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The Robust Optimization for p-hub Median Problem under Carbon Emissions Uncertainty

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Abstract

In this paper, we describe a novel and practical uncapacitated single allocation p-hub median problem. Aiming to emphasize green transportation, we take into account carbon emissions factor. The emphasis of the present study is to propose a robust optimization method, in which carbon emissions uncertainty are considered. Moreover, we study the uncertain parameters under two types of uncertainty sets. One is the box uncertainty set, and the other is the budget uncertainty set. Based on duality theory, we turn the robust optimization problem into its equivalent mixed-integer linear programming problem, which can be solved by general-purpose optimization software. To show the advantages of the proposed robust optimization method, a numerical experiment is conducted by the actual transportation problem, and the data comes from the traffic map of Hebei Province. Comparing with the deterministic model, the computational results demonstrate that the proposed robust model is more effective.

Keywords: p-hub median problem, robust optimization, carbon emission, mixed-integer linear programming

1 Introduction

The p-hub median problem is one of the most important logistic problems. It is a classical optimization problem. For a given set of nodes with pairwise traffic, p nodes are chosen as hub locations and all the traffic are routed through these hubs at a minimum cost. The p-hub median problem was first proposed by Campbell [3, 4]. Early applications of the p-hub median problems involved the design and management of telecommunication systems. Nowadays, the p-hub median problem is used in many new areas, such as air transit [8], and post systems [21], freight transportation [23].

For the classical p-hub median problem, there are a lot of literatures on the deterministic formulation. Sun et al. [18] proposed Taguchi method to find the best operating combination of controllable factors for a deterministic p-hub median problem. An integer programming formulation was devised by Garcia et al. [5] and Yaman [24] to approach the classical p-hub median problem. Parvaresh et al. [13] proposed the multiple allocation p-hub median problem under intentional disruptions as a bi-level game model. Marti et al. [10] tackled the uncapacitated r-allocation p-hub median problem, which consisted of minimizing the cost of transporting the traffics between nodes of a network through special facilities that act as transsshipment points.

In practice, most of the parameters defined in the models of the above studies on p-hub median problem are not estimated accurately or approximated by using nominal values. Thus, the parameters are usually uncertain. An optimal solution for a certain realization is not necessarily optimal for other realizations. For example, in postal and cargo services, the demand is uncertain and cannot be estimated accurately. The significance of uncertainty has motivated some researchers to address hub location problems with demand uncertainty, travel time uncertainty, and travel cost uncertainty, etc. To describe the demand uncertainty in a p-hub median problem, Talbi and Todosijevic [20] proposed several methods that gained a more innovative robust solution. They also made an empirical study of robust solution. On the basis of credibility measure theory [9], Yang et al. [25, 26, 27] dealt with p-hub center problem with uncertain travel time. Wang et al. [22] combined hub location-allocation modeling approach and equilibrium chance programming method

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to model a hub-and-spoke network design problem with fuzzy random travel times. In order to deal with the uncertainty of demand and time in a single distribution p-hub median problem without capacity limit, Ghaderi and Rahmaniani [6] took the demand and transportation time as random variables. The above research literature did not consider the uncertainty of carbon emissions. For an uncapacitated single allocation p-hub median problem, this paper considers the carbon emissions as uncertain parameters.

Robust optimization is a methodology [2] that has attracted much attention of researchers during the last few decades. The robust optimization method can be used to model the problem with uncertain parameters which have no deterministic distributions. The key to robust optimization is to establish the corresponding robust counterpart model of the original model. Iancu and Trichakis [7] extended the robust optimization framework by proposing practical methods that verify Pareto optimality. Ardestani-Jaafari and Delage [1] studied robust optimization of polyhedral uncertainty set for inventory problems. Meng et al. [11] proposed a distributionally robust optimization approach for managing elective admissions to determine some quotas in a public hospital, and the robust optimization problem was equivalent to a quadratic cone problem. The literatures [15, 19] introduced robust optimization methods with different uncertainty sets. For the single allocation p-hub median problem, Yang and Yang [28] used the robust optimization method to study the uncertainty effects of discount factors. So far, the robust optimization method has not been applied to deal with the carbon emissions uncertainty in p-hub median problem. In this paper, we propose robust optimization method to resist the carbon emissions uncertainty.

In this paper, we build a robust optimization model to describe the uncapacitated single allocation p-hub median problem under carbon emissions uncertainty. The robust counterpart model is a mixed integer linear programming model which can be solved by the method called branch and cut. This solution method is different from the heuristic algorithm used in the literatures [14, 16, 17] to solve the p-hub median problem.

The main contributions of this paper include the following three aspects: firstly, we study the uncapacitated single allocation p-hub median problem including carbon emission factors. Secondly, we develop a robust optimization method to describe this problem. The proposed robust optimization problem can be turned into its equivalent mixed-integer programming problem. Thirdly, we carry out numerical experiments in line with the actual background.

The rest of the paper is organized as follows. In Section 2, we build a deterministic mathematical model including carbon emission factors for a p-hub median problem. In Section 3, we propose the robust optimization model based on the deterministic model, and derive the robust counterpart model. The numerical experiments are carried out in Section 4. The outcomes of our numerical experiments are also discussed in Section 4. We conclude this paper in Section 5.

