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On Degrees of m-polar Fuzzy Graphs with Application

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Abstract

The concept of m-polar fuzzy set is the generalization of bipolar fuzzy set. In this paper, the concept of m-polar fuzzy set is applied to graphs and studied about the degree of a vertex in m-polar fuzzy graphs which are obtained from two given m-polar fuzzy graphs G_1 and G_2 using the operations of Cartesian product, composition, direct product, semi-strong product and strong product. We have investigated several results for finding the degree of a vertex in these graphs. Finally, an application of 3-polar fuzzy digraph is given as an example.

Keywords: m-polar fuzzy graphs, Cartesian product and composition, direct product, semi-strong and strong product, degree of a vertex, m-polar fuzzy digraph

1 Introduction

Graph theory has numerous applications to problems on computer science, electrical engineering, system analysis, operations research, economics, networking routing, transportation etc. Presently, mathematical models are developed to handle various types of system containing elements of uncertainty. A large number of these models is based on fuzzy sets. In 1965, Zadeh [30] introduced the concept of fuzzy set. Since then, the fuzzy set theory has become a growing topic of research in different fields including graph theory, decision making, social science, medical field, management, artificial intelligence department, engineering etc.

In 1975, Rosenfeld [22] introduced the concept of fuzzy graphs by considering the fuzzy relations between fuzzy sets and later on he developed the structure of fuzzy graphs. Mordeson and Nair [16] defined the complement of fuzzy graph and it was further studied by Sunitha and Kumar [27]. The concept of weak isomorphism, co-weak isomorphism and isomorphism between fuzzy graphs was introduced by Bhutani [4]. After that several researchers worked on fuzzy graphs like in [5, 3, 15, 18, 19]. Samanta and Pal introduced different types of fuzzy graphs such as fuzzy competition graphs [25, 23], fuzzy planar graphs [26], etc.

In 1994, Zhang [31, 32] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. In 2011, using the concepts of bipolar fuzzy sets, Akram [1] introduced the bipolar fuzzy graphs and defined different operations on it. Rashmanlou et al. [20, 21] studied bipolar fuzzy graphs with categorical properties. Some more work on bipolar fuzzy graphs may be found on [7, 13, 24, 28, 29].

In 2014, Chen et al. [6] introduced the notion of m-polar fuzzy set as a generalization of bipolar fuzzy set and showed that bipolar fuzzy sets and 2-polar fuzzy sets are cryptomorphic mathematical notions and that we can obtain concisely one from the corresponding one. The idea behind this is that "multipolar information" (not just bipolar information which correspond to two-valued logic) exists because data of real world problems are sometimes come from multiple agents. For example, the exact degree of telecommunication safety of mankind is a point in $[0, 1]^n$ ($n \approx 7 \times 10^9$) because different persons have been monitored different times. There are many other examples such as truth degrees of a logic formula which are based on n logic implication operators ($n \geq 2$), similarity degrees of two logic formulas which are based on n logic implication operators ($n \geq 2$), ordering results of a magazine, ordering results of a university, and inclusion degrees (accuracy measures, rough measures, approximation qualities, fuzziness measures, and decision preformation evaluations) of a rough set.

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Chen et al. [6] first introduced the concept of m-polar fuzzy graphs. Ghorai and Pal studied many properties of generalized m-polar fuzzy graphs [9], defined many operations and density of m-polar fuzzy graphs [8], studied isomorphic properties of m-polar fuzzy graphs [10] and introduced the concept of m-polar fuzzy planar graphs [11], faces and dual of m-polar fuzzy planar graphs [12]. In this paper, we study about the degree of a vertex in m-polar fuzzy graphs which are obtained from two given m-polar fuzzy graphs G_1 and G_2 using the operations of Cartesian product, composition, direct product, semi-strong product and strong product. We also investigated several results for finding the degree of a vertex in these graphs. An application of 3-polar fuzzy digraph is given as an example.

2 Preliminaries

In this section, we recall some definitions of fuzzy graphs, m-polar fuzzy sets and m-polar fuzzy relations. For further study see references [14, 17].

Definition 2.1. [16] *A fuzzy graph with V as the underlying set is a triplet $G = (V, \sigma, \mu)$, where $\sigma : V \to [0, 1]$ is a fuzzy subset of V and $\mu : V \times V \to [0, 1]$ is fuzzy relation on σ such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$.*

The underlying crisp graph of G is denoted by $G^* = (\sigma^*, \mu^*)$, where $\sigma^* = \{x \in V : \sigma(x) > 0\}$ and $\mu^* = \{(x, y) \in V \times V : \mu(x, y) > 0\}$.

Definition 2.2. [3] *A fuzzy graph $G = (V, \sigma, \mu)$ is complete if $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$.*

The purpose of this paper is to find the degree of vertices in the Cartesian product, composition, semi-strong product, strong and direct product of two m-polar fuzzy graphs based on m-polar fuzzy set which is defined below.

Throughout the paper, $[0,1]^m$ (m-power of [0,1]) is considered to be a poset with point-wise order $\leq$, where m is an natural number. "$\leq$" is defined by $x \leq y \Leftrightarrow$ for each $i = 1, 2, \ldots, m$; $p_i(x) \leq p_i(y)$ where $x, y \in [0, 1]^m$ and $p_i : [0, 1]^m \to [0, 1]$ is the i-th projection mapping.

Definition 2.3. [6] *An m-polar fuzzy set (or a $[0, 1]^m$-set) on X is a mapping $A : X \to [0, 1]^m$. The set of all m-polar fuzzy sets on X is denoted by $m(X)$.*

Definition 2.4. [9] *Let A and B are two m-polar fuzzy sets in X. Then $A \cup B$ and $A \cap B$ are also m-polar fuzzy sets in X defined by: for $i = 1, 2, \ldots, m$ and $x \in X$,*

$$p_i \circ (A \cup B)(x) = max\{p_i \circ A(x), p_i \circ B(x)\} \text{ and } p_i \circ (A \cap B)(x) = min\{p_i \circ A(x), p_i \circ B(x)\}.$$

$A \subseteq B$ if and only if $p_i \circ A(x) \leq p_i \circ B(x)$ and $A = B$ if and only if $p_i \circ A(x) = p_i \circ B(x)$.

Definition 2.5. [9] *Let A be an m-polar fuzzy set on a set X. An m-polar fuzzy relation on A is an m-polar fuzzy set B of $X \times X$ such that $B(x, y) \leq min\{A(x), A(y)\}$ for all $x, y \in X$ i.e, for each $i = 1, 2, \ldots, m$, for all $x, y \in X$, $p_i \circ B(x, y) \leq min\{p_i \circ A(x), p_i \circ A(y)\}$.*

An m-polar fuzzy relation B on X is called symmetric if $B(x, y) = B(y, x)$ for all $x, y \in X$.

For a given set V, define an equivalence relation $\sim$ on $V \times V - \{(x, x) : x \in V\}$ as follows:

$$(x_1, y_1) \sim (x_2, y_2) \Leftrightarrow \text{either } (x_1, y_1) = (x_2, y_2) \text{ or } x_1 = y_2 \text{ and } y_1 = x_2.$$

The quotient set obtained in this way is denoted by $\widetilde{V^2}$, and the equivalence class that contains the element (x, y) is denoted as xy or yx.

Throughout this paper, $G^* = (V, E)$ represents a crisp graph and $G = (V, A, B)$ represents an m-polar fuzzy graph of G^*.

