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Quasi–consensus in Second–Order Multi–agent Systems with Sampled Data Through Fuzzy Transform

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Abstract

In this paper, the problem of second–order consensus in multi–agent systems with sampled position data is handled. Due to the discrete nature of the information transmission among agents, there is an increasing interest in such kind of systems. By thinking of collecting sampled position data in a certain time interval, that is more general than considering just the sampling period as in many papers, the velocity is here approximated through fuzzy transform (or F–trasform for short). F–transform was proved to have high accuracy, even when data are not correlated, and a low computational cost. This approximation changes the autonomous system herein considered in a nonautonomous one. In light of this, we formally discuss the conditions under which a quasi–consensus can be achieved. Finally, simulation examples are given to verify and illustrate the theoretical achievements.

Keywords: nonautonomous dynamical systems, approximation techniques, fuzzy sets

1 Introduction

A large part of the current literature focuses on the case where agents are governed by first-order dynamics (e.g. [15, 18, 27]).

Anyway, second–order dynamics (e.g. see [32, 33, 35, 24, 19]) is receiving increasing attention, because in many real–world applications, the dynamics of the agents involves both position and velocity.

However, as shown in [13, 25, 35], the velocity states of agents are often unavailable or require expensive sensors, so sampled data are used to respond to this issue.

Using sampled data means to take into account the fact that the information transmission among agents is not continuous, but discrete, due to geographical constraints (such as for sensors) or unreliable communication channels in many artificial networks [31]. Yet in the perspective of energy–saving approaches to respond to the communication burden, sampled data control is becoming a promising and effective control strategy [9, 28].

As pointed out in [12], the larger sampling interval (that means less sampling data), the lesser energy consumed. Such a sampling scheme with less signals sampled can be regarded as more efficient.

Considering sampled data is beneficial even in presence of communication delay. In [30], where a first–order multi–agent system with communication delay was considered, it has been shown that the effect of delay on the agents state can be neglected if the network topology is known and every agent transmits historical data to the neighbors.

In [36], a second–order multi–agent system with communication delay in the context of sampled data was discussed. In that work, actually each agent updated its control input at the kth time by using its own and its neighbors kth sampled data; the agents dynamics was described as a continuous system with piecewise constant input.

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The main problem in multi-agent systems is to design a distributed control law for each agent, which uses only local information from its neighboring agents, such that all agents achieve a certain behavior of common interest. This is the so–called consensus problem. In recent years, many works have been proposed for solving the consensus problem from different perspectives (e.g. [3, 4]).

In particular, in [34] and [29] quasi–consensus behavior is considered in order to take into account the differences in the final relative position among agents, depending on the initial conditions. Those works were motivated by the fact that the velocity information are difficult to be directly obtained and the stored delayed position information may be used to that end. This introduces a degree of approximation which can be expressed through the concept of quasi–consensus.

In the present work, we discuss the conditions under which quasi–consensus can be achieved, if sampled position data are used through F–transform, over a certain time interval, which can be in general larger than the single sampling period, usually considered in the current literature (e.g. [35]).

F–transform was proposed by Perfilieva [21] as a fuzzy approximation technique, by formulating a functional dependency as a linear combination of basic functions. Currently, the major applications of the F–transform are in image processing, e.g. [6, 22, 7].

It should be pointed out that there are many approximation techniques in the fuzzy context (e.g. [17, 16]), but herein a fuzzy approximation technique is used in a non fuzzy context in order to handle efficiently the problem of sampling data in multi-agent systems.

In particular, introducing the F–transform approximation changes the autonomous dynamical system herein considered in a nonautonomous one. So we discuss the boundedness of the long–term solution, which can be intended as a quasi–consensus condition.

Thus, the main contribution of this work is exploiting the approximation ability of F–transform in second–order multi–agent systems with position data sampled in large intervals. Conditions for seeking consensus are formally discussed and the theoretical achievements are supported by numerical results.

The paper is structured as follows: Section 2 gives an overview on F–transform; Section 3 provides theoretical foundations; Section 4 is devoted to numerical experiments and finally Section 5 gives some conclusions.

2 F-transform: An Overview

The F–transform is an approximation technique which changes a functional problem into a problem of linear algebra.

