Journal of Uncertain Systems
Vol.11, No.4, pp.243-256, 2017
Online at: www.jus.org.uk

Designing Hub-and-Spoke Network with Uncertain Travel Times: A New Hybrid Methodology

Rui Wang, Kai Yang*, Lixing Yang
State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University
Beijing, 100044, China

Received 25 June 2017; Revised 20 August 2017

Abstract

This paper investigates a hub-and-spoke (H&S) network design problem, in which the uncertain travel times are characterized by fuzzy random variables. A new hybrid methodology by combining hub location-allocation modeling approach and equilibrium chance programming method is developed to model this problem. By analyzing the proposed model, we first handle equilibrium chance constraints and reduce them to their equivalent probability ones in some special case. After that, we adapt sample average approximation (SAA) method to probability constraint functions. Based upon the resulting SAA model, we design a simulated annealing algorithm. Finally, we conduct some numerical experiments to demonstrate the effectiveness of the proposed model and solution approach.

Keywords: hub-and-spoke network, fuzzy random variable, hub location-allocation, equilibrium chance programming, simulated annealing algorithm

1 Introduction

In a hub-and-spoke (H&S) network design, decisions regarding the location of hubs and the allocation of the spokes (non-hub nodes) to the located hubs directly affect the operational performance of freight transportation enterprises. One of the operational performance metrics is timeliness, which can be commonly characterized by the travel time between the origin-destination (O-D) pairs in the delivery network. In light of this, the freight companies strive to provide service within a predetermined delivery time requirement so as to be able to meet the time-based service guarantees. For example, FedEx has strategically designed its H&S network in order to serve the entire United States by providing overnight service to the entire nation and serving 95% of the global economy customers within 24-48 hours. Neglecting this factor may result in lost-opportunity and lost-goodwill, due to unsatisfied customers, which has a direct effect on the delivery service performance. Therefore, it is necessary to take into account the timeliness in the designing of H&S network.

During a design process in the H&S network, it is very difficult for decision makers to estimate the precise travel time which includes the transportation time on the links and the operational time spent at the hub(s). However, because of this scarcity of precise statistical travel time data, it is appropriate to use fuzzy variables to model the decision makers' experience. When asked about the travel time, a decision maker is only able to expresses his subjective judgment by the terms "about t days", which can be described as a normal fuzzy variable $n(t, \delta)$ with an average value t and a fluctuation δ. However, the average travel time t cannot be also considered deterministic since its value may vary because of traffic, weather conditions and speed variation. Under these circumstances, the probability theory can be applied to describe these kinds of objective uncertainties, in which the travel time approximately follows an uniform distribution expressed as $U[a, b]$. Therefore, the travel time between any O-D pairs need to consider both fuzziness and randomness [10, 24, 34].

To address the above issues, this paper integrates the hub location-allocation modeling approach with the equilibrium programming approach to model the H&S network design problem. The former approach selects

*Corresponding author.
Email: kaiyang@bjtu.edu.cn (K. Yang).

the location of hub facilities and allocates origin-destination (O-D) pairs to design an H&S network. The latter approach is to characterize the uncertain travel time between the O-D pairs and develops equilibrium chance constraints to satisfy delivery time requirements. By combining these above mentioned aspects, this paper present a new approach to establish a meaningful H&S network design problem with uncertain travel times. This paper intends to make the following contributions to the growing body of the H&S network design problem in the literature.

- Constructing a service-based objective function to minimize the maximum travel time between O-D pairs.
- Describing an approach to characterize uncertain travel times by possibility and probability distributions.
- Developing an equilibrium chance by credibility and probability measure to determine the decision maker's ability to meet delivery requirements.
- Proposing a parametric decomposition method to derive equivalents of mathematical model.
- Adapting a sample average approximation (SAA) method to estimate probabilistic constraints and reformulate the equivalent stochastic programming problem as its resulting SAA model.
- Developing a simulated annealing method for the resulting SAA model to achieve more reliable solutions.

The remainder of this paper is organized as follows. In Section 2, we briefly review related literature. In Section 3, we describe the modeling framework for the considered H&S network design problem. In Section 4, we present the formulation of the optimization model. In Section 5, we reduce the equilibrium chance constraints to their stochastic chance constraints in some special cases. In Section 6, we first suggest a SAA method to discretize continuous random parameters. Then, we design a simulated annealing algorithm to solve the resulting SAA model. In Section 7, some numerical experiments are conducted to demonstrate the effectiveness of the proposed model and solution approach. Finally, Section 8 draws some conclusions.

2 Literature Review

The H&S network design problem is conventionally called hub location-allocation problem, which is concerned with locating hub facilities and allocating spoke nodes to hubs. The study of hub location-allocation was formally proposed by O'Kelly [25, 26], who provided a quadratic integer programming formulation. Campbell [3] later developed a linear version known as the p-hub median problem. Skorin-Kapov et al. [30] obtained exact solutions to the p-hub median problem by developing tight linear relaxations of the formulation given by Campbell [3]. New MILP formulations of the hub location-allocation problem with fewer variables and constraints were developed by Ernst and Krishnamoorthy [7]. For a detailed review of hub location-allocation problem and its variations, see [1] and [9]. In order to solve the H&S network design problem, various heuristics based approaches have been used. These include the genetic algorithm [32], tabu search [2], ant colony optimization [27] and simulated annealing [8]. Among these solution approaches, the simulated annealing has been very successful in finding close to optimal solutions for large size problems. The simulated annealing proposed by Kirkpatrick [14] is a local search-based heuristic that is capable of escaping from being trapped into a local optimum by accepting, with small probability, worse solutions during its iterations. This feature can be an advantage, and has demonstrated considerable success in providing good solutions to many highly complicated combinatorial optimization problems as well as various real-world problems [4, 13, 15]. This paper contributes this literature by considering the uncertainty in the design of H&S networks. And it also complements this line of literature by developing a new hybrid heuristic algorithm by incorporating the SAA and the simulated annealing method.

