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A Credibilistic Optimization Approach to Single-Product Single-Period Inventory Problem with Two Suppliers

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Abstract

In this paper, a single-product single-period inventory problem is studied. A retailer can get the products from two suppliers. The uncertain proportion of supply from primary supplier is characterized by a fuzzy variable with known credibility distribution, whereas secondary supplier's supply is perfectly known in advance. In addition, the products of the reliable supplier are more expensive than the unreliable one. The retailer has to determine the optimal order quantity from the primary supplier and the optimal reserved quantity from the secondary supplier. On the basis of risk-neural criterion, the cost objective in our inventory problem is measured by Lebesgue-Stieltjes (L–S) integral. Under the triangular and trapezoidal supply modes, we analyze the properties of the proposed credibilistic inventory model, and derive its equivalent convex programming submodels. As a result, the original inventory optimization model can be solved by domain decomposition method. Finally, some numerical experiments are conducted to illustrate the effectiveness of the designed solution method, and the sensitivity of the cost parameters and fuzzy distribution parameters on solution results is also analysed.

Keywords: inventory management, credibility distribution, credibilistic optimization, domain decomposition method

1 Introduction

Inventory management is an active control program that allows the management of sales, purchases and payments. The key is to determine how to order, how much to order, and when to order. In recent years, the inventory management is playing an increasingly important role in commercial competition. One won't be able to compete with others if he/she doesn't have the ability to manage the inventory on the world market. Many researchers and practitioners have already paid more attention to the study of inventory management. In present paper, the problem of products supply between retailer and suppliers in a supply chain is studied from a new perspective.

In inventory management literature, the single supplier is considered by some researchers. Xiao and Qi [26] investigated the coordination of a supply chain with one manufacturer and two competing retailers. Qi et al. [17] studied a continuous-review inventory problem with a single supplier and a single retailer. Keren [7] studied a single supplier with random supply yield for a single-period inventory problem with deterministic demand. Sargut and Qi [19] modeled an inventory problem of a supplier and a retailer that was subject to random disruptions. The single supplier has both advantages and shortcomings. Mardan et al. [13] mentioned that although using a single supplier could bring pricing advantages to the system through paying lower supplier management costs or offering a lower unit price by discount provided by the supplier, it made the system more vulnerable against probable supplier disruption. In order to maximize the profit and minimize the risk simultaneously, Li and Chen [9] presented a two-stage bi-objective stochastic model which employed the double evaluation criteria of mean and standard deviation.

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In order to cope with uncertainties caused by supply and output, business organizations often use a secondary supplier or multiple suppliers. The study of Tomlin [21] showed that supplier contingent rerouting was an attractive strategy if disruptions were rare but long, whereas inventory mitigation was preferred if disruptions were frequent but short. Furthermore, Griri [6] developed the model in the perspective of a low risk averse retailer and quantified the risk via an exponential utility function. Babich [1] considered a two-supplier case, in which one of the suppliers had shorter lead time serving as the emergency source. Tomlin [22] and Schmitt and Snyder [20] assumed that the vision would shift to a reliable supplier, when the unreliable supplier's supply was interrupted. The situation of the multiple suppliers also was considered by some researchers, including Chen et al. [3], Dada et al. [4] and Merzifonluoglu and Feng [14].

The above single supplier and multi-supplier inventory management problems were studied in stochastic environment. In many practical situations, the exact data or probability distribution of random variable is unavailable. Especially for short life cycle and technology update fast products, due to the lack of historical data and sufficient information, the demand can only have a vague understanding in the situation. To deal with this kind of uncertainty, Petrovic et al. [15] and [16] used fuzzy logic to describe the imprecise information in an uncertain environment. Kumara et al. [8] proposed a fuzzy goal programming approach for solving the vendor selection problem with multiple objectives, in which some of the parameters were fuzzy in nature. Xu and Zhai [27] considered a two-stage supply chain coordination problem and focused on the fuzzy aspect of demand uncertainty. Yu et al. [28] proposed a single-period inventory model with fuzzy price-dependent demand, and discussed the conditions to determine the optimal pricing and inventory decisions jointly so that the expected profit could be maximized. Sang [18] concentrated on price competition between two competitive manufacturers who sold their products to a common retailer under a fuzzy decision environment. Based on credibility measure [11], Wu et al. [25] characterized incomplete information by a fuzzy variable in the agent's ability and applied the method to deal with optimal contracting problems, and Li and Liu [10] presented a new risk-neutral inventory problem with fuzzy demand, in which the expected value was adopted in the formulation of profit objective function. In present paper, we also develop a risk-neural single-product single-period inventory problem in the sense of L–S integral [2]. In our inventory problem, the uncertain proportion of supply from Supplier 1 is characterized by a fuzzy variable with known credibility distribution function [5].

This paper is organized as follows. In Section 2, after introducing some necessary notations for our inventory management problem, we model a single-product single-period inventory problem based on risk-neural criterion. In Section 3, we first discuss the equivalent deterministic programming models under the common credibility distribution functions, and design a feasible domain decomposition method to solve the equivalent convex programming submodels. In Section 4, we present a numerical example to illustrate effectiveness of the developed optimization method. Section 5 gives our conclusions in this paper.

2 Single-Product Single-Period Inventory Problem

2.1 Notations

In order to model our single-product single-period inventory problem, we employ the following notations.

Decision variables

x: order quantity from supplier 1;

y: reserved quantity from supplier 2.

Uncertain parameter

ξ: uncertain proportion of supply from Supplier 1, $0 \leq \xi \leq 1$.

Fixed parameters

d: known demand over one period;

r: reserved cost per unit;

p_1: price of each unit from supplier 1;

p_2: price of each unit from supplier 2, $p_2 > p_1$;

c_o: inventory holding (overage) cost per unit;

c_u: shortage penalty (underage) cost per unit;

a^+: the maximum value of a and 0, i.e., $a^+ = \max\{a, 0\}$;

π: cost function for retailer.

2.2 Formulation of Credibilistic Inventory Management Model

In our inventory management problem, suppose a retailer can get a product from two suppliers. Our approach is based on the assumption that the uncertain proportion of supply from Supplier 1 is characterized by a credibility distribution and retailer's demand is certain. Products of Supplier 1 are much cheaper than Supplier 2's, but the supply of Supplier 1 is uncertain in the sense that the supply production typically less than the order quantity from the retailer. Products of Supplier 2 are relatively expensive, but his supply is very reliable. The products from Supplier 2 require to be reserved before retailer ordering, because it cannot provide more than the scheduled product quantity. One great advantage of Supplier 2 is that retailer can decide his order quantity according to the product quantity provided by Supplier 1. The premise is on the basis of the already scheduled for Supplier 2. The prices of Supplier 1 and Supplier 2's unit product are p_1 and p_2, respectively. The reserved cost of per unit is r in the Supplier 2. $p_2+r > p_1$ is easy to understand, because reliable supplier's products are more expensive. Figure 1 shows the supply relationship between suppliers and retailer.