2 Deterministic Model for p-hub Median Problem

In this section, we present the deterministic model for the p-hub median problem. This formulation was originally proposed by Campbell [4], and the resulting model had fewer variables and constraints than previous formulations found in the literature. In this paper, we improve the basic model by considering carbon emissions constraint for p-hub median problem. Figure 1 shows a basic hub network with 2 hubs. Table 1 describes some sets of indexes and model parameters.

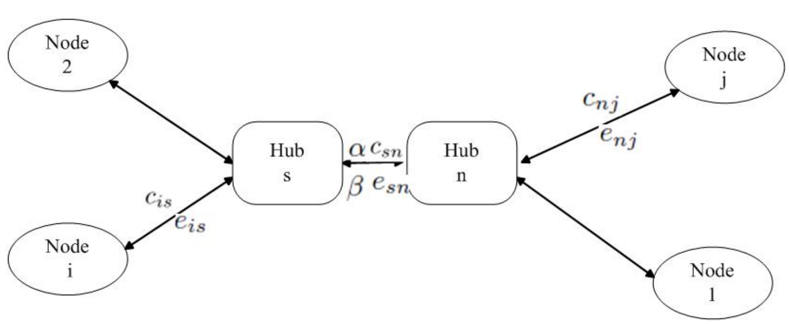


Figure 1: A basic hub network

Table 1: Notation

Notation	Description
Indices	
i, j	The element of the nodes set N
s, n	The symbols of the hubs
Parameters	
c_{is}	Transportation cost for commodity to travel from node i to hub s
c_{sn}	Transportation cost for commodity to travel from hub s to hub n
α	The discount factor for transportation between hubs
δ	The tax cost of the unit emission since carbon pollution
β	The discount factor for carbon emission between hubs
e_{is}	The amount of carbon emission from node i to hub s in the transportation
e_{sn}	The amount of carbon emission from hub s to hub n in the transportation
E^{cap}	The emission capacity for total carbon emissions during transportation
Variables	
X_{isnj}	Binary decision variable indicating the transportation across link (i, s, n, j) or not
X_{is}	Binary decision variable indicating node i is located to hub s or not

In this problem, we assume that the capacity of the hub is not limited. Moreover, hubs are completely interconnected and transports should traverse either one or two hubs. Other assumptions are as follows:

(A1) The number of hubs is predetermined (p).

(A2) The nodes cannot connect with each other.

(A3) A node can only be connected to one hub.

Based on these assumptions and notations, we propose a deterministic mathematical programming model for this problem in the following.

If node i is connected to hub s, we define $X_{is} = 1$, otherwise, we define $X_{is} = 0$. Here we deal with a single allocation problem. Then we have the constraints

$$\sum_{s=1}^{N} X_{is} = 1, \ \forall i \in N. \tag{1}$$

Constraint (1) ensures that a node is connected only to one hub.

Since the nodes cannot connect with each other, the constraints

$$X_{is} \leq X_{ss}, \ \forall i, s \in N \tag{2}$$

are introduced. These constraints indicate that the node i can be connected to s only when s is a hub, and the node i must not be connected to s when s is a node.

Considering the assumption (A1), the number of hubs to be located is predetermined (p). The following constraint

$$\sum_{s=1}^{N} X_{ss} = p \tag{3}$$

is necessary.

For the route variables, we have the following inequality constraints

$$X_{isnj} \geq X_{is} + X_{nj} - 1, \ \forall i, s, n, j \in N. \tag{4}$$

Constraint (4) ensures that the route (i, s, n, j) is a valid path in the network if and only if nodes i and j are assigned to hubs s and n, respectively.

For a valid path which from i to hub s, then from hub s to hub n, and finally from hub n to destination j, the carbon emissions is $e_{is} + \beta e_{sn} + e_{nj}$, and the transportation cost is $c_{is} + \alpha c_{sn} + c_{nj}$. Based on the policies given by the state, the total amount of carbon emissions generated during transportation is limited. This can be described by the following constraint

$$\sum_{i,j=1}^{N}\sum_{s,n=1}^{N}(e_{is}+\beta e_{sn}+e_{nj})X_{isnj}\leq E^{cap}. \tag{5}$$

The left side of constraint (5) gives the sum of carbon emissions on all transportation routes, and the right side is an allowance of the carbon emissions according to state policy.

The corresponding tax cost of the total carbon emission in this problem is

$$\delta\sum_{i,j=1}^{N}\sum_{s,n=1}^{N}(e_{is}+\beta e_{sn}+e_{nj})X_{isnj}.$$

The total transportation cost in this problem is

$$\sum_{i,j=1}^{N}\sum_{s,n=1}^{N}(c_{is}+\alpha c_{sn}+c_{nj})X_{isnj}.$$

For this optimization problem, our aim is to minimize the total cost including transportation cost and tax cost. Then the objective function of the optimization problem is

$$\sum_{i,j=1}^{N}\sum_{s,n=1}^{N}(c_{is}+\alpha c_{sn}+c_{nj}+\delta(e_{is}+\beta e_{sn}+e_{nj}))X_{isnj}. \tag{6}$$

In summary, the deterministic uncapacitated single allocation p-hub median model is shown as

$$\begin{aligned}&\min\sum_{i,j=1}^{N}\sum_{s,n=1}^{N}(c_{is}+\alpha c_{sn}+c_{nj}+\delta(e_{is}+\beta e_{sn}+e_{nj}))X_{isnj}\\&\text{s.t. constraints}\quad(1)-(5).\end{aligned} \tag{7}$$

3 Robust Optimization Model for p-hub Median Problem

3.1 Carbon Emissions Uncertainty

In this section, we apply the robust optimization framework to describe the p-hub median problem. The robust optimization specifies a suitable uncertainty set for imprecise input data and gives a solution that can ensure the feasibility for all values of uncertain parameters within the uncertainty set.

