Journal of Uncertain Systems
Vol.10, No.3, pp.181-197, 2016
Online at: www.jus.org.uk

Time Truncated Sampling Plan under Hybrid Exponential Distribution

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Received 29 July 2015; Revised 10 May 2016

Abstract

The design of acceptance sampling plans for hybrid exponential distribution under a truncated life test is considered in this paper. In this work, experimental values are treated as observed values of exponential random variables whose mean is assumed to be a fuzzy variable in the sense of Liu [25]. A new chance distribution called hybrid exponential distribution is considered in this paper and its properties are investigated. Under the chance distribution, the question of developing time truncated sampling plan is considered. For various acceptance numbers, consumer's confidence levels and values of the ratio of the fixed experimental time to the specified median life, the minimum sample sizes required to ensure the specified median life are obtained. The operating characteristic function values of the given sampling plans and associated producer's risk are presented in the fuzzy environment. The results are illustrated with examples.

Keywords: acceptance sampling plan, hybrid exponential distribution, operating characteristic function value, consumer's risk and producer's risk

1 Introduction

Designing of suitable Acceptance Sampling Plans for various situations is an important exercise in the study of Statistical Quality Control systems. Acceptance sampling plans help us to examine whether the manufactured products meet the pre-specified quality levels. They are primarily used in statistical quality control when it is not possible to perform complete inspection of the manufactured products for various reasons like, the manufactured products being destructive in nature or complete inspection may be a time consuming process. Basically, acceptance sampling plans help us to assess the quality level of the product based on sampled items. Acceptance sampling plans can be broadly classified as, "Sampling plans for Attributes" and "Sampling plans for Variables". If the quality level of the product is measured in terms of attributes like defectives or non-defectives, then the sampling plans for attributes are used. On the other hand, if the manufactured products are inspected by means of measurements like length, height, life time, etc., then sampling plans for variables used. Characteristics of an acceptance sampling plan are studied mainly with the help of probability distributions which involve certain parametric values. For example, in sampling plans for attributes, distributions like Binomial, Poisson, Hyper-Geometric, etc. play important roles. In the case of acceptance sampling plans for variables, distributions like Normal, Exponential, Gamma, Log-normal, etc. find wide applications. Several researchers have contributed to the development of sampling plans for variables under situations involving randomness. Some of them are Zimmer and Burr [40], Owen [27], Guenther [14], Aminzadeh [2], Soundararajan and Christina [37], Eric et al. [35], Geetha and Vijyaraghavan [9], etc.

It is to be noted that various types of acceptance sampling plans for variables are available in the literature like, chain sampling, continuous sampling, skip-lot sampling, time truncated sampling, tightened normal tightened sampling, reliability sampling, etc. A class of acceptance sampling plans known as time truncated sampling plan that has received the attention of many researchers is to be considered in this paper. Time truncated sampling plans have been considered by many authors under various probability distributions. Such time truncated sampling plans were developed by Epstein [8] in exponential case, Sobel and Tsichendrof [36] for Exponential distribution, Goode and

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Kao [10] for Weibull distribution, Gupta and Groll [17] for Gamma distribution, Kantam and Rosaiah [21] for Half logistic distribution, Kantam et al. [22] for Log-logistic, Rosaiah and Kantam [29] for Rayleigh, Baklizi [5] for Pareto distribution of second kind, Baklizi and El Masri [6] for Birnbaum Saunders model, and Balakrishnan et al. [7] for generalized Birnbaum-Saunders distributions. Recently, Aslam and Shahbaz [4], Tsai and Wu [39], Al-Nasser and Al-Omari [1], Singh et al. [34], Gui and Zhang [15] developed acceptance sampling plans for truncated life test for generalized exponential distribution, inverse Rayleigh distribution, generalized Rayleigh distribution, Exponentiated Freche't distribution, Compound Rayleigh distribution and Gompertz distribution.

In conventional acceptance sampling plans, a random sample is selected from the lot and the consumer decides to accept or reject the lot based on the information obtained from the sample. In life test sampling plans or time truncated acceptance sampling plans, units are subjected to life test and the number of failures up to a pre-specified time point is observed. If the number of failures reaches the acceptance number within the specified time then the inspection is stopped and the lot is rejected. On the other hand, if the number of failures is less than or equal to the acceptance number then the lot is accepted.

The quality characteristic considered under the life time experiment is a continuous variable in nature; we assume a probability distribution for the life time distribution with either mean/median as a parameter. Since the quality characteristic is a variable, there exists either a lower confidence limit or an upper confidence limit or both which establish the acceptable values of this parameter. The primary objective of time truncated acceptance sampling plan is to fix the lower confidence limit on the mean/median life of the product and make sure that the actual mean/median life of the product satisifies the consumer's confidence level with a minumum probability Invariably, the parameters involved in these probability distributions are assumed to be either known or estimated through some statistical techniques. In real life situations, finding good estimated values for parameters remains as a challenging problem. The introduction of the Fuzzy theory paved a way for an alternative solution to this problem. Various approaches for designing of sampling plans for attributes using fuzzy set theory have been considered by several researchers including Ohta and Ichihashi [26], Kanagawa and Ohta [20], Arnold [3], Grzegorzwski [11, 12, 13], Hryniewicz [18], Jamkhaneh et al. [19], Tong and Wang [38] etc. It is to be mentioned that the majority of these works are related to sampling plans for attributes and they assume the presence of fuzziness in the parameters related to the underlying distributions. While some of these works considered fuzziness in producer's risk and consumer's risk, others considered fuzziness in the submitted lot quality level.

