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Optimal Inventory Policy for Single-Period Inventory Management Problem under Equivalent Value Criterion

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Abstract

In this paper, a single-period inventory management problem with fuzzy demand is considered. The costs include the purchase cost and the lost sale penalty cost. In addition, a salvage value for the residual product will be incurred. Based on credibility measure, the retailer's profit is measured by using Lebesgue-Stieltjes (L–S) integral. In the sense of equivalent value criterion, two optimization models are proposed where the uncertain demands are characterized by discrete and continuous possibility distributions. Furthermore, the analytical expressions of the optimal order quantity are derived in the above cases. Finally, several numerical examples are performed to demonstrate the efficiency of the proposed methods.

Keywords: inventory management, fuzzy demand, equivalent value, L–S integral, optimal order quantity

1 Introduction

The single-period inventory problem is one of the classical problems in the literature on inventory management [1, 15, 16]. In real life, many products have a limited selling period and there is no chance to place a second order during the period, so the single-period inventory problem provides a very appropriate framework to make decision about the optimal order quantity. The single-period inventory problem was first studied in 1950s. The goal of the single-period inventory problem is to find the optimal order quantity by minimizing the expected cost or maximizing the expected profit of retailer under random demand. Khouja [9] and Qin et al. [15] reviewed the extensive contributions to the single-period inventory problem. The major difficulty faced by the decision maker is to forecast the uncertain demand because it usually involves various uncertainties. The most of existing literature discussed the single-period inventory models with stochastic demands, in which the probability distributions were estimated according to historical data. However, the exact probability distribution is usually unavailable because of the lack of the related historical information in many cases. That is, a decision maker may face a fuzzy environment rather than a stochastic environment. In this case, the demands are approximately estimated based on the experts' experiences or subjective judgments and characterized by fuzzy variables with some possibility distributions.

In fuzzy decision systems, based on fuzzy set theory or possibility theory, the single-period inventory problem has been studied since 1990s. For example, Petrovic et al. [14] proposed two models for fuzzy single-period inventory problem, in which the imprecise cost was described by a triangular fuzzy number and the imprecise demand was represented by a discrete fuzzy number. Ishii and Konno [7] considered the fuzzy shortage cost in the single-period inventory problem, although the demand was stochastic. In view of the fuzzy demand, Kao and Hsu [8] proposed a single-period inventory model to find the optimal order quantity of the decision maker by the method of Yager ranking index [19]. Li et al. [10] proposed a model for single-period inventory problem with stochastic demand and fuzzy inventory costs. Under fuzzy demand, Dutta et al. [4] considered the optimal inventory policy for the single-period inventory problem with reordering opportunities. Chen and Ho [3] proposed a single-period inventory problem with quantity discount. Yu et al. [20] formulated

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a single-period inventory model with fuzzy price-dependent demand. Recently, random fuzzy single-period inventory problems and fuzzy random single-period inventory problems were proposed in [6], [5] and [17]. In the former literature, the methods of handling fuzzy numbers included two cases: arithmetic defuzzification in discrete membership function and Yager's ranking method in continuous membership function. Liu and Liu [11] defined the credibility measure to measure fuzzy events, then the expected value of fuzzy variable was defined via Choquet integral. Liu and Liu [12] provided expected value operator based on the optimistic function and the pessimistic function of fuzzy variable and discussed the linearity of expected value operator. When a function was monotonic, the expected value of the function of a fuzzy variable was studied in [18, 21].

In our single-period inventory model, the retailer's profit expression is not a monotonic function with respect to fuzzy demand. Since Choquet integral is a kind of nonlinear fuzzy integrals, the computation of the expected value of the retailer's profit is usually difficult, which motivates us to study the single-period inventory management problem form a new perspective. Based on credibility measure, we measure the retailer's profit by using L–S integral and derive the analytical solutions of optimal order quantity under two different conditions. The proposed method is based on the credibility measure theory, and the advantage of the method is that it doesn't require the continuity and monotony of the function of fuzzy demand.

The remainder of this paper is organized as follows. In the next section, we describe the fuzzy single-period inventory problem and define the equivalent value profit. Section 3 derives the analytical expressions of the optimal order quantity in two cases: a discrete fuzzy demand variable and a continuous fuzzy demand variable. Several numerical examples are solved to demonstrate the validity of the proposed analysis method in Section 4. Finally, Section 5 summarizes the main results in the paper.

2 Problem Statement

Consider a single-period inventory management problem consisting of one supplier and one retailer. To begin with, the supplier determines a wholesale price for the retailer. Then the retailer makes the decision to maximize his profit based on the salvage value, the lost sale penalty cost and the forecast of fuzzy demand. In order to describe the fuzzy single-period inventory problem clearly, we give the following notations that will be used in the formulation of our including problem.

Fixed parameters

c: the purchase cost of unit product;

p: the selling price of unit product;

s: the salvage value of unit residual product;

B: the lost sale penalty cost for unit unmet demand;

π: the profit function of product;

x^+: the maximum value of x and 0, i.e., $x^+ = \max\{x, 0\}$.

