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Robust Optimal Decision for the Newsvendor Problem with Uncertain Market Demand

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Abstract

This paper considers a newsvendor problem with uncertain market demand. In the problem, the exact probability distribution is not known. The uncertain legacy loss includes the loss for overstock and the shortage penalty for stockout. The robust optimal decision is desired to minimize the expected legacy loss and the conditional value-at-risk (CVaR) about the legacy loss. Firstly three robust mean-CVaR models are built when the distribution varies in a box uncertainty set. Then the equivalent solvable forms of the three robust models are derived. Finally, the parameters' influences on the optimal order quantity and the corresponding performance are presented via numerical experiments.

Keywords: newsvendor problem, robust optimization, legacy loss, conditional value-at-risk, box uncertainty set

1 Introduction

The newsvendor problem is a famous problem of inventory management. The newsvendor's objective is to determine an optimal order quantity, which can balance the cost of ordering too many against the cost of ordering too few. In 1888 Edgeworth [8] used the newsvendor model to solve the problem of bank cash flow by taking it as a newsvendor problem. In 1950s newsvendor problem was extensively studied. With the development of the research, newsvendor models are widely used in various fields. Qin et al. [19] made a detailed review of the newsvendor problem. The newsvendor problem has become a hot topic in the current international research.

When retailers sell perishable goods for a short sales season, they often have to consider how to order the quantity of goods before the selling season to maximize their profits. For risk-neutral retailers, they usually adopt the expected value criterion to find the optimal order quantity which corresponds to the maximum profit. The profit function includes four parts: total sales revenue $r\min\{q, D\}$, total purchases cost cq, residual value $h(q-D)^+$ and shortage penalty $s(D-q)^+$. In the above parts, x^+=max$\{x, 0\}$, r is the unit selling price, c is the unit ordering cost, h is the salvage price, s is the shortage penalty price, q is the order quantity before the selling season and D is the real market demand, and normally $r > c > h$. Many previous studies obtained the optimal order quantity which maximize the expected profit.

When the order quantity is more or less than the real market demand, retailer's profit will be reduced, i.e., a certain loss will occur. In practice, retailers' aversion to loss may result in decision bias, which means that retailers' order quantity decisions do not always maximize expected profit. Due to many unpredictable disasters often occur and cause huge losses, many loss-averse retailers pay more attention to reducing their losses than increasing profits. Many researchers study the loss-averse newsvendor problem and look for the optimal order quantity for the loss-averse retailer. Legacy loss [31] is a loss definition, which is a kind of loss when the sales time is due. The legacy loss includes the loss $(c-h)(q-D)^+$ for overstock and the shortage penalty $s(D-q)^+$. If legacy loss attains its minimum, then the gap between the order decision and the realized demand is minimized. Therefore, through the above description, we know that minimizing the legacy loss can reduce the potential loss which is caused by market demand fluctuation.

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Nowadays, market demand inevitably presents a certain degree of fluctuation since it is usually unpredictable. The greater the fluctuation is, the higher risk the retailers bear when they make order decision. To reduce the impact of unpredictable market demand, researching on risk-averse models has become an important stream. The expected performance may result in an unacceptable large loss since it can not reflect the size of market fluctuation. Hence we need some other indexes. CVaR risk measure is a well-known downside risk measure. It has many advantages that other risk measures do not possess. It is coherent and easy to calculate. CVaR is the loss that has occurred, by which the loss range can be controlled. As a consequence, CVaR criterion has been widely applied both in theoretical study and in practice of newsvendor problem.

In many studies, uncertain demand of the newsvendor problem is assumed to be random and the exact probability distribution is known. There are also considerable literatures assume that the range of uncertain demand is known, but the exact distribution is unknown since it is difficult to accurately predict in the real life. For the second situation, robust optimization has been applied to choose the appropriate order quantity of newsvendor problem. Robust optimization method is also applied in supply chain network design problem under fuzzy demands and transportation costs [1] and project portfolio optimization problem with fuzzy interactive returns [17]. In order to take full advantage of the known information and make the order quantity which is close to the realized market demand as far as possible, this paper will study the newsvendor problem when the stochastic demand's probability distribution information is partially known. If the range of uncertain demand is known and the probability fluctuate in a uncertainty set, based on the expectation and CVaR of legacy loss, we will use the robust optimization method to determine an optimal order quantity of the newsvendor problem. Robust optimization method guarantees that the optimal solution is obtained in the worst case and the optimal solution is well resistant to the uncertainty.

The rest of this paper is organized as follows. In the next section, we review the literatures on loss-averse newsvendor problem, CVaR measure and robust optimization. In Section 3, we introduce the legacy loss. When the demand of newsvendor problem is discrete, this section also shows the computational method about the expectation and CVaR of the legacy loss. Section 4 builds three basic robust optimization models when the demand distribution is bounded by a box uncertainty set. The equivalent deterministic models are also derived in this section. In Section 5 we perform some numerical experiments and show the influences of parameters on the optimal decision and the cost of robustness. At last, Section 6 concludes the paper.

2 Literature Review

This section will mainly review some literatures on loss-averse newsvendor problem, CVaR risk measure and robust optimization in newsvendor problem.

Some studies of the newsvendor problem always focus on choosing an optimal order quantity to maximize the expected profit. Based on the equivalent value criterion, Guo [12] analyzed a single-period inventory problem with fuzzy demand. He determined the optimal order quantity with maximum equivalent value profit. Many loss-averse retailers pay more attention to reducing their losses than increasing profits. There are considerable literatures that study the loss-averse newsvendor problem and look for the optimal order quantity for the loss-averse retailers. Wang and Webster [26] showed that a loss-averse newsvendor may order more than a risk-neutral newsvendor when shortage penalty was not negligible. They also showed that changing wholesale price and retail price will influence the optimal order quantity of a loss-averse newsvendor. Xu et al. [31] presented a new loss definition, called legacy loss, for the loss-averse newsvendor problem, in which the loss for excess order and the shortage penalty for lost sales were considered. Tian and Guo [25] studied a single-product single-period inventory problem based on credibility theory. They determined the optimal order quantity from the primary supplier and the optimal reserved quantity from the secondary supplier to minimize the cost.