In practice transportation, many factors, such as traffic conditions and temperature changes, may cause the perturbations of the amount of carbon emissions. Hence the amount of carbon emissions in the transportation is not unchanged, it has some uncertainty. But the uncertainty is not random, and there is no specific distribution function. When we only know the range of the perturbations, we use some known uncertainty sets to describe the uncertainties of carbon emissions and apply robust optimization method to deal with the uncertain p-hub median problem. In this section, box and budget uncertainty sets are used to as much as possible to resist the uncertainty of the parameters and a robust optimization model is proposed. In this robust optimization model, e_{is}, e_{sn}, and e_{nj} depict the parameters that are exposed to uncertainty. According to the actual situation of carbon emissions, we choose box uncertainty sets to measure the perturbation of uncertain parameters e_{is} and e_{nj}, and the budget uncertainty set to measure the perturbation of e_{sn}. We shall denote perturbation vector by ζ.

We consider that the uncertain parameter e_{is} changes on the basis of the nominal value. Let $e_{is}=e_{is}^0+\zeta_{is}$, where e_{is}^0 is the nominal value, ζ_{is} is the specific perturbation value. We use a box uncertainty set to describe the perturbation. The uncertainty set of e_{is} is shown as

$$\mathcal{U}_{box}=\{e_{is}\big|e_{is}=e_{is}^0+\zeta_{is},|\zeta_{is}|\leq\theta\}. \tag{8}$$

The uncertainty of e_{nj} is also described by box uncertainty set. We denote $e_{nj} = e^0_{nj} + \zeta_{nj}$ by the nominal value e^0_{nj} and the specific perturbation value ζ_{nj}. Considering the differences between the actual parameters, we assume that the perturbation range of e_{nj} is $|\zeta_{nj}| \leq \sigma$ which is different from that of e_{is}. It can be better immune to the uncertainty.

For the uncertain parameter e_{sn}, let $e_{sn} = e^0_{sn} + \hat{e}_{sn}$, where e^0_{sn} is the nominal value and $\hat{e}_{sn}$ is the specific perturbation value. We use the budget uncertainty set to describe the perturbation of this parameter. In order to simplify the corresponding calculation, we use $\zeta_{sn} = |e_{sn} - e^0_{sn}|/\hat{e}_{sn}$ to represent the perturbation, and we know $0 \leq \zeta_{sn} \leq 1$. For the budget uncertainty set, we introduce perturbation control variable Γ_n to limit the range of perturbation. The uncertainty set of e_{sn} is shown as follows

$$\mathcal{U}_{budget} = \{e_{sn} \big| e_{sn} = e^0_{sn} \pm \zeta_{sn}\hat{e}_{sn}, 0 \leq \zeta_{sn} \leq 1, \sum_{s=1}^{N} \zeta_{sn} \leq \Gamma_n\}. \tag{9}$$

3.2 The Robust Counterpart of Uncertain Constraint

In constraints (1)-(5), only constraint (5) contains uncertain parameters. We rewrite it as the following form:

$$\sum_{i,j=1}^{N} \sum_{s,n=1}^{N} (e_{is} + \beta e_{sn} + e_{nj}) X_{isnj} \leq E_{cap}, e_{is}, e_{sn} \in \mathcal{U}_{box}, e_{nj} \in \mathcal{U}_{budget}. \tag{10}$$

Now we discuss the robust counterpart of constraint (10). For its left side, we first give a worst case formulation. Since the left side of constraint (10) is less than a constant, we can represent the worst case formulation by the following integer linear programming (ILP) problem

$$\begin{aligned} \max \ & \sum_{i,j=1}^{N} \sum_{s,n=1}^{N} (X_{isnj} e_{is} + \beta X_{isnj} e_{sn} + X_{isnj} e_{nj}) \\ \text{s.t.} \ & e_{is}, e_{sn} \in \mathcal{U}_{box} \\ & e_{nj} \in \mathcal{U}_{budget}. \end{aligned} \tag{11}$$

Substituting the uncertainty sets (8) and (9) of three kinds of uncertain parameters into the above linear programming model, we obtain the following model

$$\begin{aligned} \max \ & \sum_{i,j=1}^{N} \sum_{s,n=1}^{N} (X_{isnj}(e^0_{is} + \zeta_{is}) + \beta X_{isnj}(e^0_{sn} \pm \zeta_{sn}\hat{e}_{sn}) + X_{isnj}(e^0_{nj} + \zeta_{nj})) \\ \text{s.t.} \ & |\zeta_{is}| \leq \theta \\ & |\zeta_{nj}| \leq \sigma \\ & \sum_{s=1}^{N} \zeta_{sn} \leq \Gamma_n \\ & 0 \leq \zeta_{sn} \leq 1. \end{aligned} \tag{12}$$