Definition 2.6. [9] *An m-polar fuzzy graph of a graph $G^* = (V, E)$ is a pair $G = (V, A, B)$ where $A : V \to [0, 1]^m$ is an m-polar fuzzy set in V and $B : \widetilde{V^2} \to [0, 1]^m$ is an m-polar fuzzy set in $\widetilde{V^2}$ such that $p_i \circ B(xy) \leq min\{p_i \circ A(x), p_i \circ A(y)\}$ for all $xy \in \widetilde{V^2}$, $i = 1, 2, \ldots, m$ and $B(xy) = \mathbf{0}$ for all $xy \in (\widetilde{V^2} - E)$, ($\mathbf{0} = (0, 0, \ldots, 0)$ is the smallest element in $[0, 1]^m$).*

A is called the m-polar fuzzy vertex set of G and B is called the m-polar fuzzy edge set of G, respectively.

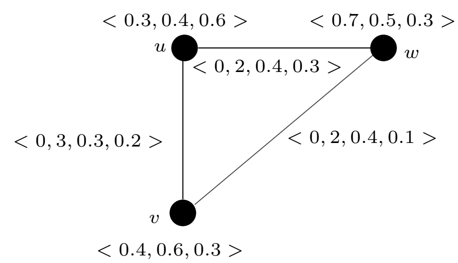

Figure 1: Example of 3-polar fuzzy graph G

Example 2.7. *Let us consider the graph $G^* = (V, E)$ where $V = \{u, v, w\}$ and $E = \{uv, vw, uw\}$. A 3-polar fuzzy graph G of G^* is shown in Fig. 1.*

Definition 2.8. [9] *The Cartesian product $G_1 \times G_2$ of two m-polar fuzzy graphs $G_1 = (V_1, A_1, B_1)$ and $G_2 = (V_2, A_2, B_2)$ of the graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively is defined as a pair $(V_1 \times V_2, A_1 \times A_2, B_1 \times B_2)$ such that for each $i = 1, 2, \ldots, m$,*

(i) $p_i \circ (A_1 \times A_2)(x_1, x_2) = p_i \circ A_1(x_1) \wedge p_i \circ A_2(x_2)$ *for all* $(x_1, x_2) \in V_1 \times V_2$.
(ii) $p_i \circ (B_1 \times B_2)((x, x_2)(x, y_2)) = p_i \circ A_1(x) \wedge p_i \circ B_2(x_2y_2)$ *for all* $x \in V_1$, *for all* $x_2y_2 \in E_2$.
(iii) $p_i \circ (B_1 \times B_2)((x_1, z)(y_1, z)) = p_i \circ B_1(x_1y_1) \wedge p_i \circ A_2(z)$ *for all* $z \in V_2$, *for all* $x_1y_1 \in E_1$.
(iv) $p_i \circ (B_1 \times B_2)((x_1, x_2)(y_1, y_2)) = 0$ *for all* $(x_1, x_2)(y_1, y_2) \in (\widetilde{V_1 \times V_2}^2 - E)$.

Definition 2.9. [9] *The composition $G_1[G_2] = (V_1 \times V_2, A_1 \circ A_2, B_1 \circ B_2)$ of two m-polar fuzzy graphs $G_1 = (V_1, A_1, B_1)$ and $G_2 = (V_2, A_2, B_2)$ of the graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively is defined as follows: for each $i = 1, 2, \ldots, m$,*

(i) $p_i \circ (A_1 \circ A_2)(x_1, x_2) = p_i \circ A_1(x_1) \wedge p_i \circ A_2(x_2)$ *for all* $(x_1, x_2) \in V_1 \times V_2$.
(ii) $p_i \circ (B_1 \circ B_2)((x, x_2)(x, y_2)) = p_i \circ A_1(x) \wedge p_i \circ B_2(x_2y_2)$ *for all* $x \in V_1$, *for all* $x_2y_2 \in E_2$.
(iii) $p_i \circ (B_1 \circ B_2)((x_1, z)(y_1, z)) = p_i \circ B_1(x_1y_1) \wedge p_i \circ A_2(z)$ *for all* $z \in V_2$, *for all* $x_1y_1 \in E_1$.
(iv) $p_i \circ (B_1 \circ B_2)((x_1, x_2)(y_1, y_2)) = p_i \circ A_2(x_2) \wedge p_i \circ A_2(y_2) \wedge p_i \circ B_1(x_1y_1)$ *for all* $(x_1, x_2)(y_1, y_2) \in E^0 - E$.
(v) $p_i \circ (B_1 \circ B_2)((x_1, x_2)(y_1, y_2)) = 0$ *for all* $(x_1, x_2)(y_1, y_2) \in (\widetilde{V_1 \times V_2}^2 - E^0)$.

Definition 2.10. [13] *Let $G_1 = (V_1, A_1, B_1)$ and $G_2 = (V_2, A_2, B_2)$ be two m-polar fuzzy graphs of the graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively such that $V_1 \cap V_2 = \emptyset$. The direct product of G_1 and G_2 is defined to be the m-polar fuzzy graph $G_1 \sqcap G_2 = (V_1 \times V_2, A_1 \sqcap A_2, B_1 \sqcap B_2)$ of the graph $G^* = (V_1 \times V_2, E)$ where $E = \{(u_1, v_1)(u_2, v_2) | u_1u_2 \in E_1, v_1v_2 \in E_2\} \subseteq \widetilde{V_1 \times V_2}^2$ and for each $i = 1, 2, \ldots, m$,*

(i) $p_i \circ (A_1 \sqcap A_2)(x_1, x_2) = p_i \circ A_1(x_1) \wedge p_i \circ A_2(x_2)$ *for all* $(x_1, x_2) \in V_1 \times V_2$.
(ii) $p_i \circ (B_1 \sqcap B_2)((x_1, x_2)(y_1, y_2)) = p_i \circ B_1(x_1y_1) \wedge p_i \circ B_2(x_2y_2)$ *for all* $x_1y_1 \in E_1$ *and* $x_2y_2 \in E_2$.
(iii) $p_i \circ (B_1 \sqcap B_2)((x_1, x_2)(y_1, y_2)) = 0$ *for all* $(x_1, x_2)(y_1, y_2) \in (\widetilde{V_1 \times V_2}^2 - E)$.

Definition 2.11. [13] *The semi-strong product of two m-polar fuzzy graphs $G_1 = (V_1, A_1, B_1)$ of $G_1^* = (V_1, E_1)$ and $G_2 = (V_2, A_2, B_2)$ of $G_2^* = (V_2, E_2)$, where it is assumed that $V_1 \cap V_2 = \emptyset$, is defined to be the m-polar fuzzy graph $G_1 \bullet G_2 = (V_1 \times V_2, A_1 \bullet A_2, B_1 \bullet B_2)$ of $G^* = (V_1 \times V_2, E)$, where $E = \{(u, v_1)(u, v_2) | u \in V_1, v_1v_2 \in E_2\} \cup \{(u_1, v_1)(u_2, v_2) | u_1u_2 \in E_1, v_1v_2 \in E_2\} \subseteq \widetilde{V_1 \times V_2}^2$ satisfying the following: for each $i = 1, 2, \ldots, m$,*

(i) $p_i \circ (A_1 \bullet A_2)(x_1, x_2) = p_i \circ A_1(x_1) \wedge p_i \circ A_2(x_2)$ *for all* $(u, v) \in V_1 \times V_2$.
(ii) $p_i \circ (B_1 \bullet B_2)((x, x_2)(x, y_2)) = p_i \circ A_1(x) \wedge p_i \circ B_2(x_2y_2)$ *for all* $x \in V_1$ *and* $x_2y_2 \in E_2$.
(iii) $p_i \circ (B_1 \bullet B_2)((x_1, x_2)(y_1, y_2)) = p_i \circ B_1(x_1y_1) \wedge p_i \circ B_2(x_2y_2)$ *for all* $x_1y_1 \in E_1$ *and* $x_2y_2 \in E_2$.
(iv) $p_i \circ (B_1 \bullet B_2)((x_1, x_2)(y_1, y_2)) = 0$ *for all* $(x_1, x_2)(y_1, y_2) \in (\widetilde{V_1 \times V_2}^2 - E)$.

