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A Stochastic Chance Constrained Closed-Loop Supply Chain Network Design Model with VaR Criterion

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Abstract

Closed-loop supply chain (CLSC) is of great significance to sustainable development, and uncertainty and risk in CLSC networks have attracted more attentions in recent years. This paper proposes a stochastic chance constrained CLSC network design model with value-at-risk (VaR) objective, in which both transportation cost and customer's demand are stochastic parameters with known joint distributions. Furthermore, based on finite discrete distributions of uncertain parameters, an equivalent deterministic mixed-integer linear programming of the original model that can be solved by CPLEX commercial software is derived. In the numerical experiment, a case study on electronic products is used to evaluate the proposed model. The computational results reveal the significance and applicability of the developed model and solution method.

Keywords: closed-loop supply chain, stochastic chance constraints, discrete joint distribution, mixed-integer linear programming

1 Introduction

A supply chain is a system of organizations, people, activities, information, and resources involved in moving a product or service from supplier to customer. A supply chain network is an evolution of the basic supply chain. Designing a supply chain network is to make the decisions to satisfy the demands of customers and minimize the sum of strategic and tactical costs. Geoffrion and Graves [7] first described a comprehensive mixed-integer programming model for the design of supply chain networks from a single-period version, and after that many scholars have developed lots of optimization models and methods to cope with this problem. For the recent developments of supply chain network design problems, the interested reader may refer to [2, 11, 26, 28, 30, 29].

Currently, with the speed of the products update and elimination accelerating, decision makers are paying an increasing interest to CLSC network design, aiming to collect and recycle used products with the objective of linking together environmental issues and business opportunities [9, 12]. As an extension of traditional supply chain, the CLSC network explicitly explores the synergy between the two flows. The forward flow deals only with supply chain activities from suppliers up to customers, while the reverse flow focuses on the activities returned form from customers [3, 10, 19]. Reverse supply chains reduce operating costs by repeatedly using products or components [24], and some researchers have attempted to design and optimize the CLSC network problem. For example, Bottani et al. [4] proposed a comprehensive analysis of the performance of the asset management process in a real-world CLSC, which was subsequently used in a multi-objective optimization procedure. Wang and Chen [27] developed a mean-standard deviation model for a CLSC problem with deteriorating products. Kadambala et al. [13] formulated a multi-objective CLSC problem based on a network-flow model measuring the time value to recover maximum assets lost due to delay at different stages of the recycle process.

Due to the complexity of the supply chain network structure and the participation of many factors, the design of CLSC network is a crucial strategic decision with important parameters such as demand and costs significantly uncertain [16]. As a result, researchers began to design CLSC model with inherent uncertainty in the network parameters. Kim et al. [14] developed a model of CLSC with returnable transport items

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(RTIs) and deteriorating items, and assumed that the return lead time of RTIs was uncertain. Kisomi et al. [15] introduced an integrated supply chain configuration model under uncertainties of customers' demands, variable costs and transportation costs. Pishvaee et al. [23] proposed a CLSC network including customers at the first and second markets, in which the quantity of returned products, demands for recovered products and transportation costs were uncertain. Mohammed et al. [20] proposed a multi-period, multi-product CLSC with two different kinds of uncertainties. The demand and returns uncertainties were considered by means of multiple scenarios; Carbon emissions due to supply chain related activities were considered as uncertain parameters by means of a set-based methodology which led to robust optimization. Cui et al. [5] and Zeballos et al. [31] studied the uncertainty of returned products in supply chain network design problem wherein Cui et al. [5] focused on demand uncertainty and Zeballos et al. [31] mainly considered the uncertainty of the quality and quantity of returned products.

The risk of CLSC network caused by the uncertain factors from every part of supply chains is one of the main concerns of both practitioners and researchers. Different attitudes towards dealing with risk have been discussed [18, 17, 21, 33], and many literatures have been presented considering stochastic programming approaches applied to CLSCs configurations under risk. Some researchers formulated the CLSC problem from the risk neutral viewpoint [1, 8, 32]. For example, Pishvaee et al. [22] proposed a programming model for an integrated forward/reverse logistics network design under uncertainty by using scenario-based stochastic approach. El-Sayed et al. [6] developed a multi-period multi-echelon forward-reverse logistics network design model under risk, in which the demands were stochastic and the total expected profit was directly affected by demand mean and return ratio for a given capacity of the network.

However, expected value approach may not be practical since CLSCs are not typically designed for the average scenario, and VaR is a useful risk measure that has been successfully applied in many application areas. More specifically, VaR is a measure of the expected loss over a given period of time in the context of a confidence level set by normal market conditions. In contrast to the expected value [25], VaR criterion is a powerful risk-aversion strategy for modeling stochastic phenomena in decision systems. The purpose of this paper is to study an integrated CLSC network design problem with stochastic optimization method based on VaR criterion. The objective function is to minimize the critical value of the total cost objective under a certain risk level. On the other hand, when the joint probability distribution is dealt with difficultly, discrete distribution often occurs in applications which can be obtained by empirical distribution or approximation of continuous probability. In this paper, we assume that the stochastic parameters obey known discrete distributions, and the VaR objective and the service level constraint are transformed into their equivalent deterministic forms. Then, the original uncertain supply chain network design model is equivalent to a deterministic mixed integer programming problem. In order to validate and verify the proposed model and method, a case study and results analysis under different risk levels are presented in the end of this paper.

The contributions of this paper are summarized as follows:

- Based on VaR criterion, a stochastic CLSC network model with chance constraints is designed to reduce the risk of the whole supply chain network.
- Uncertain customers' demands and transportation costs are assumed to be stochastic parameters and characterized by known finite discrete distributions.
- Once the chance constraints be transformed into their computationally tractable forms, an equivalent mixed-integer linear programming model of the original CLSC model is obtained.

The organization of this paper is as follows: In Section 2 we will describe the problem to be studied in this paper in full detail, and present the mathematical model of the problem. In Section 3 we discuss the equivalent deterministic forms for the objective and the service level constraint of the CLSC network. In Section 4 we apply the presented model and solving method for a real-life case study. Finally, we conclude the paper in Section 5 and suggest several areas for future research.