Since F–transform was introduced [21], several papers on the topic appeared (e.g. [5, 2]). In particular, in [2] new types of F–transforms were presented, based on B–splines, Shepard kernels, Bernstein basis polynomials and Favard-Szasz-Mirakjan type operators for the univariate case.

There are many applications of the F–transforms in image processing ([6, 22, 7]) and some others in time series analysis (e.g. [23, 20]), also by integrating the F–transform and the fuzzy tendency modeling [23] or the F–transform and fuzzy natural logic [20]. In particular, the work [20] focuses on the application of fuzzy transform (F-transform) to the analysis of time series, by assuming that the latter can be additively decomposed into trend–cycle, seasonal component and noise.

F–transforms were also used in data analysis [8] and for data compression in wireless sensor networks [10, 11]. Recently, they were used in a Picard-like scheme to solve a class of delay differential equations [26].

We briefly recall some definitions. Let $I = [a, b]$ be a closed interval and $\{t_1, t_2, \dots, t_p\}$, with $p \geq 3$, be points of I, called nodes, such that $a = t_1 < t_2 < \cdots < t_p = b$. A fuzzy partition of I is defined as a sequence $\{A_1, A_2, \dots, A_p\}$ of fuzzy sets $A_i : I \to [0, 1]$, with $i = 1, \dots, p$ such that

- $A_i(t) \neq 0$ if $t \epsilon (t_{i-1}, t_{i+1})$ and $A_i(t_i) = 1$;
- A_i is continuous and has its unique maximum at t_i;
- $\sum_{i=1}^{p} A_i(t) = 1, \quad \forall\ t \epsilon I.$

The fuzzy sets $A_1, A_2, \dots, A_p$ are called basic functions and they form an uniform fuzzy partition if the nodes are equidistant.

In general, $\overline{h} = max_i |t_{i+1} - t_i|$ is the norm of the partition. For a uniform partition $\overline{h} = (b - a)/(p - 1)$ and $t_j = a + (j - 1)\overline{h}$, with $j = 1, \dots, p$.

The fuzzy partition can be obtained by means of several basic functions, the mainly used are:

- hat functions

$$A_j(t) = \begin{cases} \frac{t_{j+1}-t}{t_{j+1}-t_j}, & t\epsilon[t_j, t_{j+1}] \\ \frac{t-t_{j-1}}{t_j-t_{j-1}}, & t\epsilon[t_{j-1}, t_j] \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

- sinusoidal shaped basic functions

$$A_j(t) = \begin{cases} \frac{1}{2}\left(\cos\frac{\pi(t_j-t)}{(t_{j+1}-t_j)}+1\right), & t\epsilon[t_j, t_{j+1}] \\ \frac{1}{2}\left(\cos\frac{\pi(t-t_j)}{(t_j-t_{j-1})}+1\right), & t\epsilon[t_{j-1}, t_j] \\ 0, & \text{otherwise}. \end{cases} \tag{2}$$

The fuzzy transform (F–transform) of a function $f(t)$ continuous on I with respect to $\{A_1, A_2, \ldots, A_p\}$ is the p–tuple $[F_1, F_2, \cdots, F_p]$ whose components are

$$F_i = \frac{\int_a^b f(t)A_i(t)dt}{\int_a^b A_i(t)dt}. \tag{3}$$

The function

$$f_{F,p} = \sum_i^p F_i A_i(t), \qquad t\epsilon I \tag{4}$$

is called inverse F–transform of f with respect to $\{A_1, A_2, \ldots, A_p\}$ and it approximates a given continuous function f on I with arbitrary precision, as stated in [21].

In many real cases, where the function f is known only at a given set of points $\{\bar{t}_1, \bar{t}_2, \ldots, \bar{t}_m\}$, with $m > p$, the discrete F–transform can be used and Eq. (3) is replaced by

$$F_i = \frac{\sum_{j=1}^m f(\bar{t}_j)A_i(\bar{t}_j)}{\sum_{j=1}^m A_i(\bar{t}_j)}, \qquad i = 1, \ldots, p. \tag{5}$$

Similarly, Eq. (4) is replaced by

$$f_{F,p}(\bar{t}_j) = \sum_i^p F_i A_i(\bar{t}_j), \qquad j = 1, \ldots, m \tag{6}$$

giving the discrete inverse F–transform.