Definition 2.12. [13] *The strong product of two m-polar fuzzy graphs $G_1 = (V_1, A_1, B_1)$ of $G_1^* = (V_1, E_1)$ and $G_2 = (V_2, A_2, B_2)$ of $G_2^* = (V_2, E_2)$ such that $V_1 \cap V_2 = \emptyset$, is defined to be the m-polar fuzzy graph $G_1 \otimes G_2 = (V_1 \times V_2, A_1 \otimes A_2, B_1 \otimes B_2)$ of $G^* = (V_1 \times V_2, E)$, where $E = \{(u, v_1)(u, v_2) | u \in V_1, v_1v_2 \in E_2\} \cup \{(u_1, w)(u_2, w) | w \in V_2, u_1u_2 \in E_1\} \cup \{(u_1, v_1)(u_2, v_2) | u_1u_2 \in E_1, v_1v_2 \in E_2\} \subseteq \widetilde{V_1 \times V_2}^2$ such that the following condition holds: for each $i = 1, 2, \ldots, m$,*

(i) $p_i \circ (A_1 \otimes A_2)(x_1, x_2) = p_i \circ A_1(x_1) \wedge p_i \circ A_2(x_2)$ *for all* $(x_1, x_2) \in V_1 \times V_2$.
(ii) $p_i \circ (B_1 \otimes B_2)((x, x_2)(x, y_2)) = p_i \circ A_1(x) \wedge p_i \circ B_2(x_2y_2)$ *for all* $x \in V_1$ *and* $x_2y_2 \in E_2$.
(iii) $p_i \circ (B_1 \otimes B_2)((x_1, x)(y_1, x)) = p_i \circ B_1(x_1y_1) \wedge p_i \circ A_2(x)$ *for all* $x \in V_2$ *and* $x_1y_1 \in E_1$.
(iv) $p_i \circ (B_1 \otimes B_2)((x_1, y_1)(x_2, x_2)) = p_i \circ B_1(x_1x_2) \wedge p_i \circ B_2(y_1y_2)$ *for all* $x_1x_2 \in E_1$ *and* $y_1y_2 \in E_2$, *and*
(v) $p_i \circ (B_1 \otimes B_2)((x_1, y_1)(x_2, x_2)) = 0$ *for all* $(x_1, y_1)(x_2, x_2) \in (\widetilde{V_1 \times V_2}^2 - E)$.

Definition 2.13. [2] *Let* $G = (V, A, B)$ *be an* m*-polar fuzzy graph of* $G^* = (V, E)$*. The open neighborhood degree of a vertex* v *in* G *is defined by* $deg(v) = (p_1 \circ deg(v), p_2 \circ deg(v), \cdots, p_m \circ deg(v))$*, where* $p_i \circ deg(v) = \sum_{\substack{u \neq v \\ uv \in E}} p_i \circ B(uv)$*,* $i = 1, 2, \ldots, m$*. If all the vertices of* G *have the same open neighborhood degree, then* G *is called regular* m*-polar fuzzy graph.*

Definition 2.14. [2] *Let* $G = (V, A, B)$ *be an* m*-polar fuzzy graph of* $G^* = (V, E)$*. The closed neighborhood degree of a vertex* v *is defined by* $deg[v] = (p_1 \circ deg[v], p_2 \circ deg[v], \cdots, p_m \circ deg[v])$*, where* $p_i \circ deg[v] = p_i \circ deg(v) + p_i \circ A(v)$*,* $i = 1, 2, \ldots, m$*. If each vertex of* G *has the same closed neighborhood degree, then* G *is called totally regular* m*-polar fuzzy graph.*

3 Degrees of Vertices in m-polar Fuzzy Graphs

Operations in m-polar fuzzy graph is a great tool to consider large m-polar fuzzy graph as a combination of small m-polar fuzzy graphs and to derive its properties from those of the smaller ones. Also, they are conveniently used in many combinatorial applications. In various situations they present a suitable construction means. For examples in partition theory we deal with complex objects. A typical such object is a fuzzy graph and fuzzy hypergraph with large chromatic number that we do not know how to compute exactly the chromatic number of these graphs. In such cases, these operations have the main role in solving problems. Hence, in this section, we study about the degree of a vertex in m-polar fuzzy graphs which are obtained from two given m-polar fuzzy graphs G_1 and G_2 using the operations of Cartesian product, composition, direct product, semi-strong product and strong product.

4 Degree of a Vertex in Cartesian Product

Now, we compute the degree of a vertex in the Cartesian product. By the definition of Cartesian product, for any vertex $(x_1, x_2) \in V_1 \times V_2$, the degree of it is denoted by $d_{G_1 \times G_2}(x_1, x_2) = (p_1 \circ d_{G_1 \times G_2}(x_1, x_2), p_2 \circ d_{G_1 \times G_2}(x_1, x_2), \cdots, p_m \circ d_{G_1 \times G_2}(x_1, x_2))$ and is defined by for $i = 1, 2, \ldots, m$,

$$\begin{aligned} p_i \circ d_{G_1 \times G_2}(x_1, x_2) &= \sum_{(x_1,x_2)(y_1,y_2) \in E} p_i \circ (B_1 \times B_2)((x_1, x_2)(y_1, y_2)) \\ &= \sum_{x_1 = y_1, x_2y_2 \in E_2} p_i \circ A_1(x_1) \wedge p_i \circ B_2(x_2y_2) \\ &\quad + \sum_{x_2 = y_2, x_1y_1 \in E_1} p_i \circ A_2(x_2) \wedge p_i \circ B_1(x_1y_1). \end{aligned}$$

Theorem 4.1. *Let* $G_1 = (V_1, A_1, B_1)$ *and* $G_2 = (V_2, A_2, B_2)$ *be two* m*-polar fuzzy graphs. If* $B_2 \subseteq A_1$ *and* $B_1 \subseteq A_2$*, then* $d_{G_1 \times G_2}(x_1, x_2) = d_{G_1}(x_1) + d_{G_2}(x_2)$ *for all* $(x_1, x_2) \in V_1 \times V_2$*.*

Proof. For each $i = 1, 2, \ldots, m$, we have

$$\begin{aligned} p_i \circ d_{G_1 \times G_2}(x_1, x_2) &= \sum_{x_1 = y_1, x_2y_2 \in E_2} p_i \circ A_1(x_1) \wedge p_i \circ B_2(x_2y_2) \\ &\quad + \sum_{x_2 = y_2, x_1y_1 \in E_1} p_i \circ A_2(x_2) \wedge p_i \circ B_1(x_1y_1) \\ &= \sum_{x_2y_2 \in E_2} p_i \circ B_2(x_2y_2) + \sum_{x_1y_1 \in E_1} p_i \circ B_1(x_1y_1) \\ &= p_i \circ d_{G_1}(x_1) + p_i \circ d_{G_2}(x_2). \end{aligned}$$