For the uncertain parameters e_{is} and e_{nj}, let ζ_{nj} and ζ_{nj} take value at endpoints, we can simplify them directly. For the uncertain parameter e_{sn}, according to the constraint maximum protected criterion, $e^0_{sn} \pm \zeta_{sn}\hat{e}_{sn} \leq e^0_{sn} + \zeta_{sn}\hat{e}_{sn}$, we introduce dual variables μ_{sn} and ν_{sn}. Using strong duality theory, we obtain problem (13) as the dual programming problem of problem (12).

$$\begin{aligned} \min \ & \sum_{i,j=1}^{N} \sum_{s,n=1}^{N} (e^0_{is} + \beta e^0_{sn} + e^0_{nj} + \theta + \sigma) X_{isnj} + \sum_{s,n=1}^{N} (\mu_{sn} + \Gamma_n \nu_{sn}) \\ \text{s.t.} \ & \mu_{sn} + \nu_{sn} \geq \beta X_{isnj}\hat{e}_{sn} \qquad \forall i, s, n, j \in N \\ & \mu_{sn} \geq 0 \qquad \forall s, n \in N \\ & \nu_{sn} \geq 0 \qquad \forall s, n \in N. \end{aligned} \tag{13}$$

As a consequence, we obtain the robust counterpart of the uncertain constraint (10) as follows:

$$\sum_{i,j=1}^{N}\sum_{s,n=1}^{N}(e_{is}^{0}+\beta e_{sn}^{0}+e_{nj}^{0}+\theta+\sigma)X_{isnj}+\sum_{s,n=1}^{N}(\mu_{sn}+\Gamma_n\nu_{sn})\leq E^{cap} \tag{14}$$

$$\mu_{sn}+\nu_{sn}\geq\beta X_{isnj}\hat{e}_{sn} \qquad \forall i,s,n,j\in N \tag{15}$$

$$\mu_{sn}\geq 0 \qquad \forall s,n\in N \tag{16}$$

$$\nu_{sn}\geq 0 \qquad \forall s,n\in N. \tag{17}$$

3.3 The Robust Counterpart of Uncertain Objective

According to the theory of robust optimization, the objective function (6) is as follows:

$$\min\sum_{i,j=1}^{N}\sum_{s,n=1}^{N}(c_{is}+\alpha c_{sn}+c_{nj}+\delta(e_{is}+\beta e_{sn}+e_{nj}))X_{isnj},e_{is},e_{sn}\in\mathcal{U}_{box},e_{nj}\in\mathcal{U}_{budget}. \tag{18}$$

We know that minimizing the objective function is equivalent to minimizing its upper bound. Therefore, introducing an auxiliary variable t, we can transform problem (18) into the following robust optimization problem

$$\begin{aligned}\min\quad & t\\ \text{s.t.}\quad & \sum_{i,j=1}^{N}\sum_{s,n=1}^{N}(c_{is}+\alpha c_{sn}+c_{nj}+\delta(e_{is}+\beta e_{sn}+e_{nj}))X_{isnj}\leq t\\ & e_{is},e_{sn}\in\mathcal{U}_{box}\\ & e_{nj}\in\mathcal{U}_{budget}.\end{aligned} \tag{19}$$

Through the above transformation, we have moved the objective function into the constraint. Hence we can obtain the robust counterparts of these new constraints by the method used in section 3.2. According to the constraints in the above model, we obtain the following problem:

$$\begin{aligned}\max\quad & \sum_{i,j=1}^{N}\sum_{s,n=1}^{N}(c_{is}+\alpha c_{sn}+c_{nj}+\delta(e_{is}+\beta e_{sn}+e_{nj}))X_{isnj}\\ \text{s.t.}\quad & e_{is},e_{sn}\in\mathcal{U}_{box}\\ & e_{nj}\in\mathcal{U}_{budget}.\end{aligned} \tag{20}$$

We substitute the uncertainty sets (8) and (9) of three kinds of uncertain parameters into the above model. Therefore the following model (21) is obtained.

$$\begin{aligned}\max\quad & \sum_{i,j=1}^{N}\sum_{s,n=1}^{N}((c_{is}+\alpha c_{sn}+c_{nj})X_{isnj}+\delta X_{isnj}(e_{is}^{0}+\zeta_{is})+\delta\beta X_{isnj}(e_{sn}^{0}\pm\zeta_{sn}\hat{e}_{sn})\\ & +\delta X_{isnj}(e_{nj}^{0}+\zeta_{nj}))\\ \text{s.t.}\quad & |\zeta_{is}|\leq\theta\\ & |\zeta_{nj}|\leq\sigma\\ & \sum_{s=1}^{N}\zeta_{sn}\leq\Gamma_n\\ & 0\leq\zeta_{sn}\leq 1.\end{aligned} \tag{21}$$

For the uncertain parameters e_{is} and e_{nj}, we can simplify them directly. For the uncertain parameter e_{sn}, according to the constraint maximum protected criterion, $e_{sn}^{0}\pm\zeta_{sn}\hat{e}_{sn}\leq e_{sn}^{0}+\zeta_{sn}\hat{e}_{sn}$, we introduce dual variables γ_{sn} and τ_{sn}. Using strong duality theory, we obtain the dual programming problem (22) of this maximization problem, which is the robust counterpart of the uncertain objective.