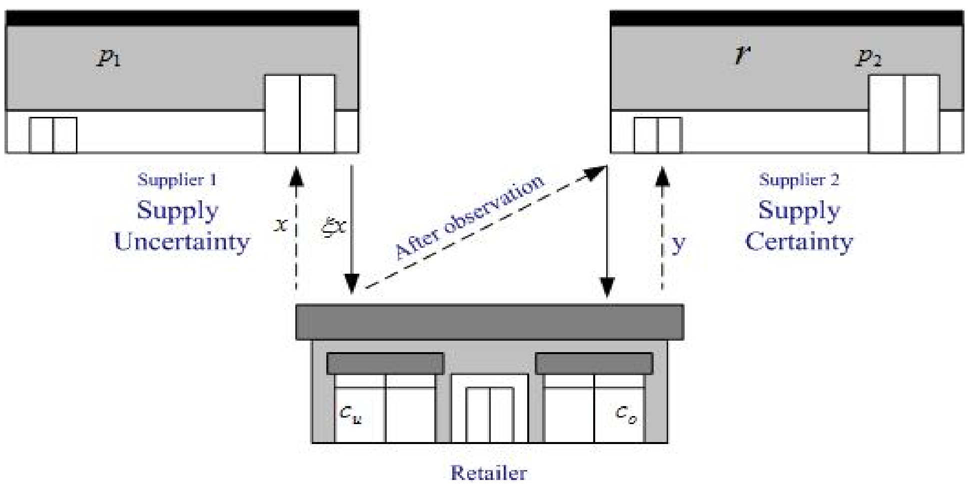


Figure 1: Supply relationship between suppliers and retailer

At the beginning of each period, retailer has to determine the optimal order quantity x from Supplier 1 and the optimal reserved quantity y from Supplier 2. The objective is to minimize the total cost of retailer. Supplier 1 can provide the quantity ξx. In this case, retailer should pay Supplier 1 the purchase cost $p_1\xi x$ and pay Supplier 2 the reservation cost ry. Note that the order quantity from Supplier 2 should not be greater than y. We can observe product quantity provided by the Supplier 1 first. When Supplier 1 cannot meet the demand, retailer can get product quantity $d-\xi x$ (if $d-\xi x < y$) or get product quantity y (if $d-\xi x \geq y$) from Supplier 2. If Supplier 1 himself can meet the demand, retailer does not need to purchase products from Supplier 2. The reservation cost paid to Supplier 2 is still ry, regardless of the number of products that retailer purchase from Supplier 2. If the product quantity provided by Supplier 1 is more than demand, retailer need increase the inventory holding cost $c_o(\xi x-d)$. When the two suppliers can not meet the need of retailer, retailer will increase the shortage penalty cost $c_u(d-\xi x-y)$.

There are four parts in the overall cost. The first part is the purchase cost $p_1\xi x$ paid to Supplier 1. The second part is the reserved cost ry and the purchase cost $p_2\min\{y,(d-\xi x)^+\}$ paid to Supplier 2. The third part is the inventory holding cost $c_o(\xi x-d)^+$. The last part is the shortage penalty cost $c_u(d-\xi x-y)^+$. As a consequence, the total cost function is represented as

$$\pi(x,y,\xi)=p_1\xi x+ry+p_2\min\{y,(d-\xi x)^+\}+c_o(\xi x-d)^+ + c_u(d-\xi x-y)^+. \tag{1}$$

The total cost function $\pi(x,y,\xi)$ is a fuzzy variable. According to [23, 24], if we denote $\Pi(x,y)$ as the equivalent value of $\pi(x,y,\xi)$, then we have the following computational formula

$$\Pi(x,y)=\int_{[0,1]}\pi(x,y,t)\mathrm{dCr}\{\xi\leq t\},$$

where the L–S measure is generated by the monotone increasing function $\mathrm{Cr}\{\xi \le t\}$ (see, [12]).

Based on the notations above, we could find the optimal order quantity and the optimal reserved quantity by solving the following credibilistic inventory optimization model

$$\begin{array}{ll} \min & \Pi(x,y) \\ \text{s.t.} & x \ge 0 \\ & y \ge 0. \end{array} \tag{2}$$

By Eq.(1), we get the following analytical expression of the cost function

$$\pi(x,y,\xi) = \begin{cases} p_1\xi x + ry + p_2 y + c_u(d - \xi x - y), & \xi x \le d \quad \textit{and} \quad d - \xi x > y \\ p_1\xi x + ry + p_2(d - \xi x), & \xi x \le d \quad \textit{and} \quad d - \xi x \le y \\ p_1\xi x + ry + c_o(\xi x - d), & \xi x > d. \end{cases} \tag{3}$$

If we denote

$$\pi_1(x,y,\xi) = p_1\xi x + ry + p_2 y + c_u(d - \xi x - y),$$

$$\pi_2(x,y,\xi) = p_1\xi x + ry + p_2(d - \xi x)$$

and

$$\pi_3(x,y,\xi) = p_1\xi x + ry + c_o(\xi x - d),$$

then one has

$$\pi(x,y,\xi) = \max\{\pi_1, \pi_2, \pi_3\}. \tag{4}$$

Given ξ, $\pi_i(x,y,\xi)$, $i = 1,2,3$, are all convex function with respect to x and y. That is, for any $\lambda \in (0,1)$, one has

$$\pi_i(x,y,\xi) \le \lambda\pi_i(x_1,y_1,\xi) + (1-\lambda)\pi_i(x_2,y_2,\xi), \tag{5}$$

where $x \in (x_1,x_2)$, $y \in (y_1,y_2)$.

Combining Eqs.(4) (5), we have

$$\pi(x,y,\xi) \le \lambda\pi(x_1,y_1,\xi) + (1-\lambda)\pi(x_2,y_2,\xi).$$

So $\pi(x,y,\xi)$ is a convex function for any given ξ. We next give an example to show the convexity of $\pi(x,y,\xi)$. Let $\widehat{\xi}$ be a realized value of ξ. We use the values of parameters involved in the model as follows: $d = 100, c_u = 12, c_o = 8, p_1 = 5, p_2 = 7, r = 1, \widehat{\xi} = 0.5$. By calculation, we have

$$\pi(x,y,0.5) = \begin{cases} -3.5x - 4y + 1200, & x < 200 \quad \textit{and} \quad 100 - 0.5x > y \\ -x + y + 700, & x < 100 \quad \textit{and} \quad 100 - 0.5x \le y \\ 6.5x + y - 800, & x \ge 200. \end{cases}$$

Figure 2 plots the image of the convex function $\pi(x,y,0.5)$. By the linearity of L–S integral, the objective function $\Pi(x,y)$ is also convex.

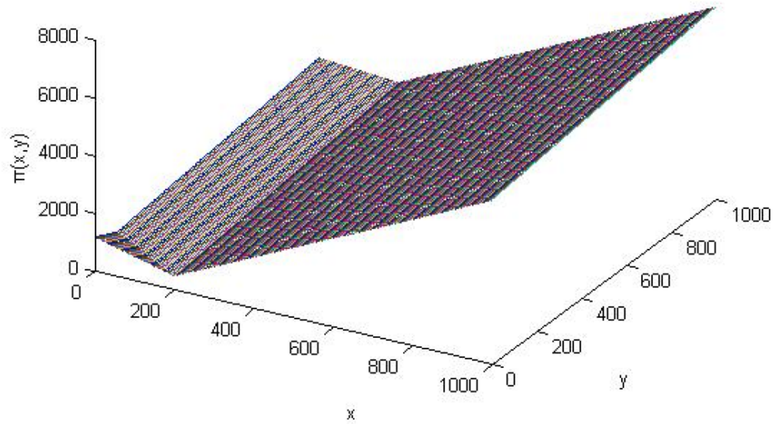


Figure 2: Image of $\pi(x, y, 0.5)$ with x and y

In the next section, we will discuss the equivalent convex programming submodels of the original credibilistic inventory model (2), and design a decomposition method to solve the obtained convex programming model.