In order to reduce the loss arising from the fluctuation of market demand, some researchers pay attention to risk control in newsvendor problem. For a two-product newsboy problem, Lau and Lau [13] found the optimal production quantities of each product to maximize the probability of achieving the profit which exceeded a predetermined target profit. Eeckhoudt et al. [9] investigated the effects of risk and risk aversion on a newsvendor's decisions. Furthermore, Choi [6] explored the multi-period risk minimization inventory models for purchasing fashion product via a mean-variance approach.

CVaR is a well-known downside risk measure. In recent years CVaR measure has been widely applied both in theoretical study and in practice of newsvendor problem. Gotoh and Takano [11] considered the minimization of the CVaR in the context of single-period newsvendor problem. For the minimization of the CVaR measures defined with two different loss functions, they provided analytical solutions or linear programming formulation. Xu et al. [32] studied the three-stage supply chain management and proposed a tri-level programming model based on the CVaR measure. In the tri-level programming model, the CVaR of expected profit of retailer, the profit of the material supplier and the profit of the manufacturer were maximized, respectively. Wu et al. [28] employed CVaR risk measure to model newsvendor problem with uncertain demand as well as a generalized version with uncertain shortage cost. Compared with the optimal order quantity under CVaR, they found that the optimal order quantity under the VaR is higher. In literature [29], based on CVaR, Wu et al. studied the effect of uncertain capacity on the inventory decisions of a risk-averse newsvendor. They found that the optimal order quantity was affected by the capacity uncertainty for the risk-averse newsvendor problem. By using CVaR to model the risk, Eskandarzadeh et al. [10] studied the production planning problem under general demand function and general distribution function of yield. Li et al. [15] studied the lead time reduction problem in a supply chain with a risk-averse retailer and a risk-neutral manufacturer for short life cycle products, and analyzed the effects of decision maker's risk aversion on the optimal decisions under the CVaR risk measure. Balancing the expected profit and CVaR in a newsvendor model setting, Xu and Li [30] investigated a risk-averse inventory model and found some conclusions about the monotonicity of optimal order quantity.

If the exact distribution of uncertain demand in the newsvendor problem was unknown, it is necessary for the decision maker to find robust solutions. The implementation of robust optimization to newsvendor problem can be traced back to Scarf et al. [23]. When only knowing mean and variance of demand, they obtained the optimal order quantity of the classical newsvendor problem. Soyster [24] illustrated that a conservative solution was got if each uncertain parameter took its worst possible value within a range. Therefore robust optimization can essentially avoid the impact of parameter uncertainty. Ever since the work of Soyster [24], robust optimization has become essential to deal with parameter uncertainty. Ben-Tal and Nemirovski [4] surveyed the main results of robust optimization as applied to uncertain linear, conic quadratic and semidefinite programming. For these cases, they obtained computationally tractable robust counterparts of uncertain problems or good approximations of these counterparts. Bertsimas and Sim [5] proposed the "budget of uncertainty" approach that had the advantage of retaining linearity over the robust counterpart. This approach addressed data uncertainty for discrete optimization and network flow problems that allowed controlling the conservatism degree of the solution. Ben-Tal et al. [3] showed that the robust counterpart of a linear optimization problem with phi-divergence uncertainty was tractable for most of the choices of phi typically. Lin and Ng [16] proposed a robust model, which was the minimax regret multi-market newsvendor model, to determine the optimal order quantity and market selection for short-life-cycle products in a single period. Wang et al. [27] studied the robust inventory financing model when the demand distribution was partly known. They discussed two demand information cases, one was the mean and variance and the other was the support of the demand distribution, and provided an explicit expression for the robust optimal policy which was robust but not conservative. For mixed integer linear programming problems with random objective coefficients, Li et al. [14] reviewed some results in the distributional analysis. When the probability distribution of the objective coefficient was incomplete and characterized through the given moment information, Li et al. discussed complexity results and conic programming models for this class of problems. Qiu et al. [20] introduced three basic models with incomplete demand information for the robust inventory decision-making problem faced by risk-averse managers with incomplete demand information. The three models are expected profit maximization, CVaR-based profit maximization and a combination of the two, respectively.

3 The Mean and CVaR of Legacy Loss

3.1 The Expected Legacy Loss

Considering a single-period newsvendor problem, we assume that the uncertain market demand is D, the unit ordering cost is c, the salvage price is h, the shortage penalty price is s, and the order quantity before the selling season is q. Therefore the loss for overstock is $(c-h)(q-D)^+$, and the shortage penalty for stockout is $s(D-q)^+$. The uncertain legacy loss [31] can be written as

$$L(q, D) = (c-h)\,(q-D)^+ + s(D-q)^+. \tag{1}$$

The mean of legacy loss is denoted as $\mathrm{E}\left[L\,(q,\,D)\right]$.