2 Closed-Loop Supply Chain Network Design under Uncertainty

2.1 Problem Description and Assumptions

In this section, the CLSC network is discussed which is a single period, multiple layer, multiple part and multiple product network that consists of both the forward flow, and the reverse flow. The forward flow includes suppliers, manufacturers, distributors and customer zones; the return flow contains collection/disassembly centers and disposal centers.

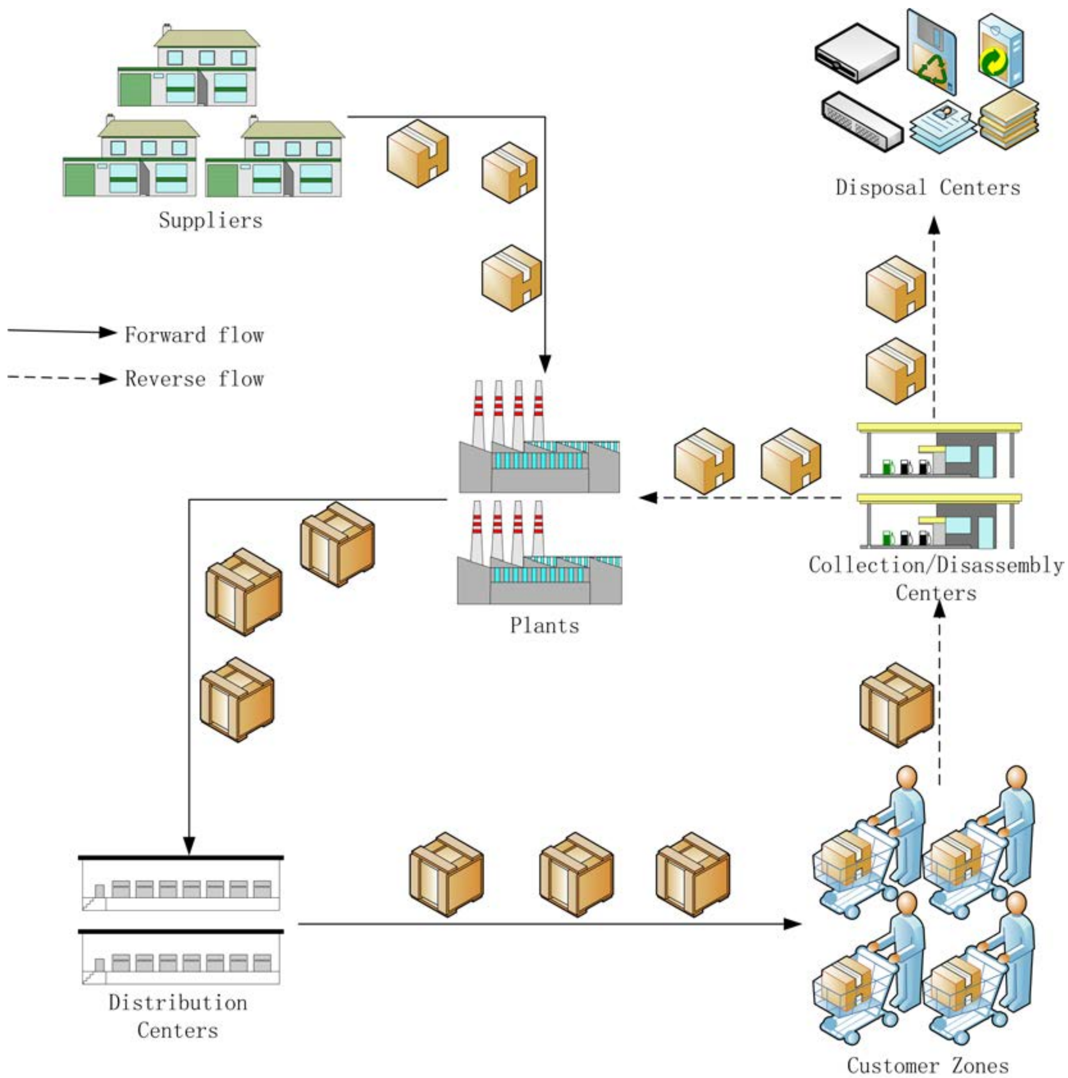

Figure 1: A closed-loop supply chain network

As it is illustrated in Figure 1, in the forward flow, manufacturers buy raw materials (parts) with different discounts from a group of potential suppliers, and then produce multiple products and send them to distribution centers. Distribution centers transfer the products from manufacturers to customer zones according to the customer's requirement. The locations of customer zones are supposed to be predetermined and fixed. In the reverse flow, the returned products are sent to collection/disassembly centers. After separation, the useful parts of the products are sent to plants and the useless parts are transported to disposal centers. Each supplier can provide different price discounts based on the number of orders for parts which are required by the manufacturers. Each product is made up of multiple parts based on the bill of materials. In addition to obtaining raw materials from suppliers, manufacturers can also obtain the raw materials from collection/disassembly centers.

As a matter of the fact, some parameters in CLSC network design problem such as demands of customers are quite uncertain. This issue is intensified in reverse flow because the quantity and quality of returned products have a higher degree of uncertainty. In addition, the transportation costs are also uncertain in the long term. Considering these uncertainties will result in more realistic supply chain models. We will develop a new class of minimum VaR models for a CLSC network design problem in this paper. Assuming that only customers' demands and transportation costs are stochastic parameters. The proposed model address a general CLSC network, which can be applied to many industries. In these industries, returned products could be disassembled for parts in collection/disassembly and then the useable parts could come back to plants as raw materials instead of buying new parts from suppliers.

The main assumptions involved in the proposed model are described below.

- Manufacturing plants and customer locations are fixed.
- With regard to the order quantity, each supplier offers dissimilar price discounts and the discount schemes are known.
- The backlogging of the unsatisfied demand isn't allowed in the network, all the returned products are fully collected to the collection/disassembly centers.
- The maximum allowable cost and the average disposal fraction are deterministic.