3 Equivalent Convex Programming Models

Based on the property of the L-S integral and Eq.(3), the equivalent value $\Pi(x, y)$ of $\pi(x, y, \xi)$ can be computed as follows

$$
\begin{aligned}
\Pi(x,y) &= \int_{[0,1]} \pi(x,y,t)\mathrm{dCr}\{\xi \le t\} \\
&= \int_{[0,1]} [p_1 tx + ry + p_2 \min\{y, (d-tx)^+\} + c_o(tx-d)^+ + c_u(d-tx-y)^+]\mathrm{dCr}\{\xi \le t\} \\
&= \int_{[0,\frac{d-y}{x})} [p_1 tx + ry + p_2 y + c_u(d-tx-y)]\mathrm{dCr}\{\xi \le t\} \\
&+ \int_{[\frac{d-y}{x},\frac{d}{x})} [p_1 tx + ry + p_2(d-tx)]\mathrm{dCr}\{\xi \le t\} \\
&+ \int_{[\frac{d}{x},1]} [p_1 tx + ry + c_o(tx-d)]\mathrm{dCr}\{\xi \le t\}.
\end{aligned} \tag{6}
$$

The Lagrangian function for model (2) is

$$
L(x, y, \lambda_1, \lambda_2) = \Pi(x, y) - \lambda_1 x - \lambda_2 y.
$$

As the first-order necessary conditions, the Karush-Kuhn-Tucker (KKT) conditions of model (2) reads

$$
\begin{cases}
(p_1+c_o)\mathrm{Cr}\{\xi \le 1\} - p_1 \int_{[0,1]} \mathrm{Cr}\{\xi \le t\}\mathrm{d}t + c_u \int_{[0,\frac{d-y}{x}]} \mathrm{Cr}\{\xi \le t\}\mathrm{d}t + p_2 \int_{[\frac{d-y}{x},\frac{d}{x})} \mathrm{Cr}\{\xi \le t\}\mathrm{d}t \\
-c_o \int_{[\frac{d}{x},1]} \mathrm{Cr}\{\xi \le t\}\mathrm{d}t + \dfrac{(p_2-c_u)(d-y)}{x}\mathrm{Cr}\{\xi \le \dfrac{d-y}{x}\} - \dfrac{(p_2+c_o)d}{x}\mathrm{Cr}\{\xi \le \dfrac{d}{x}\} - \lambda_1 = 0, \\
(c_u - p_2 - r)\mathrm{Cr}\{\xi \le 0\} + r\mathrm{Cr}\{\xi \le 1\} + (p_2 - c_u)\mathrm{Cr}\{\xi \le \dfrac{d-y}{x}\} - \lambda_2 = 0, \\
\lambda_1, \lambda_2 \ge 0, \\
x, y \ge 0.
\end{cases} \tag{7}
$$

Because the objective $\Pi(x, y)$ is a convex function, the KKT point is the global optimal solution.

3.1 Supply Mode under Triangular Credibility Distribution

Suppose the uncertain proportion of supply is a triangular fuzzy variable $\xi = (l, m, 1)$ with the following credibility distribution function [5]:

$$\mu_\xi(t) = \mathrm{Cr}\{\xi = t\} = \begin{cases} \dfrac{t-l}{2(m-l)}, & l \le t < m \\ \dfrac{1-t}{2(1-m)}, & m \le t < 1 \\ 0, & others. \end{cases}$$

In this case, the credibility of fuzzy event $\{\xi \le t\}$ is computed by

$$\mathrm{Cr}\{\xi \le t\} = \begin{cases} 0, & t < l \\ \dfrac{t-l}{2(m-l)}, & l \le t < m \\ \dfrac{1+t-2m}{2(1-m)}, & m \le t < 1 \\ 1, & t \ge 1. \end{cases} \tag{8}$$

According to Eqs.(6) and (8), the feasible region of model (2) can be decomposed into three disjoint subregions based on the values of x and y.

Case I: $m \le d/x < 1$ and $l \le (d-y)/x < m$. In this case, the objective $\Pi(x, y)$ is computed by

$$\begin{aligned} \Pi(x,y) = & \int_{[l,\frac{d-y}{x})} [p_1 tx + ry + p_2 y + c_u(d - tx - y)]\mathrm{dCr}\{\xi \le t\} \\ & + \int_{[\frac{d-y}{x},m)} [p_1 tx + ry + p_2(d - tx)]\mathrm{dCr}\{\xi \le t\} \\ & + \int_{[m,\frac{d}{x})} [p_1 tx + ry + p_2(d - tx)]\mathrm{dCr}\{\xi \le t\} \\ & + \int_{[\frac{d}{x},1]} [p_1 tx + ry + c_0(tx - d)]\mathrm{dCr}\{\xi \le t\} \\ = & \frac{c_u - p_2}{4(m-l)} \frac{y^2}{x} + [r + \frac{(c_u - p_2)l}{2(m-l)}]y \\ & + \frac{1}{4}[\frac{(c_u - p_1)l^2 + (p_1 - p_2)m^2}{m-l} + \frac{c_o + p_1 + (p_2 - p_1)m^2}{1-m}]x \\ & + \frac{(p_2 - c_u)d}{2(m-l)} \frac{y}{x} + \frac{1}{4}[\frac{(c_u - p_2)d^2}{m-l} + \frac{(p_2 + c_o)d^2}{1-m}]\frac{1}{x} \\ & + \frac{d}{2}(\frac{p_2 m - c_u l}{m-l} - \frac{p_2 m + c_o}{1-m}). \end{aligned}$$

As a consequence, model (2) reduces to the following programming submodel:

$$\begin{array}{ll} \min & \Pi_1(x,y) \\ \mathrm{s.t.} & mx \le d < x \\ & lx \le d - y < mx \\ & x \ge 0 \\ & y \ge 0, \end{array} \tag{9}$$

where

$$
\begin{aligned}
\Pi_1(x,y) = & \frac{c_u - p_2}{4(m-l)}\frac{y^2}{x} + [r + \frac{(c_u - p_2)l}{2(m-l)}]y \\
& + \frac{1}{4}[\frac{(c_u - p_1)l^2 + (p_1 - p_2)m^2}{m-l} + \frac{c_o + p_1 + (p_2 - p_1)m^2}{1-m}]x \\
& + \frac{(p_2 - c_u)d}{2(m-l)}\frac{y}{x} + \frac{1}{4}[\frac{(c_u - p_2)d^2}{m-l} + \frac{(p_2 + c_o)d^2}{1-m}]\frac{1}{x} \\
& + \frac{d}{2}(\frac{p_2 m - c_u l}{m-l} - \frac{p_2 m + c_o}{1-m}).
\end{aligned}
$$

