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T-S Fuzzy Modeling and Synchronization of Chaotic Systems

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Abstract

In this paper, we consider the fuzzy model-based designs for the complete synchronization of chaotic systems. T-S fuzzy model for chaotic systems are exactly derived. Based on the T-S fuzzy model, the fuzzy logic controllers for chaotic synchronization are designed via Linear Matrix Inequality (LMI). Lyapunov Exponent of lorenz system is calculated, one of them is positive which represents the chaotic lorenz system. Analytical and computational studies of a lorenz system have been performed by using LMI toolbox. Using this technique fuzzy controller has been designed for the complete synchronization of Lorenz's system. The qualitative and simulated results are in an excellent agreement.

Keywords: T-S fuzzy models, synchronization, fuzzy logic controller (FLC), linear matrix inequalities (LMI)

1 Introduction

Chaotic system, a nonlinear dynamical system is highly sensitive to the initial condition. This sensitivity is popularly known as the butterfly effect [1, 16]. Since Pecora and Carroll has proposed the concept of chaotic synchronization, the chaotic synchronization has become the hot subject in the field of nonlinear sciences due to its wide-scope potential application in various disciplines such as - in chemical reaction, power converters, signal process, biological system, economics and communication etc [1, 2].

Generally two chaotic systems, first one is called the master system or drive system and second one having controllers is called the slave system or response system is used for synchronization. The idea of synchronization is used the output of the slave system to control the master system using the Takai-Sugeno (T-S) fuzzy logic so that the sum of the outputs of master system and slave system tend to zero asymptotically with time [20, 24]. Synchronization of two chaotic dynamical systems is one of most important application of chaos. In secure communication, the receiver have been synchronized with transmitter to receive the massage [7, 8, 9, 10, 11, 12]. Therefore, we focus our attention for chaotic synchronization. Synchronization of chaotic Lur'e system with time delay with LMI approach is designed with sample-data controller [22]. For a class of chaotic synchronization scheme is presented through a discrete-time sliding mode control scheme [4, 6, 14, 15, 17]. LMI-based fuzzy synchronization for Chen's system is considered [5, 20, 21]. Synchonization of Chau's circuit systems via quantized-data feedback control is analysed [23]. Using the T-S fuzzy model for synchronization of Rossler and Matsumoto-Chua-Kobayashi systems is proposed [26].

Zadeh so called the father of fuzzy logic initiated the fuzzy logic theory [3, 25]. His fuzzy logic theory made a revolution in human thought and control theory and gave the new insight reasoning. A new approach in control systems analysis and design and synchronize chaotic system via Linearized Matrix Inequality (LMI)-based fuzzy logic controller have been designed [13, 18, 19]. Linearized Matrix Inequality (LMI) is a powerful tool in the field in control systems analysis and design since last two decades in LMI control systems. In the current scenario, Takagi-Sugeno (T-S) fuzzy model is widely applied to many fields because of its simple structure with local and global dynamics. Tanaka and Wang [20] established the accurate T-S fuzzy representation for many kinds of typical chaotic systems and Lian et al. [13] presented a synthesis approach for the synchronization of chaotic systems based on T-S fuzzy models.

Motivated from the above discussion, we are interested to represent the chaotic systems and its synchronization based on T.S. fuzzy modeling. Main contribution of this paper lies in four features. First, the chaotic systems are mainly redesigned by T-S fuzzy model. Second, fuzzy control methodology are used for synchronization of the obtained T-S fuzzy model. Third, Lyapunov exponent of lorenz system is obtained to justify the chaoticity of that. Fourth, numerical simulations are presented to verify the results.

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In this paper, we have devoted our research to represent a class of typical continuous-time chaotic system via T-S fuzzy models and we developed the fuzzy logic controller for synchronization of Lorenz's chaotic system. To find the feedback gain matrices, we have used LMI control toolbox. This paper is organized as follows: Section 1 is introductory in nature. Section 2 describes the system description and mathematical formulation of master-slave system. In section 3, we describe fuzzy synchronization of chaotic system. Numerical simulations are used to verify the effectiveness of synchronization technique in section 4. Finally, conclusion is given in section 5.

