots, n$. When the market demand takes value d_{ij}, the profit is $\pi_{ij} = \pi(q_i, d_{ij})$. The expected profit for the product i can be rewritten as follows.

$$E[\pi(q_i, d_i)] = \sum_{j=1}^{m} \pi_{ij} p_{ij}. \tag{3}$$

Table 1: Notations

Notations	Description
n	Total number of products
m	Total number of discrete demand
i	Index for products, $i = 1, 2, ..., n$
j	Index for discrete demands, $j = 1, 2, ..., m$
c_i	Per unit transportation cost for product i
w_i	Per unit wholesale price of product i
p_i	Per unit retail price of product i
s_i	Per unit salvage value of product i
h_i	Retailer's unit penalty cost of product i for demand that cannot be filled
C	Initial carbon emission quota from government
c_e	Unit trading price of carbon emission
$\mathcal{U}_k$	Uncertainty sets
Uncertain parameters	
d_i	Discrete market demand for product i
$\mathbf{p}_i$	Probability distribution of uncertain market demand d_i, $\mathbf{p}_i \in \mathcal{U}_1$
e_i	Carbon emission for product i during transportation, $e_i \in \mathcal{U}_2$ or $e_i \in \mathcal{U}_3$
Decision variables	
$\mathbf{q}$	Retailer's order quantities
E	Carbon emission trading quantity

We introduce two vectors $\boldsymbol{\pi}_i = (\pi_{i1}, \cdots, \pi_{ij}, \cdots, \pi_{im})^T$ and $\mathbf{p}_i = (p_{i1}, \cdots, p_{ij}, \cdots, p_{im})^T$. So the total expected profit can be rewritten as

$$\begin{aligned} \Pi(\mathbf{q}) &= \sum_{i=1}^{n} E[\pi(q_i, d_i)] \\ &= \sum_{i=1}^{n}\sum_{j=1}^{m} \pi_{ij} p_{ij} \\ &= \sum_{i=1}^{n} \boldsymbol{\pi}_i^T \mathbf{p}_i. \end{aligned} \tag{4}$$

2.2 The Carbon Emission Constraint

In this section, we consider two types of carbon policies: the cap policy and the cap-and-tarde policy. The cap policy is that retailer's carbon emission can not exceed the cap (C) during a given period.

The carbon emission of the unit product i is e_i. So we have the following constraint

$$\sum_{i=1}^{n} e_i q_i \leq C. \tag{5}$$

The cap-and-trade policy is that the retailer can trade carbon emission with the carbon trading market. When the retailer's carbon emission does not exceed the cap (C) set by government, the retailer can sell their carbon emission to the carbon trading market. When the retailer's carbon emission exceeds the cap (C), the retailer can also buy carbon emission from the carbon trading market. So we have the following constraint

$$\sum_{i=1}^{n} e_i q_i \leq C + E. \tag{6}$$

2.3 The Uncertain Cap Model

In this section, we consider the cap policy. Before the sales season, we only know the nominal value of the probability $\mathbf{p}_i$. Due to the competition of the market demand, the probability distribution is considered as an uncertain parameter, i.e., $\mathbf{p}_i \in \mathcal{U}_1$. In practice, because of the weather and traffic factors, it is difficult to obtain accurate carbon emission information. In this paper, we assume carbon emission e_i belongs to uncertainty set $\mathcal{U}_2$, i.e., $e_i \in \mathcal{U}_2$. The retailer should decide the optimal ordering quantities $\mathbf{q}$. In order to maximize profit under carbon emission constraint, we formulate the following cap model.

$$\left\{\begin{array}{l} \max\limits_{\mathbf{q}} \Pi(\mathbf{q}) \\ s.t. \ \sum\limits_{i=1}^{n} e_i q_i \leq C \\ \qquad q_i \geq 0, \quad i = 1, ..., n \end{array}\right\}_{\mathbf{p}_i \in \mathcal{U}_1, e_i \in \mathcal{U}_2.} \tag{7}$$