Uncertain parameter

ξ: the fuzzy customer demand for product.

Decision variable

Q: retailer's order quantity purchased from supplier.

To avoid trivial problems, we assume $p > c > s$ and $B > 0$ in the following discussion. When the retailer determines to order Q units products while the customer demand is ξ, the sales volume, holding quantity and shortage quantity for the retailer are denoted as $\min\{\xi, Q\}$, $\max\{Q-\xi, 0\}$ and $\max\{\xi - Q, 0\}$, respectively. Consequently, the profit for the retailer can be expressed as

$$\begin{aligned} \pi(Q,\xi) &= p\min\{\xi, Q\} + s\max\{Q-\xi, 0\} - B\max\{\xi - Q, 0\} - cQ \\ &= (p-c)\xi - (c-s)(Q-\xi)^+ - (p-c+B)(\xi-Q)^+. \end{aligned} \tag{1}$$

It is clear that the retailer's profit function $\pi(Q,\xi)$ is also a fuzzy variable. Since the credibility of fuzzy event $\{\xi \le r\}$ is a nondecreasing function with respect to r, we employ L–S integral [2] to define the equivalent value profit of $\pi(Q,\xi)$, i.e.,

$$\Pi(Q) = \int_{[0,+\infty)} \pi(Q,r)\mathrm{dCr}\{\xi \le r\}, \tag{2}$$

where the measure is induced by the credibility $\mathrm{Cr}\{\xi \leq r\}$ [13].

Based on the notations above, we find the optimal order quantity by solving the next optimization problem

$$\begin{array}{ll} \max & \Pi(Q) = \int_{[0,+\infty)} \pi(Q,r)\mathrm{dCr}\{\xi \leq r\} \\ \mathrm{s.t.} & Q \geq 0. \end{array} \tag{3}$$

In the next section, we shall discuss how to derive the optimal order quantity to problem (3) under two different conditions.

3 Model Analysis

In this section, we find the optimal order quantity by maximizing the equivalent value profit of the retailer in two cases: demand has a discrete possibility distribution and demand has a continuous possibility distribution.

3.1 Demand Follows Discrete Possibility Distribution

For a discrete fuzzy demand variable, we have the following result.

Theorem 1. *Let ξ be a discrete fuzzy demand variable with the possibility distribution* $\mathrm{Pos}\{\xi = k\} = \mu_k$, $k = 0, 1, 2, \ldots$, *i.e.,*

$$\xi \sim \begin{pmatrix} 0 & 1 & 2 & \ldots & N & \ldots \\ \mu_0 & \mu_1 & \mu_2 & \ldots & \mu_N & \ldots \end{pmatrix},$$

where $\mu_k > 0$, $k = 0, 1, 2, \ldots$. If $\sup_{k \geq 0} \mu_k = h$ with $h \in (0, 1]$, then the optimal order quantity Q_d^* can be determined by the following formula

$$Q_d^* = \min\left\{Q \mid \mathrm{Cr}\{\xi \leq Q\} \geq \frac{h(p+B-c)}{p+B-s}\right\}, \tag{4}$$

where $\mathrm{Cr}\{\xi \leq k\} = \left(\bigvee_{0\leq i\leq k} \mu_i + h - \bigvee_{i>k} \mu_i\right)/2.$

Proof. According to Eq. (1), we know the retailer's profit is equal to the following formula

$$\pi(Q,\xi) = \begin{cases} (p-c)\xi - (c-s)(Q-\xi), & \xi \leq Q \\ (p-c)\xi - (p-c+B)(\xi-Q), & \xi > Q. \end{cases} \tag{5}$$

For convenience, we denote $\Phi(k)$ as the credibility of fuzzy event $\{\xi \leq k\}$, i.e., $\Phi(k) = \mathrm{Cr}\{\xi \leq k\}$. Substituting Eq. (5) into the problem (3) leads to

$$\begin{aligned} \Pi(Q] &= \int_{[0,+\infty)} \pi(Q,r)\mathrm{dCr}\{\xi \leq r\} = \sum_{k=0}^{+\infty}[\Phi(k) - \Phi(k-1)]\pi(Q,k) \\ &= \sum_{k=0}^{Q}[\Phi(k) - \Phi(k-1)] \cdot [(p-c)k - (c-s)(Q-k)] \\ &\quad + \sum_{k=Q+1}^{\infty} [\Phi(k) - \Phi(k-1)] \cdot [(p-c)k - (p-c+B)(k-Q)]. \end{aligned} \tag{6}$$