Hence, $d_{G_1 \times G_2}(x_1, x_2) = d_{G_1}(x_1) + d_{G_2}(x_2)$. □

Example 4.2. *Let us consider the 3-polar fuzzy graphs G_1, G_2 and their Cartesian product $G_1 \times G_2$ (see Fig. 2). Here, we see that $B_2 \subseteq A_1$ and $B_1 \subseteq A_2$ in Fig. 2. So, by Theorem 4.1,*

$$\begin{aligned} p_1 \circ d_{G_1 \times G_2}(x_1, x_2) &= p_1 \circ d_{G_1}(x_1) + p_1 \circ d_{G_2}(x_2) = 0.4 + 0.3 = 0.7, \\ p_2 \circ d_{G_1 \times G_2}(x_1, x_2) &= p_2 \circ d_{G_1}(x_1) + p_2 \circ d_{G_2}(x_2) = 0.3 + 0.2 = 0.5, \\ p_3 \circ d_{G_1 \times G_2}(x_1, x_2) &= p_3 \circ d_{G_1}(x_1) + p_3 \circ d_{G_2}(x_2) = 0.2 + 0.3 = 0.5. \end{aligned}$$

So, $d_{G_1 \times G_2}(x_1, x_2) = (0.7, 0.5, 0.5)$. Also, from the Fig. 2,

$$d_{G_1 \times G_2}(x_1, x_2) = (0.3 + 0.4, 0.2 + 0.3, 0.3 + 0.2) = (0.7, 0.5, 0.5).$$

Hence, $d_{G_1 \times G_2}(x_1, x_2) = (0.7, 0.5, 0.5)$. Similarly, we can find the degrees of all other vertices in $G_1 \times G_2$. This can also be verified from the Fig. 2.

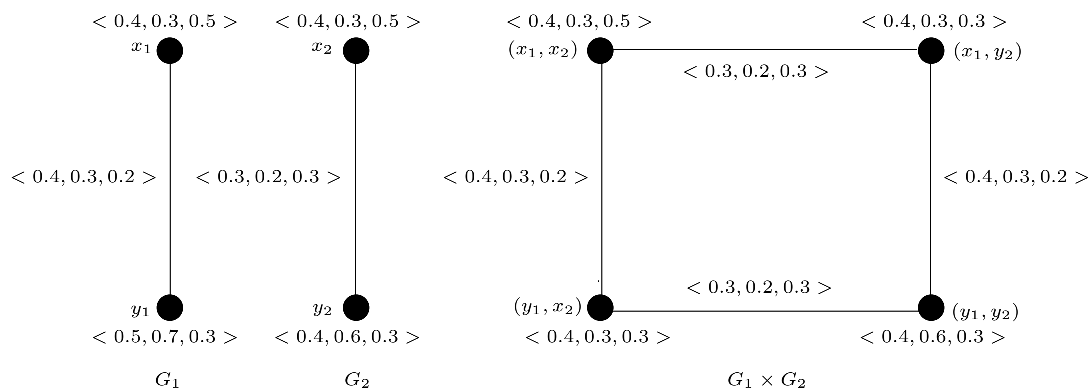

Figure 2: Cartesian product of G_1 and G_2

Theorem 4.3. *Let $G_1 = (V_1, A_1, B_1)$ and $G_2 = (V_2, A_2, B_2)$ be two m-polar fuzzy graphs such that $A_1 \subseteq B_2$. Then $B_1 \subseteq A_2$ and conversely.*

Proof. By definition of m-polar fuzzy graphs, we have $p_i \circ B_j(xy) \leq min\{p_i \circ A_j(x), p_i \circ A_j(y)\}$ for all $xy \in \widetilde{V^2}$, $i = 1, 2, \ldots, m$ and $j = 1, 2$.

Therefore, $p_i \circ B_j \leq max\{p_i \circ A_j\}$ and $min\{p_i \circ B_j\} \leq p_i \circ A_j$ for $i = 1, 2, \ldots, m$ and $j = 1, 2$. Also, since $A_1 \subseteq B_2$ therefore, $max\{p_i \circ A_1\} \leq min\{p_i \circ B_2\}$ for $i = 1, 2, \ldots, m$.

Hence, for $i = 1, 2, \ldots, m$, $p_i \circ B_1 \leq max\{p_i \circ A_1\} \leq min\{p_i \circ B_2\} \leq p_i \circ A_2$, i.e., $B_1 \subseteq A_2$. □

Theorem 4.4. *Let $G_1 = (V_1, A_1, B_1)$ and $G_2 = (V_2, A_2, B_2)$ be two m-polar fuzzy graphs.*

(i) If $A_1 \subseteq B_2$ and A_1 is constant function with $A_1(x) = (c_1, c_2, \cdots, c_m) = c$ for all $x \in V_1$, then $d_{G_1 \times G_2}(x_1, x_2) = d_{G_1}(x_1) + c d_{G_2^*}(x_2)$.

(ii) If $A_2 \subseteq B_1$ and A_2 is constant function with $A_2(x) = (k_1, k_2, \cdots, k_m) = k$ for all $x \in V_2$, then $d_{G_1 \times G_2}(x_1, x_2) = d_{G_2}(x_2) + k d_{G_1^*}(x_1)$.

Proof. (i) Because $A_1 \subseteq B_2$, by Theorem 4.3, $B_1 \subseteq A_2$. Then, for $i = 1, 2, \ldots, m$, we have

$$\begin{aligned} p_i \circ d_{G_1 \times G_2}(x_1, x_2) &= \sum_{x_1 = y_1, x_2 y_2 \in E_2} p_i \circ A_1(x_1) \wedge p_i \circ B_2(x_2 y_2) \\ &\quad + \sum_{x_2 = y_2, x_1 y_1 \in E_1} p_i \circ A_2(x_2) \wedge p_i \circ B_1(x_1 y_1) \\ &= \sum_{x_2 y_2 \in E_2} p_i \circ A_1(x_1) + \sum_{x_1 y_1 \in E_1} p_i \circ B_1(x_1 y_1) \\ &= \sum_{x_2 y_2 \in E_2} c_i + p_i \circ d_{G_1}(x_1) = c_i d_{G_2^*}(x_2) + p_i \circ d_{G_1}(x_1). \end{aligned}$$

Hence, $d_{G_1 \times G_2}(x_1, x_2) = d_{G_1}(x_1) + c d_{G_2^*}(x_2)$.