Based on the above assumptions, we will introduce the notations in next subsection.

2.2 Notations

In order to formulate the model, the notations are described as follows:

Sets

$\mathcal{I}$	Set of potential supplier center locations $i \in \mathcal{I}$
$\mathcal{J}$	Set of fixed locations for plant centers $j \in \mathcal{J}$
$\mathcal{K}$	Set of potential distribution center locations $k \in \mathcal{K}$
$\mathcal{L}$	Set of customer zones $l \in \mathcal{L}$
$\mathcal{M}$	Set of potential collection/disassembly center locations $m \in \mathcal{M}$
$\mathcal{N}$	Set of potential points for disposal center locations $n \in \mathcal{N}$
$\mathcal{P}$	Set of products $p \in \mathcal{P}$
$\mathcal{R}$	Set of parts $r \in \mathcal{R}$
$\mathcal{H}$	Set of discount segments $h \in \mathcal{H}$;

Parameters

d_{lp}	Demand product p for customer zone l
r_{lp}	Amount of return of the used product p from customer zone l
cf_i^I	Fixed cost of selecting supplier i
cf_k^K	Fixed cost of opening distribution k
cf_m^M	Fixed cost of opening collection/disassembly m
cf_n^N	Fixed cost of opening disposal n
cm_{irh}	Manufacturing cost/unit of part r by supplier i for quality discount h
cm'_{jp}	Manufacturing cost/unit of product p at plant j
$c\omega_{kp}$	Processing cost/unit of product p at distribution k
cc_{mp}	Collection/disassembly cost/unit for the returned product p at the collection/disassembly center m
cr_{mr}	Recycling cost/unit of part r sent to plant from collection/disassembly m
cd_{nr}	Disposal cost/unit of unusable returned part r at disposal center n
cp_{ijr}^I	Transportation cost of part r from supplier i to plant j
cp_{jkp}^J	Transportation cost of product p from plant j to distribution k
cp_{klp}^K	Transportation cost of product p from distribution k to customer zone l
cp_{lmp}^L	Transportation cost of product p from customer zone l to collection/disassembly m
cp_{mjr}^M	Transportation cost of part r from collection/disassembly m to plant j

cp^N_{mnr}	Transportation cost of part r from collection/disassembly m to disposal n
ρ_{irh}	Maximum quantity discount h occurs for part r offered by supplier i
ρ^*_{irh}	Slight less than δ_{irh}
s^I_{ir}	Capacity of part r for supply center i
s^J_{jp}	Capacity of product p for plant j
s^K_{kp}	Capacity of product p for distribution k
s^M_{mp}	Capacity of product p for collection/disassembly center m
s^N_{nr}	Capacity of part r for disposal center n
δ_{rp}	Quantity of part r required to produce one unit of product p
θ_r	Average disposal fraction of part r
π_{lp}	Penalty cost per unit of non-satisfied demand of product p for customer l;

Decision variables

x_{ijrh}	Quantity of part r bought from supplier i to plant j on quantity discount h
y_{jkp}	Quantity of product p sent from plant j to distribution k
z_{klp}	Quantity of product p posted from distribution k to customer zone l
o_{lmp}	Quantity of product p returned from customer zone l to collection/disassembly center m
t_{mjr}	Quantity of recycled part r shipped from collection/disassembly m to plant j
f_{mnr}	Quantity of recycled part r shipped from collection/disassembly m to disposal center n
ω_{lp}	Quantity of non-satisfied demand of product p for customer l
u_i	1 if a supplier is selected at location i, 0 otherwise
v_k	1 if a distribution center is opened at location k, 0 otherwise
c_m	1 if a collection/disassembly center is opened at location m, 0 otherwise
w_n	1 if a disposal center is opened at location n, 0 otherwise
g_{ijrh}	1 if plant j selected part r from supplier i on quantity discount h, 0 otherwise.

2.3 The Formulation of CLSC Network Model

Based on the above notations, in this subsection, we will discuss the formulation of a CLSC network design model.

2.3.1 Constraints

The following constraints play an important role in the formulation of the CLSC network design problem.

Service level constraint. We always expect the demands of all customers are satisfied as follows:

$$\sum_{k\in\mathcal{K}} z_{klp} + \omega_{lp} \geq d_{lp},\ \forall\, l \in \mathcal{L}, p \in \mathcal{P}.$$

However, it is very difficult to satisfy the customers' demands of the realisation CLSC network design problem. In real life, customers' demands are quite uncertain. That is, due to the impact of the policy environment, the cultural environment and the natural environment, customers' demands are uncertain. So we suppose that customers' demands are random parameters. The above formula can be expressed as follows:

$$\Pr\{\sum_{k\in\mathcal{K}} z_{klp} + \omega_{lp} \geq d_{lp},\ \forall\, l \in \mathcal{L}, p \in \mathcal{P}\} \geq \beta. \tag{1}$$

where $\beta \in (0, 1)$ is the service level requirement, and d_{lp} is a random variable.

Constraint (2) ensures the returned products from all customers zones are collected.

$$\sum_{m\in\mathcal{M}} o_{lmp} = r_{lp},\ \forall\, l \in \mathcal{L}, p \in \mathcal{P}. \tag{2}$$

Constraints of the movement equilibrium. Constraint (3) expresses that the raw materials required by the factory's products can be satisfied by the suppliers and collection/disassembly centers.

$$\sum_{i\in\mathcal{I}}\sum_{h\in\mathcal{H}} x_{ijrh} + \sum_{m\in\mathcal{M}} t_{mjr} = \sum_{k\in\mathcal{K}}\sum_{p\in\mathcal{P}} y_{jkp}\delta_{rp},\ \forall\, j \in \mathcal{J}, r \in \mathcal{R}. \tag{3}$$

Constraint (4) is to ensure that all products shipped from the plants are shipped to the customer territories.

$$\sum_{j\in\mathcal{J}} y_{jkp} = \sum_{l\in\mathcal{L}} z_{klp},\ \forall\, k\in\mathcal{K}, p\in\mathcal{P}. \tag{4}$$

