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A Two-Echelon Single-Period Inventory Control Problem with Market Strategies and Customer Satisfaction

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Abstract

In this research, a single-period two-echelon inventory control problem with market targeting strategies is considered. In this problem, there are several final products and raw materials with varying usage rates. The objective is to determine the order sizes of final products and raw materials before the selling period such that customers' satisfaction is reached and expected profit is maximized within an available budget. The problem is first mathematically formulated and then a modified particle swarm optimization algorithm is employed to solve the nonlinear programming problem. To validate the results obtained, a simulated annealing algorithm is provided as a benchmark. The parameters of both algorithms are calibrated using a numerical example that is given to demonstrate the application of the developed model and the solution algorithm as well.

Keywords: inventory control, single-period, market strategy, customer satisfaction, particle swarm optimization, simulated annealing

1 Introduction

Single-period inventory control models can be used to make decisions on the inventory problems of seasonal and periodical products. Before the start of the selling period in a single-period inventory problem, there is one ordering opportunity to provide raw materials required to produce final products having random demands. On one hand, if more than the demands are produced, the unsold items at the end of the period become obsolete with negligible prices. On the other hand, if less than the demands are manufactured, shortage happens that lead into yielding less profit and loosing reputation. Since the most important feature in a single period inventory problem is associated with the demand, a carful demand analysis has an important role in determining the order points of final products and raw materials.

After mid-1960 when Hadley and Whitin [15] presented a single-period inventory model, many extensions were proposed in the literature to make the model more applicable to real-world problems. Khouja [18] extensively surveyed the literature of research works on the single-period problem, where he categorized them into six groups and discussed each in detail. Some works such as the ones in Silver et al. (referred by [3]) and Lau & Lau [21] were concerned with developing objective functions, where the objective function of their models was maximization of the probability of achieving a target profit. Some research works such as the one in Dominey & Hill [12] focused on the demand, in which they assumed a Poisson probability distribution on the number of orders in stochastic lot sizes. Dey and Chakraborty [11] considered a single-period inventory problem with the fuzzy annual customer demand. The aim was to determine the optimal order quantity such that the expected profit is greater than or equal to a predetermined target. Xu & Hu [38] modeled the demand as a random fuzzy variable and used a hybrid algorithm for determining the optimal order quantity.

Instead of assuming a probability distribution for the demand, Grubbström [14] considered customers arriving in different points of time with different needs such that the compound renewal process could be applied to generate the demand. Mostard et al. [24] investigated forecasting the demand for single-period products before the beginning of

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the selling period. They applied their work in a mail-order apparel company and compared several existing and new forecasting methods. Pal et al. [26] analyzed a single-period newsvendor model to determine the optimal order quantity considering customers' balking. They developed the model without assuming any specific distribution on demand.

Some researchers like Chen & Chuang [7], Vairaktarakis [36], Abdel-Malek et al. [2], and Pasandideh et al. [27] worked on the constraints of mathematical models such as budget and shortage. Besides, in some cases, discount policies were used to purchase items. Khouja [20] formulated a single-period inventory model in the presence of sale-price discounts and Taleizadeh et al. [32,34] modeled different sale discounts at different sale quantities. Moreover, Taleizadeh and Niaki [33] derived a bi-objective model for a single-period newsboy problem with fuzzy cost and incremental discount and Lee & Lodree [23] explored various backorder cases in a newsboy problem and tried to characterize a diversity of customer responses to shortages. They used the concept of the utility theory to classify customers in terms of their willingness to wait for the supplier replenishment in case of shortages.

While in the classical single-period problem only one echelon is considered, Reyes [29] presented a mathematical model of a two-echelon supply chain problem. Chung et al. [9] proposed a model for an N-stage supply chain of the newsboy problem. Nowadays, many researchers tend to solve the single-period inventory problem strategically. Serel [30] derived an extension of the single-period inventory problem in competition environments between suppliers. In their work, there was a chance that when the first supplier could not be able to deliver the products, the second supplier would be considered. Zhang & Hua [39] employed a portfolio approach to a multi-product newsboy problem with budget constraint, in which the procurement strategy for each product was designed as a portfolio contract. Lee & Hsu [22] studied the effects of advertising on distribution-free demand with known mean and variance. They assumed three cases for demand distribution and solved the problem analytically using closed-form formulae. Bashiri et al. [6] presented a new mathematical model for strategic and tactical planning in a multiple-echelon, multiple-commodity production–distribution network. They considered different time resolutions for strategic and tactical decisions and planned an expansion of the network based on cumulative net incomes.

To name a few other recent relevant researches, Keren [19] assumed known demand, but stochastic supply called yield risks. He considered two types of risks; additive and multiplicative. His work is specially qualified for products with a high and random failure rate. Tiwari et al. [35] considered two sequential orders before the start of a selling period with an updated demand forecast after the first order in which two unrelated suppliers exist. Moreover, the mathematical formulation of Murray et al. [25] not only determines the order quantities, but also specifies the selling price of each product, which is good for pricing strategic perspectives. Wang et al. [37] brought a single period problem to food industry. They developed both single-item and multi-item single-period inventory models when market demands are assumed to be uncertain random variables. The objective of their study was to provide theoretical analysis of the models that attains optimality when demand information availability in subjective judgments leading to uncertainty along with random variation. Hnaein et al. [16] worked on replenishment planning of an assembly system with one type of finished product assembled from different types of components. The components are produced from diverse external suppliers to satisfy finished product demand. Chen et al. [8] worked on novel advances in applications of single period problem.

In addition to its real-world applicability, this research concerns with the demand issue where a targeting market is considered. To be more specific, the multi-product multi-material version of a single-period inventory problem is investigated considering random demands, discount, shortage in terms of backorder, and non-conforming items that are either manufactured or are perishable. Moreover, to avoid lost sales in case of unanticipated excessive demands, re-production during the period is allowed. In addition, customers are assumed to have different needs categorized by some criteria, so that different selling policies are considered for each customer group of the target market. Furthermore, the satisfaction guarantee policy is considered for customers who do not like the products and return them to get their money back. The returning products, just like the unsold ones, are sold with reduced prices at the end of the period. Therefore, the main contribution of this paper relates to the market segmentation policy in the single period inventory problem that is concerned with the customer relationship management (CRM) in terms of customer importance and satisfaction.

The remainder of the paper is organized as follows. In Section 2, the problem is formally defined. The mathematical formulation of the problem is derived in Section 3. In Section 4, a modified particle swarm optimization algorithm is proposed to solve the complicated problem of Section 3. Since there is no benchmark available in the literature, a simulated annealing algorithm is also developed in Section 4 to validate the results obtained. In this section, the parameters of both algorithms are calibrated to obtain solutions with better qualities. A numerical example to illustrate the applicability of the model and the solution algorithm is provided in Section 5. Finally, conclusion and some recommendations for future research come in Section 6.