Case II: $m \le d/x < 1$ and $(d-y)/x \ge m$. In this case, the objective $\Pi(x,y)$ is computed by

$$
\begin{aligned}
\Pi(x,y) = & \int_{[l,m)} [p_1 tx + ry + p_2 y + c_u(d - tx - y)]\mathrm{dCr}\{\xi \le t\} \\
& + \int_{[m,\frac{d-y}{x})} [p_1 tx + ry + p_2 y + c_u(d - tx - y)]\mathrm{dCr}\{\xi \le t\} \\
& + \int_{[\frac{d-y}{x},\frac{d}{x})} [p_1 tx + ry + p_2(d - tx)]\mathrm{dCr}\{\xi \le t\} \\
& + \int_{[\frac{d}{x},1]} [p_1 tx + ry + c_0(tx - d)]\mathrm{dCr}\{\xi \le t\} \\
= & \frac{c_u - p_2}{4(1-m)}\frac{y^2}{x} + [r + \frac{(1-2m)(p_2 - c_u)}{2(1-m)}]y \\
& + \frac{1}{4}[(m+l)(p_1 - c_u) + \frac{c_o + p_1 + (c_u - p_1)m^2}{1-m}]x \\
& + \frac{(p_2 - c_u)d}{2(1-m)}\frac{y}{x} + \frac{(c_u + c_o)d^2}{4(1-m)}\frac{1}{x} + \frac{d}{2}(c_u - \frac{c_u m + c_o}{1-m}).
\end{aligned}
$$

As a result, model (2) reduces to the following programming submodel:

$$
\begin{array}{ll}
\min & \Pi_2(x,y) \\
\text{s. t.} & mx \le d < x \\
& d - y \ge mx \\
& x \ge 0 \\
& y \ge 0,
\end{array}
\tag{10}
$$

where

$$
\begin{aligned}
\Pi_2(x,y) = & \frac{c_u - p_2}{4(1-m)}\frac{y^2}{x} + [r + \frac{(1-2m)(p_2 - c_u)}{2(1-m)}]y \\
& + \frac{1}{4}[(m+l)(p_1 - c_u) + \frac{c_o + p_1 + (c_u - p_1)m^2}{1-m}]x \\
& + \frac{(p_2 - c_u)d}{2(1-m)}\frac{y}{x} + \frac{(c_u + c_o)d^2}{4(1-m)}\frac{1}{x} + \frac{d}{2}(c_u - \frac{c_u m + c_o}{1-m}).
\end{aligned}
$$

Case III: $l \le d/x < m$ and $(d-y)/x \ge l$. In this case, the objective $\Pi(x,y)$ is computed by

$$\begin{aligned}\Pi(x,y) &= \int_{[l,\frac{d-y}{x})} [p_1 tx + ry + p_2 y + c_u(d - tx - y)]\mathrm{dCr}\{\xi \le t\} \\ &+ \int_{[\frac{d-y}{x},\frac{d}{x})} [p_1 tx + ry + p_2(d - tx)]\mathrm{dCr}\{\xi \le t\} \\ &+ \int_{[\frac{d}{x},m)} [p_1 tx + ry + c_0(tx - d)]\mathrm{dCr}\{\xi \le t\} \\ &+ \int_{[m,1]} [p_1 tx + ry + c_0(tx - d)]\mathrm{dCr}\{\xi \le t\} \\ &= \frac{c_u - p_2}{4(m-l)}\frac{y^2}{x} + [r + \frac{(c_u - p_2)l}{2(m-l)}]y \\ &+ \frac{(c_u - p_1)l^2 + (2m^2 + m - l - ml)(p_1 + c_o)}{4(m-l)}x \\ &+ \frac{(p_2 - c_u)d}{2(m-l)}\frac{y}{x} + \frac{(c_u + c_o)d^2}{4(m-l)}\frac{1}{x} - \frac{1}{2}[c_o d + \frac{(c_u l + c_o m)d}{m-l}].\end{aligned}$$

As a result, model (2) reduces to the following programming submodel:

$$\begin{array}{ll} \min & \Pi_3(x,y) \\ \mathrm{s.t.} & lx \le d < mx \\ & d - y \ge lx \\ & x \ge 0 \\ & y \ge 0, \end{array} \tag{11}$$

where

$$\begin{aligned}\Pi_3(x,y) &= \frac{c_u - p_2}{4(m-l)}\frac{y^2}{x} + [r + \frac{(c_u - p_2)l}{2(m-l)}]y \\ &+ \frac{(c_u - p_1)l^2 + (2m^2 + m - l - ml)(p_1 + c_o)}{4(m-l)}x \\ &+ \frac{(p_2 - c_u)d}{2(m-l)}\frac{y}{x} + \frac{(c_u + c_o)d^2}{4(m-l)}\frac{1}{x} - \frac{1}{2}[c_o d + \frac{(c_u l + c_o m)d}{m-l}].\end{aligned}$$

Finally, we summarize the above results in the following theorem:

Theorem 1. *If the uncertain proportion ξ of supply is a triangular fuzzy variable $\xi = (l, m, 1)$, then we have the following results:*
(i) If $m \le d/x < 1$ and $l \le (d-y)/x < m$, then model (2) reduces to submodel (9);
(ii) If $m \le d/x < 1$ and $(d-y)/x \ge m$, then model (2) reduces to submodel (10);
(iii) If $l \le d/x < m$ and $(d-y)/x \ge l$, then model (2) reduces to submodel (11).

3.2 Supply Mode under Trapezoidal Credibility Distribution

Suppose the uncertain proportion of supply is a trapezoidal fuzzy variable $\xi = (r_1, r_2, r_3, 1)$ with the following credibility distribution function [5]:

$$\mu_\xi(t) = \mathrm{Cr}\{\xi = t\} = \begin{cases} \dfrac{t - r_1}{2(r_2 - r_1)}, & r_1 \le t < r_2 \\ \dfrac{1}{2}, & r_2 \le t < r_3 \\ \dfrac{1 - t}{2(1 - r_3)}, & r_3 \le t \le 1 \\ 0, & \textit{others}. \end{cases}$$

Using the above credibility distribution function, the credibility of event $\{\xi \le t\}$ is computed by

$$\mathrm{Cr}\{\xi \le t\} = \begin{cases} 0, & t < r_1 \\ \dfrac{t - r_1}{2(r_2 - r_1)}, & r_1 \le t < r_2 \\ \dfrac{1}{2}, & r_2 \le t < r_3 \\ \dfrac{1 - 2r_3 + t}{2(1 - r_3)}, & r_3 \le t < 1 \\ 1, & t \ge 1. \end{cases} \tag{12}$$

According to Eqs.(6) and (12), the feasible region of model (2) can be decomposed into six disjoint subregions based on the values of x and y.