(ii) Similarly to the above. □

5 Degree of a Vertex in Composition

Now, we calculate the degree of a vertex in the composition of two m-polar fuzzy graphs. By the definition of composition, for any vertex $(x_1, x_2) \in V_1 \times V_2$, the degree of it is denoted by $d_{G_1[G_2]}(x_1, x_2) = (p_1 \circ d_{G_1[G_2]}(x_1, x_2), p_2 \circ d_{G_1[G_2]}(x_1, x_2), \cdots, p_m \circ d_{G_1[G_2]}(x_1, x_2))$ and is defined by for $i = 1, 2, \ldots, m$,

$$\begin{aligned} p_i \circ d_{G_1[G_2]}(x_1, x_2) &= \sum_{(x_1,x_2)(y_1,y_2)\in E} p_i \circ (B_1 \circ B_2)((x_1, x_2)(y_1, y_2)) \\ &= \sum_{x_1=y_1, x_2y_2\in E_2} p_i \circ A_1(x_1) \wedge p_i \circ B_2(x_2y_2) \\ &\quad + \sum_{x_2=y_2, x_1y_1\in E_1} p_i \circ A_2(x_2) \wedge p_i \circ B_1(x_1y_1) \\ &\quad + \sum_{x_2\neq y_2, x_1y_1\in E_1} p_i \circ A_2(x_2) \wedge p_i \circ A_2(y_2) \wedge p_i \circ B_1(x_1y_1). \end{aligned}$$

Theorem 5.1. *Let $G_1 = (V_1, A_1, B_1)$ and $G_2 = (V_2, A_2, B_2)$ be two m-polar fuzzy graphs. If $B_2 \subseteq A_1$ and $B_1 \subseteq A_2$, then $d_{G_1[G_2]}(x_1, x_2) = |V_2| d_{G_1}(x_1) + d_{G_2}(x_2)$ for all $(x_1, x_2) \in V_1 \times V_2$.*

Proof. For each $i = 1, 2, \ldots, m$, we have

$$\begin{aligned} p_i \circ d_{G_1[G_2]}(x_1, x_2) &= \sum_{x_1=y_1, x_2y_2\in E_2} p_i \circ A_1(x_1) \wedge p_i \circ B_2(x_2y_2) \\ &\quad + \sum_{x_2=y_2, x_1y_1\in E_1} p_i \circ A_2(x_2) \wedge p_i \circ B_1(x_1y_1) \\ &\quad + \sum_{x_2\neq y_2, x_1y_1\in E_1} p_i \circ A_2(x_2) \wedge p_i \circ A_2(y_2) \wedge p_i \circ B_1(x_1y_1) \\ &= \sum_{x_2y_2\in E_2} p_i \circ B_2(x_2y_2) + \sum_{x_2=y_2, x_1y_1\in E_1} p_i \circ B_1(x_1y_1) \\ &\quad + \sum_{x_2\neq y_2, x_1y_1\in E_1} p_i \circ B_1(x_1y_1) \quad (\text{Since } p_i \circ A_1 \geq p_i \circ B_2 \text{ and } p_i \circ A_2 \geq p_i \circ B_1) \\ &= p_i \circ d_{G_2}(x_2) + |V_2| p_i \circ d_{G_1}(x_1). \end{aligned}$$

Hence, $d_{G_1[G_2]}(x_1, x_2) = |V_2| d_{G_1}(x_1) + d_{G_2}(x_2)$. □

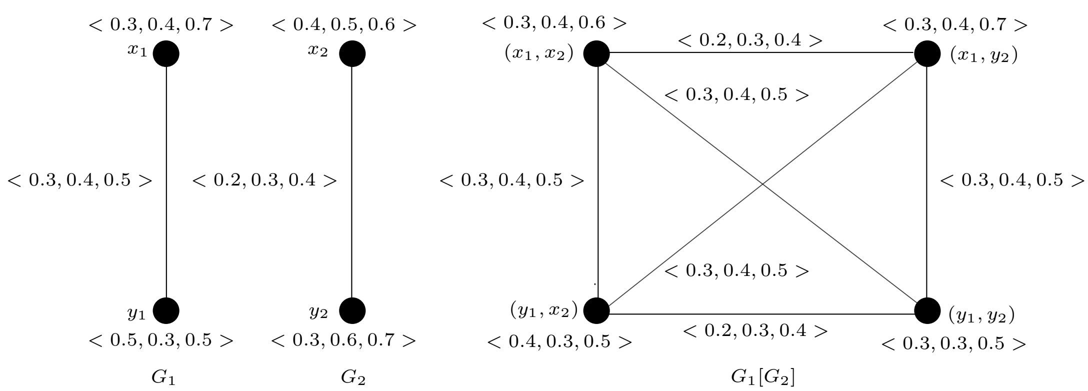

Figure 3: Composition of G_1 and G_2

Example 5.2. *Consider the 3-polar fuzzy graphs G_1, G_2 and their composition $G_1[G_2]$ (see Fig. 3). Here, $B_2 \subseteq A_1$ and $B_1 \subseteq A_2$. Therefore, by Theorem 5.1, we have*

$$\begin{aligned} p_1 \circ d_{G_1[G_2]}(x_1, x_2) &= p_1 \circ d_{G_1}(x_1)|V_2| + p_1 \circ d_{G_2}(x_2) = 0.3 \times 2 + 0.2 = 0.8, \\ p_2 \circ d_{G_1[G_2]}(x_1, x_2) &= p_2 \circ d_{G_1}(x_1)|V_2| + p_2 \circ d_{G_2}(x_2) = 0.4 \times 2 + 0.3 = 1.1, \\ p_3 \circ d_{G_1[G_2]}(x_1, x_2) &= p_3 \circ d_{G_1}(x_1)|V_2| + p_3 \circ d_{G_2}(x_2) = 0.5 \times 2 + 0.4 = 1.4. \end{aligned}$$

Therefore, $d_{G_1[G_2]}(x_1,x_2) = (0.8, 1.1, 1.4)$.
Again from Fig. 3,

$$\begin{aligned} d_{G_1[G_2]}(x_1,x_2) &= (p_1\circ d_{G_1[G_2]}(x_1,x_2), p_2\circ d_{G_1[G_2]}(x_1,x_2), p_3\circ d_{G_1[G_2]}(x_1,x_2)) \\ &= (0.3+0.2+0.3, 0.4+0.3+0.4, 0.5+0.4+0.5) \\ &= (0.8, 1.1, 1.4). \end{aligned}$$

In the same way, we can find the degree of all vertices in $G_1[G_2]$*. This can be verified from the Fig. 3.*

Theorem 5.3. *Let* $G_1 = (V_1, A_1, B_1)$ *and* $G_2 = (V_2, A_2, B_2)$ *be two* m*-polar fuzzy graphs.*

(i) If $A_1 \subseteq B_2$ *and* A_1 *is constant function with* $A_1(x) = (c_1, c_2, \cdots, c_m) = c$ *for all* $x \in V_1$*, then* $d_{G_1[G_2]}(x_1,x_2) = |V_2| d_{G_1}(x_1) + c d_{G_2^*}(x_2)$.

(ii) If $A_2 \subseteq B_1$ *and* A_2 *is constant function with* $A_2(x) = (k_1, k_2, \cdots, k_m) = k$ *for all* $x \in V_2$*, then* $d_{G_1[G_2]}(x_1,x_2) = d_{G_2}(x_2) + k|V_2| d_{G_1^*}(x_1)$.