The purpose of this paper is to study the H&S network design problems under uncertainty, which is an active research area in the literature. Marianov and Serra [20] focused on stochasticity at the hub nodes by representing hub airports as M/D/c queues and limiting through chance constraints the number of airplanes that can queue at an airport. Yang [33] presented a two-stage stochastic programming model for air freight hub location and flight route planning under seasonal demand variations. Sim [29] attempted to tackle hub location-allocation with stochastic time and utilized a chance-constrained formulation to model the minimum service-level requirement. Contreras et al. [6] studied stochastic uncapacitated hub location problem in which uncertainty is associated to demands and transportation costs. Mohammadi et al. [21] proposed a new stochastic multi-objective multi-mode transportation model for hub location-allocation problem under uncertainty. On the basis of fuzzy theory [37], some new methods have also been developed to model hub location-allocation problems. For instance, Chou [5] proposed a fuzzy multiple criteria decision-making model for evaluating and selecting the container transshipment hub port. Taghipourian et al. [31] presented a fuzzy integer linear programming approach to dynamic virtual hub location problem with the aim of minimizing the transportation cost in a network. Yang et al. [35] presented a new risk aversion hub location-allocation problem with fuzzy travel times by adopting value-at-risk criterion in the formulation of the objection function. Mohammadi and Moghaddama [22] proposed a bi-objective fuzzy hub location-allocation problem with the choice of a transportation mode over inter-hub links by incorporating a fuzzy M/M/1 queuing system. This paper differs from the above mentioned work in two aspects. First, since randomness and fuzziness often coexist in practical H&S network design problems, this paper extends the existing methods in the literature by adopting fuzzy random variables to describe uncertain travel times, characterized by both probability and possibility distributions. Second, this paper develops an equilibrium level by credibility and probability measure to determine the decision maker's ability to meet service time requirements.

In practical H&S network design problems, uncertainty may present both fuzziness and randomness. To the best of our knowledge, there are two papers in the literature dealing with the H&S network taking the mixed uncertainty into account. Mohammadi et al. [23] studied a novel sustainable hub location problem in which two new environmental-based cost functions accounting for air and noise pollution of vehicles are incorporated. To cope with hybrid uncertain data incorporated in the model, they proposed a mixed possibilistic-stochastic programming approach to construct the crisp counterpart. Yang et al. [36] addressed the planning and optimization of intermodal hub-and-spoke (IH&S) network considering mixed uncertainties in both transportation cost and travel time. In our study, we develop a new hybrid methodology to model the problem by taking in to account the hybrid uncertainty in transport process and hub operations.

3 Modeling Framework

This section presents a modeling framework for the design of H&S network with uncertain travel times. The proposed problem is represented by a graph in which nodes represent demand points and arcs represent transportation routes between the nodes.

In this modeling framework, a shipment between an origin node i and a destination node j can be traveled either through a pair of hubs (k, m) or through a single hub. The travel of shipments between each pair of O-D nodes consists of three parts (see Fig. 1). The first part is collection when shipments from the origin node i are consolidated at the origin hub k. The second part is transfer when shipments are transferred between the origin hub k and destination hub m. The third part is distribution when shipments are distributed to the destination node j. In the case of a single hub shipment ($k = m$), the travel consists of only two parts: collection and distribution.

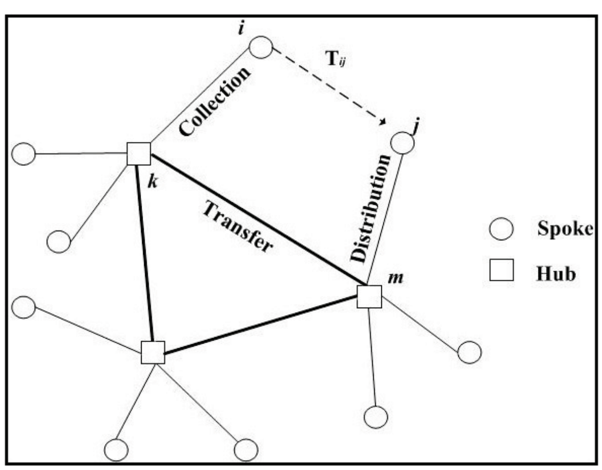

Figure 1: An example of H&S network

It is noteworthy that arrival shipments cannot be quickly transferred and need to be operated in a hub [12]. For example, arrival shipments must be unloaded, batched, break-bulked and loaded before transferring to their destinations (see Fig. 2). Hence, shipments must spend time at the hub(s), which is the sum of the operational times of each operation.

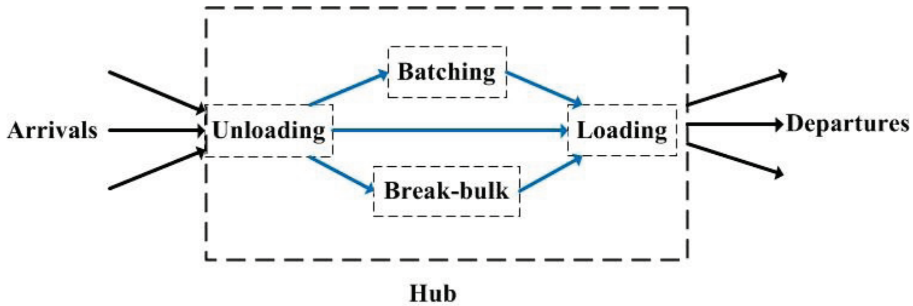

Figure 2: Hub operations

With the above analyses, the total travel time is a combination of transportation time between O-D pairs and the operational time required at the hub(s). It is widely recognized that uncertain features always exist in the H&S network, as it is often shown in the transportation process between the O-D pairs and the operation at hubs. In general, a H&S network design plan should be drawn up before the hub facilities are located in the network. That is, the transportation time and operational times cannot be determined in advance, leading to the inherent uncertainty, which is the motivation for considering the fuzzy random environment in this study.

Before formulating the mathematical model for this problem, the assumptions of this study are clarified first as follows:

(1) The hub network is complete.

(2) There is economics of scale incorporated by a discount factor for using the inter hub connections.

(3) Direct transportation between non-hub nodes is not allowed.

(4) The transportation time and the operational time are assumed to be uncertain and characterized by fuzzy random variables, governed by known possibility and probability distributions.

In the following sections, we construct the optimization model of the H&S network design problem with uncertain travel times in detail.

4 Formulation of the Optimization Model

4.1 Symbols and Parameters

Some basic symbols and parameters are listed below for the convenience of formulating the model.

N	the set of nodes in the network
Ω	the set of all samples
i, j	the spoke nodes
k, m	the hub nodes
T_{ij}	the fuzzy random transportation time between node i and node j
O_k	the fuzzy random operational time required at hub k
$T_{ij,\omega}$	the fuzzy transportation time between node i and node j for each $\omega \in \Omega$
$O_{k,\omega}$	the fuzzy operational time required at hub k for each $\omega \in \Omega$
α	the discount factor on links between hubs
β	the predetermined equilibrium level
p	the number of hubs to be selected

4.2 Decision Variables

The H&S network design needs to make decisions regarding the locations of hubs, the allocation of origin and destination nodes to the located hub, and routing the flows through the network. To facilitate the model formulation in this paper, the following decision variables are adopted:

(1) For each pair $i, k \in N$, we define the following binary decision variables,

$$X_{ik} = \begin{cases} 1, & \text{if node } i \text{ is assigned to hub } k \\ 0, & \text{otherwise.} \end{cases}$$

(2) When $i = k$, the variable X_{kk} represents the establishment or not establishment of a hub at node k.