We use the difference method to find the maximum value of $\Pi(Q)$. Let

$$\Delta\Pi(Q) = \Pi(Q+1) - \Pi(Q). \tag{7}$$

Then, $\Delta\Pi(Q)$ is the change in equivalent value profit when we switch from Q to $Q+1$. Substituting Eq. (6) into Eq. (7) leads to

$$\begin{aligned}\Delta\Pi(Q) = &-(c-s)\{[\Phi(0)-\Phi(-1)]+[\Phi(1)-\Phi(0)]+[\Phi(2)-\Phi(1)]+\cdots+[\Phi(Q)-\Phi(Q-1)]\}\\ &+(p-c+B)\{[\Phi(Q+1)-\Phi(Q)]+[\Phi(Q+2)-\Phi(Q+1)]\\ &+[\Phi(Q+3)-\Phi(Q+2)]+\cdots\}.\end{aligned}\tag{8}$$

It is clear that $\Phi(-1)=0$ due to $\xi \geq 0$. For simplification, denoting

$$S_n = [\Phi(Q+1)-\Phi(Q)]+[\Phi(Q+2)-\Phi(Q+1)]+\cdots+[\Phi(Q+n)-\Phi(Q+n-1)],$$

then we have

$$\begin{aligned}&[\Phi(Q+1)-\Phi(Q)]+[\Phi(Q+2)-\Phi(Q+1)]+[\Phi(Q+3)-\Phi(Q+2)]+\cdots\\ =&\lim_{n\to+\infty} S_n = \lim_{n\to+\infty}[\Phi(Q+n)-\Phi(Q)] = h-\Phi(Q).\end{aligned}\tag{9}$$

Substituting Eq. (9) into Eq. (8) leads to

$$\begin{aligned}\Delta\Pi(Q)] &= -(c-s)\Phi(Q)+(p-c+B)[h-\Phi(Q)]\\ &= -\Phi(Q)(p+B-s)+h(p+B-c).\end{aligned}$$

It is easy to verify that $\Delta\Pi(Q)$ is a decreasing function with respect to Q, so the graph of $\Pi(Q)$ is concave. The optimal order quantity Q_d^* is the lowest value of Q such that $\Delta\Pi(Q) \leq 0$, i.e.,

$$Q_d^* = \min\left\{Q \mid \Phi(Q) \geq \frac{h(p+B-c)}{p+B-s}\right\}.$$

According to the assumption $p > c > s$ and $B > 0$, we have $0 < h(p+B-c)/(p+B-s) < h$, where $h \in (0,1]$. In addition, $0 \leq \Phi(k) \leq h$ because $\sup\limits_{k\geq 0}\mu_k = h$. So we can determine the optimal order quantity by the following formula

$$Q_d^* = \min\left\{Q \mid \mathrm{Cr}\{\xi \leq Q\} \geq \frac{h(p+B-c)}{p+B-s}\right\}.$$

The theorem is proved. □

For a discrete fuzzy demand variable, the process to find Q_d^* is summarized as follows.

Algorithm I:
Step 1. Calculate the value $k_0 = h(p+B-c)/(p+B-s)$.
Step 2. Calculate the credibility $\Phi(k) = \mathrm{Cr}\{\xi \leq k\}$ and obtain the disjoint intervals: $I_k = (\Phi(k-1), \Phi(k)]$, $k = 0,1,2,\ldots$, where $\Phi(-1)=0$, and $\lim\limits_{k\to+\infty}\Phi(k) = h$.
Step 3. If $k_0 \in I_k$, then return the optimal order quantity $Q_d^* = k$.

3.2 Demand Follows Continuous Possibility Distribution

In this subsection, we consider the single-period inventory problem with a continuous fuzzy demand, whose possibility distribution is a continuous function. We first give the following Lemma to prove our conclusion.

Lemma 1. *Let ξ be a fuzzy variable such that the Lebesgue integral $\int_0^{+\infty}\mathrm{Cr}\{\xi \geq r\}\mathrm{d}r$ is finite. Then*

$$\lim_{r\to+\infty} r\mathrm{Cr}\{\xi \geq r\} = 0.$$

Proof. Note that the Lebesgue integral $\int_0^{+\infty}\mathrm{Cr}\{\xi \geq r\}\mathrm{d}r$ is finite. For any given $\varepsilon > 0$, there exists a real number $r_0 > 0$, when $r_0 < r/2 < r$, we have the following result

$$0 \leq \int_{\frac{r}{2}}^{r}\mathrm{Cr}\{\xi \geq x\}\mathrm{d}x \leq \varepsilon.\tag{10}$$

Since the credibility $\mathrm{Cr}\{\xi \geq r\}$ is a nonincreasing function with respect to r , then

$$\int_{\frac{r}{2}}^{r} \mathrm{Cr}\{\xi \geq x\}\mathrm{d}x \geq \frac{r}{2}\mathrm{Cr}\{\xi \geq r\} \geq 0. \tag{11}$$

According to Eqs. (10) and (11), we can get $0 \leq r\mathrm{Cr}\{\xi \geq r\}/2 \leq \varepsilon$, i.e., $0 \leq r\mathrm{Cr}\{\xi \geq r\} \leq 2\varepsilon$, Thus $\lim_{r\to+\infty} r\mathrm{Cr}\{\xi \geq r\} = 0$.