If the uncertain market demand D is discrete, we assume that $D = D_1, D_2, \ldots, D_n$, where D_i is the possible market demand quantity. For each possible demand D_i, the corresponding probability is p_i, i.e. $\Pr\{D = D_i\} = p_i$. The probability p_i, $i = 1, 2, \ldots, n$ satisfy $\sum_{i=1}^{n} p_i = 1$ and $p_i \geq 0,\ i = 1, 2, \ldots, n$. Therefore the loss-averse newsvendor's expected legacy loss is $\sum_{i=1}^{n} L_i p_i$, where $L_i = L(q,\, D_i), i = 1, 2, \ldots, n$, p_i is the probability that $L(q,\, D_i)$ takes value L_i. For convenience, we introduce two vectors $\boldsymbol{L} = (L_1,\, L_2,\, \ldots,\, L_n)^T$ and $\mathbf{p} = (p_1,\, p_2,\, \ldots,\, p_n)^T$, so the expected legacy loss can be rewritten as

$$\mathrm{E}[L(q,\, D)] = \boldsymbol{L}^T \mathbf{p}. \tag{2}$$

3.2 VaR and CVaR of Legacy Loss

Because of the fluctuation of the market demand, the accurate losses cannot be predicted. There are risks in the market. VaR and CVaR are common tools to measure the risks. For a given probability level α, the VaR of legacy loss $L(q, D)$ can be defined as

$$\mathrm{VaR}_\alpha[L(q, D)] = \inf\{z \in R | \Pr\{L(q, D) \leq z\} \geq \alpha\}.$$

This definition implies that, under a given confidence level α, the probability that $L(q, D)$ does not exceed VaR_α is not less than α. It means that the decision makers' loss is not more than the VaR value with probability α. The defects of the risk measure VaR include non-subadditivity and non-convexity.

The famous CVaR risk measure, proposed by Rockafellar and Uryasev [21], is a coherent risk measure and easy to calculate. Therefore, CVaR is more concerned and more widely used than VaR. For a given probability level α, the CVaR of $L(q, D)$ is [31]

$$\mathrm{CVaR}_\alpha[L(q, D)] = \mathrm{E}[L(q, D) | L(q, D) \geq \mathrm{VaR}_\alpha(L(q, D))]. \tag{3}$$

The main difference between VaR and CVaR is that: VaR essentially corresponds to the possible maximum loss under a confidence level, but it can not predict and control the occurrence of extreme events, which may lead to great losses, so there is a tail risk; CVaR is the loss that has occurred, by which we can control the loss range.

When the uncertain demand is continuous random variable, according to Rockafellar and Uryasev [22], we know

$$\mathrm{CVaR}_\alpha[L(q, D)] = \frac{1}{1-\alpha} \int_{L(q,D) \geq \mathrm{VaR}_\alpha(L(q,D))} L(q, D)\, \phi(y)\, \mathrm{d}y, \tag{4}$$

where $\phi(y)$ is the probability density function of D.

From the definition of CVaR, we know that it is difficult to compute CVaR directly because of the VaR parameter, which is endogenous. Rockafellar and Uryasev [22] constructed an auxiliary function to solve the calculation problem of CVaR. The function can be described as

$$\mathrm{CVaR}_\alpha[L(q, D)] = \min_{v \in R} \left\{ v + \frac{1}{1-\alpha} \mathrm{E}[L(q, D) - v]^+ \right\}.$$

For convenience, we denote the expression in braces as $\Omega_\alpha(q, v)$, i.e.,

$$\Omega_\alpha(q, v) = v + \frac{1}{1-\alpha} \mathrm{E}[L(q, D) - v]^+.$$

As a consequence,

$$\mathrm{CVaR}_\alpha[L(q, D)] = \min_{v \in R} \Omega_\alpha(q, v). \tag{5}$$

When the uncertain demand is discrete, according to Rockafellar and Uryasev [22], we know

$$\Omega_\alpha(q, v) = v + \frac{1}{1-\alpha} \sum_{i=1}^{n} p_i [L(q, D_i) - v]^+. \tag{6}$$

Under CVaR risk measure, the optimal decision will minimize $\mathrm{CVaR}_\alpha\left[L\left(q,D\right)\right]$. Thus, solving the following programming problem

$$\min_q \mathrm{CVaR}_\alpha\left[L\left(q,D\right)\right], \tag{7}$$

we can determine the optimal decision.

The function $\Omega_\alpha\left(q,v\right)$ is convex in v, so, according to Eq. (5), problem (7) can be described as

$$\min_q \mathrm{CVaR}_\alpha\left[L\left(q,D\right)\right] = \min_{q,v} \Omega_\alpha(q,v), \tag{8}$$

which implies that $(\hat{q},\hat{v})$ is the optimal solution of programming problem $\min\limits_q\ \mathrm{CVaR}_\alpha\left[L\left(q,D\right)\right]$ if and only if $\hat{q}$ minimizes $\mathrm{CVaR}_\alpha\left[L\left(q,D\right)\right]$.

4 Three Robust Mean-CVaR Models of Newsvendor Problem

4.1 Uncertainty Set

In practical problems, some fluctuations of the probability distribution of discrete demand may occur, which result in the emergence of the probability uncertainty. Ben-Haim [2] and Budescu and Du [7] have made some detailed expositions. We introduce a box uncertainty set $\mathcal{P_B}$, which is the range of probability $\mathbf{p}$. $\mathcal{P_B}$ is defined as

$$\mathbf{p} \in \mathcal{P_B} = \left\{\mathbf{p}|\mathbf{p} = \mathbf{p_0} + \boldsymbol{\zeta},\ \mathbf{e}^T\boldsymbol{\zeta} = 0,\ \boldsymbol{\mu}^- \le \boldsymbol{\zeta} \le \boldsymbol{\mu}^+\right\}, \tag{9}$$

where $\mathbf{p_0}$ is the nominal distribution, $\mathbf{e} = (1,\cdots,1)$ is a vector. Disturbance vector $\boldsymbol{\zeta}$ varies in a known support $\left[\boldsymbol{\mu}^-,\ \boldsymbol{\mu}^+\right]$. The condition $\mathbf{e}^T\boldsymbol{\zeta} = 0$ ensures that $\mathbf{p}$ meets the requirements $\mathbf{e}^T\mathbf{p} = \mathbf{e}^T\left(\mathbf{p_0} + \boldsymbol{\zeta}\right) = 1$ of the probability distribution. Obviously, when the market does not fluctuate, i.e. when $\boldsymbol{\zeta} = 0$, the equation $\mathbf{e}^T\mathbf{p_0} = 1$ holds. Since the probability must be nonnegative, we know that $\mathbf{p_0} + \boldsymbol{\zeta} \ge \mathbf{0}$.