(3) We define additional binary decision variables Y_{ikmj} that represent path in network from node i to node j through hub k first then hub m, i.e.,

$$Y_{ikmj} = \begin{cases} 1, & \text{if exists a path from node } i \text{ to } j \text{ through hub } k \text{ first then } m \\ 0, & \text{otherwise.} \end{cases}$$

4.3 Formulation of the Objective Function

Noteworthy, the above-mentioned travel times are quantified as uncertain data due to lack of knowledge in estimating precise values for these coefficients and represented in the form of fuzzy random variables. Combining the credibility and probability measure, we suggest a new equilibrium method to minimize of the maximum travel time between each O-D pair that is the sum of transportation time on the links and operational time required at the hubs specifying the prescribed equilibrium level β in the sense that

$$\min\{f \mid \Pr\{\omega \in \Omega | \mathrm{Cr}\{(\underbrace{T_{ik,\omega} + \alpha T_{km,\omega} + T_{mj,\omega}} + \underbrace{O_{k,\omega} + O_{m,\omega} * \mathrm{sign}|k-m|})Y_{ikmj} \le f\} \ge \beta\} \ge \beta,$$
$$\forall i, j, k, m \in N\},$$

where the level β is taken twice to represent different meanings, the first β on the left represents credibility level, and the second β on the right represents probability level. The value of f represents a best time guarantee that can be offered to all customers for each O-D pair. For example, UPS offers both next-day and second day services. For shipments picked up on a given day, next-day service has guaranteed delivery by the early morning of the next day, typically before 10 AM, and second-day service has guaranteed delivery by the end of the second day [29].

In general, optimizing this objective function can be formulated as solving a equilibrium chance-constrained programming model [17, 18]. As a consequence, it can be expressed as the minimisation of the delivery time requirement, Z, subject to the equilibrium chance constraints, as given below:

min f such that $\Pr\{\omega \in \Omega | \mathrm{Cr}\{(T_{ik,\omega} + \alpha T_{km,\omega} + T_{mj,\omega} + O_{k,\omega} + O_{m,\omega} * \mathrm{sign}|k-m|)Y_{ikmj} \le f\} \ge \beta\} \ge \beta, \forall i, j, k, m \in N$.

4.4 Formulation of Basic Constraints

In this section, we present the basic constraints that the H&S network design must satisfy, i.e.,

$$Y_{ikmj} \ge X_{ik} + X_{jm} - 1, \forall i, j, k, m \in N, \tag{1}$$

$$\sum_{k \in N} X_{ik} = 1, \forall i \in N, \tag{2}$$

$$X_{ik} \le X_{kk}, \forall i, k \in N, \tag{3}$$

$$\sum_{k \in N} X_{kk} = p. \tag{4}$$

Constraints (1)-(4) are the constraints for hub locations and allocations. Constraints (1) ensure that path $i \to k \to m \to j$ is a valid path in network if and only if nodes i and j are assigned to hubs k and m, respectively, i.e., X_{ik}=X_{jm}=1. Constraints (2) impose single assignment of nodes to hubs. Constraints (3) state that a spoke node i can only be assigned to an open hub at node k. Constraint (4) requires that exactly p hubs are established in the H&S network.

4.5 Mathematical Model

Based on the description above, we now present a new approach to establish a meaningful H&S network design problem with uncertain travel times. The mathematical model is formally built as follows (**M**1, for short):

$$\begin{cases} \min & f \\ \text{s.t.:} & \Pr\{\omega \in \Omega | \mathrm{Cr}\{(T_{ik,\omega} + \alpha T_{km,\omega} + T_{mj,\omega} \\ & +O_{k,\omega} + O_{m,\omega} * \mathrm{sign}|k-m|)Y_{ikmj} \le f\} \ge \beta\} \ge \beta, \forall i,j,k,m \in N \\ & Y_{ikmj} \ge X_{ik} + X_{jm} - 1, \forall i,j,k,m \in N \\ & \sum_{k\in N} X_{ik} = 1, \forall i \in N \\ & X_{ik} \le X_{kk}, \forall i,k \in N \\ & \sum_{k\in N} X_{kk} = p \\ & X_{ik} \in \{0,1\}, \forall i,k \in N \\ & Y_{ikmj} \in \{0,1\}, \forall i,j,k,m \in N. \end{cases} \tag{5}$$

Existing studies have approved that traditional H&S model is NP-hard. The proposed model **M**1 can be reduced to the traditional one if all trave times are always deterministic, so it is also an NP-hard problem. Not only the entire problem is NP-hard, even when all hubs are fixed, the allocation and routing problem in model **M**1 is still NP-hard. On the other hand, the proposed model **M**1 is a significant and non-trivial extension, because it is more flexible problem reflecting on practical situations and decision maker's philosophy of modelling hybrid uncertainty. To solve model **M**1, we may encounter the difficult of calculating the equilibrium chance constraints defining problem. To overcome these difficulties, we reduce the equilibrium chance constraints to their equivalent stochastic ones in some special cases.

5 Theoretical Analysis of the Proposed Model

To solve the proposed model **M**1, the crux is to check equilibrium chance constraints effectively. Due to twofold uncertainty involved in the equilibrium constraints, we cannot do so in general case. One method is to estimate equilibrium constraints by approximation method [16], where a continuous fuzzy random vector is approximated by a sequence of discrete fuzzy random vectors. Another alternative method is to reduce equilibrium chance constraints to probability constraints or credibility constraints. In the next section, we concentrate on reducing the equilibrium chance constraints to their equivalent stochastic ones in some special cases.

5.1 Handing Equilibrium Chance Constraints

In this section, we assume the travel times are characterized by normal fuzzy random variables, and reduce the equilibrium chance constraints to their equivalent stochastic chance constraints.

Theorem 1. *Let the transportation time T_{ij} and the operational time O_k be the normal fuzzy random variables such that for each $\omega \in \Omega$, $T_{ij,\omega} = n\left(t_{ij}(\omega), a_{ij}^2(\omega)\right)$ and $O_{k,\omega} = n\left(o_k(\omega), b_k^2(\omega)\right)$ are mutually independent fuzzy variables. Suppose t_{ij}, o_k, a_{ij} and b_k are random variables for any $i,j,k,m \in N$. Then we have*
(i) If $0 < \beta < 1/2$, then $\Pr\{\omega \in \Omega | \mathrm{Cr}\{(T_{ik,\omega} + \alpha T_{km,\omega} + T_{mj,\omega} + O_{k,\omega} + O_{m,\omega} * sign|k-m|)Y_{ikmj} \le f\} \ge \beta\} \ge \beta$ reduces to

$$\begin{aligned} &\Pr\{(t_{ik}(\omega) + \alpha t_{km}(\omega) + t_{mj}(\omega) + o_k(\omega) + o_m(\omega) * sign|k-m| \\ &\quad -\sqrt{-2\ln(2-2\beta)}(a_{ik}(\omega) + \alpha a_{km}(\omega) + a_{mj}(\omega) + b_k(\omega) + b_m(\omega) * sign|k-m|))Y_{ikmj} \le f\} \ge \beta. \end{aligned}$$