2 Problem Definition

In this paper a multi- product, multi- material, single-period inventory control problem with discount is investigated. For raw materials, it is assumed that the vendor use the cumulative discount policy. Both raw materials and final products are prepared only once at the beginning of the period. However, during the period if the final product runs out due to excessive demand, it can be produced for customers who would wait for preparation of a final product using the remaining raw material. Since the demand for a final product is random, to reduce risk of not being able to satisfy unanticipated excessive demand, it would be wise if some raw materials are kept unused for production during the period. Nonetheless, the remaining raw materials and final products at the end of the period would be useless and are sold with intangible prices. In short, the general properties of the problem are as follow:

- To produce product i, m different raw materials each with different usage quantities are required.
- Demand of a product is a random variable with a known probability distribution.
- Some customers are willing to wait for the product to be prepared during the period (shortage in terms of backorder is allowed.)
- A specific machine manufactures each product with a known percentage of nonconformity.
- Each final product has a specified fraction nonconformity. The nonconformity either can be due to the manufacturing failure rate or can be assumed for perishable items.
- Budget to buy raw materials and to produce final products is limited.
- Customers are different with different needs categorized by some criteria, so that different selling policies are considered for each group in the target market.
- If customers do not like the products that they buy, they can return them. The returned products similar to remaining ones are sold at the end of the period with discount prices.

An overall scheme of the inventory of raw materials, production process, final products, and customer groups (market strategy) is depicted in Fig. 1.

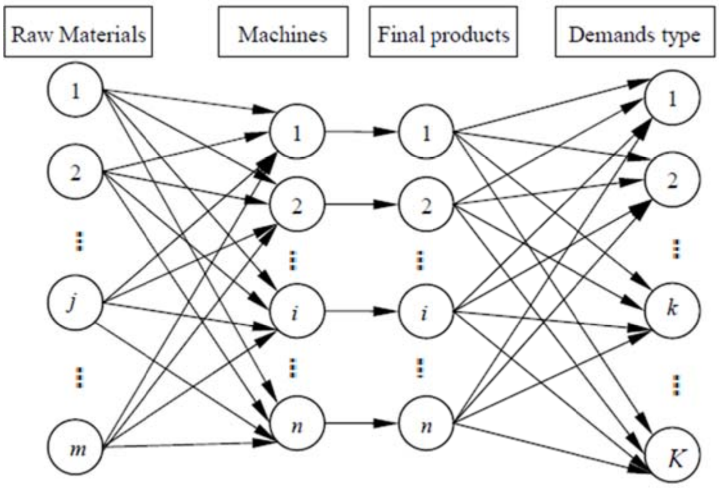

Figure 1: An overall scheme of the problem

3 Modeling

The mathematical formulation of the problem at hand is performed by first introducing the notations in Subsection 3.1 followed by cost and revenue derivations in Subsection 3.2. Then, the constraints are given in Subsection 3.3.

3.1 Notations

QR_j The quantity of *j*th raw material, $j = 1,2,\ldots,m$

η_{ij} The usage rate of *j*th raw material in ith product, $i = 1,\ldots,n$

γ_j The discount rate for *j*th raw material, ($0 < \gamma_j \le 1$)

q_{jl} The *l*th break point for cumulative discount of *j*th raw material, $l = 1,\ldots,L$

C_j The base unit cost of providing *j*th raw material

L_j The price per unit of the remaining *j*th raw material at the end of the period

h_j The holding cost per unit of the remaining *j*th raw material at the end of the period

y_{jl} A binary variable equal 1 if $q_{j,l} \leq QR_j < q_{j,l+1}$, otherwise 0

QS_{ik} The quantity of *i*th final product provided for *k*th customer group at the beginning of the period, $k = 1,\dots,K$

QS'_{ik} The quantity of *i*th final product manufactured for *k*th customer group during the period

τ_i The fraction nonconformity of *i*th final product

L'_i The price per unit of the remaining *i*th final product at the end of the period

r_{ik} The selling price per unit of *i*th final product for *k*th customer group

x_{ik} A fraction of *k*th customer group that are satisfied by *i*th final product ($0 \leq x_{ik} \leq 1$)

α_k A fraction of *k*th customer group that wait for their products to be manufactured during the period

C'_i The production cost per unit of *i*th final product

h'_i The holding cost per unit of *i*th final product that is unused until the end of the period

π_k The shortage cost per unit of a final product for *k*th customer group

D_{ik} The demand of *k*th customer group for *i*th final product (a random variable)

$f_{D_{ik}}(d_{ik})$ The probability density function for the demand of *k*th customer group for *i*th final product during the period

$F_{D_{ik}}(d_{ik})$ The cumulative distribution functions of the demand of *k*th customer group for *i*th final product during the period

ε_i A fraction of customers who return their purchased final product *i*

B The total available budget to provide raw materials and produce final products

U The profit (random variable)

$\overline{U}$ The expected profit

3.2 Deriving Profit

Four different costs are anticipated in the single-period inventory control problem; (a) cost of providing raw materials, (b) cost of transforming raw materials into final products, (c) holding cost of raw materials and final products that are unused during the period, and (d) the shortage cost. In what follows each of these costs are discussed and derived.

(a) Cost of providing raw materials

It is assumed the vendor is using the cumulative discount policy to provide raw materials. In this case, the more materials are bought, the more discount price is received. In other words the cost of providing raw materials is

$$\sum_{j=1}^{m}\sum_{l=0}^{L}\left(1-\gamma_j\right)^l C_j QR_j y_{jl} \tag{1}$$

where γ_j $\left(0 \leq \gamma_j \leq 1\right)$, is determined based on the market and the binary variable y_{jl} is defined according to the quantity purchased as

$$y_{jl} = \begin{cases} 1, & \text{if} \quad q_{jl} \leq QR_j < q_{j+1,l} \\ 0, & \text{else.} \end{cases}$$

(b) Cost of transforming raw materials into final products

The quantity of conforming product i manufactured in the period is obtained based on the machine fraction nonconformity (and/or perishable percentage) and the quantity manufactured during the period. As a result, the cost of transforming raw materials into final products in the period becomes

$$\sum_{i=1}^{n} C_i^{'} \left(\left(\frac{1}{1-\tau_i} \right) \sum_{k=1}^{K} \left(QS_{ik} + QS_{ik}^{'} \right) \right). \tag{2}$$

(c) Holding cost

The holding cost is applied to remaining raw materials and final products at the end of the period. For raw materials, the total holding cost can be obtained as

$$\sum_{j=1}^{m} h_j \left(QR_j - \sum_{i=1}^{n} \eta_{ij} \left(\frac{1}{1-\tau_i} \right) \sum_{k=1}^{K} \left(QS_{ik} + QS_{ik}^{'} \right) \right). \tag{3}$$

For final products, the holding cost is calculated by

$$\sum_{i=1}^{n} h_i^{'} \left(\sum_{k=1}^{K} \left(QS_{ik} + QS_{ik}^{'} \right) - \sum_{k=1}^{K} x_{ik} D_{ik} + \varepsilon_i \sum_{k=1}^{K} x_{ik} D_{ik} \right), \tag{4}$$

where the term $\varepsilon_i \sum_{k=1}^{K} x_{ik} D_{ik}$ is used for products that are returned using the "satisfaction guarantee" policy and remain until the end of the period.