We introduce auxiliary variable z, then model (7) can be rewritten as

$$\left\{\begin{array}{l} \max\limits_{\mathbf{q}} z \\ s.t. \ \Pi(\mathbf{q}) \geq z \\ \qquad \sum\limits_{i=1}^{n} e_i q_i \leq C \\ \qquad q_i \geq 0, \quad i = 1, ..., n \end{array}\right\}_{\mathbf{p}_i \in \mathcal{U}_1, e_i \in \mathcal{U}_2.} \tag{8}$$

3 The Robust Counterpart of Cap Model

3.1 Uncertainty Sets

In the above section, we propose uncertain model with carbon constraint. In order to deal with the uncertain parameters, we adopt robust optimization method in [4]. Assuming that each of uncertain parameters varies in a specific uncertainty set. For uncertain probability distribution $\mathbf{p}_i$, we consider the box uncertainty set

$$\mathcal{U}_{box} = \{\mathbf{p}_i | \mathbf{p}_i = \mathbf{p}_i^0 + \boldsymbol{\varphi}_i, \mathbf{e}^T \boldsymbol{\varphi}_i = 0, \boldsymbol{\rho}_i^- \leq \boldsymbol{\varphi}_i \leq \boldsymbol{\rho}_i^+\}, \tag{9}$$

where $\mathbf{p}_i^0$ denotes the nominal probability distribution and $\mathbf{e}$ is a unit vector. Perturbation vector $\boldsymbol{\varphi}_i$ takes value in a known support $[\boldsymbol{\rho}_i^-, \boldsymbol{\rho}_i^+]$. Note that the discrete market demand probability $\mathbf{p}_i$ is nonnegative, i.e., $\mathbf{p}_i^0 + \boldsymbol{\varphi}_i \geq 0$. The constraint $\mathbf{e}^T \boldsymbol{\varphi}_i = 0$ ensures that $\mathbf{p}_i$ meets the requirements $\mathbf{e}^T(\mathbf{p}_i^0 + \boldsymbol{\varphi}_i) = 1$ of the probability distribution.

The scale deviation of the uncertain carbon emission e_i, denoted by $\zeta_i = (e_i - e_i^0)/\widehat{e}_i$, takes value in $[-1, 1]$, i.e., $|\zeta_i| \leq 1$, where e_i^0 is the nominal value and $\widehat{e}_i$ is the maximum deviation for the nominal carbon emission e_i^0 of the product i. In order to control the conservativeness of the solution, adjustable parameter Γ is introduced. We consider the following box+budget uncertainty set

$$\mathcal{U}_{box+budget} = \{e_i | e_i = e_i^0 + \zeta_i \widehat{e}_i, |\zeta_i| \leq 1, \sum_{i=1}^{n} |\zeta_i| \leq \Gamma\}. \tag{10}$$

3.2 The Robust Counterpart of the Cap Model

For the uncertain cap model proposed in section 2.3, both the discrete market demand probability and the carbon emission are uncertain parameters. We consider $\mathcal{U}_1 = \mathcal{U}_{box}$ and $\mathcal{U}_2 = \mathcal{U}_{box+buget}$ in the cap model. We use the following model (11)-(14) to maximize the expected profit in the worst case, which is the robust counterpart of the cap model (8).

$$\max_{\mathbf{q}} \quad z \tag{11}$$
$$s.t. \quad \min_{\mathbf{p}_i \in \mathcal{U}_{box}} \Pi(\mathbf{q}) \geq z \tag{12}$$
$$\max_{e_i \in \mathcal{U}_{box+buget}} \sum_{i=1}^{n} e_i q_i \leq C \tag{13}$$
$$q_i \geq 0, \quad i = 1, ..., n. \tag{14}$$