Case I: $r_3 \le d/x < 1$ and $r_1 \le (d-y)/x < r_2$. In this case, the objective $\Pi(x, y)$ is computed by

$$\begin{aligned}
\Pi(x,y) =& \int_{[r_1, \frac{d-y}{x})} [p_1 tx + ry + p_2 y + c_u(d - tx - y)] \mathrm{dCr}\{\xi \le t\} \\
&+ \int_{[\frac{d-y}{x}, r_2]} [p_1 tx + ry + p_2(d - tx)] \mathrm{dCr}\{\xi \le t\} \\
&+ \int_{[r_3, \frac{d}{x})} [p_1 tx + ry + p_2(d - tx)] \mathrm{dCr}\{\xi \le t\} \\
&+ \int_{[\frac{d}{x}, 1]} [p_1 tx + ry + c_o(tx - d)] \mathrm{dCr}\{\xi \le t\} \\
=& \frac{c_u - p_2}{4(r_2 - r_1)} \frac{y^2}{x} + [r + \frac{(c_u - p_2) r_1}{2(r_2 - r_1)}] y \\
&+ \frac{1}{4} [\frac{(c_u - p_1) r_1^2 + (p_1 - p_2) r_2^2}{r_2 - r_1} + \frac{c_o + p_1 + (p_2 - p_1) r_3^2}{1 - r_3}] x \\
&+ \frac{(p_2 - c_u) d}{2(r_2 - r_1)} \frac{y}{x} + \frac{d^2}{4} [\frac{c_u - p_2}{r_2 - r_1} + \frac{p_2 + c_o}{1 - r_3}] \frac{1}{x} \\
&+ \frac{d}{2} (\frac{p_2 r_2 - c_u r_1}{r_2 - r_1} - \frac{p_2 r_3 + c_o}{1 - r_3}).
\end{aligned}$$

As a result, model (2) reduces to the following submodel:

$$\begin{array}{ll} \min & \Pi_1(x,y) \\ \text{s.t.} & r_3 x \le d < x \\ & r_1 x \le d - y < r_2 x \\ & x \ge 0 \\ & y \ge 0, \end{array} \tag{13}$$

where

$$\begin{aligned}
\Pi_1(x,y) =& \frac{c_u - p_2}{4(r_2 - r_1)} \frac{y^2}{x} + [r + \frac{(c_u - p_2) r_1}{2(r_2 - r_1)}] y \\
&+ \frac{1}{4} [\frac{(c_u - p_1) r_1^2 + (p_1 - p_2) r_2^2}{r_2 - r_1} + \frac{c_o + p_1 + (p_2 - p_1) r_3^2}{1 - r_3}] x \\
&+ \frac{(p_2 - c_u) d}{2(r_2 - r_1)} \frac{y}{x} + \frac{d^2}{4} [\frac{c_u - p_2}{r_2 - r_1} + \frac{p_2 + c_o}{1 - r_3}] \frac{1}{x} \\
&+ \frac{d}{2} (\frac{p_2 r_2 - c_u r_1}{r_2 - r_1} - \frac{p_2 r_3 + c_o}{1 - r_3}).
\end{aligned}$$

Case II: $r_3 \leq d/x < 1$ and $r_2 \leq (d-y)/x < r_3$. In this case, the objective $\Pi(x,y)$ is computed by

$$\begin{aligned}
\Pi(x,y) = &\int_{[r_1,r_2]} [p_1tx + ry + p_2y + c_u(d - tx - y)]\mathrm{dCr}\{\xi \leq t\} \\
&+ \int_{[r_3,\frac{d}{x})} [p_1tx + ry + p_2(d - tx)]\mathrm{dCr}\{\xi \leq t\} \\
&+ \int_{[\frac{d}{x},1]} [p_1tx + ry + c_0(tx - d)]\mathrm{dCr}\{\xi \leq t\} \\
&= [r + \frac{p_2 - c_u}{2}]y + \frac{(p_2 + c_o)d^2}{4(1 - r_3)}\frac{1}{x} \\
&+ \frac{1}{4}[(p_1 - c_u)(r_1 + r_2) + \frac{c_o + p_1 + (p_2 - p_1)r_3^2}{1 - r_3}]x \\
&+ \frac{d}{2}(c_u - \frac{p_2r_3 + c_o}{1 - r_3}).
\end{aligned}$$

As a result, model (2) reduces to the following submodel:

$$\begin{array}{ll}
\min & \Pi_2(x,y) \\
\text{s. t.} & r_3x \leq d < x \\
& r_2x \leq d - y < r_3x \\
& x \geq 0 \\
& y \geq 0,
\end{array} \tag{14}$$

where

$$\begin{aligned}
\Pi_2(x,y) = &[r + \frac{p_2 - c_u}{2}]y + \frac{(p_2 + c_o)d^2}{4(1 - r_3)}\frac{1}{x} \\
&+ \frac{1}{4}[(p_1 - c_u)(r_1 + r_2) + \frac{c_o + p_1 + (p_2 - p_1)r_3^2}{1 - r_3}]x \\
&+ \frac{d}{2}(c_u - \frac{p_2r_3 + c_o}{1 - r_3}).
\end{aligned}$$

Case III: $r_3 \leq d/x < 1$ and $(d-y)/x \geq r_3$. In this case, the objective $\Pi(x,y)$ is computed by

$$\begin{aligned}
\Pi(x,y) = &\int_{[r_1,r_2]} [p_1tx + ry + p_2y + c_u(d - tx - y)]\mathrm{dCr}\{\xi \leq t\} \\
&+ \int_{[r_3,\frac{d-y}{x})} [p_1tx + ry + p_2y + c_u(d - tx - y)]\mathrm{dCr}\{\xi \leq t\} \\
&+ \int_{[\frac{d-y}{x},\frac{d}{x})} [p_1tx + ry + p_2(d - tx)]\mathrm{dCr}\{\xi \leq t\} \\
&+ \int_{[\frac{d}{x},1]} [p_1tx + ry + c_0(tx - d)]\mathrm{dCr}\{\xi \leq t\} \\
&= \frac{c_u - p_2}{4(1 - r_3)}\frac{y^2}{x} + [r + \frac{(c_u - p_2)(2r_3 - 1)}{2(1 - r_3)}]y \\
&+ \frac{1}{4}[(p_1 - c_u)(r_1 + r_2) + \frac{c_o + p_1 + (c_u - p_1)r_3^2}{1 - r_3}]x \\
&+ \frac{(p_2 - c_u)d}{2(1 - r_3)}\frac{y}{x} + \frac{(c_u + c_o)d^2}{4(1 - r_3)}\frac{1}{x} \\
&+ \frac{c_ud}{2} - \frac{(c_ur_3 + c_o)d}{2(1 - r_3)}.
\end{aligned}$$

As a result, model (2) reduces to the following submodel:

$$\begin{array}{ll} \min & \Pi_3(x,y) \\ \text{s.t.} & r_3x \le d < x \\ & d - y \ge r_3 x \\ & x \ge 0 \\ & y \ge 0, \end{array} \tag{15}$$

where

$$\begin{aligned} \Pi_3(x,y) = & \frac{c_u - p_2}{4(1-r_3)}\frac{y^2}{x} + [r + \frac{(c_u-p_2)(2r_3-1)}{2(1-r_3)}]y \\ & + \frac{1}{4}[(p_1-c_u)(r_1+r_2) + \frac{c_o + p_1 + (c_u - p_1)r_3^2}{1-r_3}]x \\ & + \frac{(p_2-c_u)d}{2(1-r_3)}\frac{y}{x} + \frac{(c_u+c_o)d^2}{4(1-r_3)}\frac{1}{x} \\ & + \frac{c_u d}{2} - \frac{(c_u r_3 + c_o)d}{2(1-r_3)}. \end{aligned}$$