To study environments involving imprecise situations, Liu and Liu [24] and Liu [25] introduced a theory called credibility theory parallel to probability theory. Sampath [30, 31] and Sampath and Deepa [32] have applied chance theory developed by Liu [25] which is an integration of impreciseness and randomness in the theory of acceptance sampling for designing fuzzy acceptance sampling plans for attributes. Recently, Sampath, et al. [33] have considered the application of hybrid normal distribution (the normal distribution where the parameters involved are treated as fuzzy variables) in developing a single sampling plan for variables for situations involving both randomness and impreciseness.

A thorough review of the literature on life test sampling plans indicates that the exponential distribution plays a vital role in designing life test sampling plans under random environment. In this paper, the question of developing truncated life test sampling plan for variables using hybrid exponential distribution (exponential distribution where the parameter is treated as a fuzzy variable) is considered. Developing an acceptance sampling plan for Hybrid exponential distribution to ensure the median lifetime of the products under inspection exceeds a pre-determined quality provided by the consumer with a minimum probability is the main aim of this paper.

The rest of the paper is organized as follows. In Section 2, in order to maintain the readability of the paper we give a brief introduction to Chance theory. In Section 3, hybrid exponential distribution is developed under chance environment and its properties are discussed. The design of time truncated acceptance sampling plan for hybrid exponential distribution is considered in Section 4. Some important characteristics of the plan under chance environment are studied. In Section 5, numerical examples are given for illustrating the use of theoretical developments made in this paper. Concluding remarks are given in the final section of the paper.

2 Hybridization of Credibility and Probability Theories

The introduction of chance theory requires an understanding of the credibility theory that provides the foundation for the introduction of fuzzy variables and Probability theory.

2.1 Credibility Theory

Let Θ be a nonempty set and P be the power set of Θ. Each element of P is called an *event*. For every event A, we associate a number denoted by $Cr\{A\}$, which indicates the credibility that A will occur and that satisfying the following four axioms:

Axiom 1 (Normality) $Cr(\Theta)=1$;

Axiom 2 (Monotonicity) $Cr(A)\le Cr(B)$ whenever $A\subset B$;

Axiom 3 (Self duality) $Cr(A)+Cr(A^c)=1$ for any event A;

Axiom 4 (Maximality) $Cr(\bigcup_i A_i)=\underset{i}{Sup}\,Cr(A_i)$ for any events $\{A_i\}$ with $\underset{i}{Sup}\,Cr(A_i)<0.5$.

Credibility measure: The set function Cr is called a credibility measure if it satisfies the normality, monotonicity, self-duality and maximality axioms.

Credibility space: Let Θ be a nonempty set, P be the power set of Θ and Cr a credibility measure. Then the triplet (Θ,P,Cr) is called a credibility space.

Fuzzy variable: A fuzzy variable is a measurable function from a credibility space (Θ,P,Cr) to the set of real numbers.

Membership function: Let ξ be a fuzzy variable on the credibility space (Θ,P,Cr). Then its membership function is derived from the credibility measure by

$$\mu(x)=\left\{2Cr\left\{\xi=x\right\}\right\}\wedge 1,\, x\in\Re.$$

Credibility distribution: The credibility distribution $\Phi:\Re\rightarrow[0,1]$ of a fuzzy variable ξ is defined by

$$\Phi(x)=Cr\left\{\theta\in\Theta \mid \xi(\theta)\le x\right\}.$$

2.2 Probability Theory

Let Ω be a nonempty set and A be the power set of Ω. Each element of A is called an *event*. For every event A, we associate a number denoted by $\Pr\{A\}$, which indicates the probability that A will occur. The axioms of probability theory are as follows.

Axiom 1 (Normality) $\Pr(\Omega)=1$;

Axiom 2 (Nonnegativity) $\Pr(A)\ge 0$ for any event A;

Axiom 3 (Countable additivity) $\Pr(\bigcup_i A_i)=\sum_i \Pr(A_i)$ for every countable sequence of disjoint events $\{A_i\}$.

Probability measure: The set function $\Pr$ is called a probability measure if it satisfies the normality, non-negativity, and countable additive axioms.

Probability space: Let Ω be a nonempty set, A be the power set of Ω and $\Pr$ a probability measure. Then the triplet $(\Omega,\mathrm{A},\Pr)$ is called a probability space.

Random variable: A random variable is a measurable function from a probability space $(\Omega,\mathrm{A},\Pr)$ to the set of real numbers.

Probability distribution: The probability distribution $\Phi:\Re\rightarrow[0,1]$ of a random variable ξ is defined by

$$\Phi(x)=\Pr\left\{\omega\in\Omega \mid \xi(\omega)\le x\right\}.$$

2.3 Chance Theory

Using the above definitions related to credibility and probability spaces, Li and Liu [23] developed ideas relevant for handling situations where both impreciseness and randomness play simultaneous roles in the given system. The hybrid development based on credibility and probability space has been named as Chance theory. The following definitions are due to Li and Liu [23].

Chance space: Suppose that (Θ,P,Cr) is a credibility space and $(\Omega,\mathrm{A},\Pr)$ is a probability space. The product $(\Theta,\mathrm{P},Cr)\times(\Omega,\mathrm{A},\Pr)$ is called a chance space.

Let $(\Theta, \mathrm{P}, Cr) \times (\Omega, \mathrm{A}, \Pr)$ be a chance space. A subset $\Lambda \subset \Theta \times \Omega$ is called an event if $\Lambda(\theta) \in \mathrm{A}$ for each $\theta \in \Theta$.