(d) Shortage cost

Since the returned products are considered lost sales, the shortage cost is derived based on the usual lost sale quantities along with the quantities returned. In other words, the shortage cost is obtained as

$$\sum_{i=1}^{n} \pi_i \left(\sum_{k=1}^{K} D_{ik} - \sum_{k=1}^{K} x_{ik} D_{ik} + \varepsilon_i \sum_{k=1}^{K} x_{ik} D_{ik} \right). \tag{5}$$

3.3 Revenue

In the single period inventory problem, revenue is not only obtained by selling the final products during the period but also by selling the remaining final products and raw materials at the end of the period. Hence, the revenue is calculated as

$$\begin{aligned} &\sum_{i=1}^{n} \sum_{k=1}^{K} \left(1-\varepsilon_i \right) x_{ik} D_{ik} r_{ik} + \sum_{j=1}^{m} L_j \left[QR_j - \sum_{i=1}^{n} \eta_{ij} \left(\frac{1}{1-\tau_i} \right) \left(QS_{ik} + QS_{ik}^{'} \right) \right] \\ &+ \sum_{i=1}^{n} L_i^{'} \left[\left(QS_{ik} + QS_{ik}^{'} \right) - x_{ik} D_{ik} + \varepsilon_i x_{ik} D_{ik} \right]. \end{aligned} \tag{6}$$

Finally, the profit function can be simply derived using Eqs. (1) to (6) as

$$\begin{aligned} U\left(QR, QS, QS^{'}, D\right) = &\sum_{i=1}^{n} \sum_{k=1}^{K} \left(1-\varepsilon_i \right) x_{ik} D_{ik} r_{ik} + \sum_{j=1}^{m} L_j \left[QR_j - \sum_{i=1}^{n} \eta_{ij} \left(\frac{1}{1-\tau_i} \right) \left(QS_{ik} + QS_{ik}^{'} \right) \right] \\ &+ \sum_{i=1}^{n} L_i^{'} \left[\left(QS_{ik} + QS_{ik}^{'} \right) - x_{ik} D_{ik} + \varepsilon_i x_{ik} D_{ik} \right] \\ &- \sum_{j=1}^{m} \sum_{l=0}^{L} \left(1-\gamma_j \right)^l C_j QR_j y_{jl} + \sum_{i=1}^{n} C_i^{'} \left(\left(\frac{1}{1-\tau_i} \right) \sum_{k=1}^{K} \left(QS_{ik} + QS_{ik}^{'} \right) \right) \end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=1}^{n}\pi_i\left(\sum_{k=1}^{K}D_{ik}-\sum_{k=1}^{K}x_{ik}D_{ik}+\varepsilon_i\sum_{k=1}^{K}x_{ik}D_{ik}\right)\\
&+\sum_{i=1}^{n}h_i^{'}\left(\sum_{k=1}^{K}\left(QS_{ik}+QS_{ik}^{'}\right)-\sum_{k=1}^{K}x_{ik}D_{ik}+\varepsilon_i\sum_{k=1}^{K}x_{ik}D_{ik}\right)\\
&+\sum_{j=1}^{m}h_j\left(QR_j-\sum_{i=1}^{n}\eta_{ij}\left(\frac{1}{1-\tau_i}\right)\sum_{k=1}^{K}\left(QS_{ik}+QS_{ik}^{'}\right)\right).
\end{aligned}
\tag{7}
$$

As a result, in order to obtain the expected profit function, one can simply employ the expectation operation to get

$$
\overline{U}\left(QR,QS,QS^{'},D\right)=\sum_{i=1}^{n}\sum_{k=1}^{K}\int_0^{\infty}U\left(QR,QS,QS^{'},D_{ik}\right)f_{D_{ik}}\left(d_{ik}\right)dd_{ik}
$$

$$
=\int_0^{\infty}\left(\begin{aligned}
&\sum_{i=1}^{n}\sum_{k=1}^{K}\left(1-\varepsilon_i\right)x_{ik}d_{ik}r_{ik}+\sum_{j=1}^{m}L_j\left[QR_j-\sum_{i=1}^{n}\eta_{ij}\left(\frac{1}{1-\tau_i}\right)\left(QS_{ik}+QS_{ik}^{'}\right)\right]\\
&+\sum_{i=1}^{n}L_i^{'}\left[\left(QS_{ik}+QS_{ik}^{'}\right)-x_{ik}d_{ik}+\varepsilon_i x_{ik}d_{ik}\right]-\sum_{j=1}^{m}\sum_{l=0}^{L}\left(1-\gamma_j\right)^{l}C_jQR_jy_{jl}\\
&+\sum_{i=1}^{n}C_i^{'}\left(\left(\frac{1}{1-\tau_i}\right)\sum_{k=1}^{K}\left(QS_{ik}+QS_{ik}^{'}\right)\right)+\sum_{i=1}^{n}\pi_i\left(\sum_{k=1}^{K}d_{ik}-\sum_{k=1}^{K}x_{ik}d_{ik}+\varepsilon_i\sum_{k=1}^{K}x_{ik}d_{ik}\right)\\
&+\sum_{i=1}^{n}h_i^{'}\left(\sum_{k=1}^{K}\left(QS_{ik}+QS_{ik}^{'}\right)-\sum_{k=1}^{K}x_{ik}d_{ik}+\varepsilon_i\sum_{k=1}^{K}x_{ik}d_{ik}\right)\\
&+\sum_{j=1}^{m}h_j\left(QR_j-\sum_{i=1}^{n}\eta_{ij}\left(\frac{1}{1-\tau_i}\right)\sum_{k=1}^{K}\left(QS_{ik}+QS_{ik}^{'}\right)\right)
\end{aligned}\right)f_{D_{ik}}\left(d_{ik}\right)dd_{ik}.
\tag{8}
$$