Constraints (5)-(6) indicate that the products returned from the customer zones are completely disassembled and shipped to the plants or disposal centers.

$$\sum_{j\in\mathcal{J}} t_{mjr} = (1-\theta_r)\sum_{l\in\mathcal{L}}\sum_{p\in\mathcal{P}} o_{lmp}\delta_{rp},\ \forall\, r\in\mathcal{R}, m\in\mathcal{M}, \tag{5}$$

$$\sum_{n\in\mathcal{N}} f_{mnr} = \theta_r\sum_{l\in\mathcal{L}}\sum_{p\in\mathcal{P}} o_{lmp}\delta_{rp},\ \forall\, r\in\mathcal{R}, m\in\mathcal{M}. \tag{6}$$

Constraints of quantity discount schemes for suppliers. Constraint (7) ensures that the quantity purchased from a supplier at a specific price break is within the discount interval offered.

$$g_{ijrh}\rho_{irh-1} \le x_{ijrh} \le g_{ijrh}\rho^*_{irh},\ \forall\, i\in\mathcal{I}, j\in\mathcal{J}, r\in\mathcal{R}, h\in\mathcal{H}. \tag{7}$$

Constraint (8) ensures that only one discount level is used if part r is purchased from supplier center in location i.

$$\sum_{h\in\mathcal{H}} g_{ijrh} \le 1,\ \forall\, i\in\mathcal{I}, j\in\mathcal{J}, r\in\mathcal{R}. \tag{8}$$

Capacity constraints. Constraints (9)-(14) are based on the capacity restriction for the facilities. Constraint (9) ensures that the parts with quantity discount h from the supplier to the plant do not exceed the capacity of supplier center in location i.

$$\sum_{j\in\mathcal{J}}\sum_{h\in\mathcal{H}} x_{ijrh} \le s^I_{ir}u_i,\ \forall\, i\in\mathcal{I}, r\in\mathcal{R}. \tag{9}$$

Constraints (10)-(11) ensure that the products from the plant to the distribution center are neither capable of exceeding the capacity of the plant nor exceeding the capacity of the distribution center in location k.

$$\sum_{k\in\mathcal{K}} y_{jkp} \le s^J_{jp},\ \forall\, j\in\mathcal{J}, p\in\mathcal{P}, \tag{10}$$

$$\sum_{j\in\mathcal{J}} y_{jkp} \le v_k s^K_{kp},\ \forall\, k\in\mathcal{K}, p\in\mathcal{P}. \tag{11}$$

Constraint (12) ensures that the products from the distribution center to the customer zone do not exceed the capacity of the distribution center in location k.

$$\sum_{l\in\mathcal{L}} z_{klp} \le v_k s^K_{kp},\ \forall\, k\in\mathcal{K}, p\in\mathcal{P}. \tag{12}$$

Constraint (13) ensures that the products from the customer zone to the collection/disassembly center can not exceed the capacity of the collection/disassembly center in location m.

$$\sum_{l\in\mathcal{L}} o_{lmp} \le c_m s^M_{mp},\ \forall\, m\in\mathcal{M}, p\in\mathcal{P}. \tag{13}$$

Constraint (14) ensures that the parts from the collection/disassembly center to the disposal center can not exceed the capacity of the disposal center in location n.

$$\sum_{m\in\mathcal{M}} f_{mnr} \le w_n s^N_{nr},\ \forall\, n\in\mathcal{N}, r\in\mathcal{R}. \tag{14}$$

Constraint (15) expresses the number of parts shipped from the collection/disassembly center to the plant and the disposal center that shall not exceed the capacity of parts removed the collection/disassembly center in the location m

$$\sum_{j\in\mathcal{J}} t_{mjr} + \sum_{n\in\mathcal{N}} f_{mnr} \le c_m \sum_{p\in\mathcal{P}} s^{M}_{mp}\delta_{rp},\ \forall\, m\in\mathcal{M}, r\in\mathcal{R}. \tag{15}$$

Considering the reality of the problem, the decision variables must satisfy the following constraints,

$$g_{ijrh}, u_i, v_k, c_m, w_n \in \{0,1\},\ \forall\, i\in\mathcal{I}, j\in\mathcal{J}, r\in\mathcal{R}, h\in\mathcal{H}, k\in\mathcal{K}, m\in\mathcal{M}, n\in\mathcal{N}, \tag{16}$$

$$x_{ijrh}, y_{jkp}, z_{klp}, p_{lmp}, t_{mjr}, f_{mnr} \ge 0,\ \forall\, i\in\mathcal{I}, j\in\mathcal{J}, k\in\mathcal{K}, m\in\mathcal{M}, r\in\mathcal{R}, p\in\mathcal{P}, h\in\mathcal{H}. \tag{17}$$

2.3.2 The Objective Function

In building the objective function, we first consider the total cost, which is classified into four categories. The first category is the fixed costs of facilities,

$$TFC = \sum_{i\in\mathcal{I}} cf^{I}_{i} u_i + \sum_{k\in\mathcal{K}} cf^{K}_{k} v_k + \sum_{m\in\mathcal{M}} cf^{M}_{m} c_m + \sum_{n\in\mathcal{N}} cf^{N}_{n} w_n.$$

The second category is the processing costs,

$$\begin{aligned} TPC = &\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{r\in\mathcal{R}}\sum_{h\in\mathcal{H}} cm_{irh}x_{ijrh} + \sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}}\sum_{p\in\mathcal{P}} cm'_{jp}y_{jkp} \\ &+ \sum_{k\in\mathcal{K}}\sum_{l\in\mathcal{L}}\sum_{p\in\mathcal{P}} c\omega_{kp}z_{klp} + \sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}}\sum_{p\in\mathcal{P}} cc_{mp}o_{lmp} \\ &+ \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{J}}\sum_{r\in\mathcal{R}} cr_{mr}t_{mjr} + \sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{r\in\mathcal{R}} cd_{nr}f_{mnr}. \end{aligned}$$