4.2 Three Basic Robust Models

For the legacy loss function, we assume that the probability which corresponds to the discrete demand is uncertain. Some decision makers consider both the expected value measure and the CVaR measure of the legacy loss. When their aim is to minimize the expected legacy loss and minimize the CVaR of legacy loss, the loss-averse newsvendor problem can be formulated as the following bi-objective programming model:

$$\begin{cases} \min\limits_q & \left\{\mathrm{E}\left[L\left(q,\ D\right)\right]\right\}_{\mathbf{p}\in\mathcal{P_B}} \\ \min\limits_q & \left\{\mathrm{CVaR}_\alpha\left[L\left(q,D\right)\right]\right\}_{\mathbf{p}\in\mathcal{P_B}} \\ \text{s.t.} & q > 0. \end{cases} \tag{10}$$

The robust counterpart of problem (10) is given by

$$\begin{cases} \min\limits_q & \max\limits_{\mathbf{p}\in\mathcal{P_B}} \mathrm{E}\left[L\left(q,\ D\right)\right] \\ \min\limits_q & \max\limits_{\mathbf{p}\in\mathcal{P_B}} \mathrm{CVaR}_\alpha\left[L\left(q,D\right)\right] \\ \text{s.t.} & q > 0. \end{cases} \tag{11}$$

We adopt the constraint method to turn problem (11) into single objective model. On the one hand, if the decision maker is looking for an optimal robust solution with the minimum expected legacy loss under prescribing a maximum acceptable level A of the CVaR of legacy loss, the problem (11) can be turned into the following single objective programming model

$$\begin{cases} \min\limits_q & \max\limits_{\mathbf{p}\in\mathcal{P_B}} \mathrm{E}\left[L\left(q,\ D\right)\right] \\ \text{s.t.} & \max\limits_{\mathbf{p}\in\mathcal{P_B}} \mathrm{CVaR}_\alpha\left[L\left(q,D\right)\right] \le A \\ & q > 0, \end{cases} \tag{12}$$

which is a parametric optimization problem with parameter A.

On the other hand, if a risk aversion decision maker desires to find a robust optimal solution with the minimum CVaR of legacy loss under the condition that his acceptable expected legacy loss does not exceed B, the problem (11) can be turned into the following single objective programming model

$$\begin{cases} \min\limits_{q} & \max\limits_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \mathrm{CVaR}_{\alpha}\left[L\left(q, D\right)\right] \\ \text{s.t.} & \max\limits_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \mathrm{E}\left[L\left(q,\, D\right)\right] \leq B \\ & q > 0, \end{cases} \tag{13}$$

which is also a parametric optimization problem with parameter B.

Next the weighting method is applied to turn problem (11) into single objective model. Balancing the expected legacy loss and CVaR, we can turn the problem (11) into the following single objective programming model

$$\begin{cases} \min\limits_{q} & \lambda \max\limits_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \mathrm{E}\left[L\left(q,\, D\right)\right] + (1-\lambda) \max\limits_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \mathrm{CVaR}_{\alpha}\left[L\left(q, D\right)\right] \\ \text{s.t.} & q > 0 \end{cases} \tag{14}$$

where $\lambda \in [0, 1]$ is the weight of the expected legacy loss. According to model (14), the smaller λ means the decision maker pay more attention to predicting and controlling the occurrence of extreme events. When $\lambda = 0, 1$, model (14) degenerates into a single objective model, respectively. In the two single objective models, the decision maker only considers one objective of model (11), i.e. the decision maker only considers the expected legacy loss or the CVaR of legacy loss.

4.3 Equivalent Deterministic Programming Models

For models (12)-(14) which include $\max\limits_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \mathrm{CVaR}_{\alpha}\left[L\left(q, D\right)\right]$ and $\max\limits_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \mathrm{E}\left[L\left(q,\, D\right)\right]$ in objective or constraint function, we can derive their equivalent form.

Poojari et al. [18] provided the equivalent form of the problem $\max\limits_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \mathrm{CVaR}_{\alpha}\left[L\left(q, D\right)\right]$, which can be described as

$$\max_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \mathrm{CVaR}_{\alpha}\left[L\left(q, D\right)\right] = \max_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \min_{v\in R} \Omega_{\alpha}\left(q, v\right), \tag{15}$$

where $\Omega_{\alpha}\left(q, v\right)$ is defined as Eq. (6). $\Omega_{\alpha}\left(q, v\right)$ is concave in v and $\mathbf{p}$, so problem (15) can be rewritten as

$$\max_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \mathrm{CVaR}_{\alpha}\left[L\left(q, D\right)\right] = \min_{v\in R} \max_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \Omega_{\alpha}\left(q, v\right). \tag{16}$$

In order to facilitate the calculation of Eq. (16), a vector $\mathbf{c} = (c_1, c_2, \cdots, c_n)^T$ is introduced. Then

$$\Omega_{\alpha}\left(q, v\right) = \left\{ v + \frac{1}{1-\alpha}\mathbf{p}^T\mathbf{c} | c_i \geq L\left(q,\, D_i\right) - v, c_i \geq 0, i = 1, 2, 3, \ldots, n \right\}. \tag{17}$$