(ii) If $1/2 \le \beta \le 1$, then $\Pr\{\omega \in \Omega | \mathrm{Cr}\{(T_{ik,\omega} + \alpha T_{km,\omega} + T_{mj,\omega} + O_{k,\omega} + O_{m,\omega} * sign|k-m|)Y_{ikmj} \le f\} \ge \beta\} \ge \beta$ reduces to

$$\begin{aligned} &\Pr\{(t_{ik}(\omega) + \alpha t_{km}(\omega) + t_{mj}(\omega) + o_k(\omega) + o_m(\omega) * sign|k-m| \\ &\quad +\sqrt{-2\ln(2-2\beta)}(a_{ik}(\omega) + \alpha a_{km}(\omega) + a_{mj}(\omega) + b_k(\omega) + b_m(\omega) * sign|k-m|))Y_{ikmj} \le f\} \ge \beta. \end{aligned}$$

Proof. Since $T_{ik,\omega}$, $T_{km,\omega}$, $T_{mj,\omega}$, $O_{m,\omega}$ and $O_{k,\omega}$ are mutually independent for each $\omega \in \Omega$, according to the properties of normal fuzzy variables [19], $T_{ik,\omega} + \alpha T_{km,\omega} + T_{mj,\omega} + O_{k,\omega} + O_{m,\omega} * \mathrm{sign}|k-m|$ is a normal fuzzy variable.

Now, we consider the following credibility constraint for path $i\to k\to m\to j$

$$\mathrm{Cr}\{T_{ik,\omega}+\alpha T_{km,\omega}+T_{mj,\omega}+O_{k,\omega}+O_{m,\omega}*\mathrm{sign}|k-m|\}\leq f\}\geq\beta.$$

When $0<\beta<1/2$, we have

$$\begin{aligned}&\mathrm{Cr}\{T_{ik,\omega}+\alpha T_{km,\omega}+T_{mj,\omega}+O_{k,\omega}+O_{m,\omega}*\mathrm{sign}|k-m|\leq f\}\geq\beta\\ &\Longleftrightarrow f\geq(T_{ik,\omega}+\alpha T_{km,\omega}+T_{mj,\omega}+O_{k,\omega}+O_{m,\omega}*\mathrm{sign}|k-m|)^L_{2\beta},\end{aligned}$$

where $(T_{ik,\omega}+\alpha T_{km,\omega}+T_{mj,\omega}+O_{k,\omega}+O_{m,\omega}*\mathrm{sign}|k-m|)^L_{2\beta}$ is the left extreme point of the 2β-cut of $T_{ik,\omega}+\alpha T_{km,\omega}+T_{mj,\omega}+O_{k,\omega}+O_{m,\omega}*\mathrm{sign}|k-m|$.

Thus, $\mathrm{Cr}\{T_{ik,\omega}+\alpha T_{km,\omega}+T_{mj,\omega}+O_{k,\omega}+O_{m,\omega}*\mathrm{sign}|k-m|\leq f\}\geq\beta$ is equivalent to

$$\begin{aligned}&t_{ik}(\omega)+\alpha t_{km}(\omega)+t_{mj}(\omega)+o_k(\omega)+o_m(\omega)*\mathrm{sign}|k-m|\\ &\quad-\sqrt{-2\ln(2-2\beta)}(a_{ik}(\omega)+\alpha a_{km}(\omega)+a_{mj}(\omega)+b_k(\omega)+b_m(\omega)*\mathrm{sign}|k-m|)\leq f.\end{aligned}$$

It then follows that $\mathrm{Cr}\{(T_{ik,\omega}+\alpha T_{km,\omega}+T_{mj,\omega}+O_{k,\omega}+O_{m,\omega}*\mathrm{sign}|k-m|)Y_{ikmj}\leq Z\}\geq\beta$ can be express as

$$\begin{aligned}&(t_{ik}(\omega)+\alpha t_{km}(\omega)+t_{mj}(\omega)+o_k(\omega)+o_m(\omega)*\mathrm{sign}|k-m|\\ &-\sqrt{-2\ln(2-2\beta)}(a_{ik}(\omega)+\alpha a_{km}(\omega)+a_{mj}(\omega)+b_k(\omega)+b_m(\omega)*\mathrm{sign}|k-m|))Y_{ikmj}\leq f.\end{aligned}$$

Similarly, when $1/2\leq\beta\leq 1$, then we can obtain

$$\begin{aligned}&\mathrm{Cr}\{T_{ik,\omega}+\alpha T_{km,\omega}+T_{mj,\omega}+O_{k,\omega}+O_{m,\omega}*\mathrm{sign}|k-m|\leq f\}\geq\beta\\ &\Longleftrightarrow f\geq(T_{ik,\omega}+\alpha T_{km,\omega}+T_{mj,\omega}+O_{k,\omega}+O_{m,\omega}*\mathrm{sign}|k-m|)^R_{2-2\beta},\end{aligned}$$

where $(T_{ik,\omega}+\alpha T_{km,\omega}+T_{mj,\omega}+O_{k,\omega}+O_{m,\omega}*\mathrm{sign}|k-m|)^R_{2-2\beta}$ is the right extreme point of the $(2-2\beta)$-cut of $T_{ik,\omega}+\alpha T_{km,\omega}+T_{mj,\omega}+O_{k,\omega}+O_{m,\omega}*\mathrm{sign}|k-m|$.

As a consequence, $\mathrm{Cr}\{T_{ik,\omega}+\alpha T_{km,\omega}+T_{mj,\omega}+O_{k,\omega}+O_{m,\omega}*\mathrm{sign}|k-m|\leq f\}\geq\beta$ is equivalent to

$$\begin{aligned}&t_{ik}(\omega)+\alpha t_{km}(\omega)+t_{mj}(\omega)+o_k(\omega)+o_m(\omega)*\mathrm{sign}|k-m|\\ &\quad+\sqrt{-2\ln(2-2\beta)}(a_{ik}(\omega)+\alpha a_{km}(\omega)+a_{mj}(\omega)+b_k(\omega)+b_m(\omega)*\mathrm{sign}|k-m|)\leq f.\end{aligned}$$

It then follows that $\mathrm{Cr}\{(T_{ik,\omega}+\alpha T_{km,\omega}+T_{mj,\omega}+O_{k,\omega}+O_{m,\omega}*\mathrm{sign}|k-m|)Y_{ikmj}\leq f\}\geq\beta$ can be expressed as

$$\begin{aligned}&(t_{ik}(\omega)+\alpha t_{km}(\omega)+t_{mj}(\omega)+o_k(\omega)+o_m(\omega)*\mathrm{sign}|k-m|\\ &+\sqrt{-2\ln(2-2\beta)}(a_{ik}(\omega)+\alpha a_{km}(\omega)+a_{mj}(\omega)+b_k(\omega)+b_m(\omega)*\mathrm{sign}|k-m|))Y_{ikmj}\leq f.\end{aligned}$$

The proof of the theorem is complete. □

5.2 Equivalent Stochastic Programming Model

This section explains a parametric decomposition method to divide the proposed model **M**1 into two equivalent stochastic programming models. In this way, the steps of the proposed solving approach can be summarized as follows.