$$\begin{aligned}
\min \quad & \sum_{i,j=1}^{N}\sum_{s,n=1}^{N}(c_{is}+\alpha c_{sn}+c_{nj}+\delta(e_{is}^0+\beta e_{sn}^0+e_{nj}^0))X_{isnj}+\sum_{i,j=1}^{N}\sum_{s,n=1}^{N}\delta(\theta+\sigma)X_{isnj}\\
& +\sum_{s,n=1}^{N}(\gamma_{sn}+\Gamma_n\tau_{sn}) \qquad (22)\\
\text{s.t.} \quad & \gamma_{sn}+\tau_{sn}\geq\delta\beta X_{isnj}\hat{e}_{sn} \quad \forall i,s,n,j\in N\\
& \gamma_{sn}\geq 0 \quad \forall s,n\in N\\
& \tau_{sn}\geq 0 \quad \forall s,n\in N.
\end{aligned}$$

3.4 The Robust Counterpart Model

According to the above discussion, the uncertain uncapacitated single allocation p-hub median model is shown as

$$\begin{aligned}
\min & \sum_{i,j=1}^{N}\sum_{s,n=1}^{N}(c_{is}+\alpha c_{sn}+c_{nj}+\delta(e_{is}+\beta e_{sn}+e_{nj}))X_{isnj}\\
\text{s.t.}\ & \text{constraints} \quad (1)-(5) \qquad (23)\\
& e_{is},e_{sn}\in\mathcal{U}_{box}\\
& e_{nj}\in\mathcal{U}_{budget}.
\end{aligned}$$

Based on duality theory, combining the above results (14)-(17) and (22), the robust counterpart model of model (23) is shown as follows:

$$\begin{aligned}
\min \quad & \sum_{i,j=1}^{N}\sum_{s,n=1}^{N}(c_{is}+\alpha c_{sn}+c_{nj}+\delta(e_{is}^0+\beta e_{sn}^0+e_{nj}^0))X_{isnj}+\sum_{i,j=1}^{N}\sum_{s,n=1}^{N}\delta(\theta+\sigma)X_{isnj}\\
& +\sum_{s,n=1}^{N}(\gamma_{sn}+\Gamma_n\tau_{sn})\\
\text{s.t.} \quad & \gamma_{sn}+\tau_{sn}\geq\delta\beta\hat{e}_{sn}X_{isnj}, \forall i,s,n,j\in N \qquad (24)\\
& \mu_{sn}+\nu_{sn}\geq\beta\hat{e}_{sn}X_{isnj},\ \forall i,s,n,j\in N\\
& \sum_{i,j=1}^{N}\sum_{s,n=1}^{N}(e_{is}^0+\beta e_{sn}^0+e_{nj}^0+\theta+\sigma)X_{isnj}+\sum_{s,n=1}^{N}(\mu_{sn}+\Gamma_n\nu_{sn})\leq E^{cap}\\
& \gamma_{sn}\geq 0,\tau_{sn}\geq 0,\mu_{sn}\geq 0,\nu_{sn}\geq 0,\ \forall s,n\in N\\
& \text{constraints}(1)-(4).
\end{aligned}$$

The robust counterpart model (24) is a mixed integer linear programming model. It is because of this, that the robust counterpart model is readily solved by standard optimization packages. In the next section, we solve the corresponding mixed integer linear programming problems in the numerical experiments by CPLEX.

4 Numerical Experiments

This section consists of four parts. The following steps are taken in the following four subsections in order to describe the numerical experiments for the p-hub median model: the description of a actual p-hub median problem, computational results provided by the proposed robust optimization model, comparison study with the nominal model, influence of parameters on the optimal solution.

4.1 Problem Description

With the rapid development of "Beijing-Tianjin-Hebei", the logistics and transportation pressure of Beijing and Tianjin is growing. In order to effectively alleviate the pressure of Beijing and Tianjin, it is necessary to establish hub distribution center in Hebei province. As a part of Beijing-Tianjin-Hebei region, Hebei province is responsible to undertake this work. Therefore, based on this background, we consider to locate some hubs in Hebei prefecture level cities. The transport routes may pass through Beijing or Tianjin.

There are 11 candidate prefecture level cities in Hebei province, from which we choose 2 cities as hub distribution centers. That is N=11, p=2. The 11 prefecture level cities in Hebei province distribute as Figure 2.

Figure 2: Distribution map of 11 candidate cities in Hebei province

The aim of our research is to determine two hubs from these 11 nodes to minimize the total cost. The total cost is composed of transportation cost and carbon emission cost. We use the product of transportation distance d and the transportation cost c_d per unit distance to measure transportation cost. The transportation distance d is derived from the highway traffic map of Hebei province. Combined with the actual situation, let the transportation cost per unit distance be c_d=10¥. There is a discount factor α between the hubs because of the economies of scale. We select $\alpha = 0.75$ which is the same as that in [12].