Note that probability vector $\mathbf{p}$ belongs to the box uncertainty set defined in Eq. (9), we know $\max\limits_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \Omega_{\alpha}\left(q, v\right)$ can be written as

$$\max_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \left\{ v + \frac{1}{1-\alpha}\mathbf{p}_0^T\mathbf{c} + \frac{1}{1-\alpha}\boldsymbol{\zeta}^T\mathbf{c} \right\}, \tag{18}$$

where the third term of problem (18) is a linear problem in ζ, which can be described as

$$\begin{cases} \max\limits_{\boldsymbol{\zeta}} & \boldsymbol{\zeta}^T\mathbf{c} \\ \text{s.t.} & \mathbf{e}^T\boldsymbol{\zeta} = 0 \\ & \boldsymbol{\zeta} \geq \boldsymbol{\mu}^- \\ & \boldsymbol{\zeta} \leq \boldsymbol{\mu}^+. \end{cases} \tag{19}$$

The duality of problem (19) can be written as follows:

$$\begin{cases} \min\limits_{\boldsymbol{\eta},\varsigma,\boldsymbol{\delta}} & \left[\boldsymbol{\mu}^-\right]^T\boldsymbol{\eta} + \left[\boldsymbol{\mu}^+\right]^T\boldsymbol{\delta} \\ \text{s.t.} & \mathbf{e}\varsigma + \boldsymbol{\eta} + \boldsymbol{\delta} = \mathbf{c} \\ & \boldsymbol{\eta} \le \mathbf{0} \\ & \boldsymbol{\delta} \ge \mathbf{0}. \end{cases} \tag{20}$$

Combining (17), (18) and (20), problem (15) can be described in the following equivalent form:

$$\begin{cases} \min\limits_{v,\varsigma,\mathbf{c},\boldsymbol{\eta},\boldsymbol{\delta}} & v + \dfrac{1}{1-\alpha}(\mathbf{p}_0^T\mathbf{c} + \left[\boldsymbol{\mu}^-\right]^T\boldsymbol{\eta} + \left[\boldsymbol{\mu}^+\right]^T\boldsymbol{\delta})] \\ \text{s.t.} & c_i \ge 0,\, i = 1,2,\ldots,n \\ & c_i \ge L\,(q,\, D_i) - v,\, i = 1,2,\ldots,n \\ & \mathbf{e}\varsigma + \boldsymbol{\eta} + \boldsymbol{\delta} = \mathbf{c} \\ & \boldsymbol{\eta} \le \mathbf{0} \\ & \boldsymbol{\delta} \ge \mathbf{0} \end{cases} \tag{21}$$

where $(v, \varsigma, \mathbf{c}, \boldsymbol{\eta}, \boldsymbol{\delta}) \in R \times R \times R^n \times R^n \times R^n$.

Then we consider the equivalent form of $\max\limits_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \mathrm{E}\,[L\,(q,\, D)]$, which can be described as

$$\max_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \;\mathrm{E}\,[L\,(q,\, D)] = \boldsymbol{L}^T\mathbf{p}, \tag{22}$$

where $\boldsymbol{L} = (L_1,\, L_2, \cdots, L_n)^T$, $\mathbf{p} = (p_1,\, p_2, \cdots, p_n)^T$, $L_i = L\,(q,\, D_i)\,,\, i = 1,2,\ldots,n.$

When the distribution of discrete random demand belongs to a box uncertainty set which is defined in Eq. (9), problem (22) can be rewritten as

$$\begin{aligned} \max_{\mathbf{p}\in\mathcal{P}_{\mathcal{B}}} \;\mathrm{E}\,[L\,(q,\, D)] &= \max_{\mathbf{p_0}+\zeta\in\mathcal{P}_{\mathcal{B}}} \boldsymbol{L}^T\,(\mathbf{p_0} + \boldsymbol{\zeta}) \\ &= \boldsymbol{L}^T\mathbf{p_0} + \max_{\zeta}\left\{\boldsymbol{L}^T\boldsymbol{\zeta}|\mathbf{e}^T\boldsymbol{\zeta} = 0,\, \boldsymbol{\mu}^- \le \boldsymbol{\zeta} \le \boldsymbol{\mu}^+\right\}. \end{aligned} \tag{23}$$

The dual problem of the linear programming $\max\limits_{\zeta}\left\{\boldsymbol{L}^T\boldsymbol{\zeta}|\mathbf{e}^T\boldsymbol{\zeta} = 0,\, \boldsymbol{\mu}^- \le \boldsymbol{\zeta} \le \boldsymbol{\mu}^+\right\}$ can be described as

$$\begin{cases} \min\limits_{\gamma,\boldsymbol{\xi},\boldsymbol{\tau}} & \left[\boldsymbol{\mu}^-\right]^T\boldsymbol{\tau} + \left[\boldsymbol{\mu}^+\right]^T\boldsymbol{\xi} \\ \text{s.t.} & \mathbf{e}\gamma + \boldsymbol{\xi} + \boldsymbol{\tau} = \boldsymbol{L} \\ & \boldsymbol{\tau} \le \mathbf{0} \\ & \boldsymbol{\xi} \ge \mathbf{0}. \end{cases} \tag{24}$$

Thus the problem (22) can be described in the following equivalent form

$$\begin{cases} \min\limits_{\gamma,\boldsymbol{\xi},\boldsymbol{\tau}} & \boldsymbol{L}^T\mathbf{p_0} + \left[\boldsymbol{\mu}^-\right]^T\boldsymbol{\tau} + \left[\boldsymbol{\mu}^+\right]^T\boldsymbol{\xi} \\ \text{s.t.} & \mathbf{e}\gamma + \boldsymbol{\xi} + \boldsymbol{\tau} = \boldsymbol{L} \\ & \boldsymbol{\tau} \le \mathbf{0} \\ & \boldsymbol{\xi} \ge \mathbf{0} \end{cases} \tag{25}$$

where $(\gamma,\, \boldsymbol{\xi},\, \boldsymbol{\tau}) \in R \times R^n \times R^n$.