The lemma is proved. □

From Lemma 1, we deduce the following result.

Theorem 2. *Let ξ be a continuous fuzzy demand variable such that its credibility $\mathrm{Cr}\{\xi \leq r\}$ is a continuous function, and $\lim_{r\to+\infty} \mathrm{Cr}\{\xi \leq r\} = h$ with $h \in (0, 1]$. If Lebesgue integral $\int_0^{+\infty} \mathrm{Cr}\{\xi \geq r\}\mathrm{d}r$ is finite, then the optimal order quantity Q_c^* is determined by the following formula*

$$Q_c^* = \inf\left\{Q \mid \mathrm{Cr}\{\xi \leq Q\} = \frac{h(p + B - c)}{p + B - s}\right\}, \tag{12}$$

where $\mathrm{Cr}\{\xi \leq r\} = \{\mathrm{Pos}(\xi \leq r) + h - \mathrm{Pos}(\xi > r)\}/2$.

Proof. According to Eq. (5), the equivalent value profit in problem (3) can be obtained by the following formula.

$$\begin{aligned}\Pi(Q) &= \int_{[0,+\infty)} \pi(Q, r)\mathrm{dCr}\{\xi \leq r\} = \int_{[0,Q]} [(p - c)r - (c - s) \cdot (Q - r)]\mathrm{dCr}\{\xi \leq r\} \\ &\quad + \int_{(Q,+\infty)} [(p - c)r - (p - c + B) \cdot (r - Q)]\mathrm{dCr}\{\xi \leq r\} \\ &= (p - c)\mu - (c - s) \cdot \int_{[0,Q]} (Q - r)\mathrm{dCr}\{\xi \leq r\} - (p - c + B) \cdot \int_{(Q,+\infty)} (r - Q)\mathrm{dCr}\{\xi \leq r\},\end{aligned} \tag{13}$$

where μ is the equivalent value of ξ, i.e.,

$$\begin{aligned}\mu &= \int_{[0,+\infty)} r\mathrm{dCr}\{\xi \leq r\}] = \int_{[0,+\infty)} r\mathrm{d}(h - \mathrm{Cr}\{\xi > r\}) = -\int_{[0,+\infty)} r\mathrm{dCr}\{\xi > r\} \\ &= -r\mathrm{Cr}\{\xi > r\}([0, +\infty)) + \int_0^{+\infty} \mathrm{Cr}\{\xi > r\}\mathrm{d}r = -\lim_{r\to+\infty} r\mathrm{Cr}\{\xi > r\} + \int_0^{+\infty} \mathrm{Cr}\{\xi > r\}\mathrm{d}r.\end{aligned}$$

Note that the Lebesgue integral $\int_0^{+\infty} \mathrm{Cr}\{\xi \geq r\}\mathrm{d}r$ is finite. From Lemma 1, we have $\lim_{r\to+\infty} r\mathrm{Cr}\{\xi > r\} = 0$. Consequently, μ is a finite constant such that

$$\mu = \int_0^{+\infty} \mathrm{Cr}\{\xi > r\}\mathrm{d}r. \tag{14}$$

For simplification, denoting

$$U = \int_{[0,Q]} (Q - r)\mathrm{dCr}\{\xi \leq r\} = (Q - r)\mathrm{Cr}\{\xi \leq r\}([0, Q]) + \int_0^{Q} \mathrm{Cr}\{\xi \leq r\}\mathrm{d}r = \int_0^{Q} \mathrm{Cr}\{\xi \leq r\}\mathrm{d}r, \tag{15}$$

where $\mathrm{Cr}\{\xi \leq 0\} = 0$ due to $\xi \geq 0$. Let

$$\begin{aligned}V &= \int_{(Q,+\infty)} (r - Q)\mathrm{dCr}\{\xi \leq r\} = \int_{(Q,+\infty)} (r - Q)\mathrm{d}(h - \mathrm{Cr}\{\xi > r\}) = -\int_{(Q,+\infty)} (r - Q)\mathrm{dCr}\{\xi > r\} \\ &= -(r - Q)\mathrm{Cr}\{\xi > r\}\left((Q, +\infty)\right) + \int_Q^{+\infty} \mathrm{Cr}\{\xi > r\}\mathrm{d}r = -\lim_{r\to+\infty} r\mathrm{Cr}\{\xi > r\} + \int_Q^{+\infty} \mathrm{Cr}\{\xi > r\}\mathrm{d}r.\end{aligned} \tag{16}$$

It is clear that $\int_Q^{+\infty} \mathrm{Cr}\{\xi > r\}\mathrm{d}r$ is a function with respect to Q and $\lim_{r\to+\infty} r\mathrm{Cr}\{\xi > r\} = 0$. From the Eq. (16), we deduce the following result

$$V = \int_Q^{+\infty} \mathrm{Cr}\{\xi > r\}\mathrm{d}r. \tag{17}$$

From the Eqs. (14), (15) and (17), the equivalent value profit in Eq. (13) can be re-written as

$$\Pi(Q) = (p-c)\mu - (c-s)\int_0^Q \mathrm{Cr}\{\xi \le r\}\mathrm{d}r - (p-c+B)\int_Q^{+\infty} \mathrm{Cr}\{\xi > r\}\mathrm{d}r. \tag{18}$$