The carbon emission cost is measured by the product of the amount of carbon emitted and the unit carbon cost. According to the actual data, the fuel consumption per unit distance is about 0.12 liter, the corresponding carbon emission e is 2.7×0.12. According to the price of carbon emission in recent years, the tax cost δ of the unit carbon emission is 0.04¥/kg since carbon pollution. For transportation between hubs, there are significant reductions in the carbon emission. So there is a discount factor of the carbon emission between the hubs, and the discount factor β is assumed to be 0.65. The uncertain parameter e_{is} is evaluated in a box uncertainty set. Considering that the range of perturbation is approximately 10% of the nominal value, we take 15 as the value of the perturbation parameter θ of the carbon emissions e_{is}. Meanwhile, the perturbation parameter σ for e_{nj} is 20. The uncertain parameter e_{sn} is in a different case because it takes values in a budget uncertainty set. Considering the number of nodes, we take 10 as the value of parameter Γ_n. According to the total transportation distance of HeBei province and the government's control policy on carbon emissions, we adopt E^{cap}=5000t.

4.2 Computational Results of Robust Model

We use the robust optimization method proposed in Section 3 to model this p-hub median problem. According to the initial values of the parameters, we use CPLEX 12.6.3 software to solve model (24) on an Inter(R) Core i5-7200 (can speed up to 3.1GHz) personal computer with 4GB RAM operating under Windows 10.

For the sake of simplicity, we use 11 integers from 1 to 11 to represent city nodes: ChengDe, ZhangJiaKou, TangShan, QinHuangDao, LangFang, BaoDing, CangZhou, ShiJiaZhuang, HengShui, XingTai and HanDan, respectively. CPLEX software outputs six hub schemes, as shown in Figure 3. Comparing the objective values of different hub schemes, the optimal scheme is that nodes 5 and 8 are taken as hubs, the corresponding optimal value of the robust model is 597960.631.

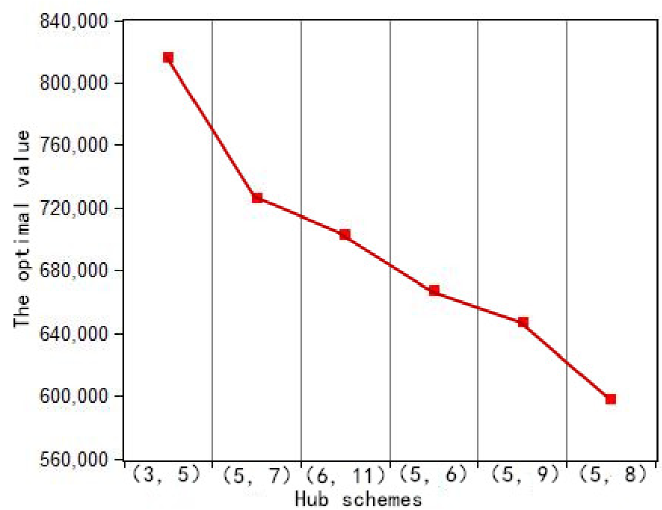


Figure 3: The objective values of the robust model under different hub schemes

When nodes 5 and 8 are selected as hubs, the optimal hub network is shown in Figure 4. As shown in Figure 4, LangFang and ShiJiaZhuang are hubs, ChengDe, ZhangJiaKou, TangShan, QinHuangDao and BaoDing are allocated to LangFang, CangZhou, HengShui, XingTai and HanDan are assigned to ShiJiaZhuang.

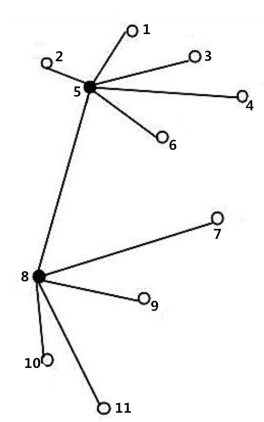


Figure 4: The optimal hub network determined by the robust model

4.3 Comparison Study with the Nominal Model

In this section we compare the computational results provided by the robust model and the nominal model. Solving the nominal model, CPLEX software outputs six hub schemes. For the sake of intuition, we add them to Figure 3 and get the comparison results shown in Figure 5.

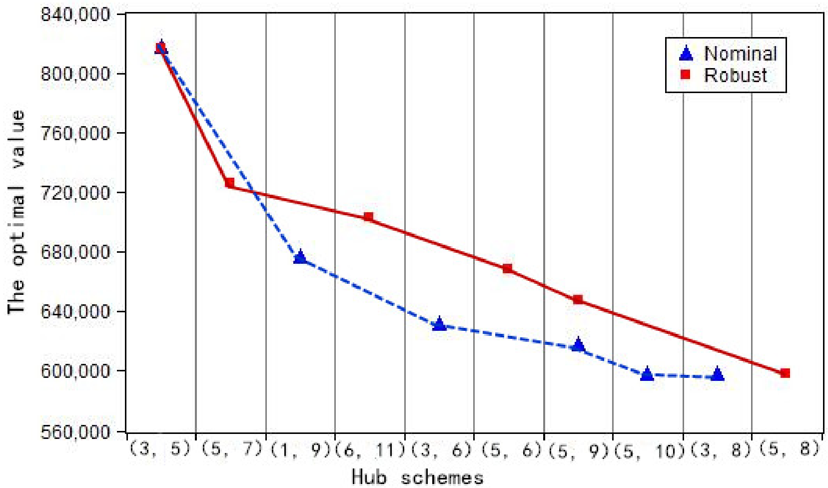


Figure 5: Comparison of the objective values of the robust model and the nominal model

Figure 5 shows that the optimal values of the robust model and the nominal model are different. The optimal value of the robust model is 597960.631, while the optimal value of the nominal model is 597779.791. The former is greater than the latter. Therefore, compared with the nominal optimal solutions, it takes more cost to perform the robust optimal solutions. The additional costs is the robust price since the robust optimal solutions must be responsible for all realizations of the uncertain parameters. In other words, we gain robust solutions as the return of increasing the cost, which is the meaning of robust feasible solution. The optimal hub network determined by the nominal model is shown in Figure 6. The optimal hub locations provided by the nominal model are 3 and 8. Then, the nodes connected to hub 3 are 1, 2, 4, 5 and 6, the nodes connected to hub 8 are 7, 9, 10 and 11.