Chance measure: Let $(\Theta, \mathrm{P}, Cr) \times (\Omega, \mathrm{A}, \Pr)$ be a chance space. Then a chance measure of an event Λ is defined as

$$Ch(\Lambda) = \begin{cases} \sup\limits_{\theta\in\Theta}\left\{Cr\{\theta\} \wedge \Pr\{\Lambda(\theta)\}\right\}, & if \sup\limits_{\theta\in\Theta}\left\{Cr\{\theta\} \wedge \Pr\{\Lambda(\theta)\}\right\} < 0.5 \\ 1-\sup\limits_{\theta\in\Theta}\left\{Cr\{\theta\} \wedge \Pr\{\Lambda^c(\theta)\}\right\}, & if \sup\limits_{\theta\in\Theta}\left\{Cr\{\theta\} \wedge \Pr\{\Lambda(\theta)\}\right\} \geq 0.5. \end{cases} \tag{1}$$

To describe a quantity with both fuzziness and randomness, the concept of hybrid variable is used. It is formally defined as follows.

Hybrid variable: A hybrid variable is a measurable function from a chance space $(\Theta, \mathrm{P}, Cr) \times (\Omega, \mathrm{A}, \Pr)$ to the set of real numbers. That is, for any Borel set B of real numbers, $\{\xi \in B\} = \{(\theta,\omega) \in \Theta\times\Omega \mid \xi(\theta,\omega) \in B\}$ is an event.

Li and Liu [23] have identified five different approaches to defining Hybrid variable. The Model IV of Li and Liu [23] will be used in our further discussion. This model is suitable for dealing with situations where the parameters involved in a given probability distribution are fuzzy by nature. The model proposed by Liu is explained below.

Let ξ be a random variable with probability density function $\varphi(x;\theta_1,\theta_2,...,\theta_n)$ where $(\theta_1,\theta_{2,},...,\theta_n)$ is a set of fuzzy parameter variables. If $\theta_1,\theta_{2,},...,\theta_n$ have membership function $\mu_1,\mu_{2,},...,\mu_n$ respectively, then for any Borel set B of real numbers, the chance $Ch(\xi \in B)$ due to Qin and Liu [28] is given by

$$Ch(\xi \in B) = \begin{cases} \sup\limits_{\theta_1,\theta_2,...,\theta_n}\left\{\min\limits_{1\leq i\leq m}\left(\dfrac{\mu_i(\theta_i)}{2}\right) \wedge \int\limits_B \varphi(x,\theta_1,\theta_2,...,\theta_n)dx\right\}, \\ \qquad if \sup\limits_{\theta_1,\theta_2,...,\theta_n}\left\{\min\limits_{1\leq i\leq m}\left(\dfrac{\mu_i(\theta_i)}{2}\right) \wedge \int\limits_B \varphi(x,\theta_1,\theta_2,...,\theta_n)dx\right\} < 0.5 \\ 1-\sup\limits_{\theta_1,\theta_2,...,\theta_n}\left\{\min\limits_{1\leq i\leq m}\left(\dfrac{\mu_i(\theta_i)}{2}\right) \wedge \int\limits_{B^c} \varphi(x,\theta_1,\theta_2,...,\theta_n)dx\right\}, \\ \qquad if \sup\limits_{\theta_1,\theta_2,...,\theta_n}\left\{\min\limits_{1\leq i\leq m}\left(\dfrac{\mu_i(\theta_i)}{2}\right) \wedge \int\limits_B \varphi(x,\theta_1,\theta_2,...,\theta_n)dx\right\} \geq 0.5. \end{cases} \tag{2}$$

Chance distribution: The chance distribution $\Phi:\Re \to [0,1]$ of a hybrid variable ξ is defined by

$$\Phi(x) = Ch\{(\theta,\omega) \in \Theta\times\Omega \mid \xi(\theta,\omega) \leq x\}.$$

Chance density function: The chance density function $\varphi:\Re \to [0,\infty)$ of a hybrid variable ξ is a function such that $\Phi(x) = \int_{-\infty}^{x}\varphi(y)dy,\ \forall \mathrm{x} \in \Re$ and $\int_{-\infty}^{\infty}\varphi(y)dy = 1$ where Φ is the chance distribution of ξ. The definitions presented are relevant for further discussion made in this paper. For more detailed and exhaustive discussion, one can refer to Li and Liu [23].

Expected value: The definition of the expected value operator of a fuzzy variable was given by Liu and Liu [24]. This definition is applicable both for continuous fuzzy variables and also discrete ones.

Let ξ be a fuzzy variable. Then the expected value of ξ is defined by

$$E(\xi) = \int\limits_0^\infty ch(\xi \geq r)dr - \int\limits_{-\infty}^0 ch(\xi \leq r)dr \tag{3}$$

provided that at least one of the two integrals is finite.

3 Hybrid Exponential Distribution

The probability density function of the random variable X having an exponential distribution with mean θ is given by $\varphi(x,\theta) = e^{-x/\theta}/\theta, x>0, \theta>0$. Here we assume θ is a fuzzy variable. Clearly the above distribution is a hybrid distribution (randomness created through the random variable X and fuzziness entering in the form impreciseness created by the parameter θ). We shall denote by ξ the hybrid variable. If μ is a membership function associated with θ then it had been shown by Qin and Liu [28], for any Borel set B of real numbers, the chance $Ch(\xi \in B)$ is given by

$$Ch(\xi \in B) = \begin{cases} \sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge \int_B \varphi(x,\theta)dx\right\}, & if \sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge \int_B \varphi(x,\theta)dx\right\} < 0.5 \\ 1-\sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge \int_{B^c} \varphi(x,\theta)dx\right\}, & if \sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge \int_B \varphi(x,\theta)dx\right\} \geq 0.5. \end{cases} \tag{4}$$

$\varphi(x,\theta)$ is an exponential probability density function. Therefore,

$$Ch(\xi \in B) = \begin{cases} \sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge \int_B \frac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\}, & if \sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge \int_B \frac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\} < 0.5 \\ 1-\sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge 1-\int_{B^c} \frac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\}, & if \sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge \int_B \frac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\} \geq 0.5. \end{cases} \tag{5}$$