2 Fuzzy Modeling and System Description of Chaotic System

Consider a continuous-time nonlinear dynamical systems as

$$\dot{x}(t) = f(x(t)), \tag{1}$$

where $x(t) \in R^n$ are state vector of the systems and $f : R^n \to R^n$ is the nonlinear function of the system. Takagi-Sugeno (T-S) fuzzy dynamic model is described by fuzzy IF-THEN rules. In the form of T-S model, the system (1) can be represented as

$$R^i : \begin{cases} IF \quad s_1(t) \quad is \quad in \quad M_{i1}, \quad s_2(t) \quad is \quad in \quad M_{i2}, \dots, \quad and \quad s_p(t) \quad is \quad in \quad M_{ip}, \quad THEN \\ \dot{x} = A_i x + b_i, \quad i = 1, 2, \dots, r, \end{cases} \tag{2}$$

where $R^i(i = 1, 2, ..., r)$ denotes the ith fuzzy rule and r is number of fuzzy rules. $s_1(t), ..., s_p(t)$ are the premise variables which consist of state vectors of the system, $M_{ij}(j = 1, 2, ..., p)$ are fuzzy sets, A_i is a constant matrix with appropriate dimension and $b_i \in R^n$ is bias term. Using the fuzzifier, the output the fuzzy system is written as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i s(t) A_i x(t) + b_i, \tag{3}$$

where

$$h_i(s(t)) = \frac{w_i(s(t))}{\sum_{i=1}^{r} w(s(t))},$$

$w_i(s(t)) = \prod_{j=1}^{p} M_{ij}(s(t)) h_i(s(t))$ is denoted as the normalized weight of the IF-THEN rules which satisfies

$$0 \le h_i(s(t)) \le 1 \quad \text{and} \quad \sum_{i=1}^{r} h_i(s(t)) = 1.$$

We consider the master system and slave system respectively in the form of T-S fuzzy dynamic model as

$$R^i : \begin{cases} IF \quad s(t) \quad is \quad in \quad M_i \quad THEN \\ \dot{x} = A_i x + b_i, \end{cases} \tag{4}$$

where $s(t)$ are the proper state vectors of the system, M_i are the fuzzy sets, $x(t) \in R^n$, A_i is the constant matrix with appropriate dimension and $b_i \in R^n$ and

$$R^i : \begin{cases} IF \quad s(t) \quad is \quad in \quad M_i \quad THEN \\ \dot{y} = A_i y + b_i + Bu(t), \end{cases} \tag{5}$$

where $y(t) \in R^n$ are the state vectors of the system, B is an input matrix, and $u(t) \in R^n$ is the fuzzy controller in the slave system.

3 Fuzzy Synchronization of Chaotic Systems

The defuzzification process of (1) is denoted as: The Master (drive) system is:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i s(t) A_i x(t) + b_i, \tag{6}$$

and the slave (response) system is:

$$\dot{y}(t) = \sum_{i=1}^{2} h_i s(t) A_i y(t) + b_i + Bu(t), \tag{7}$$

where $u(t)$ is the control input vector.

The fuzzy control rules have a linear controller in the consequent parts. The overall fuzzy controller is represented by

$$u(t) = -\frac{\sum_{i=1}^{r} w_i(s(t)) F_i x(t)}{\sum_{i=1}^{r} w_i(s(t))} = -\sum_{i=1}^{r} h_i(s(t)) F_i x(t), \tag{8}$$

where F_i is the state feedback gain matrix.

We define the error signal as

$$e(t) = x(t) - y(t). \tag{9}$$

Then the error dynamics is obtained as

$$\dot{e}(t) = \dot{x}(t) - \dot{y}(t). \tag{10}$$

We design the two fuzzy sub-controllers for synchronization [20].

Sub-controller 1

$$Control \quad Rule \quad R^i : \begin{cases} IF \quad s_1(t) \quad is \quad in \quad M_{i1}, \quad s_2(t) \quad is \quad in \quad M_{i2}, \ldots, \quad and \quad s_p(t) \quad is \quad in \quad M_{ip}, \quad THEN \\ u_1(t) = -F_i x, \quad i = 1, 2, \ldots, r. \end{cases} \tag{11}$$

Sub-controller 2

$$Control \quad Rule \quad R^i : \begin{cases} IF \quad s_1(t) \quad is \quad in \quad M_{i1}, \quad s_2(t) \quad is \quad in \quad M_{i2}, \ldots, \quad and \quad s_p(t) \quad is \quad in \quad M_{ip}, \quad THEN \\ u_2(t) = -F_i y, \quad i = 1, 2, \ldots, r. \end{cases} \tag{12}$$