Since x_{ik} is dependent on the integral interval, without loss of generality, in the integral terms in which x_{ik} exists, if the interval is changed to a proper one, then x_{ik} can be eliminated. The simplified equation the of expected profit function can be written as

$$
\begin{aligned}
\overline{U}\left(QR,QS,QS^{'},D\right)=&\sum_{i=1}^{n}\sum_{k=1}^{K}\int_0^{QS_{ik}+QS_{ik}^{'}}\left(1-\varepsilon_i\right)r_{ik}d_{ik}f_{D_{ik}}\left(d_{ik}\right)dd_{ik}\\
&+\sum_{j=1}^{m}L_j\left[QR_j-\sum_{i=1}^{n}\eta_{ij}\left(\frac{1}{1-\tau_i}\right)\sum_{k=1}^{K}\left(QS_{ik}+QS_{ik}^{'}\right)\right]\\
&+\sum_{i=1}^{n}L_i^{'}\sum_{k=1}^{K}\left[\left(QS_{ik}+QS_{ik}^{'}\right)+(\varepsilon_i-1)\int_0^{QS_{ik}+QS_{ik}^{'}}d_{ik}f_{D_{ik}}\left(d_{ik}\right)dd_{ik}\right]\\
&-\sum_{j=1}^{m}\sum_{l=0}^{L}\left(1-\gamma_j\right)^{l}C_jQR_jy_{jl}+\sum_{i=1}^{n}C_i^{'}\left(\left(\frac{1}{1-\tau_i}\right)\sum_{k=1}^{K}\left(QS_{ik}+QS_{ik}^{'}\right)\right)\\
&+\sum_{j=1}^{m}h_j\left(QR_j-\sum_{i=1}^{n}\eta_{ij}\left(\frac{1}{1-\tau_i}\right)\sum_{k=1}^{K}\left(QS_{ik}+QS_{ik}^{'}\right)\right)\\
&+\sum_{i=1}^{n}h_i^{'}\sum_{k=1}^{K}\left(\left(QS_{ik}+QS_{ik}^{'}\right)+(\varepsilon_i-1)\int_0^{QS_{ik}+QS_{ik}^{'}}d_{ik}f_{D_{ik}}\left(d_{ik}\right)dd_{ik}\right)\\
&+\sum_{i=1}^{n}\pi_i\left(\sum_{k=1}^{K}E\left(D_{ik}\right)+(\varepsilon_i-1)\sum_{k=1}^{K}\int_0^{QS_{ik}+QS_{ik}^{'}}d_{ik}f_{D_{ik}}\left(d_{ik}\right)dd_{ik}\right).
\end{aligned}
\tag{9}
$$

3.4 Constraints

The first set of constraints corresponds to the cumulative discount policy used to purchase raw materials as

$$\begin{aligned} &QR_j \ge q_{jl} y_{jl}, \qquad \forall j = 1,\dots,m \ \ \& \ \ l = 1,\dots,L \\ &QR_j < q_{jl+1} y_{jl}, \qquad \forall j = 1,\dots,m \ \ \& \ \ l = 1,\dots,L \\ &\sum_{j=1}^{m} y_{jl} = 1. \end{aligned} \tag{10}$$

In addition, since different products need different raw materials in various quantities and that the quantity of each product produced is lower than the total quantity of raw materials used to produce them, we have

$$\sum_{j=1}^{m} \frac{1}{\eta_{ij}} QR_j \ge \left(\frac{1}{1-\tau_i}\right)\left(\sum_{k=1}^{K}(QS_{ik} + QS'_{ik})\right), \qquad \forall i = 1,\dots,n. \tag{11}$$

If final products that are manufactured at the beginning of the period are sold out during the period, there will be production set up for customers who are willing to wait. In this case, the number of manufactured products during the period is less than the demand of the waiting customers. Moreover, in order to obtain the number of manufactured products during the period the following constraints are used. These constraints assure us that total number of items produced in the period is not more than the demand. These constraints are provided for all the products and customer groups as

$$\int_{QS_{ik}}^{QS_{ik}+QS'_{ik}} d_{ik}\, dd_{ik} \le \alpha_k \int_{QS_{ik}}^{\infty} d_{ik}\, dd_{ik}, \qquad \forall i = 1,\dots n, \ \& \ \ k = 1,\dots,K. \tag{12}$$

To help decision makers to apply different policies on various customers with different priorities, customers are categorized. In the following constraints if the decision maker desires to satisfy the demand of some customer groups less than 100%, he simply can insert the percentage he wants, otherwise the model obtains the percentage satisfied, optimally. These constraints calculate the least number of items the manufacturer produces each product in order to not fail to employ a desired policy for that costumer group.

$$\int_{0}^{QS_{ik}+QS'_{ik}} d_{ik}\, dd_{ik} = x_{ik}, \qquad \forall i = 1,\dots n, \ \& \ k = 1,\dots,K. \tag{13}$$

In the single period inventory problem, there are often some investors with limited budget to invest in the period. This budget is required to purchase raw materials and to transform them into final products. Thus, the budget constraint becomes

$$\sum_{j=1}^{m}\sum_{l=0}^{L}\left(1-\gamma_j\right)^l C_j QR_j y_{jl} + \sum_{i=1}^{n} C'_i \left(\frac{1}{1-\tau_i}\right)\sum_{k=1}^{K}\left(QS_{ik} + QS'_{ik}\right) \le B. \tag{14}$$

3.5 The Mathematical Model

By summing up the mathematical relations (9) to (14), the final model is

$$\begin{aligned} Max\ \overline{U} = {} & \sum_{i=1}^{n}\sum_{k=1}^{K}\int_{0}^{QS_{ik}+QS'_{ik}} \left(1-\varepsilon_i\right) r_{ik} d_{ik} f_{D_{ik}}\left(d_{ik}\right) dd_{ik} + \sum_{j=1}^{m} L_j \left[QR_j - \sum_{i=1}^{n}\eta_{ij}\left(\frac{1}{1-\tau_i}\right)\sum_{k=1}^{K}\left(QS_{ik} + QS'_{ik}\right)\right] \\ & + \sum_{i=1}^{n} L'_i \sum_{k=1}^{K}\left[\left(QS_{ik} + QS'_{ik}\right) + (\varepsilon_i - 1)\int_{0}^{QS_{ik}+QS'_{ik}} d_{ik} f_{D_{ik}}\left(d_{ik}\right) dd_{ik}\right] \\ & - \sum_{j=1}^{m}\sum_{l=0}^{L}\left(1-\gamma_j\right)^l C_j QR_j y_{jl} + \sum_{i=1}^{n} C'_i \left(\left(\frac{1}{1-\tau_i}\right)\sum_{k=1}^{K}\left(QS_{ik} + QS'_{ik}\right)\right) \\ & + \sum_{j=1}^{m} h_j \left(QR_j - \sum_{i=1}^{n}\eta_{ij}\left(\frac{1}{1-\tau_i}\right)\sum_{k=1}^{K}\left(QS_{ik} + QS'_{ik}\right)\right) \end{aligned}$$