In this paper we shall assume μ is a triangular membership function over (a,b,c). That is,

$$\mu(\theta) = \begin{cases} \frac{\theta-a}{b-a}, & if \ \mathrm{a} \leq \theta \leq b \\ \frac{\theta-b}{b-c}, & if \ \ b \leq \theta \leq c \\ 0, & otherwise. \end{cases} \tag{6}$$

For hybrid exponential distribution, the distribution function is given below

$$\Phi(t,\theta) = \begin{cases} \sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge \int_0^t \frac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\}, & if \sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge \int_0^t \frac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\} < 0.5 \\ 1-\sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge 1-\int_0^t \frac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\}, & if \sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge \int_0^t \frac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\} \geq 0.5. \end{cases} \tag{7}$$

By taking into account of this, we get the distribution function as

$$\Phi(t,\theta) = \begin{cases} \sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge 1-e^{-\frac{t}{\theta}}\right\}, & if \sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge 1-e^{-\frac{t}{\theta}}\right\} < 0.5 \\ 1-\sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge e^{-\frac{t}{\theta}}\right\}, & if \sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right) \wedge 1-e^{-\frac{t}{\theta}}\right\} \geq 0.5. \end{cases} \tag{8}$$

The following theorem gives expressions for the chance distribution considered above.

Theorem 1: The distribution function of a hybrid variable ξ which follows the hybrid exponential distribution is given below

$$Ch(\xi \leq t) = \Phi(t,\theta) = \begin{cases} 0, & if \ \ t \leq 0 \\ 1-e^{-\frac{t}{\theta_1^*}}, & if \ \ 0 < t < b*\ln(2) \\ \frac{1}{2}, & if \ \ t = b*\ln(2) \\ 1-e^{-\frac{t}{\theta_2^*}}, & if \ \ t > b*\ln(2) \end{cases}$$

where θ_1^* and θ_2^* are the solutions of $\mu(\theta)/2 = 1-e^{-t/\theta}$, $\mu(\theta)/2 = e^{-t/\theta}$, respectively.

Proof: The distribution function of hybrid exponential stated in equation (8) is

$$\Phi(t,\theta)=\begin{cases}\sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge 1-e^{-\frac{t}{\theta}}\right\}, & if\ \sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge 1-e^{-\frac{t}{\theta}}\right\} < 0.5\\ 1-\sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge e^{-\frac{t}{\theta}}\right\}, & if\ \sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge 1-e^{-\frac{t}{\theta}}\right\} \geq 0.5.\end{cases}$$

In order to find, $\Phi(t,\theta)$ over different values of t, we need to examine the behavior of $\mu(\theta)/2$ and $1-e^{-t/\theta}$ over the permissible values of t and θ. Note that, $t\in R$ and $\theta\in[a,c]$ the maximum value attained by $\mu(\theta)/2$ (which is independent of t) is $1/2$, and $1-e^{-t/\theta}$ is decreasing in θ for a given t. The curve $1-e^{-t/\theta}$ will either intersect with $\mu(\theta)/2$ depending on the choice of t. It may be noted that the curve $1-e^{-t/\theta}$ is non-decreasing in t for a given θ as shown in Figure 1 for $t_1<t_2<t_3<t_4<t_5<t_6$.

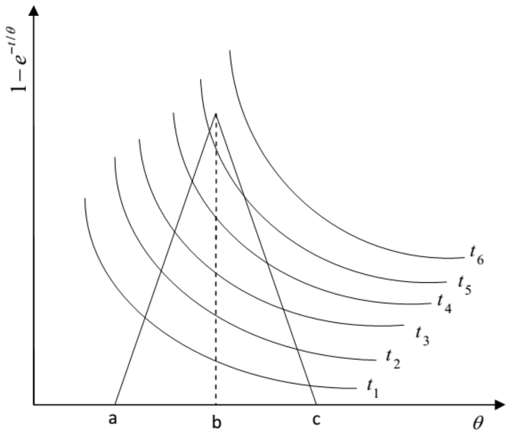

Figure 1: Impact of the value of t on the intersection of $\mu(\theta)/2$ and $1-e^{-t/\theta}$

Therefore, the curve may lie entirely above $\mu(\theta)/2$ or will intersect at two different points. The third possibility is that the curve may touch the triangle at only one point. This case will arise when $1-e^{-t/\theta}=\mu(\theta)/2$. Note that $\mu(\theta)/2=1/2$ if $\theta=b$. In this case, we have $1-e^{-t/b}=1/2$, solving for t, we get $t=b*\ln(2)$. Hence, we conclude that $1-e^{-t/\theta}$ will be greater than $1/2$ for all θ as long as $t>b*\ln(2)$ and it is less than 0.5 if $t<b*\ln(2)$. Therefore, we conclude that

$$\left(\frac{\mu(\theta)}{2}\right)\wedge 1-e^{\frac{-t}{\theta}}=\frac{\mu(\theta)}{2}\text{, for all } t>b*\ln(2).$$

This implies that

$$\underset{\theta}{Sup}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge 1-e^{\frac{-t}{\theta}}\right\}=\frac{1}{2}\text{, for all } t>b*\ln(2).$$

When $t<b*\ln(2)$, $\mu(\theta)/2$ and $1-e^{-t/\theta}$ intersect at two different points, say θ_1^* and θ_2^*. Since $\mu(\theta)/2$ is a triangle and $1-e^{-t/\theta}$ is a monotone curve (in terms of θ) we conclude that intersections will take place on different sides of the triangle as shown in Figure 2.