It is easy to show that

$$\frac{\mathrm{d}\Pi(Q)}{\mathrm{d}Q} = -(c-s)\Phi(Q) + (p-c+B)\cdot[h-\Phi(Q)].$$

It follows from the assumption $p > c > s$ and $B > 0$ that

$$\frac{\mathrm{d}^2\Pi(Q)}{\mathrm{d}Q^2} = -[(c-s)+(p-c+B)]\frac{\mathrm{d}\Phi(Q)}{\mathrm{d}Q} \le 0.$$

So $\Pi(Q)$ is a concave function with respect to Q. If there is a unique point Q that satisfies $\frac{\mathrm{d}\Pi(Q)}{\mathrm{d}Q} = 0$, then the first order condition is sufficient and necessary to determine the optimal order quantity Q_c^* that maximizes Eq. (18). As for the actual economic significance, we should select the smallest order quantity if there are multiple values of Q that satisfy the equation $\frac{\mathrm{d}\Pi(Q)}{\mathrm{d}Q} = 0$. Then the optimal order quantity can be determined by the following formula

$$Q_c^* = \inf\left\{Q \mid \mathrm{Cr}\{\xi \le Q\} = \frac{h(p+B-c)}{p+B-s}\right\}.$$

The theorem is proved. □

For a continuous fuzzy demand variable, the process to find Q_c^* is summarized as follows.

Algorithm II:
Step 1. Calculate the value $r_0 = h(p+B-c)/(p+B-s)$.
Step 2. Calculate the credibility $\Phi(r) = \mathrm{Cr}\{\xi \le r\}$ for $r \in R$.
Step 3. If $\Phi(r)$ is strictly increasing function in the neighborhood of r_0, then we have $Q_c^* = \Phi^{-1}(r_0)$. Otherwise proceed to the next step.
Step 4. Calculate the pessimistic value $\xi_{\inf}(r_0) = \inf\{r \mid \Phi(r) = r_0\}$ of ξ, and return $Q_c^* = \xi_{\inf}(r_0)$.

According to Theorem 2, we have the following conclusion to the common continuous fuzzy demand variables.

Corollary 1. *Consider problem (3), if fuzzy demand follows any one of the following distributions:*
(1) triangular possibility distribution;
(2) trapezoidal possibility distribution;
(3) normal possibility distribution;
(4) Erlang possibility distribution;
(5) exponential possibility distribution;
then the optimal order quantity Q^* is determined by the following formula

$$Q^* = \inf\left\{Q \mid \mathrm{Cr}\{\xi \le Q\} = \frac{h(p+B-c)}{p+B-s}\right\}. \tag{19}$$

Proof. We proof the corollary according to the following four cases.

Case I: Consider a bounded continuous fuzzy demand variable $|\xi| \leq M$, where $M > 0$, it is clear that $\mathrm{Cr}\{\xi \geq r\} = 0$ if $r \geq M$. Therefore, the Lebesgue integral $\int_0^{+\infty} \mathrm{Cr}\{\xi \geq r\}\mathrm{d}r$ is finite. From Theorem 2, the optimal order quantity Q^* is determined by Eq. (19) if the fuzzy demand is a triangular fuzzy demand or a trapezoidal fuzzy demand.

Case II: Consider a normal fuzzy demand variable $\eta = n(m, \sigma)$ with the possibility distribution $\mu_\eta(r) = \exp\left\{-(r-m)^2/2\sigma^2\right\}$. From the definition of credibility $\mathrm{Cr}\{\eta \geq r\}$, we deduce the following result

$$\mathrm{Cr}\{\eta \geq r\} = \begin{cases} 1 - \frac{1}{2}\exp\left\{-\frac{(r-m)^2}{2\sigma^2}\right\}, & r \leq m \\ \frac{1}{2}\exp\left\{-\frac{(r-m)^2}{2\sigma^2}\right\}, & r > m. \end{cases} \tag{20}$$

Without loss of generality, we consider the case $m > 0$. It follows from Eq. (20) that

$$\begin{aligned} \int_0^{+\infty} \mathrm{Cr}\{\eta \geq r\}\mathrm{d}r &= \int_0^m \left(1 - \frac{1}{2}\exp\left\{-\frac{(r-m)^2}{2\sigma^2}\right\}\right)\mathrm{d}r + \int_m^{+\infty} \frac{1}{2}\exp\left\{-\frac{(r-m)^2}{2\sigma^2}\right\}\mathrm{d}r \\ &= m - \int_0^m \frac{1}{2}\exp\left\{-\frac{(r-m)^2}{2\sigma^2}\right\}\mathrm{d}r + \frac{\sqrt{2\pi}\sigma}{4} \end{aligned}$$

For simplification, denoting

$$W = \int_0^m \frac{1}{2}\exp\left\{-\frac{(r-m)^2}{2\sigma^2}\right\}\mathrm{d}r.$$

It is clear that $W > 0$, so

$$0 < \int_0^{+\infty} \mathrm{Cr}\{\eta \geq r\}\mathrm{d}r < m + \frac{\sqrt{2\pi}\sigma}{4}. \tag{21}$$

From Eq. (21), we know $\int_0^{+\infty} \mathrm{Cr}\{\eta \geq r\}\mathrm{d}r$ is finite. From Theorem 2, the optimal order quantity Q^* is determined by Eq. (19) if η is a normal fuzzy demand variable.