$$+\sum_{i=1}^{n} h_i' \sum_{k=1}^{K}\left(\left(QS_{ik}+QS_{ik}'\right)+(\varepsilon_i-1)\int_0^{QS_{ik}+QS_{ik}'} d_{ik} f_{D_{ik}}\left(d_{ik}\right) dd_{ik}\right)$$
$$+\sum_{i=1}^{n} \pi_i \left(\sum_{k=1}^{K} E\left(D_{ik}\right)+(\varepsilon_i-1)\sum_{k=1}^{K}\int_0^{QS_{ik}+QS_{ik}'} d_{ik} f_{D_{ik}}\left(d_{ik}\right) dd_{ik}\right)$$

$$\begin{aligned}
s.t.\quad & QR_j \ge q_{jl} y_{jl}, \qquad \forall j=1,\ldots,m \;\;\&\;\; l=1,\ldots,L\\
& QR_j < q_{jl+1} y_{jl}, \qquad \forall j=1,\ldots,m \;\;\&\;\; l=1,\ldots,L\\
& \sum_{j=1}^{m} y_{jl} = 1\\
& \sum_{j=1}^{m} \frac{1}{\eta_{ij}} QR_j \ge \left(\frac{1}{1-\tau_i}\right)\left(\sum_{k=1}^{K}(QS_{ik}+QS_{ik}')\right), \qquad \forall i=1,\ldots,n\\
& \int_{QS_{ik}}^{QS_{ik}+QS_{ik}'} d_{ik} dd_{ik} \le \alpha_k \int_{QS_{ik}}^{\infty} d_{ik} dd_{ik}\;, \qquad \forall i=1,\ldots n, \;\&\;\; k=1,\ldots,K\\
& \int_0^{QS_{ik}+QS_{ik}'} d_{ik} dd_{ik} = x_{ik}, \qquad \forall i=1,\ldots n, \;\&\; k=1,\ldots,K\\
& \sum_{j=1}^{m}\sum_{l=0}^{L}\left(1-\gamma_j\right)^l C_j QR_j y_{jl} + \sum_{i=1}^{n} C_i' \left(\frac{1}{1-\tau_i}\right)\sum_{k=1}^{K}\left(QS_{ik}+QS_{ik}'\right) \le B\\
& QR_j \ge 0, \qquad QS_{ik} \ge 0, \qquad QS_{ik}' \ge 0, \qquad y_{il} = \left\{0,1\right\},\\
& \forall i=1,\ldots,n \;\;,\;\; j=1,\ldots,m \;\;,\;\; k=1,\ldots,K \;\;,\;\; l=1,\ldots,L.
\end{aligned}$$

As seen, the above model is a non-linear integer problem. The model is hard enough to solve due to its two main characteristics, i.e. having integer decision variables along with non-linear objective function and constraints. The complexity of the model grows as the numbers of integer decision variables and constraints become large in even medium-size problems. The above characteristics justify the use of a meta-heuristic method to solve the problem.

4 A Solution Algorithm

Since the non-linear integer mathematical problem modeled in Section 3 is hard to be solved analytically, the particle swarm optimization (PSO) algorithm is modified in this research to find a near-optimum solution. In this section, after providing the basis of the PSO algorithm in Subsection 4.1, the modified PSO algorithm is proposed in Subsection 4.2. In Subsection 4.3, the parameters of the proposed algorithm are tuned using a regression approach and solving a quadratic mathematical model. Besides, in order to validate the results obtained, a simulated annealing approach is proposed in Subsection 4.4 to serve as a benchmark.

4.1 The Particle Swarm Optimization Algorithm

PSO is a population-based algorithm first introduced by Kennedy & Eberhart [17]. Kennedy et al. [18] discussed social and computational concepts of PSO. This algorithm starts by a random population of solutions, named particles. Each particle is first assigned a randomized velocity and then it iteratively searches the problem space to find a solution. In each iteration, the objective function value based on each solution is the particle's current location [4]. The movement of each particle to another location determined by using some aspect of its current location, the location of the best fitness achieved so far across the whole population (global best fitness), and the previous best position of the particle. Eventually it is likely, the swarm become close to an optimum fitness of the objective function. The particle swarm is more than just a collection of particles. Particles by itself have no power to solve any problem; it is their interactions that make it a powerful procedure [4]. The following parameters are used to explain the PSO algorithm in detail.

t Iteration number

w The inertia weight as a coefficient of the particle's current location to update its velocity; it can be interpreted as the fluidity of the medium in which a particle moves

w_d A coefficient of *w* that gradually reduces w to a much lower value

c_1 Determines the magnitude of the random force in the direction of the last best particle location

c_2 Determines the magnitude of the random force in the direction of the global best location

a_1 A uniform random variable in [0,1] as a coefficient of the best location of a specific particle till the last iteration; this coefficient allows random possible moves of particles

a_2 A uniform random variable in interval [0,1] as a coefficient of the best global location; this coefficient also allows random possible moves of particles

O_{Best} Objective value of the best location of a specific particle till the last iteration

O_{Best}^{Global} Objective value of the global best location

$\vec{P}_{variable}^{t}$ Position vector of iteration t for a specific variable (in this paper QR, QS, QS')

$\vec{P}_{variable}^{Best}$ Position vector of the best location of a specific particle till the last iteration

$\vec{P}_{variable}^{GBest}$ Position vector of the global best location

$MaxV_{variable}$ Upper bound of velocity for a specific variable that limits the magnitude of particle movements

In PSO, the velocity of each particle is iteratively updated so that particles randomly move toward the best location and eventually the global best location of all particles found until the last iteration. Assuming a D-dimensional search space, the algorithm of implementing PSO is as follows [26]:

1. Initialize a population array of particles with random positions and velocities on D dimensions of the search space.
2. For each particle, evaluate the desired optimization fitness function in D variables.
3. Compare particle's fitness evaluation with its O_{Best}. If the current value is better than O_{Best}, then replace O_{Best} with the current value and set $\vec{P}_{variable}^{Best}$ equal to the current location.
4. Change the velocity and position of the particles according to the following equations:

$$\vec{V}_{variable}^{t} = w\vec{V}_{variable}^{t-1} + c_1(a_1)(\vec{P}_{variable}^{Best} - \vec{P}_{variable}^{t-1}) + c_2(a_2)(\vec{P}_{variable}^{GBest} - \vec{P}_{variable}^{t-1}), \tag{15}$$

$$\vec{P}_{variable}^{t} = \vec{P}_{variable}^{t-1} + \vec{V}_{variable}^{t}. \tag{16}$$

5. If a stopping criterion is met, stop. Otherwise, go to Step 2.

4.2 The Modified PSO Algorithm

The steps involved in the modified PSO algorithm of this research to find near-optimum solutions of the problem at hand are shown in Algorithm (1) as follows:

Algorithm (1)

I. Initialization

I.1. Set the algorithm parameters:

$$MaxV_{variable}, \qquad w, \qquad w_d, \qquad c_1, \qquad c_2, \qquad population = 30.$$

II. Produce the first particles randomly

II.1. Calculate the maximum quantity of raw materials using

$$Max\ QR_j = 0.75\left(\frac{B.\sum_{i=1}^{n}\eta_{ij}}{C_j.\sum_{j=1}^{m}\sum_{i=1}^{n}\eta_{ij}}\right), \qquad \forall j = 1,\ldots,m. \tag{17}$$

In Eq. (17), 75 percent of the maximum quantity of raw materials is calculated so that some budget remains for production line.