Evidently,

$$\sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge 1-e^{-\frac{t}{\theta}}\right\}=1-e^{-\frac{t}{\theta^*_1}} \tag{9}$$

where θ_1^* is the solution of $\mu(\theta)/2=1-e^{-t/\theta}$. Therefore, we have $\Phi(t,\theta)=1-e^{-t/\theta_1^*}$ for all $t<b*\ln(2)$.

For all $t>b*\ln(2)$,

$$\Phi(t,\theta)=\sup_{\theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge\int_t^{\theta}\frac{1}{\theta}e^{-\frac{t}{\theta}}dt\right\}=\sup_{\theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge e^{\frac{-t}{\theta}}\right\}.$$

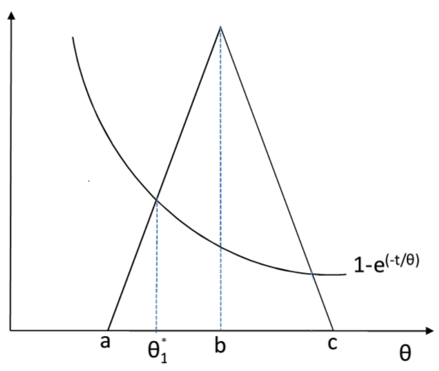

Figure 2: Intersection of $\mu(\theta)/2$ and $1-e^{-t/\theta}$

Since $e^{-t/\theta}$ is strictly increasing in θ for a given t, by following the lines of earlier arguments, we understand the scenario prevailing in this case, will be as shown in Figure 3.

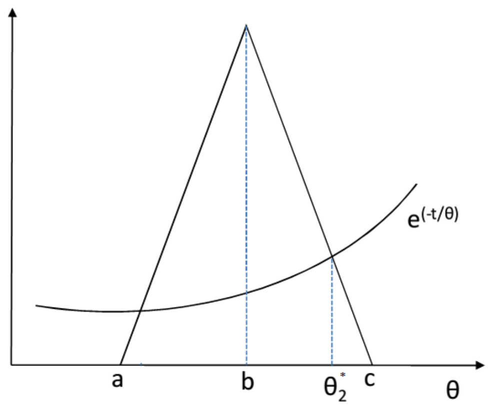

Figure 3: Intersection of $\mu(\theta)/2$ and $e^{-t/\theta}$

Evidently,

$$\sup_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge e^{-\frac{t}{\theta}}\right\} = e^{-\frac{t}{\theta^*_2}} \tag{10}$$

where θ_2^* is the solution of $\mu(\theta)/2 = e^{-t/\theta}$. Therefore, we have $\Phi(t,\theta) = 1-e^{-t/\theta_2^*}$ for all $t > b*\ln(2)$.

Thus we have proved Theorem 1.

Expected Value of Hybrid Exponential Distribution

Since the chance variable corresponding to hybrid exponential distribution assumes only non-negative values, the expected value of the exponential hybrid variable ξ is calculated using the formula,

$$E(\xi) = \int_0^\infty 1-\Phi(t,\theta)dt \tag{11}$$

The value of the integral given in (11) cannot be theoretically computed. Hence it is decided to investigate the value of the above integral on making use of trapezoidal rule for numerical integration. The interval of integration is partitioned into 100 sub intervals where the lower limit of the interval considered for integration is taken as zero and the upper limit is determined by a very large value whose value is closer to zero. It may be noted that the integrand is a decreasing function.

Table 1 furnished below gives the expected value of hybrid exponential distribution for different choices of a, b, c were a and c are determined by fixing the value of b and defining $a = b-\varepsilon$ and $c = b+\varepsilon$. Four different choices were used for ε, say, 0.05, 0.1, 0.15 and 0.2.

The quality of product whose life time has skewed distribution can be more meaningfully assessed using the median of the distribution rather than its mean [16]. Hence, it is worthwhile to determine the value of median in hybrid exponential distribution. The median of the hybrid exponential distribution denoted by θ_m^h is obtained by solving

$$\Phi(t,\theta) = \frac{1}{2}.$$

Table 1: Expected values of hybrid exponential distribution

b	ε=0.05	0.1	0.15	0.2
1	0.956571	0.981155	1.005553	1.029842
2	1.913142	1.962311	2.011106	2.059684
3	2.869713	2.943466	3.01666	3.089526
4	3.826284	3.924622	4.022213	4.119368
5	4.782855	4.905777	5.027766	5.14921
6	5.739426	5.886933	6.033319	6.179052
7	6.695997	6.868088	7.038873	7.208894
8	7.652568	7.849243	8.044426	8.238736
9	8.609139	8.830399	9.049979	9.268578
10	9.56571	9.811554	10.05553	10.29842
11	10.53	10.79954	11.06786	11.33434
12	11.48658	11.78109	12.07322	12.36429
13	12.44309	12.76262	13.07862	13.39359
14	13.39951	13.74363	14.08477	14.42354
15	14.35636	14.72482	15.09083	15.45336
16	15.31292	15.70552	16.09637	16.48254
17	16.26952	16.68632	17.1017	17.51249
18	17.22617	17.66736	18.10748	18.54293
19	18.18277	18.64833	19.11321	19.57293
20	19.13937	19.62926	20.11835	20.60276

Table 2: Median and mean of hybrid exponential distribution

B	Median	Mean	Ratio
1	0.6928	1.037387	0.667834
2	1.38516	2.075775	0.667297
3	2.07935	3.101884	0.670352
4	2.77423	4.140722	0.669986
5	3.45907	5.179359	0.667857
6	4.1583	6.204763	0.670179
7	4.85943	7.243632	0.670856
8	5.54113	8.26846	0.670152
9	6.23725	9.307648	0.670121
10	6.92084	10.34655	0.668903

From Theorem 1, it can be seen that, the median of the hybrid exponential distribution is $\theta_m = b*\ln(2)$. It may be recalled that in the case of crisp exponential distribution, the median is a constant multiplied by the mean of the distribution. That is, we have $\theta_m = \theta*\ln(2)$. Now, we shall examine whether there exists any such relationship between the expected value and median in the case of hybrid exponential distribution. Towards this, for different values of b, the ratio of the median value to the expectation of hybrid distribution were computed and it was found that in the case of hybrid exponential distribution $\theta_m^h = (0.67)E[\xi]$. Table 2 gives the values of median, expectation and ratio of median to the expectation for hybrid distribution for different values of b.