In the case of $0<\beta<1/2$, according to the discussion in Theorem 1, we can transform problem (5) to the following stochastic programming model:

$$\begin{cases}\min & f\\ \text{s.t.:} & \Pr\{(t_{ik}(\omega)+\alpha t_{km}(\omega)+t_{mj}(\omega)+o_k(\omega)+o_m(\omega)*\mathrm{sign}|k-m|-\sqrt{-2\ln(2-2\beta)}(a_{ik}(\omega)\\ & +\alpha a_{km}(\omega)+a_{mj}(\omega)+b_k(\omega)+b_m(\omega)*\mathrm{sign}|k-m|))Y_{ikmj}\leq f\}\geq\beta,\forall i,j,k,m\in N\\ & \text{Constraints (1)}-(4).\end{cases}\tag{6}$$

In the case of $1/2 \leq \beta \leq 1$, problem (5) can be converted into the following stochastic programming model:

$$\begin{cases} \min & f \\ \text{s.t.:} & \Pr\{(t_{ik}(\omega) + \alpha t_{km}(\omega) + t_{mj}(\omega) + o_k(\omega) + o_m(\omega) * \text{sign}|k-m| + \sqrt{-2\ln(2-2\beta)}(a_{ik}(\omega) \\ & +\alpha a_{km}(\omega) + a_{mj}(\omega) + b_k(\omega) + b_m(\omega) * \text{sign}|k-m|))Y_{ikmj} \leq f\} \geq \beta, \forall i,j,k,m \in N \\ & \text{Constraints } (1)-(4). \end{cases} \tag{7}$$

Models (6) and (7) are stochastic programming problems. The conventional methods for solving probabilistic constrained problems involving continuous distributions are based on the derivation of deterministic equivalent formulations of the original problem. As we know, this conversion is usually hard to perform. In the current development, we assume that $\boldsymbol{\xi}_{ikmj} = (t_{ik}, t_{km}, t_{mj}, o_k, o_m, a_{ik}, a_{km}, a_{mj}, b_k, b_m)$ are random vectors described by joint probability density function, and the components of random vector are not necessary mutually independent, i.e., we allow for the interactions among uncertain travel times. In this case, we cannot turn the probabilistic constraints into their respective deterministic equivalents. One line of approach is to approximate the random parameters by a discrete one and let the discretization be finer and finer, hoping that the solutions of the approximate problem with a finite number of scenarios will converge to the optimal solution of the original problem. In the next section, we will present a solution method that integrates a sampling strategy, the SAA method, coupled with a simulated annealing algorithm to solve equivalent stochastic programming problems.

6 Solution Method

In practical situation, it is usually assumed that the equilibrium level $\beta \geq 1/2$. So, in the next section, we only discuss the solution method for equivalent stochastic programming problem (7). Furthermore, a same approach could be used in the case of $0 < \beta < 1/2$. The methodology incorporates a sampling technique, known as the SAA method, coupled with a simulated annealing algorithm.

6.1 SAA Method to Stochastic Function

The sample average approximation (SAA) method is an approach for solving stochastic programming problems by using Monte Carlo simulation. The SAA method has been well used in stochastic environment [11, 28]. In the SAA computational procedure, we first generate ω^s_{ikmj} from a probability space $(\Omega, \Sigma, \Pr)$ and produce random samples $\boldsymbol{\xi}^s_{ikmj} = \xi(\omega^s_{ikmj})$ for $s \in S_{ikmj}$. Equivalently, we generate random samples $\boldsymbol{\xi}^s_{ikmj}$ for $s \in S_{ikmj}$ according to the probability distribution of $\boldsymbol{\xi}_{ikmj}$.

We now compute probabilistic constraints according to the SAA method proposed above. For any $i,j,k,m \in N$, we denote by S_{ikmj} the finite set of scenarios characterizing the probability distribution of the random vector $\boldsymbol{\xi}_{ikmj}$. Let $\boldsymbol{\xi}^s_{ikmj} = \left(\hat{t}^s_{ik}, \hat{t}^s_{km}, \hat{t}^s_{mj}, \hat{o}^s_k, \hat{o}^s_m, \hat{a}^s_{ik}, \hat{a}^s_{km}, \hat{a}^s_{mj}, \hat{b}^s_k, \hat{b}^s_m\right)$ be the deterministic vector representing the joint realizations of the components $\boldsymbol{\xi}_{ikmj}$ under scenario s, $s \in S_{ikmj}$. The probabilities associated with scenarios are denoted by p^s_{ikmj}, $s \in S_{ikmj}$, where $p^s_{ikmj} > 0$ and $\sum_{s \in S_{ikmj}} p^s_{ikmj} = 1$.

In case of $1/2 \leq \beta \leq 1$, we consider the following probabilistic constraints:

$$\begin{aligned} &\Pr\{(t_{ik}(\omega) + \alpha t_{km}(\omega) + t_{mj}(\omega) + o_k(\omega) + o_m(\omega) * \text{sign}|k-m| \\ &\quad + \sqrt{-2\ln(2-2\beta)}(a_{ik}(\omega) + \alpha a_{km}(\omega) + a_{mj}(\omega) + b_k(\omega) + b_m(\omega) * \text{sign}|k-m|))Y_{ikmj} \leq f\} \geq \beta. \end{aligned}$$

By introducing a "big enough" constant M, one has

$$\begin{aligned} &(\hat{t}^s_{ik}(\omega) + \alpha \hat{t}^s_{km}(\omega) + \hat{t}^s_{mj}(\omega) + \hat{o}^s_k(\omega) + \hat{o}^s_m(\omega) * \text{sign}|k-m| + \sqrt{-2\ln(2-2\beta)}(\hat{a}^s_{ik}(\omega) \\ &\quad + \alpha \hat{a}^s_{km}(\omega) + \hat{a}^s_{mj}(\omega) + \hat{b}^s_k(\omega) + \hat{b}^s_m(\omega) * \text{sign}|k-m|))Y_{ikmj} - M \leq f, s \in S_{ikmj}. \end{aligned}$$

In addition, we introduce a vector Z_{ikmj} of binary variables whose components Z^s_{ikmj}, $s \in S_{ikmj}$ take value 0 if the corresponding constraint has to be satisfied and 1 otherwise.