Case III: Consider an Erlang fuzzy demand variable ζ with the possibility distribution

$$\mu_\zeta(r) = (\frac{r}{k\rho})^k \exp\{k - \frac{r}{\rho}\},$$

where $r \geq 0, \rho > 0$ and $k \in \mathrm{N}^+$. For any $r \geq 0$, we have

$$\mathrm{Cr}\{\zeta \geq r\} = \begin{cases} 1 - \frac{1}{2}\left(\frac{r}{k\rho}\right)^k \exp\left\{k - \frac{r}{\rho}\right\}, & 0 \leq r \leq k\rho \\ \frac{1}{2}\left(\frac{r}{k\rho}\right)^k \exp\left\{k - \frac{r}{\rho}\right\}, & r > k\rho. \end{cases}$$

Then, we have

$$\begin{aligned} \int_0^{+\infty} \mathrm{Cr}\{\zeta \geq r\}\mathrm{d}r &= \int_0^{k\rho} \left(1 - \frac{1}{2}\left(\frac{r}{k\rho}\right)^k \exp\left\{k - \frac{r}{\rho}\right\}\right)\mathrm{d}r + \int_{k\rho}^{+\infty} \frac{1}{2}\left(\frac{r}{k\rho}\right)^k \exp\left\{k - \frac{r}{\rho}\right\}\mathrm{d}r \\ &= k\rho - \frac{1}{2}\int_0^{k\rho} \left(\frac{r}{k\rho}\right)^k \exp\left\{k - \frac{r}{\rho}\right\}\mathrm{d}r + \frac{1}{2}\int_{k\rho}^{+\infty} \left(\frac{r}{k\rho}\right)^k \exp\left\{k - \frac{r}{\rho}\right\}\mathrm{d}r \\ &= \rho\left(k + 1 + \sum_{n=1}^{k} \frac{(k-1)!}{(k-n)!}\left(\frac{1}{k}\right)^{n-1} - \frac{1}{2}(k-1)!\left(\frac{1}{k}\right)^{k-1}\exp(k)\right). \end{aligned} \tag{22}$$

From Eq. (22), we know $\int_0^{+\infty} \mathrm{Cr}\{\zeta \geq r\}\mathrm{d}r$ is finite. From Theorem 2, the optimal order quantity Q^* is determined by Eq. (19) if ζ is an Erlang fuzzy demand variable where $\rho > 0$ and $k \in \mathrm{N}^+$.

Case IV: Consider an exponential fuzzy variable τ with the possibility distribution

$$\mu_\tau(r) = \frac{r}{\rho}\exp\left\{1 - \frac{r}{\rho}\right\}.$$

It is clear that an exponential fuzzy variable is a special case of an Erlang fuzzy variable.

The corollary is proved. □

4 Numerical Examples

In the following example, we employ Algorithm I to solve the single-period inventory problem with a discrete fuzzy demand variable.

Example 1. Suppose the selling price, the salvage value of unit residual product and the costs of unit product are $p = 4, s = 1, c = 3, B = 5$, respectively. Demand is estimated to be "about 10 products". The possibility distribution μ_k defined on the discrete domain $\{6, 7, \ldots, 14\}$ is given in Table 1.

Table 1: The possibility distribution of demand

ξ	6	7	8	9	10	11	12	13	14
μ	0	0.25	0.5	0.75	1	0.75	0.5	0.25	0

It is clear that $h = \max_{6 \le k \le 14} \mu_k = 1$. By calculation, $h(p + B - c)/(p + B - s) = 0.75$. According to Table 1, we can obtain the credibility of fuzzy event $\{\xi \le r\}$,

$$\mathrm{Cr}\{\xi \le r\} = \begin{cases} 0, & r < 7 \\ 0.125, & 7 \le r < 8 \\ 0.25, & 8 \le r < 9 \\ 0.375, & 9 \le r < 10 \\ 0.625, & 10 \le r < 11 \\ 0.75, & 11 \le r < 12 \\ 0.875, & 12 \le r < 13 \\ 1, & 13 \le r < +\infty. \end{cases}$$

So we can achieve the disjoint intervals: $I_7 = (0, 0.125]$, $I_8 = (0.125, 0.25]$, $I_9 = (0.25, 0.375]$, $I_{10} = (0.375, 0.625]$, $I_{11} = (0.625, 0.75]$, $I_{12} = (0.75, 0.875]$, $I_{13} = (0.875, 1]$. Because $0.75 \in I_{11}$, the optimal order quantity is $Q^* = 11$.