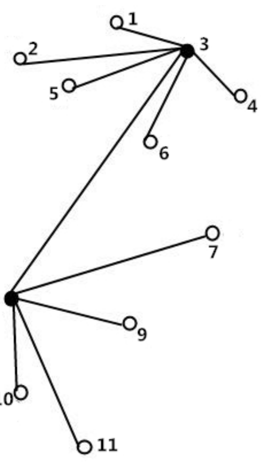


Figure 6: The hub network determined by the nominal model

Figures 4 and 6 show that the robust optimization approach can provide a different hub network design from that provided by the corresponding deterministic approach. From Figures 4 and 6, we can see the difference between the robust solution and the nominal solution. This is in line with the actual situation: compared with node 3 (TangShan), node 5 (LangFang) is in the central area of Beijing-Tianjin-Hebei region and more suitable to be a hub. In this sense, our robust model outperforms the nominal model.

4.4 The Influence of Adjustable Parameter Variation

The robust model has three perturbation control parameters θ, σ, Γ_n. When the perturbation control parameters take different values, different robust formulations are derived. In the following study, we analyze the effects of perturbation control parameters on the optimal value and hub network.

The uncertain parameters e_{is} and e_{nj} are described by box uncertainty sets. The perturbation control parameters are θ and σ. We limit θ to be within $[15, 25]$, σ to be within $[20, 40]$. Our analysis shows that the changes of θ and σ do not affect the optimal hub network. The location of the hub does not change, but the optimal value changes accordingly, as shown in Figure 7. As we can see from Figure 7, the perturbation control parameters θ and σ impact the robust optimal value. The optimal value of the robust model increases linearly as θ and σ increase.

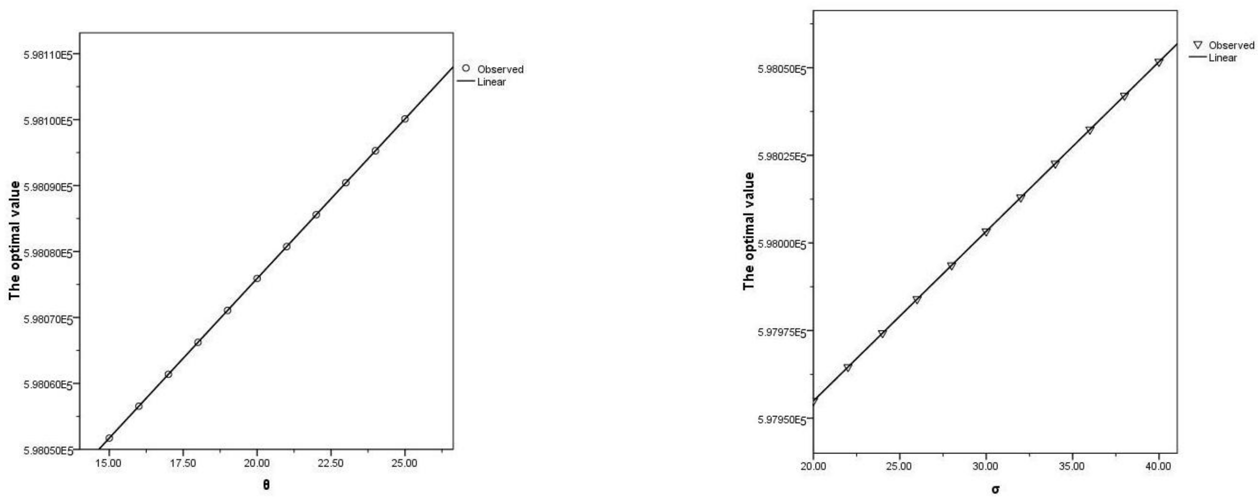


Figure 7: The optimal values under $\theta \in [15, 25]$ and $\sigma \in [20, 40]$

In the robust model, the uncertain parameter e_{sn} is described by a budget uncertainty set. Now we analyze the influence of the perturbation control parameter Γ_n on the optimal hub network. We take values of Γ_n from 4 to 10. Figure 8 shows the optimal values for different Γ_n.