According to the definition of probability measure, problem (7) can be approximated by the following SAA model (**M**2, for short):

$$\left\{\begin{array}{ll} \min & f \\ \text{s.t.:} & (\hat{t}^s_{ik}(\omega)+\alpha\hat{t}^s_{km}(\omega)+\hat{t}^s_{mj}(\omega)+\hat{o}^s_k(\omega)+\hat{o}^s_m(\omega)*\text{sign}|k-m|+\sqrt{-2\ln(2-2\beta)}(\hat{a}^s_{ik}(\omega) \\ & +\alpha\hat{a}^s_{km}(\omega)+\hat{a}^s_{mj}(\omega)+\hat{b}^s_k(\omega)+\hat{b}^s_m(\omega)*\text{sign}|k-m|))Y_{ikmj}-M\cdot Z^s_{ikmj}\le f, \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad \forall i,j,k,m\in N, s\in S_{ikmj} \\ & \sum_{s\in S_{ikmj}} p^s_{ikmj}Z^s_{ikmj}\le(1-\beta), \forall i,k,m,j\in N \\ & Z^s_{ikmj}\in\{0,1\}, \forall i,k,m,j\in N, s\in S_{ikmj} \\ & \text{Constraints } (1)-(4). \end{array}\right. \tag{8}$$

It is observed that $\sum_{s\in S_{ikmj}} p^s_{ikmj}Z^s_{ikmj}\le(1-\beta)$, for any $i,k,m,j\in N$, define a binary knapsack constraint ensuring that the violation of the stochastic constraints is limited to $(1-\beta)$.

In the reformulation introduced above, the number of constraints is replicated in the number of scenarios. Even for small size problems, the SAA model **M**2 is still a very large mixed-integer problem. Thus, traditional optimization methods such as linear programming plus branch and bound can become useless in terms of computing times. Furthermore, when using these methods, the number of branches is likely to increase dramatically because of the capacity constraints. Under this consideration, we focus our attention on heuristic solution approaches, which will be described in the next section.

6.2 Simulated Annealing Algorithm

This section proposes a simulated annealing algorithm to solve the SAA model **M**2 to optimality.

Initialize $(T_0, T_F, \nu, N_{non-improving}, I_{iter})$
Step 1: Randomly generate the initial solution X.
Step 2: Let $T=T_0$; $I=0$; F_{best}=obj(X); $X_{best}=X$;
Step 3: $I=I+1$;
Step 4: Generate a new solution $X^{'}$ based on X
If (Exist a group contains only a single node)
{
Generate a new solution $X^{'}$ from X by hybrid operation;
}
Else
{
Generate r =random$(0,1)$;
Case $0<r\le 0.4$: Generate a new solution $X^{'}$ from X by shift operation;
Case $0.4<r\le 1$: Generate a new solution $X^{'}$ from X by exchange operation;
}
Step 5: If Δ=obj$(X^{'})$-obj$(X)\le 0$ { Let $X=X^{'}$}
Else
{
Generate r =random$(0,1)$;
If $r<\exp(-\Delta/KT)$ let $X=X^{'}$}
}
Step 6: If (obj(X)<F_{best}) {$X_{best}=X$; F_{best}=obj(X); $N=0$; }
Step 7: If $I=I_{iter}$
{
$T=\nu T$; $I=0$; $N=N+1$;
}
Else { Go to **Step 3**;}
Step 8: If $T<T_F$ or $N=N_{non-improving}$ { Terminate the simulated annealing algorithm;}
Else { Go to **Step 3**;}

7 Numerical Experiments

In order to evaluate the performance of the proposed model and the proposed approach, this section conducts some numerical experiments based on a generated data set. The simulated annealing algorithm has been coded in C++ programming language. All numerical tests are carried out on a personal computer (SONY with Intel(R) Core(TM) Duo CPU @ 3.53Ghz and RAM 4.00GB), using the Microsoft Windows 7 operating system.

7.1 Test Problems Generation

This study randomly generates 10 nodes data set on the plane. For this data set, the x and y coordinates are randomly generated from the square region $[0,100]\times[0,100]$. Suppose that there is a link connecting every pair of nodes. We set the Euclidean distance $D_{ij}=\sqrt{(x_i-x_j)^2+(y_i-y_j)^2}$ for $i,j=1,2,\ldots,10$. To obtain fuzzy random instances, the travel times are assumed as uncertain parameters. The normal fuzzy random distributions are used for these uncertain parameters. Specifically, we assume that the transportation times and operational times are characterized by $T_{ik,\omega}=(t_{ik}(\omega),a_{ik}^2(\omega))$, $T_{km,\omega}=(t_{km}(\omega),a_{km}^2(\omega))$ and $T_{mj,\omega}=(t_{mj}(\omega),a_{jm}^2(\omega))$, $O_{k,\omega}=(o_k(\omega),b_k^2(\omega))$, and $O_{m,\omega}=(o_m(\omega),b_m^2(\omega))$, respectively. In addition, the random parameter $\boldsymbol{\xi}_{ikmj}=(t_{ik},t_{km},t_{mj},o_k,o_m,a_{ik},a_{km},a_{mj},b_k,b_m)$ has a 10-dimensional joint uniform distribution defined as:

$$U=\{(x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9,x_{10})|\gamma D_{ik}\le x_1\le \pi D_{ik},\gamma D_{km}\le x_2\le \pi D_{km},\gamma D_{mj}\le x_3\le \pi D_{mj},\\ 5\le x_4\le 10, 5\le x_5\le 10, 0\le x_6\le 1, 0\le x_7\le 1, 0\le x_8\le 1, 0\le x_9\le 1, 0\le x_{10}\le 1\},$$

where $\gamma=0.75$, $\pi=1.25$.

For the generated data set, we consider several experiments by varying the hub number value $p\in\{2,3\}$. For each hub number value, we provide results with two levels of inter-hub transportation discount $\alpha=0.2$ and 0.8. Each instance has been solved for three different equilibrium levels $\beta\in\{0.85,0.90,0.95\}$. Recall that during the SAA procedure, we need to generate sample points to discrete continuous random parameters. For simplicity, we set $S_{ikmj}=S$ for any $i,k,m,j=1,2,\ldots,10$. We test the proposed simulated annealing algorithm by using different sample size $S\in\{50,100,150,200,250,300\}$. The result of experiments indicates that we use a sample size $S=200$ to obtain good estimations of the optimal value of original problem.