The third category is the transportation costs between the facilities in the network flow,

$$\begin{aligned} TTC = &\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{r\in\mathcal{R}}\sum_{h\in\mathcal{H}} cp^{I}_{ijr}x_{ijrh} + \sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}}\sum_{p\in\mathcal{P}} cp^{J}_{jkp}y_{jkp} \\ &+ \sum_{k\in\mathcal{K}}\sum_{l\in\mathcal{L}}\sum_{p\in\mathcal{P}} cp^{K}_{klp}z_{klp} + \sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}}\sum_{p\in\mathcal{P}} cp^{L}_{lmp}o_{lmp} \\ &+ \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{J}}\sum_{r\in\mathcal{R}} cp^{M}_{mjr}t_{mjr} + \sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{r\in\mathcal{R}} cp^{N}_{mnr}f_{mnr}. \end{aligned}$$

The fourth category is penalty costs of the network,

$$PC = \sum_{l\in\mathcal{L}}\sum_{p\in\mathcal{P}} \pi_{lp}\omega_{lp}.$$

Based on the above statement, we get the total cost of the CLSC as follows:

$$TC = TFC + TPC + TTC + PC.$$

We define the objective of the problem as minimizing the value of the risk, i.e., the total cost does not exceed the predetermined maximum allowable cost φ. So the objective function of CLSC network design problem is written as:

$$\min \varphi. \tag{18}$$

As a matter of the fact, fluctuations in fuel prices have a significant impact on transportation costs and the uncertainty of fuel prices will continue in the current economic climate. Thus, the transportation costs are uncertain in the long term. Considering these uncertainties will result in more realistic supply chain models. Assuming that transportation costs are stochastic parameters, we find the minimum φ such that the following constraint (19) for a prespecified small tolerance,

$$\Pr\{TC \le \varphi\} \ge 1-\alpha \tag{19}$$

where $cp^{I}_{ijr}, cp^{J}_{jkp}, cp^{K}_{klp}, cp^{L}_{lmp}, cp^{M}_{mjr}, cp^{N}_{mnr}$ are random variables. And $\alpha \in (0,1)$ is the small tolerance of the VaR.

Based on the above description, we can obtain the following stochastic model for a CLSC network design problem:

$$\begin{array}{ll} \min & \varphi \\ \text{s.t.} & \Pr\{TC \le \varphi\} \ge 1-\alpha \\ & \text{constraints (1)-(17).} \end{array} \tag{20}$$

Obviously, the stochastic CLSC network design model is very difficult to handle and cannot been solved by conventional optimization method when the random variables obey the general probability distribution. In order to convert the above stochastic programming problem into a solvable form, we assume random parameters obey the discrete probability distributions and discuss the computational issue of the stochastic programming in the next section.

3 Analysis of CLSC Network Model

In order to solve the proposed stochastic programming CLSC model (20), we first assume that the discrete distribution of transportation costs

$$cp = \left(cp^{I}_{111}, ..., cp^{I}_{ijr}, cp^{J}_{111}, ..., cp^{J}_{jkp}, cp^{K}_{111}, ..., cp^{K}_{klp}, cp^{L}_{111}, ..., cp^{L}_{lmp}, cp^{M}_{111}, ..., cp^{M}_{mjr}, cp^{N}_{111}, ..., cp^{N}_{mnr}\right)$$

is characterized by

$$cp \sim \begin{pmatrix} \hat{cp}^1 & \hat{cp}^2 & ... & \hat{cp}^{\mathrm{S}} \\ p_1 & p_2 & ... & p_S \end{pmatrix},$$

where $\hat{cp}^{\mathrm{s}} = \left(\hat{cp}^{I,s}_{111}, ..., \hat{cp}^{I,s}_{ijr}, \hat{cp}^{J,s}_{111}, ..., \hat{cp}^{J,s}_{jkp}, \hat{cp}^{K,s}_{111}, ..., \hat{cp}^{K,s}_{klp}, \hat{cp}^{L,s}_{111}, ..., \hat{cp}^{L,s}_{lmp}, \hat{cp}^{M,s}_{111}, ..., \hat{cp}^{M,s}_{mjr}, \hat{cp}^{N,s}_{111}, ..., \hat{cp}^{N,s}_{mnr}\right)$ is the sth scenario, and $p_s > 0, s = 1,2,...,S$, for the sth scenario such that $\sum_{s=1}^{S} p_s = 1$.

We defined a binary vector $\boldsymbol{\sigma}$ whose components $\sigma_s, s = 1,2,...,S$, take 1 if the corresponding set of objective function has to be satisfied and 0 otherwise. And for each scenario s, a large enough number M is introduced so that the objective function of stochastic model with minimum VaR can be converted to the following equivalent deterministic form:

$$\begin{array}{ll} \min & \varphi \\ \text{s.t.} & \sum_{i\in\mathcal{I}} cf^{I}_i u_i + \sum_{k\in\mathcal{K}} cf^{K}_k v_k + \sum_{m\in\mathcal{M}} cf^{M}_m c_m + \sum_{n\in\mathcal{N}} cf^{N}_n w_n \\ & + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{r\in\mathcal{R}}\sum_{h\in\mathcal{H}} (\hat{cp}^{I,s}_{ijr} + cm_{irh}) x_{ijrh} + \sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}}\sum_{p\in\mathcal{P}} (\hat{cp}^{J,s}_{jkp} + cm'_{jp}) y_{jkp} \\ & + \sum_{k\in\mathcal{K}}\sum_{l\in\mathcal{L}}\sum_{p\in\mathcal{P}} (\hat{cp}^{K,s}_{klp} + c\omega_{kp}) z_{klp} + \sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}}\sum_{p\in\mathcal{P}} (\hat{cp}^{L,s}_{lmp} + cc_{mp}) o_{lmp} \\ & + \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{J}}\sum_{r\in\mathcal{R}} (\hat{cp}^{M,s}_{mjr} + cr_{mr}) t_{mjr} + \sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{r\in\mathcal{R}} (\hat{cp}^{N,s}_{mnr} + cd_{nr}) f_{mnr} \\ & + \sum_{l\in\mathcal{L}}\sum_{p\in\mathcal{P}} \pi_{lp}\omega_{lp} \le \varphi + (1-\sigma_s)M, s = 1,2,...,S \\ & \sum_{s=1}^{S} \sigma_s p_s \ge 1-\alpha \\ & \sigma_s \in \{0,1\}, s = 1,2,...,S, \end{array} \tag{21}$$

where $\sum_{s=1}^{S} \sigma_s p_s \geq 1-\alpha$ ensures that the obedience of stochastic service level constraints is not less than $1-\alpha$.