4 Design of Time Truncated Acceptance Sampling Plan for Hybrid Exponential Distribution

In time truncated acceptance sampling plans, n items from the lot are inspected over a given period of time, say, t. The lot is accepted if the number of observed failures till time point t does not exceed a pre-specified acceptance number c, and the test is terminated with rejection of the lot if the number of failures observed before the time period t exceeds the acceptance number c. The inspection time t is a pre-specified quantity. The sampling plan should use a carefully chosen value for t. It is usually taken as a multiple of a targeted median life time of the product, say, θ_m^0. That is, we take $t = a\theta_m^0$, where a is also a pre-determined quantity which indicates the number of cycles needed to guarantee specified median life time of the product. This is based on the reasoning that inspection over various cycles where the number of cycles is made dependent on the given median life time will ensure a minimum quality level expressed interms of a desired median life time θ_m^0.

It may be noted that arriving at a decision based on a time truncated acceptance sampling plan is equivalent to taking a decision while testing the null hypothesis $H_0 : \theta_m \geq \theta_m^0$ against the alternative hypothesis $H_1 : \theta_m < \theta_m^0$ at level of significance $1 - P^*$, which is nothing but the consumer's risk. Here, θ_m denotes the actual median life time which is in general unknown. The constant P^* referred to as consumer's confidence level is the lower bound for rejecting a bad lot. Here, we have

$$P\left[\text{rejecting a lot} / \theta_m \geq \theta_m^0\right] \leq 1 - P^* \qquad \text{(level of significance- consumer's risk)} \tag{12}$$

and

$$P\left[\text{rejecting a lot} / \theta_m < \theta_m^0\right] \geq P^* \quad \text{(consumer's confidence level)} \tag{13}$$

The parameters of the time truncated acceptance sampling plan are the number of items n to be drawn from the lot, an acceptance number c, the time ratio t/θ_m^0, where θ_m^0 the specified median life time and t is the pre-assigned testing time. Symbollically, the sampling plan is denoted by the triplet $(n, c, t/\theta_m^0)$. Any set of values for the parameters of a time truncated acceptance sampling plan is expected to satisfy the conditions stated in (12) and (13). Here, we restrict ourselves to those sampling plans satisfying inequality related to consumer's risk. It may be noted that several set of plan parameter values satisfying this requirement. Hence, we look for a sampling plan by fixing the inspection time t, median life time θ_m^0 and acceptance number c for a given P^*. When these values are fixed, one can find several n for which the consumer's risk inequality is satisfied. Hence, we look for a small positive integer n such that

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \leq 1 - P^* \tag{14}$$

where p is the probability that an item fails before the time t. In our study, where we integrate randomness and impreciseness, instead of probability p, we use the chance of an item fails before time t. This chance value is computed using the chance distribution obtained earlier, namely,

$$Ch(\xi \le t) = \Phi(t,\theta) = \begin{cases} 0, & if\ t \le 0 \\ 1 - e^{-\frac{t}{\theta_1^*}}, & if\ 0 < t < b*\ln(2) \\ \frac{1}{2}, & if\ t = b*\ln(2) \\ 1 - e^{-\frac{t}{\theta_2^*}}, & if\ t > b*\ln(2) \end{cases}$$

where θ_1^* and θ_2^* are the solutions of $\mu(\theta)/2 = 1 - e^{-t/\theta}$ and $\mu(\theta)/2 = e^{-t/\theta}$.

To be precise, p used in (14) is computed using the relation $p = \Phi\left(t, \theta_m^0\right)$ where θ_m^0 is the desired median life time and $t = a\theta_m^0$ where a is a pre-specified constant. It may be noted that the cumulative distribution function of crisp exponential distribution and hence the chance distribution function is monotonically decreasing in median. Hence from the inequality (14), we observe, if the number of failures less than or equal to c then the chance of the event $\left[\Phi\left(t,\theta_m\right)) \le \Phi\left(t,\theta_m^o\right)\right]$ will be P^*. This ensures that $\theta_m \ge \theta_m^0$, where θ_m is the true or actual median life time.

The desired quality level expressed in terms of the median life time θ_m^0 can be uniquely determined by the expected value of the hybrid exponential distribution through the relation $\theta_m^0 = \theta_0 \ln\left(2\right)$, where θ_0 is the expected value of the hybrid exponential distribution. Hence, taking $t = a\theta_m^0$ is equivalent to $t = a \times \theta_0 \ln(2)$. It is clear that chance value p depends on the inspection duration t. As mentioned above, in our study, we take $t = a\theta_m^0$ where θ_m^0 is the median of the chance distribution, and the value a is a pre-specified constant. We assign different values for a in this work pursuing the lines of earlier similar investigations done under crisp situation.