In the simulated annealing algorithm procedure, parameter settings may have great influence on the computational results. Thus, a set of pilot experiments with the following combinations of parameters is conducted to identify the best combination of parameters for the proposed simulated annealing procedure:

$\nu=0.950,0.965,0.970,$

$I_{iter}=100,200,300,400,500,600,1000,$

$K=0.1,0.2,0.3,0.4,0.5.$

From the results of the pilot study, we observe that setting $\nu=0.970$, $I_{iter}=500$, and $K=0.3$ gave the best results. Therefore, these parameter values are used for further computational studies. Other parameters used in the experiment are: $T_0=1000$, $T_F=1$, and $N_{non-improving}=100$. Since $T_0\nu^{227}=1000\times 0.970^{227}<1=T_F$, the current temperature will fall below the final temperature after 227 temperature reductions. Thus, all the experiments are terminated after 227 iterations, or when X_{best} is not improved in 100 successive reductions in temperature. Each numerical experiment is performed 10 times.

7.2 Computational Results

Table 1 summarizes the computational results by the simulated annealing algorithm for the generated data set. The meanings of the column headings are as follows: the first column gives the hub number value p; column 2 presents the discount factor α; column 3 shows the different equilibrium level β; the columns under "Optimal" report the objective function value and the average CPU time requirement in seconds. Table 1 provides strong evidence that the proposed solution approach can solve the instances in a reasonable amount of time. These results serve as a useful reference for the decision makers in designing the H&S network topography.

Table 1: Results from simulated annealing algorithm

p	α	β	Optimal Objective value	CPU(sec)
2	0.2	0.85	109.064093	176
		0.90	110.867878	327
		0.95	113.958821	102
	0.8	0.85	134.076156	115
		0.90	137.372783	266
		0.95	139.894049	171
3	0.2	0.85	87.438718	285
		0.90	89.913589	181
		0.95	92.1402508	137
	0.8	0.85	129.518291	144
		0.90	133.401290	162
		0.95	136.607421	177

7.3 Sensitivity Analysis

In order to recognize the most significant parameter of the proposed model, we carry out sensitivity analysis in this subsection. This study complements our analytical results and gives us additional managerial insights and interpretations.

In order to extract some important managerial insights, the sensitivity of objective value with respect to the equilibrium level β and the number of hubs p in the network are investigated in Figs. 3 and 4, respectively. Fig. 3 illustrates the impact of the equilibrium level on the objective value for the problem with $\alpha = 0.2$ and $p = 3$. It can be seen that the objective value increases as the equilibrium level increases. This is an expected result because as the equilibrium level gets higher, the decision maker would choose a relatively larger delivery time requirement to meet customers for the service-level guarantee. Fig. 4 demonstrates the impact of varying p on the objective value for the problem with $\alpha = 0.2$ and $\beta = 0.85$. As p increases, the objective value is decreased. This can be explained as follows: by using a higher number of hubs, more flow units benefit from economies of scales of hub-to-hub links to be transferred to their destination with lower travel time. Therefore, with the method proposed in this paper, the decision maker can make better decisions.

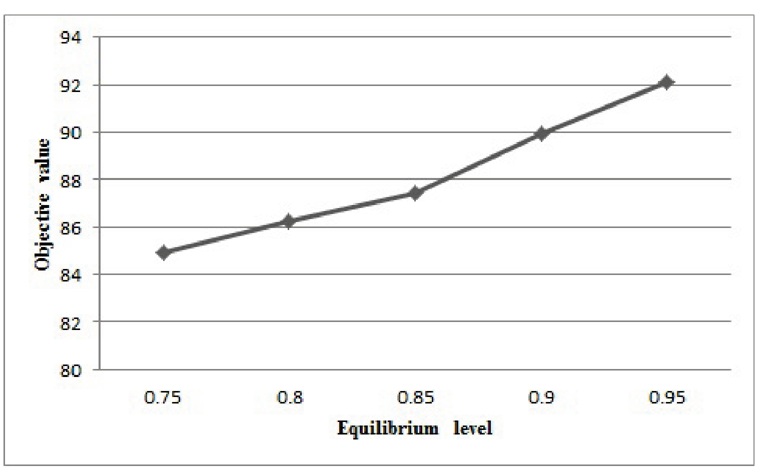

Figure 3: Objective value vs. equilibrium level

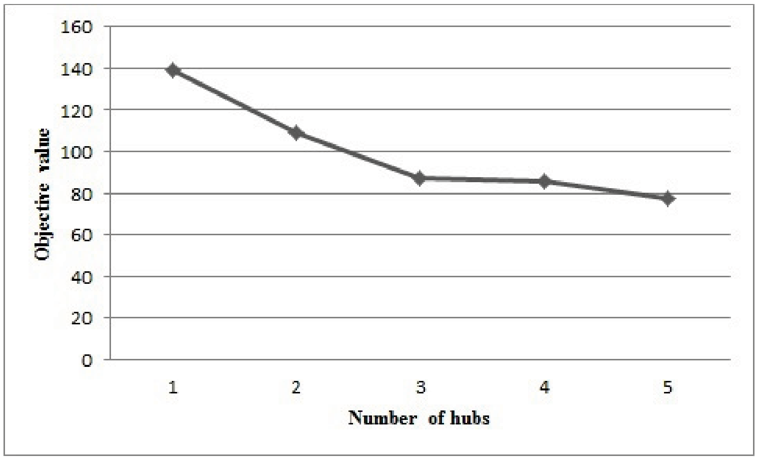

Figure 4: Objective value vs. number of hubs

8 Conclusions

In this paper, we have addressed a new type of H&S network design problem, where the travel times are characterized by fuzzy random travel times with known possibility and probability distributions. The major conclusions include the following several aspects:

(i) We developed a novel hybrid methodology to model the considered H&S network design problem and described an equilibrium level by credibility and probability measure to determine the decision maker's ability to meet service time requirements.

(ii) We proposed a parametric decomposition method to reduce the equilibrium chance constraints to their equivalent stochastic ones for normal fuzzy random travel times.

(iii) We adapted SAA method to probability constraint functions. Furthermore, we designed a simulated annealing algorithm to solve the resulting SAA model.

(iv) We conducted some numerical experiments based on a generated data set. Computational results were presented to better validate the performance of the proposed model and solution approach.

Acknowledgements

This research was supported by the National Natural Science Foundation of China under Grant No. 71422002, No. 71621001, No. 71701012 and No. 71631007, and the Fundamental Research Funds of Beijing Jiaotong University under Grant No. 2015RC075.

References

[1] Alumur, S., and B.Y. Kara, Network hub location problems: the state of the art, *European Journal of Operational Research*, vol.190, pp.1–21, 2008.

[2] Calik, H., Alumur, S., Kara, B., and O. Karasan, A tabu-search based heuristic for the hub covering problem over incomplete hub networks, *Computers & Operations Research*, vol.36, pp.3088–3096, 2009.

[3] Campbell, J.F., Integer programming formulations of discrete hub location problems, *European Journal of Operational Research*, vol.72, pp.387–405, 1994.