Next, we discuss the equivalent deterministic form of the probabilistic constraint (1). Suppose that demand vector d follows a finite discrete distribution expressed in the following form

$$d=(d_{11},...,d_{lp}) \sim \begin{pmatrix} \hat{d}^1 & \hat{d}^2 & ... & \hat{d}^{\mathrm{T}} \\ q_1 & q_2 & ... & q_T \end{pmatrix},$$

where $\hat{d}^{\mathrm{t}}=(\hat{d}^{\mathrm{t}}_{11},...,\hat{d}^{\mathrm{t}}_{lp})$ is the tth realization of demand, $q_t>0$ for the tth scenario such that $\sum_{t=1}^{T} q_t=1$. we define a binary vector $\boldsymbol{\tau}$, where the elements are made up of τ_t, take 1 if the corresponding set of objective function has to be satisfied and 0 otherwise. Meanwhile, for each scenario t, we introduce a large enough number N. The probabilistic constraint (1) is replaced by

$$\begin{aligned}
&\sum_{k\in\mathcal{K}} z_{klp}+\omega_{lp} \geq \hat{d}^{\mathrm{t}}_{lp}-(1-\tau_t)N,\ \forall\, l\in\mathcal{L}, p\in\mathcal{P}, t=1,2,...,T\\
&\sum_{t=1}^{T} \tau_t q_t \geq \beta\\
&\tau_t\in\{0,1\}, t=1,2,...,T.
\end{aligned} \tag{22}$$

Based on the above description, the stochastic programming problem is equivalent to the following mixed integer linear programming problem:

$$\begin{aligned}
\min\quad & \varphi\\
\text{s. t.}\quad & \sum_{i\in\mathcal{I}} cf_i^I u_i+\sum_{k\in\mathcal{K}} cf_k^K v_k+\sum_{m\in\mathcal{M}} cf_m^M c_m+\sum_{n\in\mathcal{N}} cf_n^N w_n\\
&+\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{r\in\mathcal{R}}\sum_{h\in\mathcal{H}}(\hat{cp}_{ijr}^{I,s}+cm_{irh})x_{ijrh}+\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}}\sum_{p\in\mathcal{P}}(\hat{cp}_{jkp}^{J,s}+cm'_{jp})y_{jkp}\\
&+\sum_{k\in\mathcal{K}}\sum_{l\in\mathcal{L}}\sum_{p\in\mathcal{P}}(\hat{cp}_{klp}^{K,s}+c\omega_{kp})z_{klp}+\sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}}\sum_{p\in\mathcal{P}}(\hat{cp}_{lmp}^{L,s}+cc_{mp})o_{lmp}\\
&+\sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{J}}\sum_{r\in\mathcal{R}}(\hat{cp}_{mjr}^{M,s}+cr_{mr})t_{mjr}+\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{r\in\mathcal{R}}(\hat{cp}_{mnr}^{N,s}+cd_{nr})f_{mnr}\\
&+\sum_{l\in\mathcal{L}}\sum_{p\in\mathcal{P}}\pi_{lp}\omega_{lp}\leq\varphi+(1-\sigma_s)M, s=1,2,...,S\\
&\sum_{s=1}^{S}\sigma_s p_s\geq 1-\alpha\\
&\sum_{k\in\mathcal{K}} z_{klp}+\omega_{lp} \geq \hat{d}^{\mathrm{t}}_{lp}-(1-\tau_t)N,\ \forall\, l\in\mathcal{L}, p\in\mathcal{P}, t=1,2,...,T\\
&\sum_{t=1}^{T}\tau_t q_t\geq\beta\\
&\sigma_s,\tau_t\in\{0,1\}, s=1,2,...,S, t=1,2,...,T\\
&\text{constraints (2)-(17).}
\end{aligned} \tag{23}$$

In next section, a numerical example will be presented and solved to demonstrate the applicability of the proposed methodology as well as to validate the results obtained.

4 Numerical Example

4.1 Problem Description of Numerical Example

In this section, the performance of the proposed model will be illustrated via a numerical example. Due to the increasing demand for electronic products and the rapid development of electronic technology, the average time for Chinese users to replace electronic products is shortened, resulting in a large number of electronic products being discarded. Only a few obsolete electronic products have been recycled. These obsolete electronic products become invisible killers of environmental pollution, and many precious metals in electronic products are also wasted. Therefore, it is necessary to consider the closed-loop design of the supply chain network of electronic products.

Firstly, we will introduce the hypothetical data of consumer electronics industry in the numerical example. The scale of the computational experiment is as follows: there are three potential suppliers that offer six parts to three plants for the production of three electronic products, and each supplier can provide three different price discounts based on the order quantity for multiple parts. The production is used to satisfy customers that are located at six locations through five potential distribution centers. In the reverse chain, returned products are collected at three potential collection/disassembly centers. After inspection carried out at potential collection/disassembly centers, recyclable products and scrap products are separated. Scrapped products are sent to two disposal centers and recyclable products are sent to three plants.