3 Methodology and Properties

3.1 Basic Graph Theory and Notations

Let $G = (v, E, A)$ be a weighted graph, where $v = (v_1, \ldots, v_N)$ is the nonempty set of nodes/agents, $E \subseteq v \times v$ is the set of edges/arcs, by recalling that $(v_i, v_j) \in E$ means that there is an edge from node i to node j. The topology of a weighted graph G can be represented by the $N \times N$ adjacency matrix $\mathbf{A}$, with elements $a_{ij} > 0$ if $(v_j, v_i) \in E$, otherwise $a_{ij} = 0$. Throughout this paper, it is assumed that $a_{ii} = 0$ and the topology is fixed, that is $\mathbf{A}$ is time-invariant. Let $d_i = \sum_{j=1}^N a_{ij}$ denote the weighted in-degree of node i and $\mathbf{D} = diag(d_1, \ldots, d_N)$ the $N \times N$ in-degree matrix. The graph Laplacian matrix is $\mathbf{L} = \mathbf{D} - \mathbf{A}$. It satisfies the diffusion property $\sum_{j=1}^N L_{ij} = 0$. For an undirected graph G the matrix $\mathbf{A}$ (and consequently $\mathbf{L}$) is symmetric.

The set of neighbors of node i is denoted as $N_i = \{j|(v_j, v_i) \in E\}$. The cardinality of the set N_i represents the degree of the node i. If node j is a neighbor of node i, then node i can get information from node j, not necessarily vice versa for directed graph (digraph). For undirected graphs, the neighbor is in mutual relation.

A path between nodes v_i and v_j is a sequence of edges with distinct nodes. An undirected graph G is connected if there is a path between any pair of distinct nodes in it.

We recall a well–known Lemma [14].

Lemma 1. *Let G be a graph on N vertices with Laplacian $\mathbf{L}$. Denote the eigenvalues of $\mathbf{L}$ by $\lambda_1, \ldots, \lambda_N$, satisfying $\lambda_1 \leq \cdots \leq \lambda_N$. Then $\lambda_1 = 0$ and the N-sized vector $\mathbf{1} = [1, \cdots, 1]^T$ its eigenvector. Besides, if G is connected $\lambda_2 > 0$.*

Remark 2. *If $\mathbf{q}$ is a vector with all elements equal to a real constant, then $\mathbf{Lq} = \mathbf{0}$, in force of the diffusion property.*

3.2 Problem Formulation

The second–order consensus protocol in multi–agent dynamical systems is commonly described in the linear range by (e.g. [33])

$$\dot{x}_i = v_i \tag{7}$$

$$\dot{v}_i = -\alpha \sum_{j=1}^{N} L_{ij} x_j - \beta \sum_{j=1}^{N} L_{ij} v_j \tag{8}$$

which substantially expresses the dynamics of coupled oscillators, and where $x_i \in \mathbb{R}^n$, α and β are real constants.

In what follows, we assume $\beta = c\alpha$ (e.g. [24]), with $c \in \mathbb{R}$.

In real situations, agents usually communicate with each other at discrete time. Therefore, in order to utilize less information (saving energy for example in presence of digital sensors), it is desirable to use sampled data instead of the current data.

Let T be a time interval with m discrete values $t_1 < \cdots < t_m$. We consider the sampled position data $x(t_k)$, with $k = 1, \ldots, m$.

So, by considering that $v_j(t_k) = [x_j(t_{k+1}) - x_j(t_k)]/h$, one has

$$\dot{v}_i(t) = -\alpha \sum_{j=1}^{N} L_{ij} x_j - c\frac{\alpha}{h} \sum_{j=1}^{N} L_{ij} \sum_{l=1}^{p} \gamma_l \sum_{k=1}^{m-1} A_l(t_k) d_j(t_k) A_l(t) \tag{9}$$

with $\gamma_l = (\sum_{k=1}^{m-1} A_l(t_k))^{-1}$ and $d_j = x_j(t_{k+1}) - x_j(t_k)$.

For the remainder of this work, $\mathbf{I}_s$ will denote the $s \times s$ identity matrix.