Case IV: $r_2 \le d/x < r_3$ and $r_1 \le (d-y)/x < r_2$. In this case, the objective $\Pi(x,y)$ is computed by

$$\begin{aligned} \Pi(x,y) = & \int_{[r_1, \frac{d-y}{x})} [p_1tx + ry + p_2y + c_u(d - tx - y)]\mathrm{dCr}\{\xi \le t\} \\ & + \int_{[\frac{d-y}{x}, r_2]} [p_1tx + ry + p_2(d-tx)]\mathrm{dCr}\{\xi \le t\} \\ & + \int_{[r_3,r_4]} [p_1tx + ry + c_o(tx-d)]\mathrm{dCr}\{\xi \le t\} \\ = & \frac{c_u - p_2}{4(r_2-r_1)}\frac{y^2}{x} + [r + \frac{(c_u-p_2)r_1}{2(r_2-r_1)}]y \\ & + \frac{1}{4}[\frac{(c_u-p_1)r_1^2 + (p_1-p_2)r_2^2}{r_2-r_1} + \frac{(c_o+p_1)(1-r_3^2)}{1-r_3}]x \\ & + \frac{(p_2-c_u)d}{2(r_2-r_1)}\frac{y}{x} + \frac{(c_u-p_2)d^2}{4(r_2-r_1)}\frac{1}{x} \\ & + \frac{(p_2r_2 - c_ur_1)d}{2(r_2-r_1)} - \frac{c_o d}{2}. \end{aligned}$$

As a result, model (2) reduces to the following submodel:

$$\begin{array}{ll} \min & \Pi_4(x,y) \\ \text{s.t.} & r_2x \le d < r_3x \\ & r_1x \le d - y < r_2 x \\ & x \ge 0 \\ & y \ge 0, \end{array} \tag{16}$$

where

$$\begin{aligned} \Pi_4(x,y) = & \frac{c_u - p_2}{4(r_2-r_1)}\frac{y^2}{x} + [r + \frac{(c_u-p_2)r_1}{2(r_2-r_1)}]y \\ & + \frac{1}{4}[\frac{(c_u-p_1)r_1^2 + (p_1-p_2)r_2^2}{r_2-r_1} + \frac{(c_o+p_1)(1-r_3^2)}{1-r_3}]x \\ & + \frac{(p_2-c_u)d}{2(r_2-r_1)}\frac{y}{x} + \frac{(c_u-p_2)d^2}{4(r_2-r_1)}\frac{1}{x} \\ & + \frac{(p_2r_2 - c_ur_1)d}{2(r_2-r_1)} - \frac{c_o d}{2}. \end{aligned}$$

Case V: $r_2 \le d/x < r_3$ and $(d-y)/x \ge r_2$. In this case, the objective $\Pi(x,y)$ is computed by

$$\begin{aligned}\Pi(x,y) = &\int_{[r_1,\frac{d-y}{x})}[p_1tx+ry+p_2y+c_u(d-tx-y)]\mathrm{dCr}\{\xi\le t\}\\ &+\int_{[\frac{d-y}{x},r_2]}[p_1tx+ry+p_2(d-tx)]\mathrm{dCr}\{\xi\le t\}\\ &+\int_{[r_3,r_4]}[p_1tx+ry+c_o(tx-d)]\mathrm{dCr}\{\xi\le t\}\\ &=\frac{(p_1-c_u)(r_1+r_2)+(p_1+c_o)(1+r_3)}{4}x\\ &+(r+\frac{p_2-c_u}{2})y+\frac{(c_u-c_o)d}{2}.\end{aligned}$$

As a result, model (2) reduces to the following submodel:

$$\begin{array}{ll}\min & \Pi_5(x,y)\\ \text{s.t.} & r_2x\le d<r_3x\\ & d-y\ge r_2x\\ & x\ge 0\\ & y\ge 0,\end{array}\tag{17}$$

where

$$\begin{aligned}\Pi_5(x,y)=&\frac{(p_1-c_u)(r_1+r_2)+(p_1+c_o)(1+r_3)}{4}x\\ &+(r+\frac{p_2-c_u}{2})y+\frac{(c_u-c_o)d}{2}.\end{aligned}$$

Case VI: $r_1 \le d/x < r_2$ and $(d-y)/x \ge r_1$. In this case, the objective $\Pi(x,y)$ is computed by

$$\begin{aligned}\Pi(x,y) = &\int_{[r_1,\frac{d-y}{x})}[p_1tx+ry+p_2y+c_u(d-tx-y)]\mathrm{dCr}\{\xi\le t\}\\ &+\int_{[\frac{d-y}{x},\frac{d}{x})}[p_1tx+ry+p_2(d-tx)]\mathrm{dCr}\{\xi\le t\}\\ &+\int_{[\frac{d}{x},r_2]}[p_1tx+ry+c_o(tx-d)]\mathrm{dCr}\{\xi\le t\}\\ &+\int_{[r_3,1]}[p_1tx+ry+c_o(tx-d)]\mathrm{dCr}\{\xi\le t\}\\ &=\frac{c_u-p_2}{4(r_2-r_1)}\frac{y^2}{x}+[r+\frac{(c_u-p_2)r_1}{2(r_2-r_1)}]y\\ &+\frac{1}{4}[\frac{(c_u-p_1)r_1^2+(p_1+c_o)r_2^2}{r_2-r_1}+(c_o+p_1)(1+r_3)]x\\ &+\frac{(p_2-c_u)d}{2(r_2-r_1)}\frac{y}{x}+\frac{(c_u+c_o)d^2}{4(r_2-r_1)}\frac{1}{x}\\ &-\frac{d}{2}[c_o+\frac{c_ur_1+c_or_2}{r_2-r_1}].\end{aligned}$$

As a result, model (2) reduces to the following submodel:

$$\begin{array}{ll}\min & \Pi_6(x,y)\\ \text{s.t.} & r_1x\le d<r_2x\\ & d-y\ge r_1x\\ & x\ge 0\\ & y\ge 0,\end{array}\tag{18}$$

where

$$\begin{aligned}\Pi_6(x,y) = & \frac{c_u - p_2}{4(r_2 - r_1)}\frac{y^2}{x} + [r + \frac{(c_u - p_2)r_1}{2(r_2 - r_1)}]y \\ & + \frac{1}{4}[\frac{(c_u - p_1)r_1^2 + (p_1 + c_o)r_2^2}{r_2 - r_1} + (c_o + p_1)(1 + r_3)]x \\ & + \frac{(p_2 - c_u)d}{2(r_2 - r_1)}\frac{y}{x} + \frac{(c_u + c_o)d^2}{4(r_2 - r_1)}\frac{1}{x} \\ & - \frac{d}{2}[c_o + \frac{c_u r_1 + c_o r_2}{r_2 - r_1}].\end{aligned}$$

We summarize the above results in the following theorem:

Theorem 2. *If the uncertain proportion ξ of supply is a trapezoidal fuzzy variable $\xi = (r_1, r_2, r_3, 1)$, then we have the following results:*
(i) If $r_3 \le d/x < 1$ and $r_1 \le (d-y)/x < r_2$, then model (2) reduces to summodel (13);
(ii) If $r_3 \le d/x < 1$ and $r_2 \le (d-y)/x < r_3$, then model (2) reduces to summodel (14);
(iii) If $r_3 \le d/x < 1$ and $(d-y)/x \ge r_3$, then model (2) reduces to summodel (15);
(iv) If $r_2 \le d/x < r_3$ and $r_1 \le (d-y)/x < r_2$, then model (2) reduces to summodel (16);
(v) If $r_2 \le d/x < r_3$ and $(d-y)/x \ge r_2$, then model (2) reduces to summodel (17);
(vi) If $r_1 \le d/x < r_2$ and $(d-y)/x \ge r_1$, then model (2) reduces to summodel (18).