By combining these two subcontrollers, we construct the overall fuzzy controller as

$$u(t) = u_1(t) + u_2(t),$$

$$u(t) = -\sum_{i=1}^{r} h_i s(t) F_i y(t) + \sum_{i=1}^{r} h_i s(t) F_i x(t). \tag{13}$$

Using the equation (11), the equation (8) is rewritten as

$$\dot{e}(t) = \sum_{i=1}^{r} h_i s(t)(A_i - BF_i)x(t) - \sum_{i=1}^{r} h_i s(t)(A_i - BF_i)y(t). \tag{14}$$

Using the LMI, there exists the feedback gains matrices such that

$$((A_1 - BF_1) - (A_i - BF_i))^T \times ((A_1 - BF_1) - (A_i - BF_i)) = 0. \tag{15}$$

Then the overall error system (8) is linearized as

$$\dot{e}(t) = Ge(t)$$

via the fuzzy controller (11), where

$$G = A_1 - BF_1 = A_i - BF_i$$

and the Hurwitz matrix $G < 0$. Then the error system is asymptotically stable.

Theorem 1. *If there exist feedback gains $F_i, i = 1, 2, \ldots, r$, such that the error system can be linearized as $\dot{e}(t) = Ge(t)$ and the the Hurwitz matrix $G = A_i - BF_i$, then the error system is asymptotically stable [20] and the response system (5) can synchronize the drive chaotic system (4) under fuzzy logic controller.*

Remark 1. *If B is nonsingular matrix, then the system is exactly linearized using*

$$F_i = B^{-1} \times (A_i - G).$$

If B is not nonsingular matrix, then some approximation technique can be utilized for controller design. In this paper, we have assumed that B is nonsingular matrix for the simplicity.

4 Numerical Simulation

The Master Lorenz's systems is described as:

$$\begin{cases} \dot{x}_1 = a*(x_2 - x_1) \\ \dot{x}_2 = x_1 * x_3 + c*x_1 - x_2 \\ \dot{x}_3 = x_1 * x_2 - b*x_3 \end{cases} \tag{16}$$

where $x = [x_1, x_2, x_3]^T$ are state vectors, $x \in [-d, d]$ and $d > 0$ when the initial condition $x_1(0) = -0.1, x_2(0) = -0.2, x_3(0) = 3$ and the parameter $a = 10$, $b = 8/3$ and $c = 26$.

The Slave Lorenz's systems is described as:

$$\begin{cases} \dot{y}_1 = a*(y_2 - y_1) + u_1 \\ \dot{y}_2 = y_1 * y_3 + c*y_1 - y_2 + u_2 \\ \dot{y}_3 = y_1 * y_2 - b*y_3 + u_3 \end{cases} \tag{17}$$

where $y = [y_1, y_2, y_3]^T$ are state vectors, $y \in [-d, d]$ and $d > 0$ when the initial condition $y_1(0) = 0.5, y_2(0) = 0.7, y_3(0) = 1.5$, the parameter $a = 10$, $b = 8/3$ and $c = 26$ and $u(t) = [u_1(t), u_2(t), u_3(t)]^T$ is the control input vector.

We have computed the Lyapunov exponent of lorenz system, when $t = 300$. We have $\lambda_1 = 0.81433$, $\lambda_2 = -0.0003306$ and $\lambda_3 = -14.4776$. One of these Lyapunov exponent is positive, which represent that lorenz system is chaotic. It is shown in Fig.1.