So, in compact form we have

$$\dot{\mathbf{x}} = \mathbf{v} \tag{10}$$

$$\dot{\mathbf{v}} = -\alpha \mathbf{L} \otimes \mathbf{I}_n \mathbf{x} - c\frac{\alpha}{h} \mathbf{u}(t) \tag{11}$$

where

$$\mathbf{u}(t) = \mathbf{L} \otimes \mathbf{I}_n \mathbf{D} \overline{\mathbf{A}} \mathbf{\Gamma} \mathbf{A}(t), \tag{12}$$

being $\mathbf{D}$ the $nN \times (m-1)$ matrix, of which the ith row is $(d_i(t_1), \cdots, d_i(t_{m-1}))$, $\overline{\mathbf{A}}$ is the $(m-1) \times p$ matrix of which the ith row is $(A_1(t_i), \cdots, A_1(t_i))$, Γ is the diagonal matrix with p non-null elements γ_l, $\mathbf{A}(t)$ the vector of the basic functions $A_l(t)$. Note that $p < m$ (see Section II).

Definition 3. *The multi-agent system is said to achieve quasi-consensus if for any initial condition*

$$\lim_{t\to\infty} \|x_i - x_j\| = c_{ij} \tag{13}$$

$$\lim_{t\to\infty} \|v_i - v_j\| = 0 \tag{14}$$

for any $i, j = 1, \ldots, N$, with $i \neq j$, and where c_{ij} are constants. If $c_{ij} = 0$, then the quasi-consensus is called consensus.

Let $\overline{x}(t) = (\sum_{i=1}^{N} x_i(t))/N$, $\overline{v}(t) = (\sum_{i=1}^{N} v_i(t))/N$ be the average consensus states of position and velocity, respectively.

In order to express the distance between the generic x_i and $\overline{x}$, and similarly between v_i and $\overline{v}$, we introduce the following error vectors

$$\chi = \mathbf{M} \otimes \mathbf{I}_n \mathbf{x}, \qquad \eta = \mathbf{M} \otimes \mathbf{I}_n \mathbf{v} \tag{15}$$

where $\mathbf{M} = \mathbf{I}_N + \mathbf{M}_c$, being $\mathbf{M}_c$ the $N \times N$ matrix with all elements equal to $-1/N$.

So in concise notation, Eqs. (10-11) become (see Remark 2)

$$\dot{\mathbf{e}} = \mathbf{H}\mathbf{e} - c\frac{\alpha}{h}\mathbf{w}(t) \tag{16}$$

where $\mathbf{e}^T = [\chi, \eta]^T$, $\mathbf{w}^T(t) = [\mathbf{0}, \mathbf{M} \otimes \mathbf{I}_n \mathbf{u}(t)]^T$ and

$$\mathbf{H} = \begin{pmatrix} \mathbf{0} & \mathbf{I}_{Nn} \\ -\alpha \mathbf{L} \otimes \mathbf{I}_n & \mathbf{0} \end{pmatrix}. \tag{17}$$

In the next subsection we state the properties of the error dynamics system above.

3.3 Properties

Assumption 4. *The matrix* $\mathbf{H}$ *is nonsingular.*

Remark 5. *If the graph is connected, then the assumption 4 holds true. Since as a consequence of Lemma 1, the matrix* $\mathbf{L}$ *has full column–rank [13], then according to Theorem 2.1 in [1] the matrix* $\mathbf{H}$ *is nonsingular.*

In order to establish boundedness for the vector error function $\mathbf{e}(t)$, we rewrite Eq. (16) in a discretized form by means of finite differences. So we get

$$\mathbf{e}_{i+1} = \mathbf{P}\mathbf{e}_i - c\alpha \mathbf{w}_i \tag{18}$$

where $\mathbf{P} = \mathbf{I}_{2Nn} + h\mathbf{H}$.

Let $\mathbf{e}_\infty$ denote the error for $t \to \infty$ and $\mathbf{e}_0$ the error at $t = 0$.

Throughout the paper, $\rho(\mathbf{P})$ will denote the spectral radius of the matrix $\mathbf{P}$. Besides, in what follows regarding inequalities, the component–wise convention is assumed.