Proof. (i) Because $A_1 \subseteq B_2$, by Theorem 4.3, $B_1 \subseteq A_2$. Now for $i = 1, 2, \ldots, m$ we have

$$\begin{aligned} p_i\circ d_{G_1[G_2]}(x_1,x_2) &= \sum_{x_1=y_1, x_2y_2\in E_2} p_i\circ A_1(x_1)\wedge p_i\circ B_2(x_2y_2) \\ &\quad + \sum_{x_2=y_2, x_1y_1\in E_1} p_i\circ A_2(x_2)\wedge p_i\circ B_1(x_1y_1) \\ &\quad + \sum_{x_2\neq y_2, x_1y_1\in E_1} p_i\circ A_2(x_2)\wedge p_i\circ A_2(y_2)\wedge p_i\circ B_1(x_1y_1) \\ &= \sum_{x_2y_2\in E_2} p_i\circ A_1(x_1) + \sum_{x_2=y_2, x_1y_1\in E_1} p_i\circ B_1(x_1y_1) \\ &\quad + \sum_{x_2\neq y_2, x_1y_1\in E_1} p_i\circ B_1(x_1y_1) \\ &= \sum_{x_2y_2\in E_2} c_i + |V_2| \sum_{x_1y_1\in E_1} p_i\circ B_1(x_1y_1) \\ &= c_i d_{G_2^*}(x_2) + |V_2| p_i\circ d_{G_1}(x_1). \end{aligned}$$

Hence, $d_{G_1[G_2]}(x_1,x_2) = |V_2| d_{G_1}(x_1) + c d_{G_2^*}(x_2)$.
(ii) Similarly to the above. □

6 Degree of a Vertex in Direct Product

Degree of a vertex in the direct product is as follows. By definition of direct product for any vertex $(x_1,x_2) \in V_1\times V_2$, the degree of (x_1,x_2) is denoted by $d_{G_1\sqcap G_2}(x_1,x_2) = (p_1\circ d_{G_1\sqcap G_2}(x_1,x_2), p_2\circ d_{G_1\sqcap G_2}(x_1,x_2), \cdots, p_m\circ d_{G_1\sqcap G_2}(x_1,x_2))$ and is defined by for $i = 1, 2, \ldots, m$,

$$\begin{aligned} p_i\circ d_{G_1\sqcap G_2}(x_1,x_2) &= \sum_{(x_1,x_2)(y_1,y_2)\in E} p_i\circ (B_1\sqcap B_2)((x_1,x_2)(y_1,y_2)) \\ &= \sum_{x_1y_1\in E_1, x_2y_2\in E_2} p_i\circ B_1(x_1y_1)\wedge p_i\circ B_2(x_2y_2). \end{aligned}$$

Theorem 6.1. *Let* $G_1 = (V_1, A_1, B_1)$ *and* $G_2 = (V_2, A_2, B_2)$ *be two* m*-polar fuzzy graphs. If* $B_1 \subseteq B_2$*, then* $d_{G_1\sqcap G_2}(x_1,x_2) = d_{G_1}(x_1)$*. Also, if* $B_2 \subseteq B_1$*, then* $d_{G_1\sqcap G_2}(x_1,x_2) = d_{G_2}(x_2)$ *for all* $(x_1,x_2) \in V_1\times V_2$.

Proof. Let $B_1 \subseteq B_2$ i.e., $p_i\circ B_2 \geq p_i\circ B_1$ for each $i = 1, 2, \ldots, m$. Then we have

$$\begin{aligned} p_i\circ d_{G_1\sqcap G_2}(x_1,x_2) &= \sum_{x_1y_1\in E_1, x_2y_2\in E_2} p_i\circ B_1(x_1y_1)\wedge p_i\circ B_2(x_2y_2) \\ &= \sum_{x_1y_1\in E_1} p_i\circ B_1(x_1y_1) = p_i\circ d_{G_1}(x_1) \quad \text{for } i = 1, 2, \ldots, m. \end{aligned}$$

Hence, $d_{G_1 \sqcap G_2}(x_1, x_2) = d_{G_1}(x_1)$.

Similarly, if $B_2 \subseteq B_1$ then $d_{G_1 \sqcap G_2}(x_1, x_2) = d_{G_2}(x_2)$. □

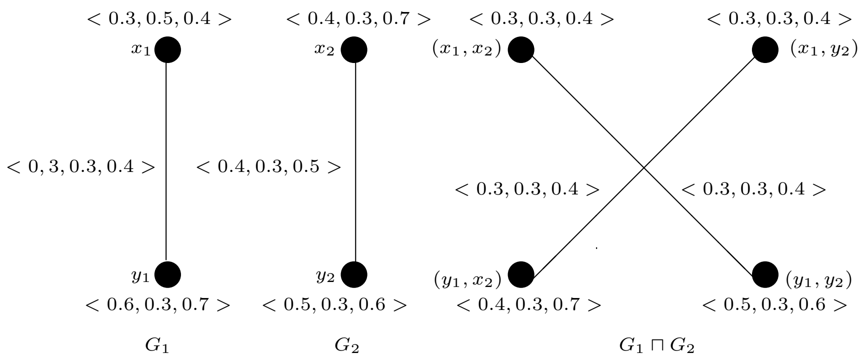

Figure 4: The direct product of G_1 and G_2

Example 6.2. *In this example we consider the direct product of two 3-polar fuzzy graphs and calculate the degree of vertices in the direct product. Let us now consider the 3-polar fuzzy graphs G_1, G_2 and their direct product $G_1 \sqcap G_2$ (see Fig. 4). Here, we see that $p_i \circ B_2 \geq p_i \circ B_1$ for $i = 1, 2, 3$ i.e., $B_1 \subseteq B_2$. Hence by Theorem 6.1, we have*

$$p_1 \circ d_{G_1 \sqcap G_2}(x_1, x_2) = p_1 \circ d_{G_1}(x_1) = 0.3,$$
$$p_2 \circ d_{G_1 \sqcap G_2}(x_1, x_2) = p_2 \circ d_{G_1}(x_1) = 0.3,$$
$$p_3 \circ d_{G_1 \sqcap G_2}(x_1, x_2) = p_3 \circ d_{G_1}(x_1) = 0.4.$$

So, $d_{G_1 \sqcap G_2}(x_1, x_2) = (0.3, 0.3, 0.4)$. Similarly, we can find the of all other vertices in $G_1 \sqcap G_2$. This can also be verified from Fig. 4.

7 Degree of a Vertex in Semi-strong Product

Next, we consider the semi-strong product of two m-polar fuzzy graphs and calculate the degrees of vertices of it. For any vertex vertex $(x_1, x_2) \in V_1 \times V_2$ in the semi-strong product $G_1 \bullet G_2$, the degree of (x_1, x_2) is denoted by $d_{G_1 \bullet G_2}(x_1, x_2) = (p_1 \circ d_{G_1 \bullet G_2}(x_1, x_2), p_2 \circ d_{G_1 \bullet G_2}(x_1, x_2), \cdots, p_m \circ d_{G_1 \bullet G_2}(x_1, x_2))$ and is defined by for $i = 1, 2, \ldots, m$,

$$\begin{aligned} p_i \circ d_{G_1 \bullet G_2}(x_1, x_2) &= \sum_{(x_1,x_2)(y_1,y_2) \in E} p_i \circ (B_1 \bullet B_2)((x_1, x_2)(y_1, y_2)) \\ &= \sum_{x_1 = y_1, x_2 y_2 \in E_2} p_i \circ A_1(x_1) \wedge p_i \circ B_2(x_2 y_2) \\ &\quad + \sum_{x_1 y_1 \in E_1, x_2 y_2 \in E_2} p_i \circ B_1(x_1 y_1) \wedge p_i \circ B_2(x_2 y_2). \end{aligned}$$

Theorem 7.1. *Let $G_1 = (V_1, A_1, B_1)$ and $G_2 = (V_2, A_2, B_2)$ be two m-polar fuzzy graphs. If $B_1 \subseteq B_2 \subseteq A_1$, then $d_{G_1 \bullet G_2}(x_1, x_2) = d_{G_1}(x_1) + d_{G_2}(x_2)$ for all $(x_1, x_2) \in V_1 \times V_2$.*