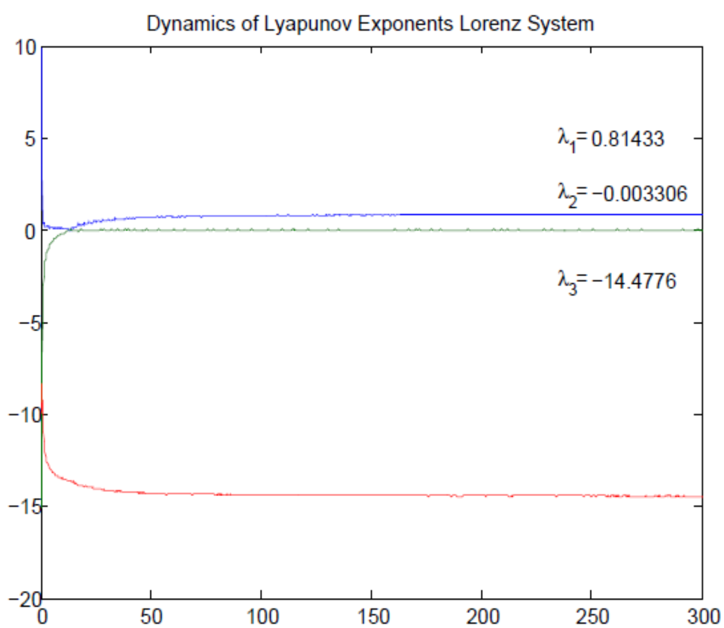

Figure 1: Lyapunov exponent of chaotic lorenz system

The chaotic motion of lorenz system is shown in Fig.2.

We have the following T-S fuzzy model of master (Lorenz's) system under

$$R^1 : \begin{cases} IF \quad s(t) \quad is \quad in \quad M_1 \quad THEN \\ \dot{x} = A_1 x + b_1 \end{cases} \tag{18}$$

and

$$R^2 : \begin{cases} IF \quad s(t) \quad is \quad in \quad M_2 \quad THEN \\ \dot{x} = A_2 x + b_2, \end{cases} \tag{19}$$

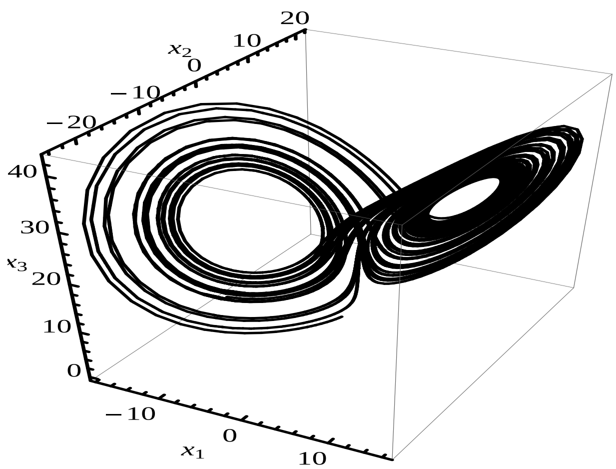

Figure 2: 3 Dimensional phase portrait of chaotic lorenz system (without controller)

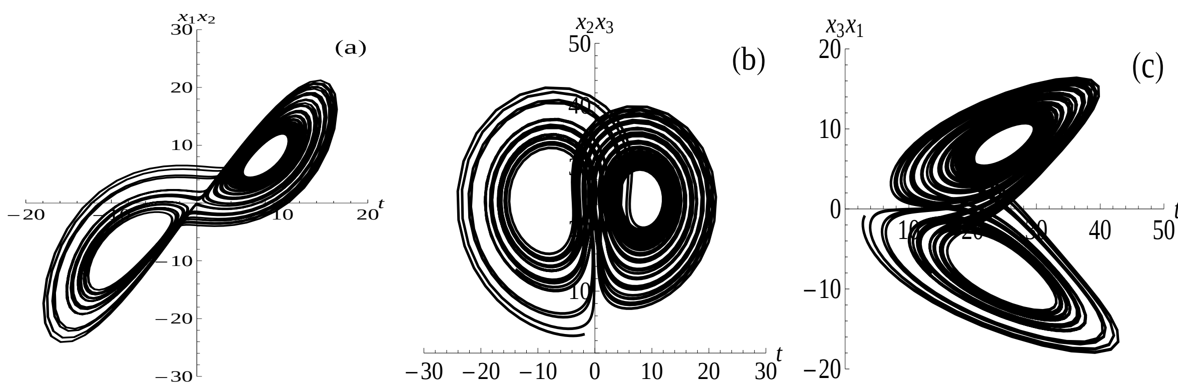

Figure 3: 2-D Phase portrait of chaotic lorenz system (without controller)

where

$$A_1 = \begin{bmatrix} -a & a & 0 \\ c & -1 & d \\ 0 & -d & -b \end{bmatrix},\ A_2 = \begin{bmatrix} -a & a & 0 \\ c & -1 & -d \\ 0 & d & -b \end{bmatrix},\ b_1 = b_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

$$M_1(x) = \frac{1}{2}(1 + \frac{x}{d}),\ M_2(x) = \frac{1}{2}(1 - \frac{x}{d}).$$

The constant $d = 50$.