The above model (11)-(14) has a complicated structure and is a semi-infinite programming problem. Because the problem is computationally intractable, we apply duality theory to transform constraints (12) and (13) into tractable constraints. For the left side of constraint (12), we have

$$\begin{aligned} \min_{\mathbf{p}_i \in \mathcal{U}_{box}} \Pi(q) &= \sum_{i=1}^{n} \boldsymbol{\pi}_i^T \mathbf{p}_i^0 + \min_{\boldsymbol{\varphi}_i} \sum_{i=1}^{n} \boldsymbol{\pi}_i^T \boldsymbol{\varphi}_i \\ &= \sum_{i=1}^{n} \boldsymbol{\pi}_i^T \mathbf{p}_i^0 + \min_{\boldsymbol{\varphi}_i} \{ \sum_{i=1}^{n} \boldsymbol{\pi}_i^T \boldsymbol{\varphi}_i \big| \mathbf{e}^T \boldsymbol{\varphi}_i = 0, \boldsymbol{\rho}_i^- \leq \boldsymbol{\varphi}_i \leq \boldsymbol{\rho}_i^+ \}. \end{aligned} \tag{15}$$

According to duality theory, linear programming problem $\min_{\boldsymbol{\varphi}_i} \{ \sum_{i=1}^{n} \boldsymbol{\pi}_i^T \boldsymbol{\varphi}_i \big| \mathbf{e}^T \boldsymbol{\varphi}_i = 0, \boldsymbol{\rho}_i^- \leq \boldsymbol{\varphi}_i \leq \boldsymbol{\rho}_i^+ \}$ can be given as (16).

$$\begin{aligned} \max_{\gamma_i, \boldsymbol{\xi}_i, \boldsymbol{\nu}_i} \quad & \sum_{i=1}^{n} (\boldsymbol{\rho}_i^+ \boldsymbol{\nu}_i + \boldsymbol{\rho}_i^- \boldsymbol{\xi}_i) \\ s.t. \quad & \mathbf{e}\gamma_i + \boldsymbol{\xi}_i + \boldsymbol{\nu}_i = \boldsymbol{\pi}_i, \quad i = 1, \ldots, n \\ & \boldsymbol{\nu}_i \leq 0, \quad i = 1, \ldots, n \\ & \boldsymbol{\xi}_i \geq 0, \quad i = 1, \ldots, n, \end{aligned} \tag{16}$$

where $(\gamma_i, \boldsymbol{\xi}_i, \boldsymbol{\nu}_i) \in R \times R^m \times R^m$. For the left side of the constraint (13), we have

$$\max_{e_i \in \mathcal{U}_{box+buget}} \sum_{i=1}^{n} e_i q_i = \sum_{i=1}^{n} e_i^0 q_i + \max_{\zeta_i} \{ \sum_{i=1}^{n} \zeta_i \widehat{e}_i q_i \big| |\zeta_i| \leq 1, \sum_{i=1}^{n} |\zeta_i| \leq \Gamma \}. \tag{17}$$

Using duality theory, linear programming problem (17) can be written as

$$\begin{aligned} \min_{\theta, r_i} \quad & \sum_{i=1}^{n} e_i^0 q_i + \theta\Gamma + \sum_{i=1}^{n} r_i \\ s.t. \quad & \theta + r_i \geq \widehat{e}_i |q_i|, \quad i = 1, \cdots, n \\ & \theta \geq 0, \ r_i \geq 0, \quad i = 1, \cdots, n. \end{aligned} \tag{18}$$

According to the above analysis, the robust counterpart of the carbon cap model (8) is

$$\begin{aligned} \max_{\mathbf{q}, \theta, r_i, \gamma_i, \boldsymbol{\xi}_i, \boldsymbol{\nu}_i} \quad & z \\ s.t. \quad & \sum_{i=1}^{n} (\boldsymbol{\pi}_i^T \mathbf{p}_i^0 + \boldsymbol{\rho}_i^+ \boldsymbol{\nu}_i + \boldsymbol{\rho}_i^- \boldsymbol{\xi}_i) \geq z \\ & \sum_{i=1}^{n} e_i^0 q_i + \theta\Gamma + \sum_{i=1}^{n} r_i \leq C \\ & \mathbf{e}\gamma_i + \boldsymbol{\xi}_i + \boldsymbol{\nu}_i = \boldsymbol{\pi}_i, \quad i = 1, \ldots, n \\ & \theta + r_i \geq \widehat{e}_i q_i, \quad i = 1, \ldots, n \\ & q_i, \boldsymbol{\xi}_i, \theta, r_i \geq 0, \quad i = 1, \ldots, n \\ & \boldsymbol{\nu}_i \leq 0, \quad i = 1, \ldots, n. \end{aligned} \tag{19}$$