II.2. Generate a uniform random variable in $[0, MaxQR_j]$ for each raw material j.

II.3. Using the random quantity of raw material generated in Step II.2, determine the quantity of the final product i to be produced within the budget as

$$QS_i = \min\left\{0.25\left(\frac{B}{n.C_i^{'}}\right), \min_j\left\{\frac{QR_j}{\eta_{ij}}\right\}\right\}, \qquad \forall i = 1,\ldots,n. \tag{18}$$

II.4. To determine the quantities of final products at the beginning of the period and their quantities produced during the period for each customer groups, use

$$QS_{ik} = \frac{(1-\alpha)QS_i}{K}, \qquad QS_{ik}^{'} = \frac{\alpha QS_i}{K}. \tag{19}$$

II.5. As far as constraint (12) remains valid, subtract one unit from $QS_{ik}^{'}$. In case $QS_{ik}^{'}$ becomes zero, subtract one unit from QS_{ik} (this step is optional).

II.6. In order to determine the discount level, calculate y_{jl} using QR_j and then determine l.

II.7. If the decision maker did not determine x_{ik}, calculate it using QS_{ik} and $QS_{ik}^{'}$.

III. Random particle fitness

III.1. Set the global best objective function (O_{Best}^{Global}) value as negative infinity.

III.2. Calculate the fitness of the random particle produced in Step (II).

III.3. Set the best objective (O_{Best}) as a current fitness of the particle and set the best position ($P_{variable}^{Best}$) as the current position of the particle.

III.4. If $O_{Best} > O_{Best}^{Global}$, then replace O_{Best}^{Global} and $P_{variable}^{GBest}$ by O_{Best} and $P_{variable}^{Best}$.

IV. Optimizing the particles

IV.1. Update the particle velocity using following formulae

$$\begin{aligned} V_{QR}^{t} &= w.V_{QR}^{t-1} + c_1(a_1)(P_{QR}^{Best} - P_{QR}^{t-1}) + c_2(a_2)(P_{QR}^{GBest} - P_{QR}^{t-1}), \\ V_{QS}^{t} &= w.V_{QS}^{t-1} + c_1(a_1)(P_{QS}^{Best} - P_{QS}^{t-1}) + c_2(a_2)(P_{QS}^{GBest} - P_{QS}^{t-1}). \end{aligned} \tag{20}$$

IV.2. If the updated velocities are more than upper bound of velocities, change them by

$$\begin{aligned} \left|V_{QR}^{t}\right| > \left|MaxV_{QR}\right| &\Rightarrow \begin{cases} \text{if } \ V_{QR}^{t} > 0 \Rightarrow V_{QR}^{t} = MaxV_{QR} \\ \text{if } \ V_{QR}^{t} < 0 \Rightarrow V_{QR}^{t} = -MaxV_{QR}, \end{cases} \\ \left|V_{QS}^{t}\right| > \left|MaxV_{QS}\right| &\Rightarrow \begin{cases} \text{if } \ V_{QS}^{t} > 0 \Rightarrow V_{QS}^{t} = MaxV_{QS} \\ \text{if } \ V_{QS}^{t} < 0 \Rightarrow V_{QS}^{t} = -MaxV_{QS}. \end{cases} \end{aligned} \tag{21}$$

IV.3. Update the particles position using

$$\begin{aligned} P_{QR}^{t} &= P_{QR}^{t-1} + V_{QR}^{t}, \\ P_{QS}^{t} &= P_{QS}^{t-1} + V_{QS}^{t}, \\ P_{QS'}^{t} &= P_{QS'}^{t-1} + V_{QS'}^{t}. \end{aligned} \tag{22}$$

V. Checking the constraints

V.1. Check Constraint (11); if it is not satisfied, inverse the velocity vector of QR_j for each j based on which Constraint (11) is not valid.

V.2. Check Constraint (12); if it is not satisfied, inverse the velocity vector of $QS_{ik}^{'}$ for each i and k based on which Constraint (11) is not valid.

V.3. Check Constraint (13); if it is not satisfied for x_{ik}, then inverse the velocity vector for each QS_{ik} based on which the Constraint (13) is not valid.

V.4. Check Constraint (14); if it is not satisfied, set the value of the corresponding objective function a negative number (such as –1 or –big M or another optional negative number).

VI. The particle fitness

VI.1. If Constraint (14) is valid; calculate the objective value of the corresponding particle.

VI.2. If the calculated objective value of the particle is better than O_{Best}, update O_{Best} by the current objective value of the particle and subsequently update the best position (P_{Best}).

VI.3. If $O_{Best} > O_{Best}^{Global}$, then replace O_{Best}^{Global} and $P_{variable}^{GBest}$ by O_{Best} and $P_{variable}^{Best}$.

VII. Stopping criterion

VII.1. Multiply w by w_d, i.e., $w^t = w^{t-1} \times w_d$.

VII.2. If the number of iterations is less than 1000, go to Step (IV), otherwise stop and print the results.

4.3 Parameter Calibration

One of the parameters is the population size that usually affects the performance of a population-based meta-heuristic algorithm. This parameter is often set empirically based on the problem complexity. As indicated by Poli et al. [28], the range of 20–50 is quite common. For the problem under investigation, this parameter is set 30 empirically based on a trial and error procedure. However, the other parameters used in the modified PSO algorithm are tuned via a regression approach and solving a quadratic mathematical model. To do this, important parameters with significant impacts are first discussed. Then, these parameters are randomly changed 30 times, each in a range proposed in the literature, using a sample of size 5. Finally, the mean of the objective functions for each sample is treated as the performance of the algorithm in that sample.

The parameters c_1 and c_2 that are often called acceleration coefficients have significant impacts on the performance of PSO. While the values of c_1 and c_2 in early PSO applications were adopted $c_1 = c_2 = 2$, in this article $c_1 = c_2 = c$, where c changes in the range [1,3].

One of the techniques to control the magnitude of the velocity is to define bounds so that each velocity is kept within the range [$-MaxV_{variable}$, $+MaxV_{variable}$]. Here, the bounds are examined in the range [2,5].