Table 3 gives the minimum values of n satisfying the inequality (14), for P^* = 0.75, 0.90, 0.95, 0.99 and t/θ_m^0 =0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712. These choices were motivated by the works of Gupta and Groll [17], Kantam [22], Tsai and Wu [39], Balakrishnan et al. [7], and Aslam and Shabaz [4]. It is well known that, if the sample size n is large and p is very small then binomial is approximated by Poisson distribution with mean $\theta_0 = np$. Table 4 gives the minimum values of n under Poisson approximation for the same set of values used in Table 3.

Operating Characteristic (OC) Functions of the Time Truncated Acceptance Sampling Plan

The operating characteristic (OC) function describes the efficiency of acceptance sampling plans. It calculates the efficiency of a statistical hypothesis test which is designed to accept or reject a lot / product. The OC function for the above time truncated sampling plan $\left(n, c, t/\theta_m^0\right)$ is defined as

$$\text{OC}(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \tag{15}$$

where $p = \Phi\left(t,\theta\right)$. It is treated as a function of the lot quality parameter. OC(p) is a decreasing function of p which decreases when θ_m decreases. For the given time truncated acceptance sampling plan, the OC function values have been computed for different combinations of P^* and θ_m/θ_m^0 and they are listed in Table 5.

Producer's Risk

In the usual frame work, the producer's risk is the probability of rejection of a lot when $\theta_m \ge \theta_m^0$. For a given value of the producer's risk, say, α , in the given sampling plan, one may be interested in knowing the minimum value of the median ratio θ_m/θ_m^0 that will ensure the producer's risk to be at most α . The value of θ_m/θ_m^0 is the smallest positive number for which p satisfies the following inequality

$$\sum_{i=c+1}^{n} \binom{n}{i} p^i (1-p)^{n-i} \le \alpha \tag{16}$$

Table 3: Minimum sample size necessary to assert the median life to exceed a given value, θ_m^0 with probability P^* and corresponding acceptance number, c, using binomial probabilities

P*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	4	3	2	2	1	1	1	1
	1	7	5	4	4	3	2	2	2
	2	11	8	6	5	4	4	3	3
	3	14	10	8	7	6	5	5	4
	4	17	12	10	9	7	6	6	5
	5	20	15	12	10	8	7	7	6
	6	23	17	14	12	10	8	8	8
	7	26	19	16	14	11	10	9	9
	8	29	21	17	15	12	11	10	10
	9	32	24	19	17	14	12	11	11
	10	36	26	21	18	15	13	12	12
0.9	0	6	4	3	3	2	2	1	1
	1	10	7	6	5	4	3	3	2
	2	14	10	8	7	5	4	4	4
	3	17	12	10	8	6	6	5	5
	4	21	15	12	10	8	7	6	6
	5	24	17	14	12	9	8	7	7
	6	28	20	16	14	11	9	9	8
	7	31	22	18	15	12	11	10	9
	8	34	25	20	17	13	12	11	10
	9	38	27	22	19	15	13	12	11
	10	41	30	24	21	16	14	13	12
0.95	0	7	5	4	3	2	2	2	1
	1	12	8	7	5	4	3	3	3
	2	16	11	9	7	6	5	4	4
	3	20	14	11	9	7	6	5	5
	4	23	17	13	11	9	7	7	6
	5	27	19	15	13	10	9	8	7
	6	31	22	17	15	12	10	9	8
	7	34	24	20	17	13	11	10	9
	8	38	27	22	18	14	12	11	11
	9	41	29	24	20	16	14	12	12
	10	44	32	26	22	17	15	14	13
0.99	0	11	8	6	5	3	3	2	2
	1	16	11	9	7	5	4	4	3
	2	21	14	11	9	7	6	5	4
	3	25	17	14	11	9	7	6	6
	4	29	20	16	14	10	8	7	7
	5	33	23	18	15	12	10	9	8
	6	37	26	21	17	13	11	10	9
	7	41	29	23	19	15	12	11	10
	8	44	31	25	21	16	14	12	11
	9	48	34	27	23	18	15	13	13
	10	52	37	29	25	19	16	15	14

Table 4: Minimum sample size necessary to assert the median life to exceed a given value, θ_m^0 with probability P^* and corresponding acceptance number, c, using Poisson probabilities

P*	*c*	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	4	3	3	3	2	2	2	2
	1	8	6	5	5	4	4	3	3
	2	12	9	7	6	5	5	5	5
	3	15	11	9	8	7	6	6	6
	4	18	14	11	10	8	8	7	7
	5	21	16	13	12	10	9	8	8
	6	25	18	15	13	11	10	10	9
	7	28	21	17	15	13	11	11	11
	8	31	23	19	17	14	13	12	12
	9	34	25	21	19	15	14	13	13
	10	37	28	23	20	17	15	14	14
0.9	0	7	5	4	4	3	3	3	3
	1	11	9	7	6	5	5	5	5
	2	15	12	10	9	7	7	6	6
	3	19	14	12	11	9	8	8	7
	4	23	17	14	13	10	10	9	9
	5	27	20	16	15	12	11	10	10
	6	30	22	19	16	14	12	12	11
	7	34	25	21	18	15	14	13	13
	8	37	28	23	20	17	15	14	14
	9	40	30	25	22	18	17	16	15
	10	44	33	27	24	20	18	17	17
0.95	0	9	7	6	5	4	4	4	4
	1	14	10	9	8	6	6	6	5
	2	18	14	11	10	8	8	7	7
	3	22	17	14	12	10	9	9	9
	4	26	20	16	14	12	11	10	10
	5	30	22	19	16	14	12	12	11
	6	34	25	21	18	15	14	13	13
	7	38	28	23	20	17	15	15	14
	8	41	31	25	22	19	17	16	16
	9	45	33	28	24	20	18	17	17
	10	48	36	30	26	22	20	19	18
0.99	0	13	10	8	7	6	6	5	5
	1	19	14	12	11	9	8	8	7
	2	24	18	15	13	11	10	10	9
	3	29	21	18	16	13	12	11	11
	4	33	25	20	18	15	14	13	13
	5	37	28	23	20	17	15	15	14
	6	42	31	26	23	19	17	16	16
	7	46	34	28	25	20	19	18	17
	8	49	37	30	27	22	20	19	19
	9	53	40	33	29	24	22	21	20
	10	57	42	35	31	26	23	22	22