According to problem (21) and problem (25), model (12) can be written as the following solvable form:

$$\begin{cases} \min\limits_{q,v,\varsigma,\mathbf{c},\boldsymbol{\eta},\boldsymbol{\delta},\gamma,\boldsymbol{\xi},\boldsymbol{\tau}} & \boldsymbol{L}^T\mathbf{p_0} + \left[\boldsymbol{\mu}^-\right]^T\boldsymbol{\tau} + \left[\boldsymbol{\mu}^+\right]^T\boldsymbol{\xi} \\ \text{s.t.} & v + \dfrac{1}{1-\alpha}(\mathbf{p}_0^T\mathbf{c} + \left[\boldsymbol{\mu}^-\right]^T\boldsymbol{\eta} + \left[\boldsymbol{\mu}^+\right]^T\boldsymbol{\delta}) \leq A \\ & c_i \geq 0,\, i = 1, 2, \ldots, n \\ & c_i \geq L\,(q,\, D_i) - v,\, i = 1, 2, \ldots, n \\ & \mathbf{e}\varsigma + \boldsymbol{\eta} + \boldsymbol{\delta} = \mathbf{c} \\ & \boldsymbol{\eta} \leq \mathbf{0} \\ & \boldsymbol{\delta} \geq \mathbf{0} \\ & \mathbf{e}\gamma + \boldsymbol{\xi} + \boldsymbol{\tau} = \boldsymbol{L} \\ & \boldsymbol{\tau} \leq \mathbf{0} \\ & \boldsymbol{\xi} \geq \mathbf{0}. \end{cases} \tag{26}$$

Similarly, model (13) can be written as the following solvable form

$$\begin{cases} \min\limits_{q,v,\varsigma,\mathbf{c},\boldsymbol{\eta},\boldsymbol{\delta},\gamma,\boldsymbol{\xi},\boldsymbol{\tau}} & v + \dfrac{1}{1-\alpha}(\mathbf{p}_0^T\mathbf{c} + \left[\boldsymbol{\mu}^-\right]^T\boldsymbol{\eta} + \left[\boldsymbol{\mu}^+\right]^T\boldsymbol{\delta} \\ \text{s.t.} & \boldsymbol{L}^T\mathbf{p_0} + \left[\boldsymbol{\mu}^-\right]^T\boldsymbol{\tau} + \left[\boldsymbol{\mu}^+\right]^T\boldsymbol{\xi} \leq B \\ & c_i \geq 0,\, i = 1, 2, \ldots, n \\ & c_i \geq L\,(q,\, D_i) - v,\, i = 1, 2, \ldots, n \\ & \mathbf{e}\varsigma + \boldsymbol{\eta} + \boldsymbol{\delta} = \mathbf{c} \\ & \boldsymbol{\eta} \leq \mathbf{0} \\ & \boldsymbol{\delta} \geq \mathbf{0} \\ & \mathbf{e}\gamma + \boldsymbol{\xi} + \boldsymbol{\tau} = \boldsymbol{L} \\ & \boldsymbol{\tau} \leq \mathbf{0} \\ & \boldsymbol{\xi} \geq \mathbf{0}. \end{cases} \tag{27}$$

The single objective programming model (14) can be written as the following solvable form

$$\begin{cases} \min\limits_{q,v,\varsigma,\mathbf{c},\boldsymbol{\eta},\boldsymbol{\delta},\gamma,\boldsymbol{\xi},\boldsymbol{\tau}} & \lambda(\boldsymbol{L}^T\mathbf{p_0} + \left[\boldsymbol{\mu}^-\right]^T\boldsymbol{\tau} + \left[\boldsymbol{\mu}^+\right]^T\boldsymbol{\xi}) + (1-\lambda)[v + \dfrac{1}{1-\alpha}(\mathbf{p}_0^T\mathbf{c} + \left[\boldsymbol{\mu}^-\right]^T\boldsymbol{\eta} + \left[\boldsymbol{\mu}^+\right]^T\boldsymbol{\delta})] \\ \text{s.t.} & c_i \geq 0,\, i = 1, 2, \ldots, n \\ & c_i \geq L\,(q,\, D_i) - v,\, i = 1, 2, \ldots, n \\ & \mathbf{e}\varsigma + \boldsymbol{\eta} + \boldsymbol{\delta} = \mathbf{c} \\ & \boldsymbol{\eta} \leq \mathbf{0} \\ & \boldsymbol{\delta} \geq \mathbf{0} \\ & \mathbf{e}\gamma + \boldsymbol{\xi} + \boldsymbol{\tau} = \boldsymbol{L} \\ & \boldsymbol{\tau} \leq \mathbf{0} \\ & \boldsymbol{\xi} \geq \mathbf{0}. \end{cases} \tag{28}$$

5 Numerical Experiments

In this section, an order quantity problem of loss-averse calendar retailer is considered. We will do some numerical experiments to demonstrate our proposed method.

5.1 Problem Description

A loss-averse retailer named "century culture" is selling a calendar. As we know, calendar selling cycle is very short. Only at the end of the previous year and the beginning of the next year sales are very good. In the ordinary time it is very difficult to sell out the calendar and new calendar will appear after this cycle. Because of the market demand's uncertainty, compared with obtaining greater profits, the retailer pay more attention to reducing the loss. So the proposed method in this paper can be applied to determine the appropriate order quantity for this problem.

The following data is provided to the retailer. The selling price $r = 9$, the order cost $c = 4$, the salvage price $h = 2$, the shortage penalty price $s = 1$. The uncertain daily market demand D of calendar is a discrete random variable, the set of possible demand quantities is $\{44, 46, 49, 51, 54, 57, 59\}$. The corresponding nominal distribution is $\mathbf{p_0}$=(0.1, 0.12, 0.16, 0.22, 0.15, 0.14, 0.11)^T. Random disturbance $\boldsymbol{\zeta} \in \left[\boldsymbol{\mu}^-, \boldsymbol{\mu}^+\right]$ is in a box uncertainty set, where $\boldsymbol{\mu}^- = -\mathbf{0.1}$ and $\boldsymbol{\mu}^+ = \mathbf{0.1}$.