In the following examples, we employ Algorithm II to solve the single-period inventory problem with a continuous fuzzy demand variable.

Example 2. Consider a single-period inventory problem with a trapezoidal fuzzy demand, i.e., $\xi = (10, 14, 16, 20)$. Suppose the selling price, the purchase cost of unit product and the salvage value of unit product are $p = 12, c = 10, s = 4$, respectively. In addition, let the lost sale penalty cost for unit unmet demand B be a variable.

In this case, one has

$$\mathrm{Cr}\{\xi \le r\} = \begin{cases} 0, & r < 10 \\ \frac{r}{8} - \frac{5}{4}, & 10 \le r < 14 \\ \frac{1}{2}, & 14 \le r < 16 \\ \frac{r}{8} - \frac{3}{2}, & 16 \le r < 20 \\ 1, & 20 \le r. \end{cases}$$

It is clear that $\lim_{r \to +\infty} \mathrm{Cr}\{\xi \le r\} = h = 1$. From Theorem 2 and Corollary 1, we discuss the following three cases.

Case I: In the case that $r_0 = p + B - c/(p + B - s) < 0.5$, i.e., $p + B - c < c - s$, we have $Q^* = 10 + 8r_0$.

Case II: In the case that $r_0 = p + B - c/(p + B - s) = 0.5$, i.e., $p + B - c = c - s$, we have $Q^* = \xi_{\inf}(r_0) = 14$.

Case III: In the case that $r_0 = p + B - c/(p + B - s) > 0.5$, i.e., $p + B - c > c - s$, we have $Q^* = 12 + 8r_0$.

In the case that B is variable, the corresponding results are provided in Table 2, where the maximum profit is determined by Eq. (18).

Example 3. Consider a single-period inventory problem with a normal fuzzy demand $\eta = n(15, 2)$. Suppose the selling price, the purchase cost of unit product and the lost sale penalty cost for unit unmet demand are $p = 12, c = 10, B = 4$, respectively. Assume that the salvage value of unit product s is variable.

Table 2: The optimal order quantity and maximum profit

Condition	The lost sale penalty cost	The optimal order quantity	The maximum profit
	$B = 0$	12	22
$p + B - c < c - s$	$B = 1$	12.7	19
	$B = 2$	13.2	16.4
	$B = 3$	13.6	14.1
$p + B - c = c - s$	$B = 4$	14	12
	$B = 5$	16.3	11.1
$p + B - c > c - s$	$B = 8$	17	9
	$B = 12$	17.6	7.2
	$B = 16$	18	6

In this case, one has

$$\mathrm{Cr}\{\eta \le r\} = \begin{cases} \frac{1}{2}\exp\{-\frac{(r-15)^2}{8}\}, & r \le 15 \\ 1 - \frac{1}{2}\exp\{-\frac{(r-15)^2}{8}\}, & r > 15. \end{cases}$$

Since $\Phi(r) = \mathrm{Cr}\{\eta \le r\}$ is strictly increasing function with respect to r, we have

$$Q^* = \Phi^{-1}\left(\frac{h(p + B - c)}{p + B - s}\right),$$

where $h = 1$. From Theorem 2 and Corollary 1, we discuss the following three cases.
Case I: In the case that $r_0 = p + B - c/(p + B - s) < 0.5$, i.e., $p+B-c < c-s$, we have $Q^* = 15-\sqrt{-8\ln 2r_0}$.
Case II: In the case that $r_0 = p + B - c/(p + B - s) = 0.5$, i.e., $p + B - c = c - s$, we have $Q^* = 15$.
Case III: In the case that $r_0 = p + B - c/(p + B - s) > 0.5$, i.e., $p + B - c > c - s$, we have $Q^* = 15 + \sqrt{-8\ln 2(1 - r_0)}$.
In the case that s is variable, the corresponding results are provided in Table 3.

Table 3: The optimal order quantity and maximum profit

Condition	The salvage value	The optimal order quantity	The maximum profit
	$s = 1$	13.66	3.27
$p + B - c < c - s$	$s = 2$	13.89	4.73
	$s = 3$	14.2	7.02
$p + B - c = c - s$	$s = 4$	15	16
	$s = 5$	15.87	23.93
$p + B - c > c - s$	$s = 6$	16.34	24.49
	$s = 8$	17.36	21.68
	$s = 9$	18.17	19.16

From Tables 1 and 2, we can see that the optimal order quantity is an increasing function with respect to the lost sale penalty cost B or the salvage value s. We adopt "underage" cost ($c_u = p + B - c$) and "overage" cost ($c_o = c - s$) perspective to explain the single-period inventory management problem. By means of the examples above, we should order relatively lower quantity when $c_u \le c_o$, and vice-versa. The above results are logical in economics.