The fuzzy Lorenz's system is

$$\dot{x}(t) = \begin{pmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \\ \dot{x_3}(t) \end{pmatrix} = \sum_{i=1}^{2} m_i s(t) A_i x(t) + b_i. \tag{20}$$

The T-S fuzzy model of slave (Lorenz's) system is

$$\dot{y}(t) = A_i y(t) + b_i + Bu(t). \tag{21}$$

By defuzzification process,

$$\dot{y}(t) = \begin{pmatrix} \dot{y_1}(t) \\ \dot{y_2}(t) \\ \dot{y_3}(t) \end{pmatrix} = \sum_{i=1}^{2} h_i s(t) A_i x(t) + b_i + Bu(t). \tag{22}$$

Here, $\Sigma h_i s(t) = \Sigma m_i s(t) = 1$. We define the error signal as

$$e(t) = x(t) - y(t). \tag{23}$$

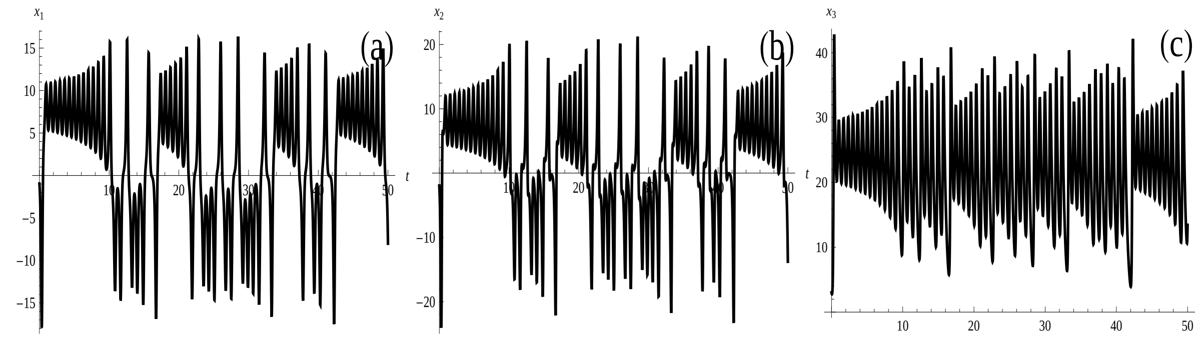

Figure 4: Time series graphs of chaotic lorenz system (without controller)

Then the error dynamics is obtained as

$$\dot{e}(t) = \dot{x}(t) - \dot{y}(t), \tag{24}$$

$$\dot{e}(t) = \sum_{i=1}^{2} m_i s(t) A_i x(t) + b_i - \sum_{i=1}^{2} h_i s(t) A_i y(t) - b_i - Bu(t). \tag{25}$$

We design the two fuzzy sub-controllers for synchronization [20]

Sub-controller 1

$$Control \quad Rule \quad R^i : \begin{cases} IF \quad s_1(t) \quad is \quad in \quad M_{i1}, \quad s_2(t) \quad is \quad in \quad M_{i2}, \dots, \quad and \quad s_p(t) \quad is \quad in \quad M_{ip}, \quad THEN \\ u_1(t) = -F_i x, \quad i = 1, 2. \end{cases} \tag{26}$$

Sub-controller 2

$$Control \quad Rule \quad R^i : \begin{cases} IF \quad s_1(t) \quad is \quad in \quad M_{i1}, \quad s_2(t) \quad is \quad in \quad M_{i2}, \dots, \quad and \quad s_p(t) \quad is \quad in \quad M_{ip}, \quad THEN \\ u_2(t) = -F_i y, \quad i = 1, 2. \end{cases} \tag{27}$$

By combining these two sub-controllers, we construct the overall fuzzy controller as

$$u(t) = u_1(t) + u_2(t).$$

$$u(t) = -\sum_{i=1}^{2} m_i s(t) F_i x(t) + \sum_{i=1}^{2} h_i s(t) F_i y(t) \tag{28}$$

such that

$$\lim_{t \to \infty} e(t) = 0.$$

The design is to determine the feedback gain matrices F_i. By substituting u_i in (24), we have

$$\dot{e}(t) = \sum_{i=1}^{2} h_i s(t) A_i - BF_i y(t) - \sum_{i=1}^{2} h_i s(t) A_i - BF_i x(t). \tag{29}$$