4 The Extension of Uncertain Cap Model

4.1 The Uncertain Cap-and-Trade Model

In this section, we consider the cap-and-trade policy. In the cap-and-trade model, c_e and E are carbon trading price and carbon emission quantity, respectively. When the carbon trading quantity $E > 0$, the retailer will buy a certain amount of carbon emission quota from the carbon trading market. When the carbon trading quantity $E = 0$, the retailer needn't trade with the carbon trading market. When the carbon trading quantity $E < 0$, the retailer will sell a certain amount of carbon emission quota to the carbon trading market. In terms of constraints, we consider a new uncertainty set

$$\mathcal{U}_3 = \mathcal{U}_{box+ball} = \{e_i \big| e_i = e_i^0 + \zeta_i \widehat{e}_i, |\zeta_i| \leq 1, \sqrt{\sum_{i=1}^{n} \zeta_i^2} \leq \Omega\}. \tag{20}$$

The cap-and-trade model is as follows,

$$\left\{ \begin{array}{ll} \max\limits_{\mathbf{q},E} & \Pi(\mathbf{q}) - c_e E \\ s.t. & \sum\limits_{i=1}^{n} e_i q_i \leq C + E \\ & q_i \geq 0, \quad i = 1, ..., n \end{array} \right\}_{\mathbf{p}_i \in \mathcal{U}_1, e_i \in \mathcal{U}_3.} \tag{21}$$

We introduce auxiliary variable f, then model (21) can be rewritten as

$$\left\{ \begin{array}{ll} \max\limits_{\mathbf{q},E} & f \\ s.t. & \Pi(\mathbf{q}) - c_e E \geq f \\ & \sum\limits_{i=1}^{n} e_i q_i \leq C + E \\ & q_i \geq 0, \quad i = 1, ..., n \end{array} \right\}_{\mathbf{p}_i \in \mathcal{U}_1, e_i \in \mathcal{U}_3.} \tag{22}$$

4.2 The Robust Counterpart of the Cap-and-Trade Model

For the proposed uncertain cap-and-trade model (22), the probability distribution of discrete demand and the carbon emission during transportation are regarded as uncertain parameters. We consider $\mathcal{U}_1 = \mathcal{U}_{box}$, $\mathcal{U}_3 = \mathcal{U}_{box+ball}$ in the cap-and-trade model. The robust counterpart of model (22) is

$$\max_{\mathbf{q},E} \quad f \tag{23}$$

$$s.t. \quad \min_{\mathbf{p}_i \in \mathcal{U}_{box}} \Pi(\mathbf{q}) - c_e E \geq f \tag{24}$$

$$\max_{e_i \in \mathcal{U}_{box+ball}} \sum_{i=1}^{n} e_i q_i \leq C + E \tag{25}$$

$$q_i \geq 0, \quad i = 1, ..., n. \tag{26}$$

In order to obtain the optimal solution, we transform the above robust counterpart into a computationally tractable formulation. The key is to deal with the constraints (24) and (25). Considering the left side of the constraint (24), we have

$$\begin{aligned} \min_{\mathbf{p}_i \in \mathcal{U}_{box}} \Pi(\mathbf{q}) - c_e E &= \sum_{i=1}^{n} \boldsymbol{\pi}_i^T \mathbf{p}_i^0 - c_e E + \min_{\boldsymbol{\varphi}_i} \sum_{i=1}^{n} \boldsymbol{\pi}_i^T \boldsymbol{\varphi}_i \\ &= \sum_{i=1}^{n} \boldsymbol{\pi}_i^T \mathbf{p}_i^0 - c_e E + \min_{\boldsymbol{\varphi}_i} \{ \sum_{i=1}^{n} \boldsymbol{\pi}_i^T \boldsymbol{\varphi}_i \big| \mathbf{e}^T \boldsymbol{\varphi}_i = 0, \boldsymbol{\rho}_i^- \leq \boldsymbol{\varphi}_i \leq \boldsymbol{\rho}_i^+ \}. \end{aligned} \tag{27}$$