5 Conclusions

In this paper, a single-period inventory problem with fuzzy demand was analysed based on the equivalent value criterion. The major results of the paper are summarized as follows.

(i) Based on the credibility measure, we defined the equivalent value profit of retailer by using L–S integral.

(ii) In the sense of the equivalent value profit, the analytical expressions of the optimal order quantity were derived in two cases: a discrete fuzzy demand variable and a continuous fuzzy demand variable. Since common continuous fuzzy demand variables satisfy the condition of Theorem 2, it is easy to determine the optimal order quantity in the cases.

(iii) We provided some numerical examples to demonstrate the efficiency of the proposed model. The computational results conformed with the economic significance.

When the possibility distribution function of fuzzy demand variable is neither discrete nor continuous, the optimal order quantity needs to be analyzed in our further research.

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References

[1] Arrow, K.J., Harris, T., and J. Marshak, Optimal inventory policy, *Econometrica*, vol.19, no.3, pp.250–272, 1951.

[2] Carter, M., and B.V. Brunt, *The Lebesgue-Stieltjes Integral*, Springer-Verlag, New York, 2000.

[3] Chen, S.P., and Y.H. Ho, Optimal inventory policy for the fuzzy newsboy problem with quantity discounts, *Information Sciences*, vol.228, pp.75–89, 2013.

[4] Dutta, P., Chakraborty, D., and A.R. Roy, An inventory model for single-period products with reordering opportunities under fuzzy demand, *Computers and Mathematics with Applications*, vol.53, no.10, pp.1502–1517, 2007.

[5] Hu, J.S., Liu, F.X., Xu, Y.J., Xu, R.Q., and Y. Zhou, Newsboy problem with random fuzzy demand, *Operations Research and Management Science*, vol.3, pp.67–71, 2012.

[6] Hu, J.S., Zheng, H., Xu, R.Q., Ji, Y.P., and C.Y. Guo, Supply chain coordination for fuzzy random newsboy problem with imperfect quality, *International Journal of Approximate Reasoning*, vol.51, no.7, pp.771–784, 2010.

[7] Ishii, H., and T. Konno, A stochastic inventory problem with fuzzy shortage cost, *European Journal of Operational Research*, vol.106, no.1, pp.90–94, 1998.

[8] Kao, C., and W.K. Hsu, A single-period inventory model with fuzzy demand, *Computers and Mathematics with Applications*, vol.43, no.6, pp.841–848, 2002.

[9] Khouja, B., The single period (news-vendor) problem: literature review and suggestions for future research, *Omega*, vol.27, pp.537–553, 1999.

[10] Li, L., Kabadi, S.N., and K.P.K. Nair, Fuzzy models for single-period inventory problem, *Fuzzy Sets and Systems*, vol.132, no.3, pp.273–289, 2002.

[11] Liu, B., and Y.K. Liu, Expected value of fuzzy variable and fuzzy expected value model, *IEEE Transactions on Fuzzy Systems*, vol.10, no.4, pp.445–450, 2002.

[12] Liu, Y.K., and B. Liu, Expected value operator of random fuzzy variable and random fuzzy expected value models, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol.11, no.2, pp.195–215, 2003.

[13] Liu, Y.K., and Y. Liu, Measure generated by joint credibility distribution function, *Journal of Uncertain Systems*, vol.8, no.3, pp.239–240, 2014.

[14] Petrovic, D., Petrovic, R., and M. Vujosevic, Fuzzy models for the newsboy problem, *International Journal of Production Economics*, vol.45, no.1, pp.435–441, 1996.

[15] Qin, Y., Wang, R.X., Vakharia, A.J., Chen, Y.W., and M.H. Seref, The newsvendor problem: review and directions for future research, *European Journal of Operational Research*, vol.213, no.2, pp.361–374, 2011.

[16] Silver, E.A., Pyke, D.F., and R. Peterson, *Inventory Management and Production Planning and Scheduling*, Wiley, New York, 1998.

[17] Xu, Y., and J. Hu, Random fuzzy demand newsboy problem, *Physics Procedia*, vol.25, pp.924–931, 2012.

[18] Xue, F., Tang, W.S., and R.Q. Zhao, The expected value of a function of a fuzzy variable with a continuous membership function, *Computers and Mathematics with Applications*, vol.55, no.6, pp.1215–1224, 2008.

[19] Yager, R.R., A procedure for ordering fuzzy subsets of the unit interval, *Information Sciences*, vol.24, no.2, pp.143–161, 1981.

[20] Yu, Y., Zhu, J., and C.W. Wang, A newsvendor model with fuzzy price-dependent demand, *Applied Mathematical Modelling*, vol.37, no.5, pp.2644–2661, 2013.

[21] Zhu, Y.G., and X.Y. Ji, Expected values of functions of fuzzy variables, *Journal of Intelligent and Fuzzy Systems*, vol.17, no.5, pp.471–478, 2006.