Using the LMI, there exists the feedback gains matrices such that

$$((A_1 - BF_1) - (A_2 - BF_2))^T \times ((A_1 - BF_1) - (A_2 - BF_2)) = 0. \tag{30}$$

We synchronize between master-slave Lorenz's systems. We take the initial condition for master and slave systems respectively as $x(0) = \begin{bmatrix} -0.1 & -0.2 & 3 \end{bmatrix}^T$, $y(0) = \begin{bmatrix} 0.5 & 0.7 & 1.5 \end{bmatrix}^T$.

We choose the input matrix B as identity matrix. Computed by LMI toolbox we obtain the feedback gains using closed-loop eigenvectors in Matlab Commands (eigenvalues, ρ = ([-2;-2+12*sqt(3)*i;-2-12*sqt(3)*i]) with above initial conditions as following:

$$F_1 = \begin{bmatrix} 8 & -13.46 & 0 \\ -24.54 & -1 & -50 \\ 0 & 50 & 0.67 \end{bmatrix}, \; F_2 = \begin{bmatrix} 8 & -13.46 & 0 \\ -24.54 & -1 & 50 \\ 0 & -50 & 0.67 \end{bmatrix}. \tag{31}$$

Thus, the overall error system (24) is linearized as

$$\dot{e}(t) = Ge(t) \tag{32}$$

via the fuzzy controller (25) where

$$G = A_1 - BF_1 = A_2 - BF_2 = \begin{bmatrix} -18.00 & 23.46 & 0.00 \\ 52.54 & 0.00 & 100 \\ 0.00 & -100 & -3.34 \end{bmatrix}. \tag{33}$$