Proof. Let $B_1 \subseteq B_2 \subseteq A_1$ i.e., $p_i \circ A_1 \geq p_i \circ B_2 \geq p_i \circ B_1$ for each $i = 1, 2, \ldots, m$. Then, for $i = 1, 2, \ldots, m$ and $(x_1, x_2) \in V_1 \times V_2$,

$$\begin{aligned} p_i \circ d_{G_1 \bullet G_2}(x_1, x_2) &= \sum_{x_1 = y_1, x_2 y_2 \in E_2} p_i \circ A_1(x_1) \wedge p_i \bullet B_2(x_2 y_2) \\ &\quad + \sum_{x_1 y_1 \in E_1, x_2 y_2 \in E_2} p_i \circ B_1(x_1 y_1) \wedge p_i \bullet B_2(x_2 y_2) \\ &= \sum_{x_2 y_2 \in E_2} p_i \circ B_2(x_2 y_2) + \sum_{x_1 y_1 \in E_1} p_i \circ B_1(x_1 y_1) \\ &= p_i \circ d_{G_2}(x_2) + p_i \circ d_{G_1}(x_1). \end{aligned}$$

Hence, $d_{G_1 \bullet G_2}(x_1, x_2) = d_{G_1}(x_1) + d_{G_2}(x_2)$ for all $(x_1, x_2) \in V_1 \times V_2$. □

Example 7.2. *Consider the 3-polar fuzzy graphs G_1, G_2 and their semi-strong product $G_1 \bullet G_2$ (see Fig. 5). Here, we see that $p_i \circ A_1 \geq p_i \circ B_2 \geq p_i \circ B_1$ for $i = 1, 2, 3$ i.e., $B_1 \subseteq B_2 \subseteq A_1$. Hence by Theorem 7.1, we have*

$$\begin{aligned}
p_1 \circ d_{G_1 \bullet G_2}(x_1, x_2) &= p_1 \circ d_{G_1}(x_1) + p_1 \circ d_{G_2}(x_2) = 0.2 + 0.2 = 0.4, \\
p_2 \circ d_{G_1 \bullet G_2}(x_1, x_2) &= p_2 \circ d_{G_1}(x_1) + p_2 \circ d_{G_2}(x_2) = 0.2 + 0.3 = 0.5, \\
p_3 \circ d_{G_1 \bullet G_2}(x_1, x_2) &= p_3 \circ d_{G_1}(x_1) + p_3 \circ d_{G_2}(x_2) = 0.3 + 0.4 = 0.7.
\end{aligned}$$

So, $d_{G_1 \sqcap G_2}(x_1, x_2) = (0.4, 0.5, 0.7)$.

Again from the Fig. 5, we see that $d_{G_1 \sqcap G_2}(x_1, x_2) = (0.2 + 0.2, 0.2 + 0.3, 0.3 + 0.4) = (0.4, 0.5, 0.7)$. Similarly, we can find the degrees of all vertices in $G_1 \bullet G_2$ which can be verified from the figure also.

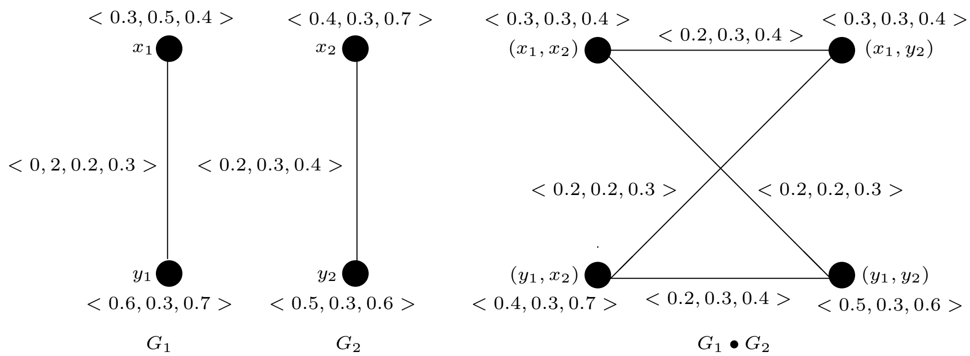

Figure 5: Semi-strong product of G_1 and G_2

8 Degree of a Vertex in Strong Product

Finally, we compute the degree of a vertex in strong product of m-polar fuzzy graphs. By definition of strong product for any vertex $(x_1, x_2) \in V_1 \times V_2$ in the strong product $G_1 \otimes G_2$, the degree of (x_1, x_2) is denoted by $d_{G_1 \otimes G_2}(x_1, x_2) = (p_1 \circ d_{G_1 \otimes G_2}(x_1, x_2), p_2 \circ d_{G_1 \otimes G_2}(x_1, x_2), \cdots, p_m \circ d_{G_1 \otimes G_2}(x_1, x_2))$ and is defined by for $i = 1, 2, \ldots, m$,

$$\begin{aligned}
p_i \circ d_{G_1 \otimes G_2}(x_1, x_2) &= \sum_{(x_1, x_2)(y_1, y_2) \in E} p_i \circ (B_1 \otimes B_2)((x_1, x_2)(y_1, y_2)) \\
&= \sum_{x_1 = y_1, x_2 y_2 \in E_2} p_i \circ A_1(x_1) \wedge p_i \circ B_2(x_2 y_2) \\
&\quad + \sum_{x_2 = y_2, x_1 y_1 \in E_1} p_i \circ A_2(x_2) \wedge p_i \circ B_1(x_1 y_1) \\
&\quad + \sum_{x_1 y_1 \in E_1, x_2 y_2 \in E_2} p_i \circ B_1(x_1 y_1) \wedge p_i \circ B_2(x_2 y_2).
\end{aligned}$$

Theorem 8.1. *Let $G_1 = (V_1, A_1, B_1)$ and $G_2 = (V_2, A_2, B_2)$ be two m-polar fuzzy graphs. If $B_2 \subseteq A_1$, $B_1 \subseteq A_2$ and $B_1 \subseteq B_2$, then $d_{G_1 \otimes G_2}(x_1, x_2) = |V_2| d_{G_1}(x_1) + d_{G_2}(x_2)$ for all $(x_1, x_2) \in V_1 \times V_2$.*

Proof. For $i = 1, 2, \ldots, m$ and $(x_1, x_2) \in V_1 \times V_2$ we have

$$\begin{aligned}
p_i \circ d_{G_1 \otimes G_2}(x_1, x_2) &= \sum_{(x_1, x_2)(y_1, y_2) \in E} p_i \circ (B_1 \otimes B_2)((x_1, x_2)(y_1, y_2)) \\
&= \sum_{x_1 = y_1, x_2 y_2 \in E_2} p_i \circ A_1(x_1) \wedge p_i \circ B_2(x_2 y_2) \\
&\quad + \sum_{x_2 = y_2, x_1 y_1 \in E_1} p_i \circ A_2(x_2) \wedge p_i \circ B_1(x_2 y_2)
\end{aligned}$$

$$
\begin{aligned}
&+ \sum_{x_1y_1 \in E_1, x_2y_2 \in E_2} p_i \circ B_1(x_1y_1) \wedge p_i \circ B_2(x_2y_2) \\
&= \sum_{x_2y_2 \in E_2} p_i \circ B_2(x_2y_2) + \sum_{x_2=y_2, x_1y_1 \in E_1} p_i \circ B_1(x_1y_1) \\
&+ \sum_{x_1y_1 \in E_1} p_i \circ B_1(x_1y_1) \\
&= p_i \circ d_{G_2}(x_2) + |V_2| p_i \circ d_{G_1}(x_1).
\end{aligned}
$$

