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How to Predict Nesting Sites?

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Abstract

How to predict nesting sites? Usually, all we know is the past nesting sites, and the fact that the birds select a site which is optimal for them (in some reasonable sense), but we do not know the exact objective function describing this optimality. In this paper, we propose a way to make predictions in such a situation.

Keywords: nesting sites, linear discriminant analysis

1 Formulation of the Biological Problem

We observe nesting sites for a certain bird species. Our goals (see, e.g., [4, 8]) are:

- to analyze which criteria are important for selecting nesting sites, and
- to come up with formulas that would enable us to predict nesting sites.

2 Reformulating the Problem in Precise Terms

General description. Let $v_1, \ldots, v_n$ be parameters that may influence the selection of the nesting site: e.g., elevation, hydrology, vegetation level, etc. For each geographic location $\vec{x}$, we record the values of all these variables $v_1(\vec{x}), \ldots, v_n(\vec{x})$.

Main assumption. We assume that the birds select a nesting site based on the values of (some of) these quantities. Namely, a bird tries to maximize the value of some objective function $F(v_1, \cdots, v_n)$ depending on these values.

Simplifying assumption. Let us start with the simplest case, when the objective function is linear, i.e., when

$$F(v_1, \cdots, v_n) = \sum_{i=1}^{n} w_i \cdot v_i \quad (1)$$

for some weights w_i.

We assume that each year, each of the observed nesting sites $\vec{x}_j$ has the largest possible value of this objective function among all locations within the corresponding *Voronoi cell* C_j (see, e.g., [2, 3, 5] and references therein) – i.e., among all locations $\vec{x}$ which are closer to $\vec{x}_j$ than to any other nesting location.

Under this assumption, we would like to find the weights $w_1, \ldots, w_n$ that best explain the observed nesting sites.

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3 Analysis of the Problem

The fact that on the cell C_j, the linear function (1) attains its largest value at the site $\vec{x}_j$, means that

$$\sum_{i=1}^{n} w_i \cdot v_i(\vec{x}_j) \geq \sum_{i=1}^{n} w_i \cdot v_i(\vec{x}) \text{ for all } \vec{x} \in C_j.$$

In other words, we should have

$$\vec{w} \cdot \vec{a}(\vec{x}) \stackrel{\text{def}}{=} \sum_{i=1}^{n} w_i \cdot a_i(\vec{x}) \geq 0, \tag{2}$$

where we denoted $\vec{w} = (w_1, \cdots, w_n)$, $\vec{a}(\vec{x}) = (a_1(\vec{x}), \cdots, a_n(\vec{x}))$, and $a_i(\vec{x}) \stackrel{\text{def}}{=} v_i(\vec{x}_j) - v_i(\vec{x})$.

Similarly, we should have $w \cdot (-a(\vec{x})) \leq 0$ for all $\vec{x}$.

4 How Can We Solve This Problem?

This can be reduced to a known problem. From the mathematical viewpoint, this problem is similar to a *linear discriminant analysis* (see, e.g., [1, 6, 7]), when:

- we have two sets A and B and
- we need to select a hyperplane that separates them, i.e., a vector $\vec{w}$ for which $\vec{w} \cdot \vec{a} \geq 0$ for all $a \in A$ and $\vec{w} \cdot \vec{b} \leq 0$ for all $b \in B$.

In our case:

- A is the set of all the vectors $\vec{a}(\vec{x})$, and
- B is the set of all the vectors $-\vec{a}(\vec{x})$.

How to solve our problem. The standard way of solving this problem is to compute the mean $\vec{\mu}$ of all the vectors $\vec{a} \in A$, the covariance matrix Σ, and then to take $\vec{w} = \Sigma^{-1}\vec{\mu}$. So, in our case, we should do the following:

- compute all the vectors $\vec{a}(x)$ with components $a_i(\vec{x}) = v_i(\vec{x}_j) - v_i(\vec{x})$, where $\vec{x} \in C_j$; let N be the total number of such vectors;
- compute the average $\vec{\mu} = \sum_{\vec{x}} \vec{a}(\vec{x})/N$ of these vectors;
- compute the corresponding covariance matrix Σ with components

$$\Sigma_{ik} = \frac{1}{N} \cdot \sum_{\vec{x}} (a_i(\vec{x}) - \mu_i) \cdot (a_k(\vec{x}) - \mu_k); \tag{3}$$

- compute the desired weights as $\vec{w} = \Sigma^{-1}\vec{\mu}$, i.e., as a solution to a linear system $\Sigma\vec{w} = \vec{\mu}$.

5 Auxiliary Question: How Can We Gauge the Quality of the Resulting Prediction

To gauge the quality of the resulting prediction, for each cell C_j, we compute the location $\vec{c}_j$ at which the weighted combination $\vec{w} \cdot \vec{v}(\vec{x})$ attains its maximum. The mean square distance between these predicted nesting sites $\vec{c}_j$ and the actual nesting sites $\vec{x}_j$ can serve as a natural measure of prediction accuracy.

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