According to duality theory, linear programming problem $\min_{\boldsymbol{\varphi}_i}\{\sum_{i=1}^{n}\boldsymbol{\pi}_i^T\boldsymbol{\varphi}_i \big| \mathbf{e}^T\boldsymbol{\varphi}_i = 0, \boldsymbol{\rho}_i^- \leq \boldsymbol{\varphi}_i \leq \boldsymbol{\rho}_i^+\}$ can be written as (16). For the left side of the constraint (25), one has

$$\max_{e_i\in\mathcal{U}_{box+ball}} \sum_{i=1}^{n} e_i q_i = \sum_{i=1}^{n} e_i^0 q_i + \max_{\zeta_i}\{\sum_{i=1}^{n}\zeta_i\widehat{e}_i q_i \big| |\zeta_i| \leq 1, \sqrt{\sum_{i=1}^{n}\zeta_i^2} \leq \Omega\}. \tag{28}$$

Using the conic duality theory, the second order cone programming problem $\max_{\zeta_i}\{\sum_{i=1}^{n}\zeta_i\widehat{e}_i q_i \big| |\zeta_i| \leq 1, \sqrt{\sum_{i=1}^{n}\zeta_i^2} \leq \Omega\}$ can be written as

$$\begin{aligned} &\min_{\omega_i,z_i}\{\sum_{i=1}^{n}|\omega_i| + \Omega\sqrt{\sum_{i=1}^{n}z_i^2} : \omega_i + z_i = -\widehat{e}_i q_i\} \\ &= \min_{z_i}\{\sum_{i=1}^{n}|-\widehat{e}_i q_i - z_i| + \Omega\sqrt{\sum_{i=1}^{n}z_i^2}\}. \end{aligned} \tag{29}$$

Overall, we get the following robust counterpart of the cap-and-trade model (22).

$$\begin{aligned} \max_{\mathbf{q},E,\gamma_i,\boldsymbol{\xi}_i,\boldsymbol{\nu}_i,z_i} \quad & f \\ s.t. \quad & \sum_{i=1}^{n}(\boldsymbol{\pi}_i^T\mathbf{p}_i^0 + \boldsymbol{\rho}_i^+\boldsymbol{\nu}_i + \boldsymbol{\rho}_i^-\boldsymbol{\xi}_i) - c_e E \geq f \\ & \sum_{i=1}^{n} e_i^0 q_i + \sum_{i=1}^{n}|-\widehat{e}_i q_i - z_i| + \Omega\sqrt{\sum_{i=1}^{n}z_i^2} \leq C + E \\ & \mathbf{e}\gamma_i + \boldsymbol{\xi}_i + \boldsymbol{\nu}_i = \boldsymbol{\pi}_i, \quad i = 1,\ldots,n \\ & \boldsymbol{\nu}_i \leq 0, \quad i = 1,\ldots,n \\ & \boldsymbol{\xi}_i \geq 0, \quad i = 1,\ldots,n \\ & q_i \geq 0, \quad i = 1,\ldots,n. \end{aligned} \tag{30}$$

5 Numerical Experiments

In this section, we give a numerical example to illustrate the effectiveness and efficiency of the proposed multi-product distributionally robust optimization models. We use CPLEX 12.6 on personal computer (Dell with Intel(R) Core(TM) i3-4150 3.50 GHz CPU and RAM 4.00 GB) by using the Microsoft Windows 10 operating system to solve all mathematical models.