So far, we have turned the original inventory management model (2) into its equivalent submodels under two supply modes. In the next subsection, a domain decomposition method is designed to solve model (2).

3.3 Domain Decomposition Method

If the supply mode is under triangular credibility distribution, then the solution process of the proposed inventory model (2) is summarized as follows.

Step 1. Decompose the feasible region of model (2) into the following three disjoint subregions:
(i) $m \le d/x < 1$ and $l \le (d-y)/x < m$;
(ii) $m \le d/x < 1$ and $(d-y)/x \ge m$;
(iii) $l \le d/x < m$ and $(d-y)/x \ge l$.

Step 2. Solve submodels (9) (10) and (11) by Lingo software in three subregions, and denote local optimal solutions and optimal values by (x_i, y_i) and Π_i, respectively.

Step 3. Compare three local optimal values Π_i at (x_i, y_i), $i = 1, 2, 3$, and find the minimum value $\Pi^*(x^*, y^*) = \min_{1\le i\le 3} \Pi_i(x_i, y_i)$.

Step 4. Report (x^*, y^*) as the global optimal solution of model (2) and Π^* as the optimal value.

4 Numerical Experiments

In this section, we present an application example about inventory management problem. The retailer's optimal strategies will be obtained by the proposed credibilistic optimization method. In our example, the uncertain proportion of supply from unreliable supplier is characterized by a triangular credibility distribution. We analyze the sensitivity of the cost parameters and fuzzy distribution parameters on the optimal solutions and the optimal values.

4.1 Problem Statement

We consider an inventory problem about haze mask. Haze is the most serious in winter. Before this season comes, the haze mask retailer needs to store the products. The retailer can get the products from two suppliers A and B. The retailer orders the number of haze masks based on overage cost, shortage cost and his estimate on output of A. According to the past experiences, the market demand $d = 100$ for haze mask is stable in the season. The quantity provided by A is uncertain, whereas B is able to deliver the exact quantity reserved.

The unit prices of their products are $p_1 = \$5$ and $p_2 = \$7$, respectively. The retailer can decide the amount of the products ordered from B according to the quantity A provided, but B requires to be reserved before retailer ordering. The reserved cost per unit is $r = \$1$. Each unsold unit at the end of the period is charged an overage cost of $c_o = \$8$ and each unit of unmet demand is charged a shortage cost of $c_u = \$12$. Before winter comes, the retailer needs to determine the optimal order quantity from A and the optimal reserved quantity from B. To model this inventory problem, we characterize the uncertain proportion of A's supply of the haze mask by a triangular fuzzy variable $\xi = (l, m, 1)$, where the parameters l and m represent the uncertainty degrees about supply quantity from A.

Figure 3 plots the surface of $\Pi(x, y)$ for $l = 0.5$ and $m = 0.8$ in Case I, from which we find that $\Pi(x, y)$ is a convex function. Using Lingo software, we obtain the optimal solutions $x^* = 106$, $y^* = 34$ and the corresponding optimal value $\Pi(106, 34) = 582.7110$.

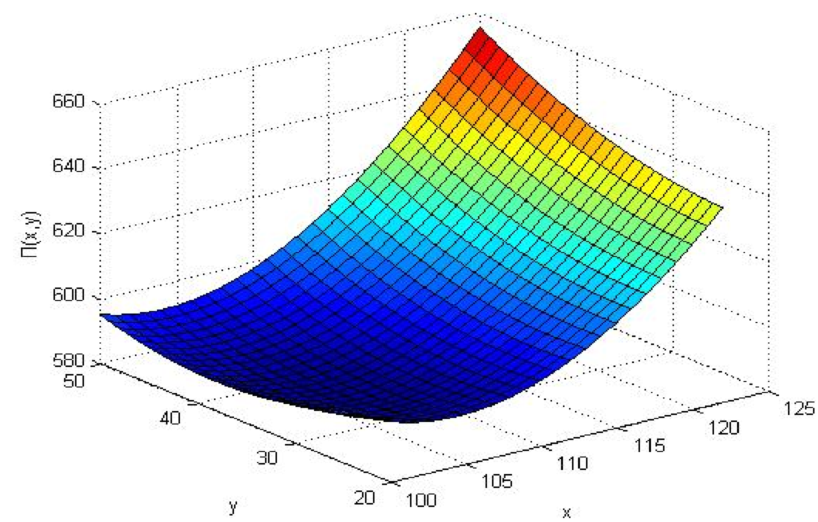


Figure 3: Graphical representation of the objective function $\Pi(x, y)$ in Case I

4.2 The Effects of Various Cost Parameters

We now observe the effects of the model parameters on the optimal solutions and the optimal values. In our numerical experiments, we only change the value of one parameter, and the values of other parameters are fixed. From Table 1, we observe that as the purchase price p_1 increases, the optimal order quantity x^* from A decreases, while the optimal reserved quantity y^* from B increases. From Table 2, we observe the opposite results occur when the price p_2 increases.

Table 1: Effects of the parameter p_1 on the optimal decisions

p_1	3	3.5	4	4.5	5
x^*	111	110	109	107	106
y^*	31	31	32	33	34
$\Pi(x^*, y^*)$	414.1804	457.0947	499	541.3709	582.7110
p_1	5.5	6	6.5	7	7.5
x^*	105	104	103	102	100
y^*	35	35	36	36	38
$\Pi(x^*, y^*)$	623.6022	664.0554	704.0466	743.6389	782.7500

Table 2: Effects of the parameter p_2 on the optimal decisions

p_2	4.5	5	5.5	6	6.5
x^*	100	102	103	104	105
y^*	42	40	39	37	36
$\Pi(x^*, y^*)$	534.7500	545.2696	555.3082	564.9096	574.0258
p_2	7	7.5	8	8.5	9
x^*	106	107	108	109	110
y^*	34	32	29	26	23
$\Pi(x^*, y^*)$	582.7110	590.9287	598.6716	605.8392	612.3182

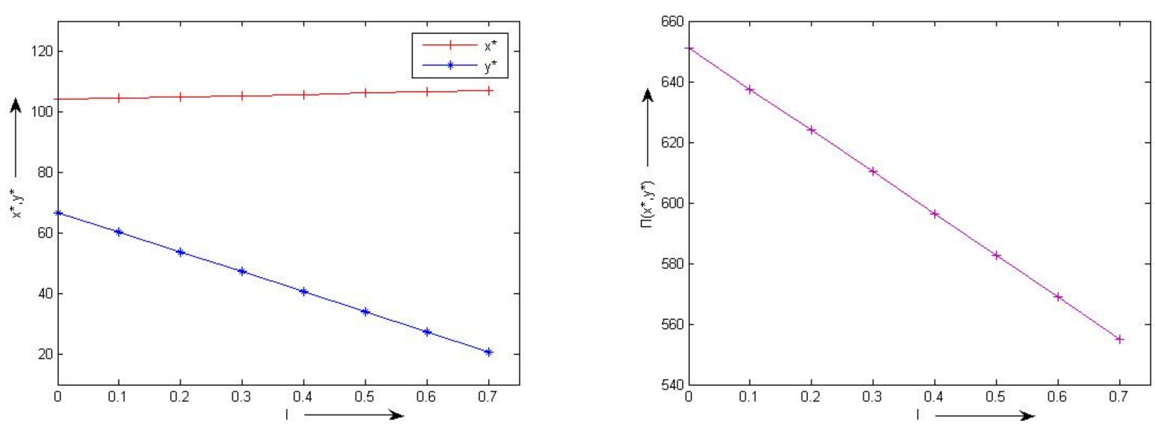
Table 3 indicates the effects of the overage cost c_o and the underage cost c_u. In order not to add too much extra cost, the optimal order quantity x^* is decreasing and the optimal reserved quantity y^* is increasing with respect to c_o or c_u. The underage cost c_u has negligible influence on the optimal order quantity x^*. In two cases, the total cost is increasing with respect to c_o and c_u.