Another important parameter is "inertia weight." Some researchers have found that the best performance could be obtained by initially setting w to some relatively high value (e.g., 0.9), which allows particles move in low viscosity to perform extensive exploration. However, after some iterations w will gradually reduce to have more exploitation so that homing into local optima would be better (e.g. 0.4). Nonetheless, it is possible to start values of $w > 1$, which would make the swarm unstable, provided that the value is reduced sufficiently to bring the swarm in a stable region [31]. In this research, the inertia weight is changed in the range [0.4,1.2] and w_d takes values in [0.8,1].

Evaluating the mean objective function 30 times, each with a sample of 5 instances with different parameter settings, the experimental results of employing the modified PSO are given in Table 1. These results are obtained using a numerical example given in Section 5. Using the results in Table 1, the quadratic regression with four variables (X_{MaxV}, X_w, X_{w_d}, and X_c) is first fitted for the mean objective value ($\overline{Y}$). Then, the combination of the variables that maximizes $\overline{Y}$ is selected to be the calibrated values of the parameters. The fitted response is

$$\begin{aligned}\overline{Y} = &-11600000X_{MaxV}^2 + 8800000X_{MaxV}X_w - 289700000X_{MaxV}X_{w_d} - 5900000X_{MaxV}X_c - 353900000X_w^2 \\ &+ 71300000X_wX_{w_d} - 67200000X_wX_c + 5402100000X_{w_d}^2 + 297300000X_{w_d}X_c - 169300000X_c^2 \\ &+ 355900000X_{MaxV} + 505400000X_w - 8847400000X_{w_d} + 697000000X_c + 2203800000.\end{aligned} \tag{23}$$

Hence, the quadratic mathematical model is

$$\begin{aligned}Max\ \overline{Y} = &-11600000X_{MaxV}^2 + 8800000X_{MaxV}X_w - 289700000X_{MaxV}X_{w_d} - 5900000X_{MaxV}X_c \\ &- 353900000X_w^2 + 71300000X_wX_{w_d} - 67200000X_wX_c + 5402100000X_{w_d}^2 \\ &+ 297300000X_{w_d}X_c - 169300000X_c^2 + 355900000X_{MaxV} + 505400000X_w \\ &- 8847400000X_{w_d} + 697000000X_c + 2203800000 \\ s.t.\ \ & 2 \le X_{MaxV} \le 5 \\ & 0.4 \le X_w \le 1.2 \\ & 0.8 \le X_{w_d} \le 1.0 \\ & 1 \le X_c \le 3.\end{aligned} \tag{24}$$

Solving the model, the calibrated PSO parameters are determined as:

$X^*_{MaxV} = 2, \quad X^*_{w} = 1.2, \quad X^*_{w_d} = 0.8, \quad X^*_{c} = 3$.

Table 1: Experimental results

No.	$Max\ V$	w	w_d	c	Objective Mean
1	3	0.750	1.000	1.500	211,580,000
2	5	1.016	0.826	1.836	145,650,000
3	4	1.041	0.986	2.549	220,660,000
4	5	0.935	0.970	2.607	211,130,000
5	3	0.768	0.996	1.271	-95,850,000
6	3	0.535	0.886	2.938	184,820,000
7	2	0.826	0.858	1.723	-2,470,000
8	5	1.151	0.938	2.263	118,080,000
9	5	1.048	0.821	1.284	-58,940,000
10	2	0.644	0.837	2.821	143,000,000
11	5	0.449	0.963	2.698	264,910,000
12	2	0.513	0.948	2.109	180,280,000
13	2	0.751	0.876	2.828	252,960,000
14	5	0.808	0.977	1.121	-130,110,000
15	2	0.712	0.807	1.039	-240,700,000
15	3	1.185	0.839	2.347	55,310,000
17	4	0.602	0.908	1.513	49,160,000
18	4	0.418	0.861	1.259	-98,630,000
19	5	0.628	0.909	1.470	-47,930,000
20	5	0.751	0.941	1.142	-208,110,000
21	4	1.154	0.992	1.321	-27,610,000
22	5	1.081	0.960	1.650	64,570,000
23	4	0.618	0.881	1.613	119,210,000
24	4	0.832	0.896	1.260	-89,050,000
25	2	0.532	0.853	2.219	148,200,000
26	5	0.527	0.977	1.062	-176,830,000
27	4	0.805	0.908	1.021	-171,410,000
28	3	0.577	0.805	1.776	50,150,000
29	5	0.983	0.868	1.131	-203,520,000
30	5	1.039	0.981	2.856	236,650,000

4.4 The Simulated Annealing Algorithm

SA works by analogy between the annealing process and the optimization of mathematical models. In this algorithm, it is assumed that an optimization problem is analogous to the arrangement status of molecules in a particular object in a specific temperatures, and the objective is to achieve the most regular crystal lattice configuration by minimizing the free energy of the system. If the cooling process is sufficiently slow, the final arrangement of molecules would have superiority [13,10].

In SA, in order to improve the objective function value, a new solution is determined in each iteration. If this new solution is better than the previous solution, the new solution will be replaced with the previous solution. Further, if it is also better than the best solution achieved till now, these two solutions will be changed [5]. However, if this new solution is not better than the previous solution, it will be accepted and replaced with the previous solution with some probability in the hope of escaping local optimal. The probability of this non-improved replacing depends on a temperature parameter, which is decreasing in each iteration of the Algorithm (2).

The steps involved in the SA algorithm developed in this paper to validate the results obtained by PSO are shown in Algorithm (2):

Algorithm (2)

1) Choose the maximum iterations, $It = 500$.
2) Choose the initial temperature $T_0 = 100$; and set the temperature as $T = T_0$.
3) Choose the number of iterations at each temperature, $N_t = 5$.
4) Set the final temperature ($T_f = 0.001$) and set the reduction constant T_d of temperature as $\left(T_f / T_0\right)^{(1/It)}$.
5) Produce twenty initial random solutions (similar to PSO initial random population).
6) Set the maximum objective values of solutions as the best solution.
7) Create twenty new solutions as follows:
   produce an integer random number in range [1,2]. If it is 1 use Eq. (25), if it is 2 use Eq. (26) as

$$\begin{aligned}&\text{New solution} = \text{Previous Solution} + \\ &\qquad \text{R} \times 2.5 \times (\text{a}_1 \times (\text{ A Random Sultion} - \text{ Previous Solution}))\end{aligned} \tag{25}$$

$$\begin{aligned}&\text{New Solution} = \text{Previous Solution} + \\ &\qquad \text{R} \times 2.5 \times (\text{a}_1 \times (\text{ Best Solution} - \text{ Previous Solution}))\end{aligned} \tag{26}$$

   where R is the direction of changing initially set equal to 1 and in iterations of the algorithm it can be changed into -1 or 1. a_1 is a uniform random number in [0, 1]. The number 2.5 has been obtained empirically so that it makes the new solution to be generated suitably.
8) Check all constraints (except constraint 14). If they are satisfied go to Step 9, otherwise set $R = -R$.
9) Check the new solution with its lower bound. If it is infeasible (for example, when *QS* becomes negative), set the new solution equal to its lower bound.
10) Check Constraint (14). If it is satisfied, calculate the objective value of the new solution, otherwise set the corresponding objective value as –big M.
11) Calculate Δ by

$$\Delta = \frac{(\text{Objective value of the new solution } - \text{ objective value of the previous solution})}{\left|\text{objective value of the previous solution}\right|} . \tag{27}$$