5.1 Problem Description

We assume that the retailer orders three kinds of fruits from three suppliers, then sells them during the sales season. According to realistic statistics, market demands of three fruits are chosen randomly from the intervals [80, 120], [100, 140], [130, 170], respectively. And the corresponding probabilities are given in Table 2. Perturbation vector $\boldsymbol{\varphi}_i$ is in the interval $[\boldsymbol{\rho}_i^-, \boldsymbol{\rho}_i^+]$, and the uncertainty level $\boldsymbol{\rho}_i^+$ ($\boldsymbol{\rho}_i^+ = -\boldsymbol{\rho}_i^-$) takes value in [0, 0.12]. Note that the unit carbon emission of different types of fruits are different. Meanwhile, to analyze the effect of uncertain carbon emission on the decision, we assume that the maximum deviation of uncertain carbon emission is $\widehat{e}_i = 0.3e_i^0$, where e_i^0 is the nominal value given in Table 3. Γ takes value in [0, 3], and Ω takes value in [0, 1.7]. When $\boldsymbol{\rho}_i^+ = \Gamma = \Omega = 0$, the nominal optimal value is equal to the robust optimal value. The optimal solution without carbon constraint is $\mathbf{q} = (111, 130, 149)$. Due to $\sum_{i=1}^{n} e_i^0 q_i = 1915kg$, the cap of carbon emission is not binding on the model when $C \geq 1915kg$. In this paper, assuming $C = 1700kg$, we consider the cap of carbon emission is binding. The conservative levels Γ and Ω of carbon emission are under different carbon policies, respectively. For the sake of simplicity, $\boldsymbol{\rho}_i^+$ is denoted as $\boldsymbol{\rho}$. The values of other parameters are set in Table 3.

Table 2: The nominal probability p_{ij} of discrete demand for product i

p_{ij}	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
$i=1$	0.21	0.18	0.22	0.15	0.24
$i=2$	0.15	0.22	0.25	0.18	0.2
$i=3$	0.17	0.26	0.24	0.12	0.21

Table 3: The values of model parameters

i	$p_i(¥/unit)$	$w_i(¥/unit)$	$c_i(¥/unit)$	$s_i(¥/unit)$	$h_i(¥/unit)$	$e_i^0(kg/unit)$
1	65	27	3	10	5	2
2	70	34	6	15	7.5	5
3	55	26	9	10	5	7

5.2 Computational Results and Analysis

For convenience, we define the robust value as the objective value calculated by taking the nominal optimal solution into the robust model. The feasibility indicates that when the robust value exists, it is feasible to take the nominal optimal solution into the robust model.

5.2.1 The Influence of Adjustable Parameters in the Cap Model

In this section, we discuss the impact of adjustable parameters $\boldsymbol{\rho}$ and Γ on the optimal order quantities and optimal value of the cap model. After calculation, the nominally optimal value under the nominal optimal solution $\mathbf{q} = (102, 110, 135)$ is $z = 8887.93$.

Case I: We assume that the probability distribution of market demand is uncertain and the carbon emission is deterministic, i.e., $\Gamma = 0$.

Table 4 shows the optimal order quantities for the retailer under various value of uncertainty level. All the nominal optimal solutions are feasible in the robust model. Robust optimal value and robust value just decrease as the uncertainty level $\boldsymbol{\rho}$ increases. This result is shown in Figure 1. Meanwhile, robust optimal value is larger than robust value. As the uncertainty level $\boldsymbol{\rho}$ increases, the robust value declines faster than the robust optimal value.

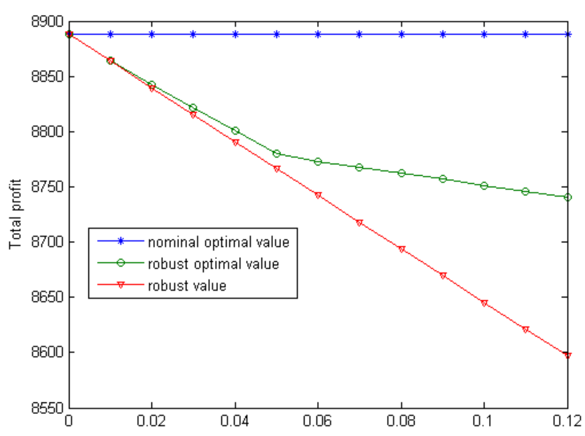

Figure 1: The total profits of nominal and robust cap models under different $\boldsymbol{\rho}$

Table 4: The calculation results under different $\boldsymbol{\rho}$

$\boldsymbol{\rho}$	optimal order quantity	robust optimal value	feasibility	robust value
0.01	101,112,134	8864.20	yes	8863.68
0.04	100,111,135	8800.43	yes	8790.93
0.05	93,111,137	8780.23	yes	8766.68
0.12	87,112,138	8741.03	yes	8596.93

Case II: We assume that the carbon emission is uncertain and the probability distribution of market demand is also uncertain but the uncertain level $\boldsymbol{\rho}$ is certain. We choose $\boldsymbol{\rho} = 0.1$. The maximum profit under the nominally optimal decision $\mathbf{q} = (87, 112, 138)$ is $z = 8751.48$.