Table 3: Effects of the parameters c_o and c_u on the optimal decisions

c_o	4	6	8	10	12	14	16
x^*	109	107	106	105	105	104	104
y^*	33	33	34	34	34	35	35
$\Pi(x^*, y^*)$	580.2346	581.6886	582.7110	583.5417	584.1369	584.6093	584.9939
c_u	9	11	13	15	17	19	21
x^*	106	106	106	106	106	106	106
y^*	16	31	36	39	40	41	42
$\Pi(x^*, y^*)$	573.1780	581.1182	583.7755	585.0931	585.9201	586.4642	586.8195

4.3 The Effects of Fuzzy Distribution Parameters

Figure 4 (a) shows the tendency of the optimal solutions when l varies its values. When l increases its values, the optimal order quantity x^* increases and the optimal reserved quantity y^* decreases. Increasing the values of l means to increase the minimum ratio of quantity A provided. That is, due to A's reliability increasing, the order quantity from A appropriately increases and the reserved quantity from B faster reduces. Figure 4 (b) illustrates that as l increases, the total cost decreases.

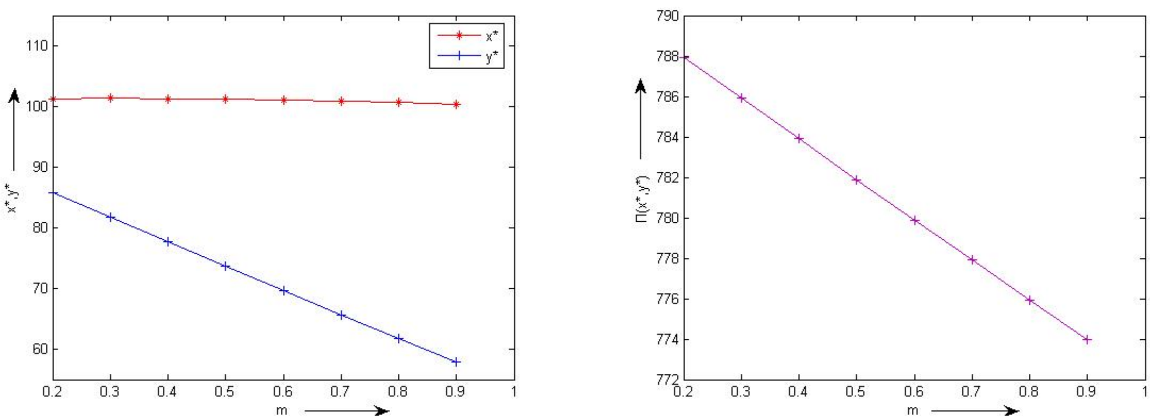


(a) (b)

Figure 4: Impacts of parameter l on the optimal decisions

In order to observe the impacts of the parameter m, we set the value of l as 0.1. Figure 5 (a) shows the tendency of the optimal solutions when m varies its values. When m varies its values from 0.2 to 0.3, the optimal order quantity x^* increases. However, when m varies its values from 0.3 to 0.9, the optimal order quantity x^* decreases slowly. The optimal reserved quantity y^* always decreases. Increasing the values of m means that the maximum possible amount of products A provided is closer to the number of products retailers ordered. In order to meet the demand and minimize cost, when the order quantity x^* increases to a value, x^* would decrease slowly. Figure 5 (b) shows that the total cost always decreases.

In Cases II and III, we can get the similar results as shown in Figure 6. In Case II, the optimal solutions are $x^* = 105$, $y^* = 16$ with the optimal value $\Pi(105, 16) = 597.0893$. In Case III, the optimal solutions are $x^* = 125$, $y^* = 22$ with the optimal value $\Pi(125, 22) = 630.0083$.



(a) (b)

Figure 5: Impacts of parameter m on the optimal decisions

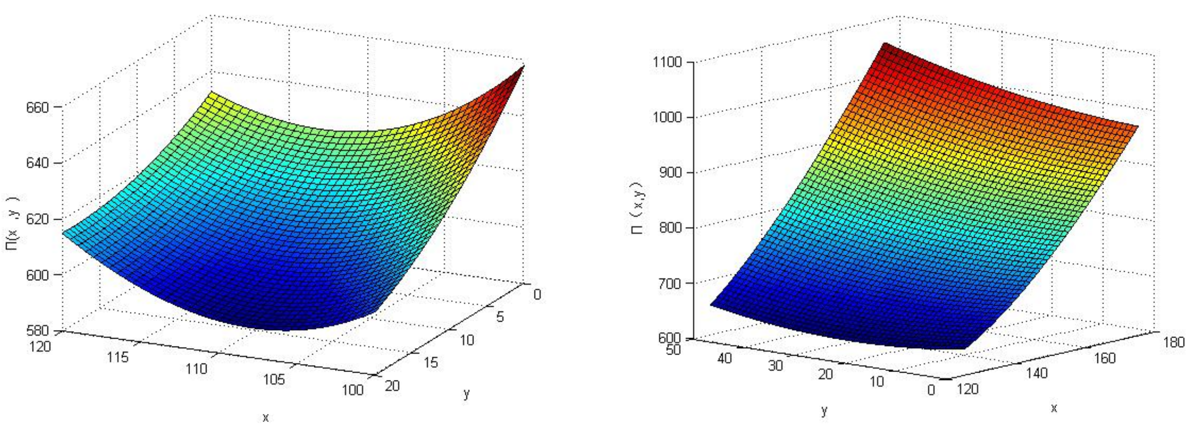


Figure 6: Graphical representations of the objective function $\Pi(x, y)$ in Cases II and III

Comparing the computational results in Cases I, II and III, we find that in Case I the retailer's cost is the least. As a consequence, the haze mask retailer could adopt the decision that he orders 106 from A and reserves 34 from B at the beginning of the reason. The global minimum cost is 582.7110.

5 Conclusions

In this paper, a single-product single-period inventory model was studied. The retailer faced supply uncertainty from the primary supplier, but the secondary supplier was reliable. The main new results of this paper are summarized as follows.

Firstly, the uncertain proportion of supply was characterized by a credibility distribution function. At the same time, the providing quantity relied on the order quantity. Based on risk-neural criterion, we developed the equivalent value model to minimize the total costs incurred in our inventory management problem.

Secondly, under the triangular and trapezoidal supply modes, we analyzed the properties of our inventory management model, and derived its equivalent convex programming submodels. As a result, a domain decomposition method was designed to solve the proposed inventory management model.

Thirdly, some numerical experiments were conducted to illustrate the effectiveness of the designed decomposition method, and the sensitivity of the cost parameters and fuzzy distribution parameters on solution results was also analysed.

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