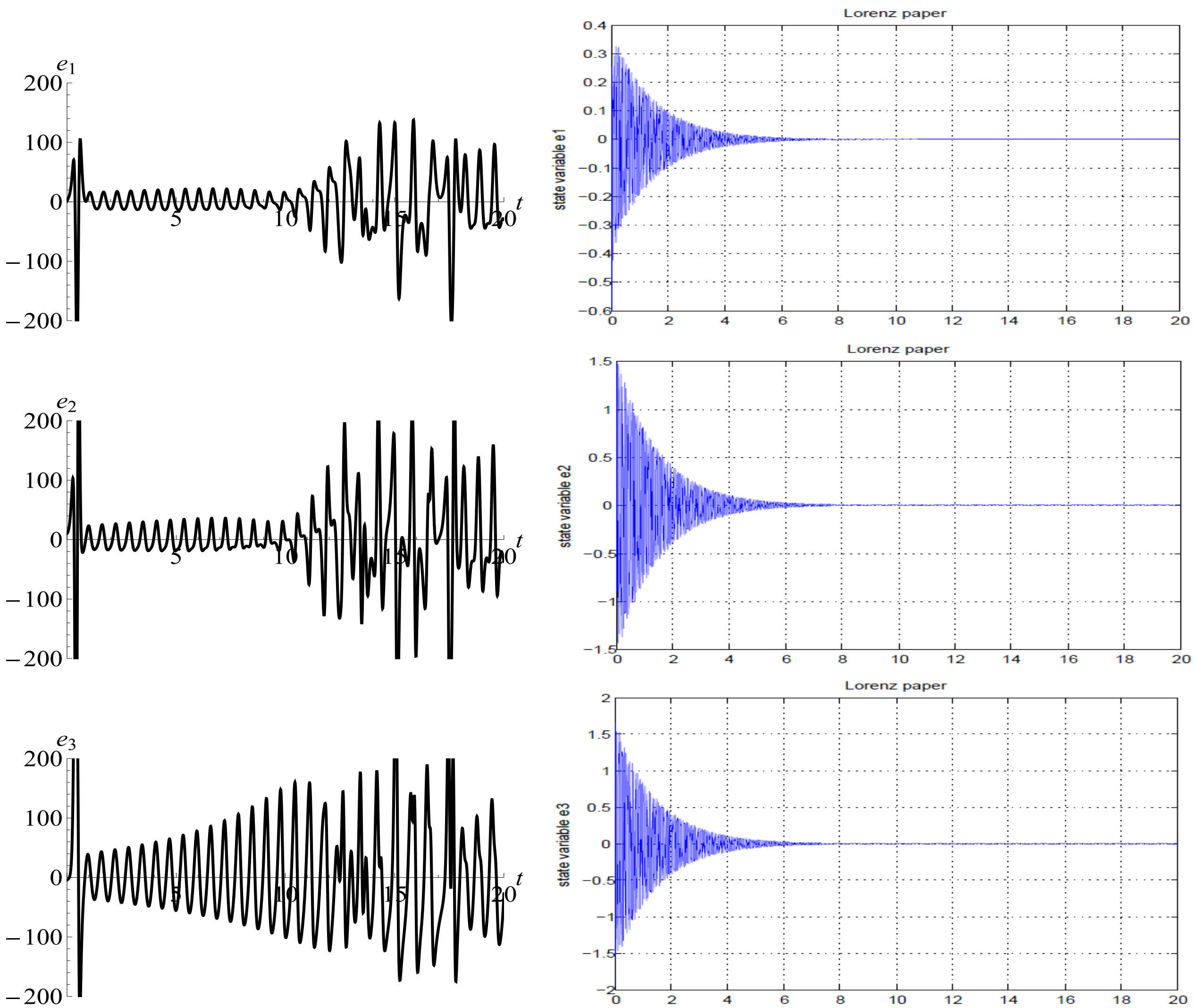

Figure 5: Time series of error dynamics (left side) and its synchronization of error dynamics (right side) in component wise-e_1, e_2, e_3

Simulation: At initial condition for master and slave lorenz systems

$$(x_1(0) = -0.1, x_2(0) = -0.2, x_3(0) = 3)^T; \; (y_1(0) = 0.5, y_2(0) = 0.7, y_3(0) = 1.5)^T.$$

Fig.2 shows the 3-D phase portrait chaotic lorenz systems. Fig.3 (a)-(c) is shown as the 2-D phase portrait of chaotic lorenz system in $x_1 - x_2$ component, in $x_2 - x_3$ component and in $x_3 - x_1$ component with respect to time, respectively. Similarly Fig.4 (a)-(c) is shown as time-series of lorenz system.

At the initial condition $e(0) = (e_1(0) = -0.6, e_2(0) = -0.9, e_3(0) = 1.5)$, Fig.5 is shown as error dynamics (left side) in component wise-e_1, e_2, e_3 and its synchronization of error dynamics (right side)in component wise-e_1, e_2, e_3, respectively.

Here, all eigen value of Hurwitz matrix, G have negative real parts. Hurwitz stability criterion demonstrates that error system is asymptotically stables. The stabilization of error system means that $e_1(t) \to 0$, $e_2(t) \to 0$ and $e_3(t) \to 0$. in componentwise. It is shown in the Fig.5.

5 Conclusion

In this paper, we have presented and investigated a classical chaotic system which is highly sensitive to the initial condition, via T-S fuzzy model. and synchronization methodology of chaotic lorenz system. This methodology provides the new insight in control systems analysis and design using LMI techniques. T-S fuzzy modeling and this methodology with the help of LMI technique is an effective and fruitful results for synchronization of lorenz system. We have found the lyapunov exponent of lorenz system in which one value of lyapunov exponent is positive and other two are negative. This shows that lorenz system is chaotic.

In numerical solution, the feedback gain common matrices computed by LMI using closed-loop eigenvectors in Matlab Commands (eigenvalues,$\rho = ([-2; -2+12*sqt(3)*i; -2-12*sqt(3)*i])$ with initial conditions

$$(x_1(0) = -0.1, x_2(0) = -0.2, x_3(0) = 3)^T; (y_1(0) = 0.5, y_2(0) = 0.7, y_3(0) = 1.5)^T$$

have been obtained. We have obtained the Hurwitz matrix $G < 0$, which shows that error system of two identical chaotic systems is asymptotically stable with time. It confirms the stability of fuzzy control system and satisfies the linear matrix inequalities (LMIs). This results demonstrate the efficient fruitfulness of the feedback and T.S. fuzzy control theory application to the synchronization for two identical lorenz systems.

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