In Table 5, when the conservative level of carbon emission is increased, the changes of the optimal order quantities of product 1 and product 2 are not obvious. But the optimal order quantities of product 3 are significantly decreased due to the higher carbon emission per unit product during transportation. So, under the current conditions, the retailer should choose to order low carbon products. Meanwhile, because the conservative level of carbon emission has a perturbation, the nominal optimal solution is no longer the optimal solution and even infeasible. Therefore, to avoid greater losses, the retailer should consider the influence of the perturbation parameter of uncertain carbon emission. Robust optimal value decreases as conservative level of carbon emission increases, which is shown in Figure 2.

Table 5: The calculation results under different Γ

Γ	optimal order quantity	robust optimal value	feasibility
0.5	87,112,120	8301.48	no
1	87,112,106	7951.48	no
1.5	88,113,96	7718.48	no
2	88,111,88	7486.23	no
2.5	86,112,85	7413.48	no
3	86,112,82	7338.48	no

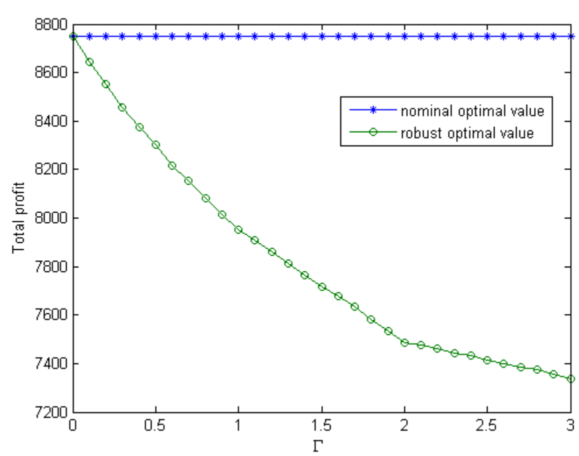

Figure 2: The total profits of nominal and robust cap models under different Γ

5.2.2 The Influence of Adjustable Parameters in the Cap-and-Trade Model

Here, we discuss the influence of adjustable parameter $\boldsymbol{\rho}$ and Ω on the optimal order quantities, carbon emission trading quantity and optimal value of the cap-and-trade model. By calculation, the nominal optimal solution is $\mathbf{q} = (101, 110, 140)$, and the nominal optimal value is $f = 8898.73$.

Case I: We assume that the probability of discrete demand is fluctuating and the carbon emission is deterministic, i.e., $\Omega = 0$.

When the uncertainty level $\boldsymbol{\rho} \leq 0.05$, it is observed that robust optimal order quantities remains unchange which is shown in Table 6, and the changes of robust optimal value and robust value are not obvious. In Figure 3, when the uncertainty level $\boldsymbol{\rho} > 0.05$, it is easy to see that the robust value declines faster than the robust optimal value. In addition, for the cap-and-trade model, robust optimal value is larger than robust value. Meanwhile, both robust optimal value and robust value of the cap-and-trade model are higher than the cap model.

Table 6: The calculation results under different $\boldsymbol{\rho}$

$\boldsymbol{\rho}$	optimal order quantity	E	robust optimal value	feasibility	robust value
0.04	101,110,140	32	8804.13	yes	8804.12
0.05	101,110,140	37	8781.73	yes	8780.47
0.08	88,112,140	16	8763.33	yes	8709.52
0.12	87,112,140	14	8742.02	yes	8614.92