12) If Δ is positive, replace the new solution with the previous solution. Otherwise, generate a uniform random number. If this random number is bigger than $e^{\Delta/T}$, replace the new solution with the previous one. Else, do nothing.
13) If $N_t = 5$, go to step 14, otherwise set $N_t = N_t + 1$ and go to Step 6.
14) Update the temperature. $T = T \times T_d$.
15) If $It = 500$, stop and print the best solution. Otherwise, go to Step 6.

Note that a similar procedure to the one used for the PSO algorithm is employed to calibrate the parameters of the SA as well.

5 Numerical Examples

In order to demonstrate the application of the proposed model and to investigate the performance of the proposed solution methodology, a numerical example is provided in this section. Suppose there are 3 different products in a target market consisting of 3 different categories of customers. Let the random demand during the period follows a normal distribution with known mean (μ_{ik}) and variance (σ_{ik}^2) for each product and each customer group. These products are manufactured using 9 different types of raw materials with different usages. The total budget available for this investment opportunity is 15×10^8 currency units. For each raw material, the vendor proposes a cumulative discount with one break point at the quantity of 10,000. Table 2 shows the corresponding data of raw materials and Table 3 contains final product data. In Table 4, the usage rates of the raw materials used in final products are given and Table 5 contains the data corresponding to customer groups. The data of final products and customer groups are shown in Table 6.

Table 2: Data on raw materials

j	1	2	3	4	5	6	7	8	9
C_j	1000	200	100	100	2000	100	10000	100	200
γ_j	0.2	0.1	0.05	0.05	0.1	0.05	0.1	0.05	0.05
L_j	500	0	0	0	400	0	2000	0	0
h_j	10	5	5	2	20	5	25	5	2

Table 3: Data on final products

i	1	2	3
τ_i	0.01	0.01	0.001
$L_i^{'}$	600	500	2000
$h_i^{'}$	15	30	50
$C_i^{'}$	700	500	600
ε_i	0.01	0.001	0.001

Table 4: The usage rate of *j*th raw material in *i*th product

η_{ij}	1	2	3	4	5	6	7	8	9
1	1	2	2	1	0	0	0	0	0
2	0	0	0	0	1	1	0	0	0
3	0	0	0	0	0	0	1	2	1

Table 5: Data on customer groups

K	1	2	3
α_k	0.4	0.2	0.1
π_k	200	100	100

Table 6: Data on final products and customer groups

	i/k	1	2	3
r_{ik}	1	3800	6700	17,000
	2	3500	6500	17,000
	3	4500	7000	17,000
x_{ik}	1	0.5	0	0
	2	0.7	0	0
	3	0.8	0	0
μ_{ik}	1	30,000	20,000	500
	2	10,000	20,000	200
	3	10,000	10,000	1000
σ_{ik}	1	1000	1000	50
	2	1000	1000	50
	3	800	800	50

The proposed parameter-tuned PSO algorithm is employed on the numerical illustration using the MATLAB software 50 times, where the best objective value achieved is 2.9322×10^8. The best result is summarized in Table 7. Note that since a high cost is associated with product 3, its production has no economic justification. As a result, the fractions of customer groups that are satisfied with product 3 are zeros. Similarly, to validate the results obtained, the SA described in Section 4 is utilized as well to find the best objective value of 2.5954×10^8. The best solution of the SA algorithm is shown in Table 8. As it can be seen, PSO provides a better near-optimum solution.

Table 7: The summarized PSO solution

QR_j	QR_1	QR_2	QR_3	QR_4	QR_5	QR_6	QR_7	QR_8	QR_9
	77,750	155,500	155,500	77,750	57,730	57,730	0	0	0
QS_{ik}	QS_{11}	QS_{12}	QS_{13}	QS_{21}	QS_{22}	QS_{23}	QS_{31}	QS_{32}	QS_{33}
	0	0	2,400	11,631	11,631	11,631	0	0	0
QS'_{ik}	QS'_{11}	QS'_{12}	QS'_{13}	QS'_{21}	QS'_{22}	QS'_{23}	QS'_{31}	QS'_{32}	QS'_{33}
	32,431	32,388	9,753	11,130	11,126	0	0	0	0
x_{ik}	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}
	0.9925	0.9915	0.9964	0.9971	0.9971	0.9793	0	0	0

Table 8: The summarized SA solution

QR_j	QR_1	QR_2	QR_3	QR_4	QR_5	QR_6	QR_7	QR_8	QR_9
	78,960	157,930	157,930	78,960	73,700	73,550	0	0	0
QS_{ik}	QS_{11}	QS_{12}	QS_{13}	QS_{21}	QS_{22}	QS_{23}	QS_{31}	QS_{32}	QS_{33}
	34,669	31,601	11,905	23,655	23,655	23,655	0	0	0
QS'_{ik}	QS'_{11}	QS'_{12}	QS'_{13}	QS'_{21}	QS'_{22}	QS'_{23}	QS'_{31}	QS'_{32}	QS'_{33}
	0	0	0	683	0	0	0	0	0
x_{ik}	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}
	1.0000	0.9453	0.9914	1.0000	0.9999	1.0000	0	0	0

Thirty numerical examples in larger scale were also solved, based on which PSO showed to be the better algorithm to solve all. While the second part of these examples is the same as the one of the above examples, the first part is twice the size. The detailed solutions of the examples are not shown here to save spaces. Nonetheless, for a typical example of large-size problems, the best value of the objective function found by PSO and SA are 3.678×10^{12} and 3.4934×10^{12}, respectively.

6 Conclusion

In this research, a single-period inventory problem with market targeting consideration was investigated in which there were several final products and several raw materials with different usage rates. The objective was to determine the order size of both final products and raw materials before the selling period such that customer demands are satisfied and within an available budget, the expected profit is maximized. The problem was first mathematically formulated and then a modified parameter-tuned particle swarm optimization algorithm was utilized to find a near-optimum solution of the nonlinear programming problem. Finally, a numerical example was given to not only clarify the application of both the developed model and the solution algorithm, but also to validate the results obtained by employing a parameter-tuned simulated annealing algorithm.

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