Next, we introduce some parameters of the model, including fixed costs for opening facilities, processing costs, capacities of facilities, and other related parameter values. The fixed costs ($) of raw material suppliers, distribution centers, collection/disassembly centers and disposal centers are chosen randomly in the intervals (8000000, 1000000), (5000000, 7500000), (4200000, 6500000), (7000000, 9000000), respectively. Manufacturing cost ($) /unit of part is chosen randomly from the interval (200, 350). Manufacturing cost/unit of product is chosen randomly from the interval (500, 650). Processing cost ($) /unit of product is chosen randomly from the interval (200, 350). Collection/disassembly cost/unit for the returned product is chosen randomly from the interval (100, 150). Recycling cost ($) /unit of part and disposal cost ($) /unit of unusable returned part are chosen randomly from the interval (300, 450). Capacities of supply centers, plants, distribution centers, collection/disassembly centers and disposal centers are chosen randomly in the intervals (3500, 6500), (800, 2500), (400, 1000), (350, 800), (3500, 4500), respectively. Return of the used product from each customer zone is chosen randomly from the interval (200, 350). When $h = 1$, maximum quantity discount for each part is chosen randomly from the interval (500, 700). When $h = 2$, maximum quantity discount for each part is chosen randomly from the interval (1000, 1500). So, we define that the supplier will not offer a discount when x_{ijr1} is in the range of 1 to ρ^*_{ir1}; if x_{ijr2} is in the range of ρ_{ir1} to ρ^*_{ir2}, the supplier will provide 80% discount; if x_{ijr3} exceeds ρ^*_{ir2}, the supplier will offer 70% discount. Average disposal fraction is 0.4 for all parts. Penalty cost per unit of non-satisfied demand is chosen randomly from the interval (6000, 8000). Table 1 indicates the number of parts needed to produce one unit of product.

Table 1: The number of part r required for produce one unit of product p

δ_{rp}	part1	part2	part3	part4	part5	part6
Product1	1	2	5	3	3	3
Product2	1	1	2	3	4	2
Product3	1	2	3	5	2	3

Moreover, we consider the random parameters with the discrete distributions. The discrete distributions structure is determined in the following way. We uniformly select the transportation costs ($) from the interval (4.5, 14.5), and the probability of the scene is $1/S$. We uniformly select the customers' demands from the interval (145, 280), and the probability of the scene is $1/T$. Finally, we can get the discrete distributions of cp and d. The numerical example is solved using MATLAB R2014a and ILOG CPLEX 12.6.3 MIP solver on an Inter(R) Core i5-7200 (can speed up to 2.50 GHz) personal computer with 8.0 GB RAM operating under Windows 10.

4.2 Results Analysis

According to the parametric design in the above subsection, we give the following calculation results and results analysis.

- **Case 1: The calculation results under uncertain transportation costs**

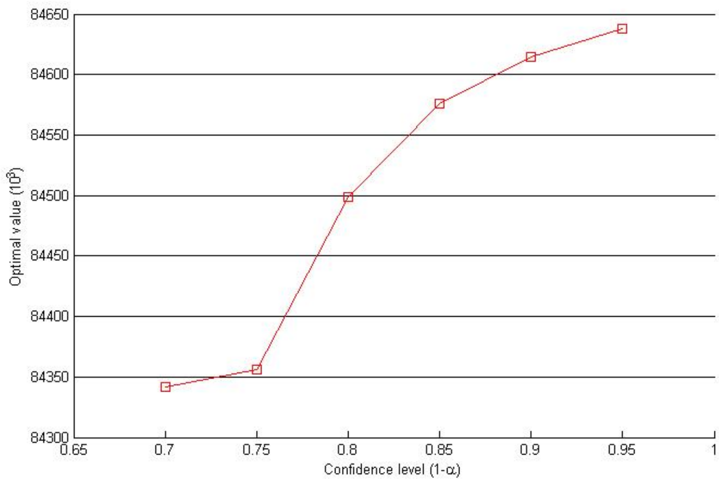

Figure 2: The optimal values under different confidence levels

Table 2: The computational results under different confidence levels

Confidence level $(1-\alpha)$	0.95	0.9	0.85	0.8	0.75	0.7
Optimal value	84637877.60	84614690.00	84576044.00	84498752.00	84355888.07	84342007.67

In this part of the numerical experiment, the values of customers' demands are obtained in a way that take the mean value $\bar{d}_{lp}$ from $\hat{d}^1_{lp}$ to $\hat{d}^{\mathrm{T}}_{lp}$, where $\hat{d}^1_{lp}, ..., \hat{d}^{\mathrm{T}}_{lp} \in \{\hat{d}^{\mathrm{t}}_{lp}, l \in \mathcal{L}, p \in \mathcal{P}, t = 1, 2, ..., T\}$. And the transportation costs are random variables with discrete distributions. Table 2 shows the objective function values of the stochastic model under different confidence levels. We can clearly see from Table 2 that the transportation costs have a great impact on the total cost. As illustrated in Figure 2, with the confidence level increases, the total cost under average customers' demands is also increasing. Therefore, if decision makers want to reduce the risk under uncertain transportation costs, they will need to increase their investment in the entire CLSC. It can guarantee the effectiveness of the financial control in their operations.

- **Case 2: The calculation results under uncertain demands**

Table 3: The calculation results under different service levels

Service level (β)	0.95	0.9	0.85	0.8	0.75	0.7
Optimal value	85784373.57	85418287.62	85291010.91	85159039.07	85042167.52	84927668.04

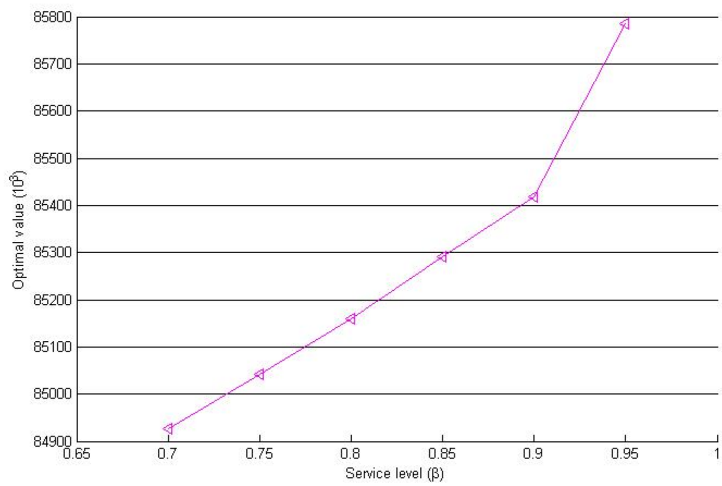

Figure 3: The optimal values under different service levels

We take the mean value $\bar{cp}^I_{ijr}$ from $\hat{cp}^{I,1}_{ijr}$ to $\hat{cp}^{I,S}_{ijr}$, where $\hat{cp}^{I,1}_{ijr}, ..., \hat{cp}^{I,S}_{ijr} \in \{\hat{cp}^{I,s}_{ijr}, i \in \mathcal{I}, j \in \mathcal{J}, r \in \mathcal{R}, s = 1, ..., S\}$. Similarly, we can obtain the mean vector of cp as follows:

$$\bar{cp} = (\bar{cp}^I_{ijr}, \bar{cp}^J_{jkp}, \bar{cp}^K_{klp}, \bar{cp}^L_{lmp}, \bar{cp}^M_{mjr}, \bar{cp}^N_{mnr}).$$