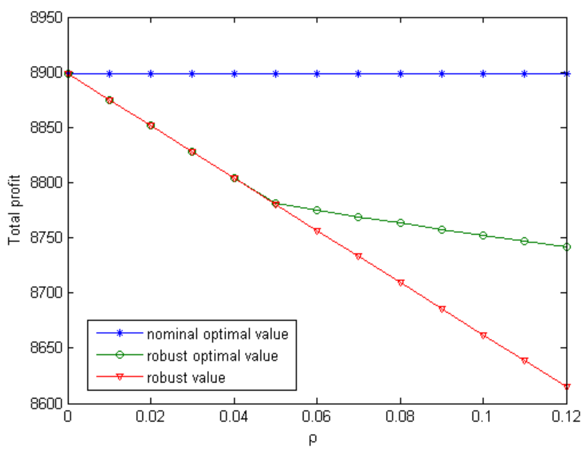

Figure 3: The total profits of nominal and robust cap-and-trade models under different $\boldsymbol{\rho}$

Case II: We assume that the conservative level of carbon emission Γ is fluctuating and the demand probability changes in a certain interval of uncertainty. We choose $\boldsymbol{\rho} = 0.1, K = 1700$, and $c_e = 3.5$. The optimal decision is $\mathbf{q} = (87, 112, 140)$, $E = 14$, The maximum profit is $f = 8752.48$.

In Table 7, as the conservative level of carbon emission increases, the optimal order quantities of product 1 are unchanged, the change of the optimal order quantities of product 2 is not obvious, and the optimal order quantities of product 3 are obviously decreased. When the conservative level of carbon emission is increased, the retailer choose to sell carbon emission trading quantity to maximize profit. Figure 4 shows the variation of robust optimal value and robust value as the conservative level of carbon emission increases. It is observed that the robust optimal value remains unchanged when $\Omega \geq 1.5$.

Table 7: The calculation results under different Ω

Ω	optimal order quantity	E	robust optimal value	feasibility	robust value
0.2	87,112,31	-711.48	8566.67	yes	8512.64
0.4	87,112,15	-789.51	8439.77	yes	8272.81
0.6	87,112,10	-789.7	8315.41	yes	8032.98
0.8	87,112,7	-781.91	8191.92	yes	7793.15
1	87,111,5	-761.19	8069.40	yes	7553.32
1.2	87,111,3	-748.62	7975.41	yes	7315.05
1.4	87,111,2	-739.19	7917.39	yes	7120.19
1.6	87,111,0	-752.3	7913.28	yes	6998.75

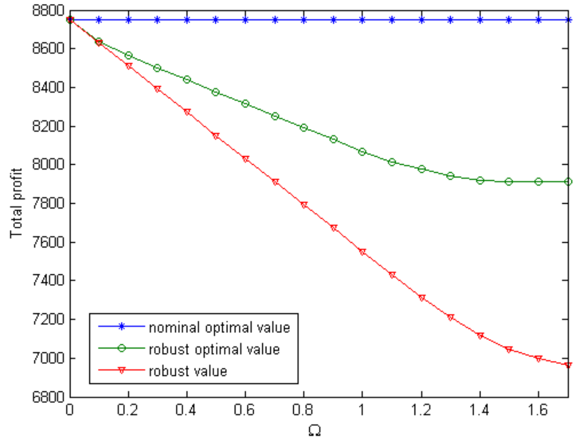

Figure 4: The total profit of nominal and robust cap-and-trade models under different Ω

6 Conclusions

In this paper, we studied the distributionally robust multi-product newsvendor problem. The main conclusions include the following several aspects.

(1) Under uncertain probability distribution and carbon emission, we proposed two multi-product newsvendor models with the cap and cap-and-trade policies, respectively.

(2) Considering the box uncertainty set,the intersection of box and budget uncertainty set and the intersection of box and ball uncertainty set in different models, we used the robust optimization method to deal with the corresponding uncertain parameters. Furthermore, we built the robust counterparts of the corresponding multi-product newsvendor models and found their robust optimal solutions, respectively, which can immunize against uncertainty in our newsvendor problem.

(3) The computational results demonstrate that our robust optimal solution is the best uncertainty-immunized solution that we can associate with our uncertain multi-product newsvendor problem.

Extension to considering the multi-period newsvendor model under carbon policy is an interesting research direction.

Acknowledgments

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