Theorem 6. *If* $\rho(\mathbf{P}) \leq 1$*, then the following error bound holds true*

$$\mathbf{e}_\infty \leq \mathbf{e}_0 + c\frac{\alpha}{h}\mathbf{H}^{-1}\boldsymbol{\Delta} \tag{19}$$

where $\boldsymbol{\Delta} = (\mathbf{M} \otimes \mathbf{I}_n)(\mathbf{L} \otimes \mathbf{I}_n)\mathbf{D}\overline{\mathbf{A}}\boldsymbol{\Gamma}$.

Proof. From Eq. (18), it follows that

$$\mathbf{e}_{i+1} = \mathbf{P}^{i+1}\mathbf{e}_0 - c\alpha \sum_{k=1}^{i} \mathbf{P}^k \mathbf{w}_{i-k} \leq \mathbf{P}^{i+1}\mathbf{e}_0 - c\alpha \sum_{k=1}^{i} \mathbf{P}^k \boldsymbol{\Delta} \tag{20}$$

where $\boldsymbol{\Delta}$ is the upper bound for $\mathbf{w}_i$, by recalling that the maximum value of the basic functions $A_j(t_i)$ is 1 for any i.

The conclusion is readily achieved, by noticing that on the right-hand side of Eq. (20) there is a geometric series of matrices. $\square$

Let $\overline{\mathbf{c}}$ be a vector of real constants.

Remark 7. *If* $\|\mathbf{e}_\infty\| = \|\overline{\mathbf{c}}\|$*, then quasi–consensus is achieved.*

We state the following Theorem.

Theorem 8. *Suppose that the hypotheses of Theorem 6 are satisfied. Quasi–consensus implies that*

$$|\frac{c\alpha}{h}| \leq \frac{\overline{\sigma}(\mathbf{H})}{\|\Delta\|}\|\mathbf{e}_0\| \tag{21}$$

where $\overline{\sigma}(\mathbf{H})$ *is the maximum singular value of the matrix* $\mathbf{H}$.

Proof. By imposing that

$$\mathbf{e}_0 + c\frac{\alpha}{h}\mathbf{H}^{-1}\boldsymbol{\Delta} = \bar{\mathbf{c}}, \tag{22}$$

it follows that

$$|\frac{c\alpha}{h}|\|\Delta\| \leq \|\mathbf{H}(\mathbf{e}_0 - \bar{\mathbf{c}})\|. \tag{23}$$

So the conclusion readily holds. □

4 Numerical Experiments

4.1 First Example

As a first example application we consider a case with $N = 5$, $n = 1$ (i.e. $x_j \in \mathbf{R}$, $j = 1, \ldots, 5$) and $m = 30$, $p = 12$ in a unit time interval. Besides, we have $c\alpha/h = 0.078$. We use sinusoidal shaped basic functions. The Laplacian matrix for the undirected graph we considered is

$$\mathbf{L} = \begin{pmatrix} 0.5 & -0.2 & -0.3 & 0 & 0 \\ -0.2 & 1.1 & 0 & -0.8 & -0.1 \\ -0.1 & 0 & 1.5 & -1 & -0.4 \\ 0 & -0.6 & -0.6 & 1.2 & 0 \\ 0 & -0.4 & -0.2 & 0 & 0.6 \end{pmatrix}. \tag{24}$$

Under the hypothesis of Theorem 7 (i.e. $\rho(\mathbf{D}) \leq 1$), one can observe from Fig. 1 that both position and velocity errors are bounded. In particular, with regard to velocities, since the initial conditions are close to zero, the trajectories lie close to zero.