Then we make the mean vector of transportation costs as the transportation costs in this part of the experiment. And customers' demands are random variables with discrete distributions. Table 3 shows the objective function values of the stochastic model under different service levels. According to Table 3, the impact of customers' demands on the total cost is also significant. In the case of uncertain transportation costs, the total cost sensitivity is strong, and when customers' demands are uncertain, the sensitivity to the total cost is also very strong (see Figure 3). In Figure 3 the total cost under the average transportation costs increases with the service level increasing. Therefore, if decision makers want to reduce the risk under uncertain customers' demands, they will need to increase their investment in the entire CLSC. It also can guarantee the effectiveness of the financial control in their operations.

- **Case 3: The calculation results under uncertain transportation costs and demands**

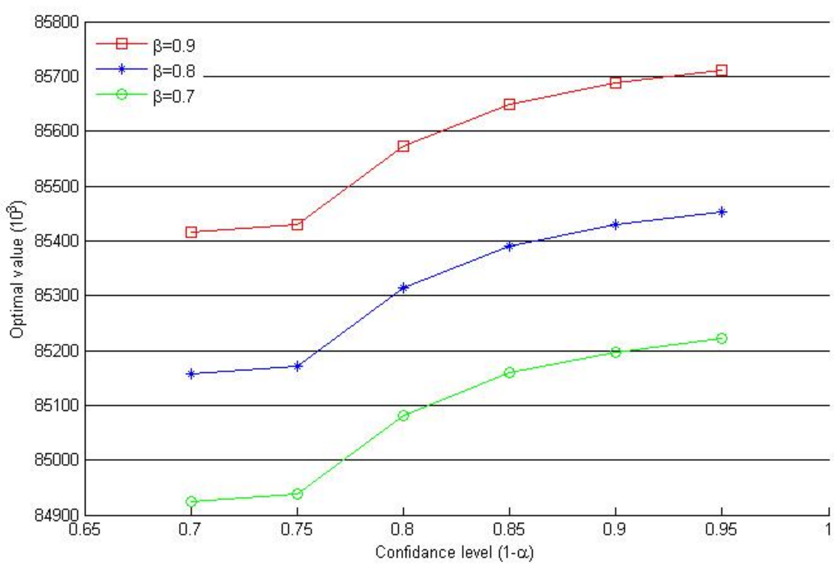

Figure 4: The optimal values under different confidence levels and different service levels

Table 4: The computational results under different confidence levels and different service levels

Service level (β)	Confidence level $(1-\alpha)$					
	0.95	0.9	0.85	0.8	0.75	0.7
0.9	85711411.60	85688224.00	85649578.00	85572286.00	85429292.21	85415370.38
0.8	85452159.60	85428972.00	85390326.00	85313114.00	85170097.23	85156170.13
0.7	85220787.60	85197600.00	85158954.00	85081662.00	84938743.29	84924842.55

In case 3, let transportation costs and customers' demands be random variables with discrete distribution. In reality, the decision makers often prefer the high confidence level and high service level, which causes the high values meaningless for the CLSC network design problem. Thus, we just consider the confidence level of 0.7, 0.75, 0.85, 0.9, 0.95 and the service level of 0.9, 0.8, 0.7 in the numerical experiment. Table 4 shows the computational results of stochastic model when the parameters α and β take different values. The following statement is an analysis of the change in the VaR at five significant levels and three service levels. In Figure 4, it can be seen that the total cost is increasing when confidence levels and service levels increase at the same time. This confirms that the risk of the entire CLSC network decreases when the total cost of the chain increases. From Table 4, we can more accurately see that the change in service levels has a greater impact on the total cost than the confidence levels. In other words, customers' demands have a greater impact on the total cost than transportation costs. Therefore, decision makers can adjust the parameters α and β according to the maximum risk that they can bear, so as to ensure the effectiveness of financial control in operation and improve the efficiency of enterprises.

5 Conclusions

In this paper, on the basis of the CLSC problem with quantity discount, a stochastic model for minimizing the value of risk is proposed. In the proposed model customers' demands and transportation costs were assumed to be random variables. If the random variables were characterized by general probability distributions, the developed stochastic CLSC problem would be difficult to handle by conventional optimization methods. So we assumed that the random variables followed finite joint discrete distributions, and derived the equivalent forms of the probabilistic level constrains. Subsequently, a tractable mixed-integer linear programming model of the original problem was formulated by introducing auxiliary binary variables. In order to test and verify the effectiveness of the proposed modeling idea, a CLSC network design example about electronic products was proposed in the numerical experiment. By analyzing the computational results of the numerical experiment, the stochastic model of CLSC network design problem was more practical than the general CLSC network model. Therefore, it can be concluded that the proposed stochastic model can be used as a powerful tool in practical cases.

Anyway, our modeling effort and analysis come with limitations. In this model, we assume the customers' demands and transportation costs be the uncertain parameters, but there are other inherent uncertain parameters in the CLSC network design, such as the amount of return products and average disposal fraction, which are not considered. These parameters are very important and have a large degree of uncertainty in the CLSC network. In addition, it may be infeasible to employ CPLEX solver for quite large instances of the CLSC problem. So some efficient meta-heuristic algorithms can be used to help us do further research in the future.

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