5.2 Computational Results by Minimizing the Expected Legacy Loss

If the retailer desires an optimal robust solution with the minimum expected legacy loss under prescribing a maximum acceptable level A of the CVaR of legacy loss, the problem described in subsection 5.1 can be built as model (12), model (26) is its equivalent form. When we fix confidence level $\alpha = 0.9$ and change the given maximum acceptable level A (we limit $9 \le A \le 15$), the optimal order quantities with the minimum expected legacy loss and their performances are presented in Table 1.

Table 1: The optimal order quantity and the expected performance under $\alpha = 0.9$

A	q_0^*	$\mathrm{E}_N[L(q)]$	q_0'	$\mathrm{E}_B[L(q)]$
9	51	5.41	50	8.52
10	49	5.13	52	9.81
11	49	5.13	47	8.71
12	49	5.13	50	8.52
13	49	5.13	47	8.71
14	49	5.13	47	8.71
15	49	5.13	47	8.71

In Table 1, q_0^* and $\mathrm{E}_N[L(q)]$ represent the optimal order quantity and the minimum expected legacy loss under the nominal demand distribution, q_0' and $\mathrm{E}_B[L(q)]$ represent the optimal order quantity and the minimum expected legacy loss under the box uncertainty demand distribution, respectively. For the given maximum acceptable CVaR of legacy loss, we find that the changes of the optimal order quantity q_0^* and the expected legacy loss $\mathrm{E}_N[L(q)]$ are not large under the nominal demand distribution. When $A \ge 10$, the optimal order quantity q_0^* and the expected legacy loss $\mathrm{E}_N[L(q)]$ are not changed. But under the box uncertainty demand distribution, when $9 \le A \le 12$, the changes of q_0' and $\mathrm{E}_B[L(q)]$ are very obvious. When $A = 10$, the expected legacy loss is 9.81, while the expected legacy loss is 8.52 in the case of $A = 12$. When $13 \le A \le 15$, q_0' and $\mathrm{E}_B[L(q)]$ are not changed.

When we fix the maximum acceptable level $A = 9$, the results for the optimal order quantity and the expected legacy loss under different values of α are shown in Table 2. Under nominal distribution, when $\alpha \le 0.9$, q_0^* and $\mathrm{E}_N[L(q)]$ are not changed. Under the box uncertainty demand distribution, the optimal order quantity $q_0' = 50$ when $\alpha \ge 0.8$, which is higher than $q_0' = 47$ obtained when confidence level $\alpha < 0.8$; but $\mathrm{E}_B[L(q)] = 8.52$ when confidence level $\alpha \ge 0.8$, which is lower than $\mathrm{E}_B[L(q)] = 8.71$ obtained when confidence level $\alpha < 0.8$.

Table 2: The optimal order quantity and the expected performance under $A = 9$

α	q_0^*	$\mathrm{E}_N(L(q))$	q_0'	$\mathrm{E}_B(L(q))$
0.9	51	5.41	50	8.52
0.85	49	5.13	50	8.52
0.8	49	5.13	50	8.52
0.75	49	5.13	47	8.71
0.7	49	5.13	47	8.71

According to the data in Tables 1 and 2, the optimal order quantity q_0' under the box uncertainty demand distribution given by model (12) is more sensitive to parameters' change than the optimal order quantity q_2^*

under the nominal distribution. Tables 1 and 2 also show that the minimum expected legacy loss $\mathrm{E}_N[L(q)]$ under the nominal demand distribution is always lower than the minimum expected legacy loss $\mathrm{E}_B[L(q)]$ under the box uncertainty demand distribution, it is the cost of robust.

5.3 Computational Results by Minimizing the CVaR

If the retailer desires to find a robust optimal solution with the minimum CVaR of legacy loss under the condition that his expected legacy loss does not exceed B, we build the problem described in subsection 5.1 as model (13), the equivalent form is model (27). Now we observe the influence of parameter B on the optimal order decision and CVaR performance. Under confidence level $\alpha = 0.9$ and the acceptable expected legacy loss $B = 8, 8.5, 9, 9.5, 10$, the optimal order quantity and its performance are reported in Table 3.

Table 3: The optimal order quantity and the CVaR performance under $\alpha = 0.9$

B	q_1^*	$C_N(L(q))$	q_1'	$C_B(L(q))$
8	51	14	54	20
8.5	51	14	54	20
9	51	14	54	20
9.5	49	10	51	14
10	49	10	51	14

In Table 3, q_1^* and $C_N(L(q))$ represent the optimal order quantity and the minimum CVaR about legacy loss under the nominal demand distribution, q_1' and $C_B(L(q))$ represent the optimal order quantity and the minimum CVaR about legacy loss under the box uncertainty demand distribution, respectively. The computational results reported in Table 3 show that: for different maximum acceptable expected legacy loss, compared with q_1' and $C_B(L(q))$ under the box uncertainty distribution, both q_1^* and $C_N(L(q))$ under the nominal distribution are lower; q_1^*, $C_N(L(q))$, q_1' and $C_B(L(q))$ are not increasing with the increase of B.

Next we observe the influence of parameter α on the optimal order decision and CVaR performance. When we fix the maximum acceptable expected legacy loss $B = 10$, under different values of α, the computational results for the optimal order quantity and the CVaR are shown in Table 4. From these computational results we find that: q_1^* is always lower than q_1', $C_N(L(q))$ is also lower than $C_B(L(q))$; q_1^* is not decreasing with the decrease of α, q_1' does not change with the change of α; both $C_N(L(q))$ and $C_B(L(q))$ are increasing with respect to α.