This shows that, $d_{G_1 \otimes G_2}(x_1, x_2) = |V_2| d_{G_1}(x_1) + d_{G_2}(x_2)$. □

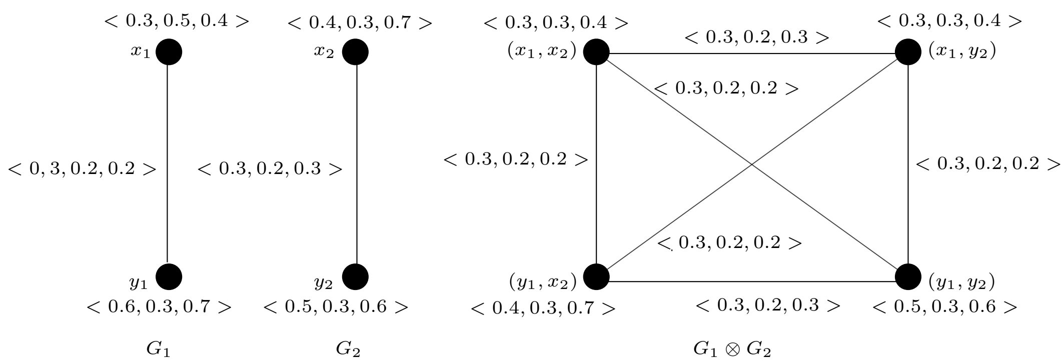

Figure 6: Strong product of G_1 and G_2

Example 8.2. *Let us consider the 3-polar fuzzy graphs G_1, G_2 and their strong product $G_1 \otimes G_2$ (see Fig.6). Here, $p_i \circ A_1 \geq p_i \circ B_2$, $p_i \circ A_2 \geq p_i \circ B_1$, and $p_i \circ B_1 \leq p_i \circ B_2$ for $i = 1, 2, 3$ i.e., $B_2 \subseteq A_1$, $B_1 \subseteq A_2$ and $B_1 \subseteq B_2$. Hence by Theorem 8.1, we have*

$$
\begin{aligned}
p_1 \circ d_{G_1 \otimes G_2}(x_1, x_2) &= p_1 \circ d_{G_2}(x_2) + |V_2| p_1 \circ d_{G_1}(x_1) = 0.3 + 2 \times 0.3 = 0.9, \\
p_2 \circ d_{G_1 \otimes G_2}(x_1, x_2) &= p_2 \circ d_{G_2}(x_2) + |V_2| p_2 \circ d_{G_1}(x_1) = 0.2 + 2 \times 0.2 = 0.6, \\
p_3 \circ d_{G_1 \otimes G_2}(x_1, x_2) &= p_3 \circ d_{G_2}(x_2) + |V_2| p_3 \circ d_{G_1}(x_1) = 0.3 + 2 \times 0.2 = 0.7.
\end{aligned}
$$

So, $d_{G_1 \otimes G_2}(x_1, x_2) = (0.9, 0.6, 0.7)$. Again, from the Figure we see that, $d_{G_1 \otimes G_2}(x_1, x_2) = (0.3+0.3+0.3, 0.2+0.2+0.2, 0.3+0.2+0.2) = (0.9, 0.6, 0.7)$. Similarly, we can find the degrees of all vertices in the strong product from the Theorem 8.1 as well as from the Fig. 6 also.

9 Application of m-polar Fuzzy Graphs

A directed graph (or digraph) is a graph whose edges have direction and called arcs (edges). Arrows on arcs are used to encode the directional information: an arc from vertex x to the vertex y indicates that one may move from x to y but not from y to x.

We write $xy \in E$ to mean $x \to y \in E$, and if $e = xy \in E$, we say x and y are adjacent such that x is a starting node and y is an ending node.

Definition 9.1. *An m-polar fuzzy digraph of a digraph $G^* = (V, E)$ is a pair $G = (V, A, B)$, where $A : V \to [0,1]^m$ is an m-polar fuzzy set on V and $B : \widetilde{V^2} \to [0,1]^m$ is an m-polar fuzzy set in $\widetilde{V^2}$ such that $p_i \circ B(xy) \leq min\{p_i \circ A(x), p_i \circ A(y)\}$ for all $xy \in \widetilde{V^2}$, for each $i = 1, 2, \ldots, m$ and $B(xy) = \mathbf{0}$ for all $xy \in (\widetilde{V^2} - E)$. B need not be symmetric i.e., $B(xy) \neq B(yx)$.*

Graph models have broad application in many disciplines of mathematics, social sciences, natural sciences and computer sciences. In studies of group behavior, it is inspected that many people can influence thinking of others. A digraph can be use to model such behavior and this graph is called an influence graph. We will present the influence of a person in a social group on Gtalk. Let $V = \{Asit, Sankar, Kartik, Prabir, Shakti\}$ be the set of five persons in a social group. The influence degree depends on the legitimate prevailing, unity

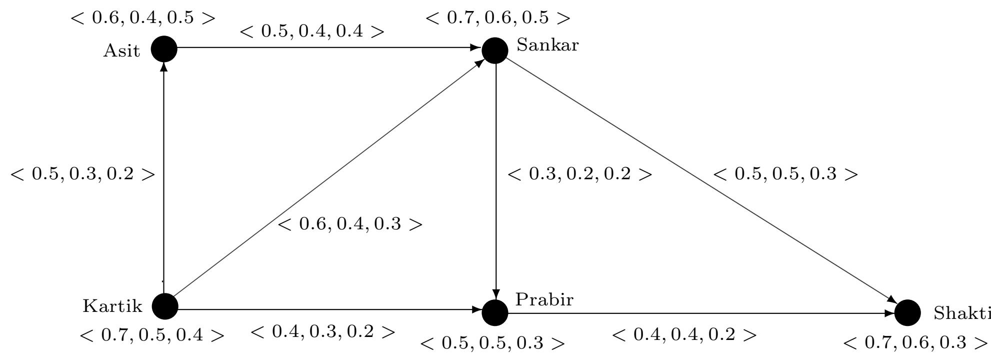

Figure 7: 3-polar fuzzy influence graph

building, and appealing to values. Then, we have a 3-polar fuzzy influence graph $G = (V, A, B)$, where vertices represent the person of a social group and edges represent the influence of a person on other.

From the above Fig. 7, we see that Kartik influence Asit, Sankar and Prabir. Kartik's 60% hold on Asit is due to legitimate prevailing, 40% is due to unity building, 50% is due to appealing to values. His 70% hold on Sankar is due to legitimate prevailing, 60% is due to unity building, 50% is due to appealing to values. Similarly, for Prabir also. Asit influence Sankar, Sankar influence Shakti and Prabir. So, we observe that Kartik is the most influential person in the group.

10 Conclusions

The theory of graph is an extremely useful tool in solving the combinatorial problems in different areas including algebra, number theory, geometry, topology, operation research, optimization, computer science, etc. The m-polar fuzzy models are the generalization of fuzzy models and give more precision, flexibility, and comparability to the system as compared to the classical and fuzzy models. So, in this paper we study about the degree of a vertex in Cartesian product, composition, direct product, semi-strong product and strong product of two m-polar fuzzy graphs. Our next plan is to extend our research work on m-polar fuzzy intersection graphs, m-polar fuzzy interval graphs, traversal of m-polar fuzzy hypergraphs, etc.

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