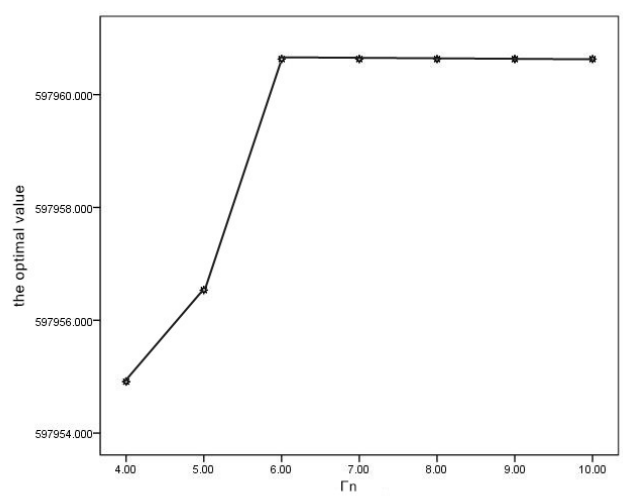
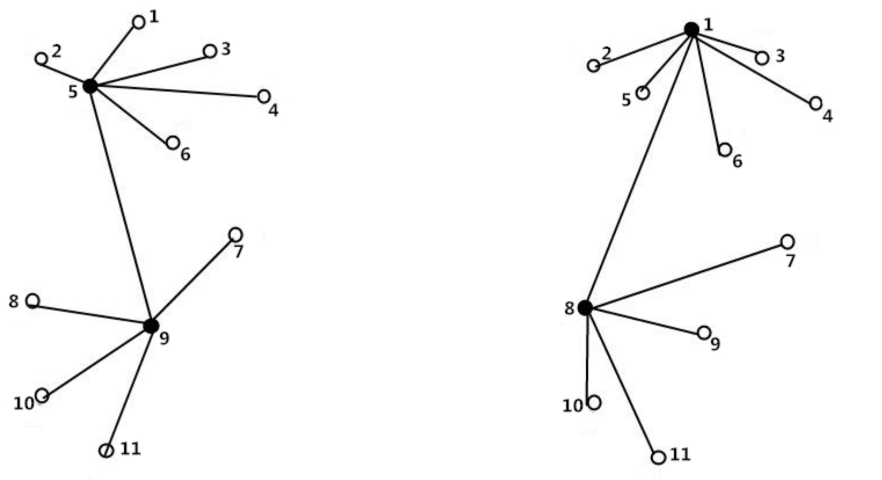


Figure 8: The optimal values under different Γ_n

Figure 9 shows the optimal hub locations and the optimal network in the cases of $\Gamma_n = 4$ and $\Gamma_n = 6$. When $\Gamma_n = 4$, the optimal hub locations provided by the robust model are 5 and 9. Then, the nodes connected to hub 5 are 1, 2, 3, 4 and 6, the nodes connected to hub 9 are 7, 8, 10 and 11. In the cases of $\Gamma_n = 6$, The optimal hub locations given by the robust model are 1 and 8, and the nodes 2, 3, 4, 5 and 6 are connected to hub 1, the nodes 7, 9, 10 and 11 are connected to hub 8. The specific results for other values of Γ_n are shown in Table 2.



(a) The hub network under $\Gamma_n = 4$ (b) The hub network under $\Gamma_n = 6$

Figure 9: The influence of Γ_n on hub network

Table 2: The results of the robust model under different Γ_n

Γ_n	The hub location	The optimal value
4	(5,9)	597954.911
5	(2,8)	597956.535
6	(1,8)	597960.631
7	(5,8)	597960.631
8	(5,8)	597960.631
9	(5,8)	597960.631
10	(5,8)	597960.631

Table 2 shows that the perturbation control parameter Γ_n does not affect the optimal value greatly, but it affects the optimal solution. When perturbation control parameters $\Gamma_n = 7, 8, 9, 10$, the optimal solutions of the robust model are unchanged.

In summary, the robust model can resist the uncertainties of some parameters, and small data uncertainty deserves significant attention. As the number of uncertain parameters increase, the impact of these uncertain parameters on the optimal decision will increase. Therefore, changing the value of the parameter may cause the optimal decision to change.

5 Conclusions

In this paper, we propose a robust optimization method for uncertain p-hub median problem which emphasizes green transportation. We describe carbon emission quantity as uncertain parameters which takes values in box and budget uncertainty sets, but precise distributions are not available. Based on dual theory, we turn the robust optimization problem into its equivalent mixed-integer programming problem which is computationally tractable. We discuss the influence of adjustable parameter variation on robust solution. The numerical experiments show that the robust optimal solution can resist data uncertainty. Therefore, it is reasonable that we use robust model to against the prediction error caused by uncertainty.

We apply the proposed robust optimization method to an actual p-hub median problem. In the problem, there are 11 candidate prefecture level cities in Hebei province, from which we choose 2 cities as hub distribution centers. The numerical experiments show that the robust model suggests LangFang, rather than TangShan, to be a hub. In reality, LangFang city is in the central area of Beijing-Tianjin-Hebei region, and ShiJiaZhuang has been the hub of logistics transportation in HeBei province. This result shows that the optimal decision of the robust model has higher quality than that of the nominal model. It further proves the effectiveness and application value of our robust optimization method. The proposed robust optimization method in this paper can be applied to other actual p-hub median problem.

In our proposed p-hub median model, cost is taken as the objective to be optimized. However, for the transport of certain perishable products, transportation time is also an important criterion that can not be ignored. Multiple assignments p-hub median can make the network more flexible and efficient. Modeling multiple objectives and multiple assignments p-hub median problem using robust optimization method deserves to be studied in future. Furthermore, we will study the application of heuristic algorithm in solving the robust p-hub median problem.

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