[4] Candalino, T.J., Kobza, J.E., and S.H. Jacobson, Designing optimal aviation baggage screening strategies using simulated annealing, *Computers & Operations Research*, vol.31, pp.1753–1767, 2004.

[5] Chou, C., Application of FMCDM model to selecting the hub location in the marine transportation: a case study in southeastern Asia, *Mathematical and Computer Modelling*, vol.51, pp.791–801, 2010.

[6] Contreras, I., Cordeau, J.F., and G. Laporte, Stochastic uncapacitated hub location, *European Journal of Operational Research*, vol.212, pp.518–528, 2011.

[7] Ernst, A.T., and M. Krishnamoorthy, Exact and heuristic algorithms for the uncapacitated multiple allocation p-hub median problem, *European Journal of Operational Research*, vol.104, pp.100–112, 1998.

[8] Ernst, A.T., and M. Krishnamoorthy, Solution algorithms for the capacitated single allocation hub location problem, *Annals of operations Research*, vol.86, pp.141–159, 1998.

[9] Farahani, R.Z., Hekmatfar, M., Arabani, A.B., and E. Nikbakhsh, Hub location problems: a review of models, classification, techniques and application, *Computers & Industrial Engineering*, vol.64, pp.1096–1109, 2013.

[10] Feng, Y.Q., Wu, W.C., Zhang, B.M., and J.W. Gao, Transmission line maintenance scheduling considering both randomness and fuzziness, *Journal of Uncertain Systems*, vol.5, no.4, pp.243–256, 2011

[11] Golenko-Ginzburg, D., Gonik, A., and Z. Laslo, Resource constrained scheduling simulation model for alternative stochastic network projects, *Mathematics and Computers in Simulation*, vol.63, pp.105–117, 2003.

[12] Ishfaq, R., and C.R. Sox, Intermodal logistics: the interplay of financial, operational and service issues, *Transportation Research Part E*, vol.46, pp.926–949, 2010.

[13] Kim, K.H., and K.C. Moon, Berth scheduling by simulated annealing, *Transportation Research Part B*, vol.37, pp.542–560, 2003.

[14] Kirkpatrick, S., Gelatt, C.D., and M.P. Vecchi, Optimization by simulated annealing, *Science*, vol.220, pp.671–680, 1983.

[15] Lee, D.H., Cao, Z., and Q. Meng, Scheduling of two-transtainer systems fo loading outbound containers in port container terminals with simulated annealing algorithm, *International Journal of Production Economics*, vol.107, pp.115–124, 2007.

[16] Liu, Y.K., The approximation method for two-stage fuzzy randomprogramming with recourse, *IEEE Transactions on Fuzzy Systems*, vol.15, pp.1197–1208, 2007.

[17] Liu, Y.K., and B. Liu, Fuzzy random variable: a scalar expected value operator, *Fuzzy Optimization and Decision Making*, vol.2, pp.143–160, 2003.

[18] Liu, Y.K., and B. Liu, Fuzzy random programming with equilibrium chance constraints, *Information Sciences*, vol.15, pp.363–395, 2005.

[19] Liu, Y.K., and J. Gao, The independence of fuzzy variables with applications to fuzzy random optimization, *International Journal of Uncertainty Fuzziness Knowledge-Based Systems*, vol.15, pp.1–20, 2007.

[20] Marianov, V., and D. Serra, Location models for airline hubs behaving as M/D/c queues, *Computers & Operations Research*, vol.30, pp.983–1003, 2003.

[21] Mohammadi, M., Jolai, F., and R. Tavakkoli-Moghaddam, Solving a new stochastic multi-mode p-hub covering location problem considering risk by a novel multi-objective algorithm, *Applied Mathematical Modelling*, vol.37, pp.10053–10073, 2013.

[22] Mohammadi, M., and R.T. Moghaddama, Design of a fuzzy bi-objective reliable p-hub center problem, *Journal of Intelligent & Fuzzy Systems*, vol.30, no.5, pp.2563–2580, 2016.

[23] Mohammadi, M., Torabi, S.A., and R. Tavakkoli-Moghaddam, Sustainable hub location under mixed uncertainty, *Transportation Research Part E*, vol.62, pp.89–115, 2014.

[24] Nematian, J., A new method for multi-objective linear programming models with fuzzy random variables, *Journal of Uncertain Systems*, vol.6, no.1, pp.38–50, 2012.

[25] O'Kelly, M.E., The location of interesting hub facilities, *Transportation Science*, vol.20, pp.92–106, 1986.

[26] O'Kelly, M.E., A quadratic integer program for the location of interacting hub facilities, *European Journal of Operational Research*, vol.32, pp.393–404, 1987.

[27] Randall, M., Solution approaches for the capacitated single allocation hub location problem using ant colony optimization, *Computational Optimization and Applications*, vol.39, pp.239–261, 2008.

[28] Sarper, H., Monte Carlo simulation for analysis of the optimum value distribution in stochastic mathematical programs, *Mathematics and Computers in Simulation*, vol.35, pp.469–480, 1993.

[29] Sim, T., Lowe, T.J., and B.W. Thomas, The stochastic p-hub center problem with service-level constraints, *Computers & Operations Research*, vol.36, pp.3166–3177, 2009.

[30] Skorin-Kapov, D., Skorin-Kapov, J., and M.E. O'Kelly, Tight linear programming relaxations of uncapacitated p-hub median problems, *European Journal of Operational Research*, vol.94, pp.582–593, 1996.

[31] Taghipourian, F., Mahdavi, I., Mahdavi-Amiri, N., and A. Makui, A fuzzy programming approach for dynamic virtual hub location problem, *Applied Mathematical Modelling*, vol.36, pp.3257–3270, 2012.

[32] Topcuoglu, H., Corut, F., Ermis, M., and G. Yilmaz, Solving the uncapacitated hub location using genetic algorithms, *Computers & Operations Research*, vol.32, pp.967–984, 2005.

[33] Yang, T.H., Stochastic air freight hub location and freight routes planning, *Applied Mathematical Modelling*, vol.33, pp.4424–4430, 2009.

[34] Yang, L., Gao, Z., and K. Li, Railway freight transportation planning with mixed uncertainty of randomness and fuzziness, *Applied Soft Computing*, vol.11, pp.778–792, 2011.

[35] Yang, K., Liu, Y., and G. Yang, Solving fuzzy p-hub center problem by genetic algorithm incorporating local search, *Applied Soft Computing*, vol.13, pp.2624–2632, 2013.

[36] Yang, K., Yang, L.X., and Z.Y. Gao, Planning and optimization of intermodal hub-and-spoke network under mixed uncertainty, *Transportation Research Part E*, vol.13, pp.2624–2632, 2016.

[37] Zadeh, L.A., Fuzzy sets, *Information and Control*, vol.8, pp.338–353, 1965.