Table 4: The optimal order quantity and the CVaR performance under $B = 10$

α	q_1^*	$C_N(L(q))$	q_1'	$C_B(L(q))$
0.9	49	10	51	14
0.85	50	7.16	51	12.83
0.8	50	5.36	51	9.625

According to the data in Tables 3 and 4, the minimum expected legacy loss $C_N(L(q))$ under the nominal demand distribution is always lower than the minimum expected legacy loss $C_B(L(q))$ under the box uncertainty demand distribution, it is the cost of robust.

5.4 Computational Results by Minimizing the Combination of Expected Legacy Loss and CVaR

In this subsection, we build the problem described in subsection 5.1 as model (14), model (28) is its equivalent form. In this subsection, we will analyze the influences of weight λ and confidence level α on the optimal order quantity and performance of model (14).

Firstly the influences of weight λ on the optimal order quantity and performance is analyzed. Fixing the confidence level $\alpha = 0.9$, corresponding to 11 different values of weight λ, Table 5 lists the optimal order quantities and their performance, respectively. In Table 5, q_2^* and $P_N(L(q))$ represent the optimal order quantity and the minimum mean-CVaR performance about legacy loss under the nominal demand distribution,

q_2' and $P_B(L(q))$ represent the optimal order quantity and the minimum mean-CVaR performance about legacy loss under the box uncertainty demand distribution.

Table 5: The optimal order quantity and the mean-CVaR performance under $\alpha = 0.9$

λ	q_2^*	$P_N(L(q))$	q_2'	$P_B(L(q))$
0	49	5.7	46	9.88
0.1	50	5.64	46	8.45
0.2	50	5.59	48	8.10
0.3	49	5.52	48	8.07
0.4	49	5.47	46	7.9
0.5	49	5.41	46	7.03
0.6	49	5.35	49	6.65
0.7	49	5.30	49	6.28
0.8	49	5.24	49	6.02
0.9	49	5.18	49	5.53
1	49	5.13	49	5.16

From the computational results in Table 5, we can conclude that: for the fixed confidence level $\alpha = 0.9$, when $\lambda = 0, 0.1, 0.2, 0.3, 0.4, 0.5$, the optimal order quantity q_2^* is larger than q_2', when $\lambda = 0.6, 0.7, 0.8, 0.9, 1$, under both the nominal demand distribution and the box uncertainty demand distribution, the optimal order quantity q_2^* and q_2' always take the same value 49; both $P_N(L(q))$ and $P_B(L(q))$ are decreasing with the increase of λ; for any one of 11 different values of weight λ, $P_N(L(q))$ is less than $P_B(L(q))$.

When we fix the weight $\lambda = 0.5$, the influence of confidence level α on the order quantity and performance is presented in Table 6. Changing α does not affect the order quantity q_2^* under the nominal demand distribution, but q_2' is not decrease with the decrease of α under the box uncertainty demand distribution. Both $P_N(L(q))$ and $P_B(L(q))$ is increase with the decrease of α.

Table 6: The optimal order quantity and the mean-CVaR performance under $\lambda = 0.5$

α	q_2^*	$P_N(L(q))$	q_2'	$P_B(L(q))$
0.95	49	5.26	49	6.80
0.9	49	5.41	49	7.03
0.85	49	5.58	51	7.29
0.8	49	5.77	51	7.58

Tables 5 and 6 show that the optimal order quantity q_2' under the box uncertainty demand distribution is more sensitive to the parameters' change than the optimal order quantity q_2^* under the nominal distribution. According to the data in Tables 5 and 6, the minimum expected legacy loss $P_N(L(q))$ under the nominal demand distribution is always lower than the minimum expected legacy loss $P_B(L(q))$ under the box uncertainty demand distribution, it is the cost of robust.

According to Tables 1-6, for the given values of parameters, the optimal order quantities under the box uncertainty demand distribution are usually different from that under the nominal demand distribution; even if the optimal order quantities are same under both distributions, the corresponding performance are different; the optimal order quantity under the box uncertainty demand distribution is more sensitive to the parameters' change than that under the nominal demand distribution; the minimum expected legacy loss under the nominal demand distribution is always lower than that under the box uncertainty demand distribution, it means the cost of robustness.

6 Conclusions

In this paper, we studied the robust order decision of a single period newsvendor problem, in which the random demand is discrete and the probability distribution is imprecise. In the modeling process, both the mean measure and the CVaR measure were considered. The major conclusions include the following three aspects:

(*i*) For the newsvendor problem we built three robust optimization models with a box uncertainty set, which are minimizing the expected legacy loss under a given maximum acceptable level of the CVaR of legacy loss; minimizing the CVaR of legacy loss under a given maximum acceptable expected legacy loss; minimizing the combination of the expected legacy loss and the CVaR of legacy loss. According to their own preferences, the decision makers can choose one among these three models to obtain the robust optimal decisions.

(*ii*) The equivalent linear programming models of these proposed robust optimization models were obtained by using duality theory. Through the equivalent models, the optimal robust order quantity decision can be made easily and conveniently.

(*iii*) An order quantity problem of loss-averse calendar retailer was modeled by the proposed robust optimization methods, respectively. The computational results of numerical experiments provided the robust optimal order quantity, the nominal optimal order quantity and their performances. These data showed that the robust optimal order quantity and the nominal optimal order quantity are usually different, the former is more sensitive to the parameters' change than the latter, and the cost of robustness exists. The proposed robust optimization methods help the retailer to make decisions when the exact probability distribution of random demand is absent.

In our future research, we will extend our robust optimization models to multi-period newsvendor problem. Another idea is to develop new robust optimization models for newsvendor problems.

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