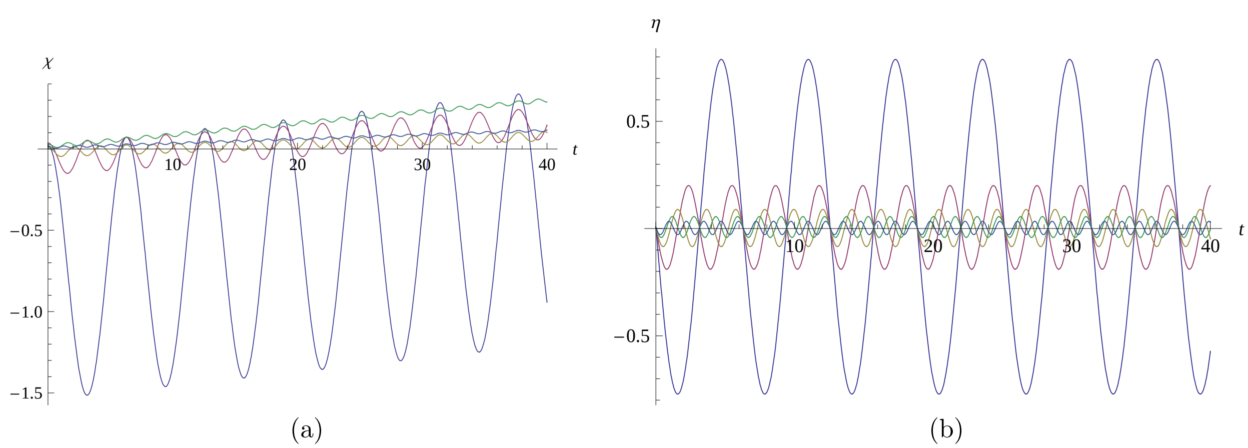

Figure 1: First example: (a) position errors, (b) velocity errors

4.2 Second Example

In this example, we fix $N = 4$, $n = 2$ (i.e. $x_j \in \mathbf{R}^2$, $j = 1, \ldots, 4$) and $m = 30$, $p = 12$ in a unit time interval. Besides, we fix $c\alpha/h = 0.9$. We use sinusoidal shaped basic functions. The Laplacian matrix for the undirected graph we considered is

$$\mathbf{L} = \begin{pmatrix} 1.5 & -0.5 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 2 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}. \tag{25}$$

Even for this case, we found $\rho(\mathbf{D}) \leq 1$. Fig. 2 shows the behavior of position and velocity errors. This example shows clearly (even more than the previous one) that the error trajectories are enveloped, that is the error bound holds.

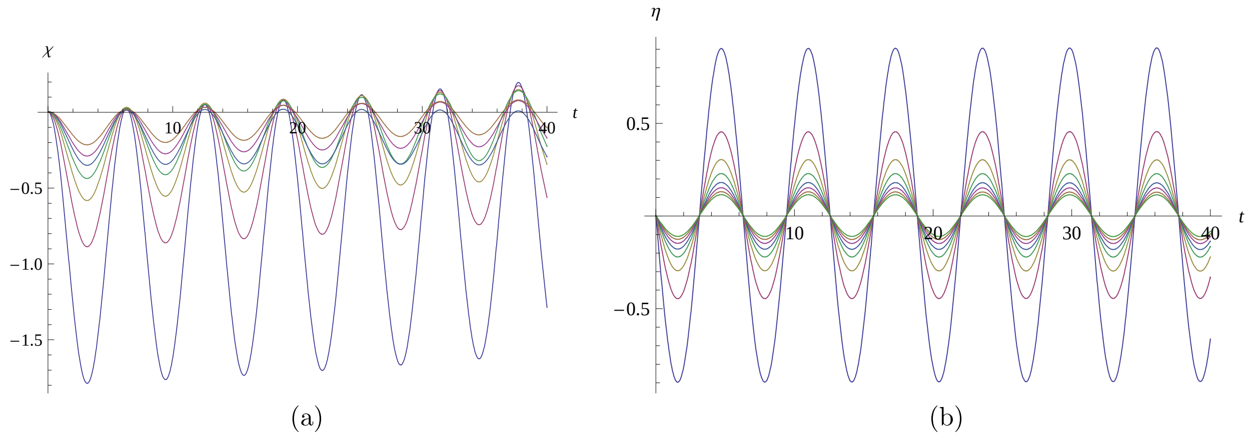

Figure 2: Second example: (a) position errors, (b) velocity errors

5 Conclusions

In this paper, we formally discussed the conditions under which quasi–consensus in a multi–agent system can be achieved, in presence of a certain number of sampled position data. Sampled position data are commonly used in order to approximate velocities, due to the fact that real systems are not continuous, but discrete, and detecting velocities would be more difficult and expensive than gathering position data. We used F–transform to approximate velocities in a certain time interval, where sampled position data are available. Approximating velocities motivated the discussion about quasi–consensus, that is the boundedness of the differences between the final relative positions of agents. Finally, we provided some numerical examples, which show a good agreement with the theoretical achievements.

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