Table 5: Operating characteristic values of given sampling plan for hybrid exponential distribution

P*	t/θ_m^0	c	n	θ/θ_m^0 =2	4	6	8	10	12
0.75	0.628	2	11	0.39657	0.79295	0.91176	0.95515	0.97428	0.98394
	0.942	2	8	0.36833	0.7749	0.90254	0.95004	0.9712	0.98195
	1.257	2	6	0.40825	0.79828	0.91416	0.95639	0.975	0.98439
	1.571	2	5	0.41773	0.8026	0.9161	0.9574	0.97558	0.98476
	2.356	2	4	0.36225	0.765	0.89637	0.9463	0.96883	0.98037
	3.141	2	4	0.19312	0.61879	0.81199	0.8964	0.9375	0.95958
	3.927	2	3	0.36476	0.75545	0.8892	0.94163	0.96575	0.97826
	4.712	2	3	0.25829	0.66847	0.83898	0.91181	0.94695	0.96575
0.9	0.628	2	14	0.22863	0.66723	0.84378	0.91636	0.95048	0.96839
	0.942	2	10	0.20983	0.64798	0.83226	0.90946	0.94611	0.96548
	1.257	2	8	0.1928	0.62903	0.8206	0.90236	0.94159	0.96245
	1.571	2	7	0.16072	0.5903	0.79604	0.88719	0.93182	0.95586
	2.356	2	5	0.17368	0.60208	0.80268	0.89104	0.93419	0.95742
	3.141	2	4	0.19312	0.61879	0.81199	0.8964	0.9375	0.95958
	3.927	2	4	0.09726	0.48059	0.71646	0.83445	0.89636	0.93124
	4.712	2	4	0.04735	0.36225	0.61873	0.765	0.8476	0.89637
0.95	0.628	2	16	0.15282	0.58297	0.79191	0.88481	0.93035	0.95491
	0.942	2	11	0.15474	0.5848	0.79295	0.88542	0.93072	0.95515
	1.257	2	9	0.12729	0.5457	0.76681	0.86883	0.91988	0.94776
	1.571	2	7	0.16072	0.5903	0.79604	0.88719	0.93182	0.95586
	2.356	2	6	0.07691	0.45141	0.69723	0.82242	0.88858	0.92599
	3.141	2	5	0.0642	0.41794	0.66942	0.80272	0.8748	0.91617
	3.927	2	4	0.09726	0.48059	0.71646	0.83445	0.89636	0.93124
	4.712	2	4	0.04735	0.36225	0.61873	0.765	0.8476	0.89637
0.99	0.628	2	21	0.05108	0.39356	0.65215	0.79155	0.86743	0.91113
	0.942	2	15	0.04104	0.36209	0.62481	0.77179	0.85346	0.9011
	1.257	2	11	0.05241	0.39604	0.65386	0.79263	0.86814	0.91161
	1.571	2	9	0.05361	0.39841	0.65557	0.79375	0.86888	0.91212
	2.356	2	7	0.03222	0.326	0.59043	0.74576	0.83452	0.88724
	3.141	2	6	0.01952	0.26529	0.52865	0.69729	0.79846	0.86047
	3.927	2	5	0.02216	0.27464	0.5369	0.70323	0.80265	0.86345
	4.712	2	4	0.04735	0.36225	0.61873	0.765	0.8476	0.89637

equivalently,

$$\sum_{i=0}^{c}\binom{n}{i}p^{i}\left(1-p\right)^{n-i}\geq 1-\alpha. \tag{17}$$

Hence, for a given sampling plan $\left(n,c,t/\theta_m^0\right)$ with specified confidence level P^*, the minimum value of θ_m/θ_m^0 satisfying the inequality (16) are worked out, and they are presented in Table 6.

Table 6: Minimum ratio of true median life to specified median life for the acceptance of a lot with producer's risk of 0.05

P*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	48.98	55.1	49.02	61.26	45.94	61.24	76.56	91.87
	1	11.45	11.85	12.24	15.3	16.21	12.42	15.52	18.62
	2	7.65	8	7.57	7.49	8.25	10.99	8.55	10.26
	3	5.72	5.8	5.87	6.16	7.45	7.49	9.37	7.36
	4	4.76	4.72	5	5.43	5.65	5.8	7.26	5.92
	5	4.18	4.45	4.47	4.35	4.61	4.82	6.02	5.05
	6	3.8	3.96	4.11	4.15	4.72	4.17	5.22	6.26
	7	3.53	3.61	3.85	4	4.12	4.63	4.65	5.57
	8	3.32	3.36	3.38	3.53	3.69	4.17	4.22	5.07
	9	3.16	3.34	3.26	3.48	3.83	3.81	3.89	4.67
	10	3.14	3.16	3.17	3.16	3.51	3.53	3.63	4.36
0.9	0	73.46	73.46	73.52	91.89	91.87	122.48	76.56	91.87
	1	16.77	17.18	19.37	19.76	22.94	21.6	27.01	18.62
	2	9.97	10.32	10.68	11.4	11.23	10.99	13.74	16.49
	3	7.11	7.19	7.74	7.34	7.45	9.93	9.37	11.24
	4	6.04	6.17	6.3	6.25	6.91	7.53	7.26	8.7
	5	5.15	5.18	5.45	5.59	5.57	6.15	6.02	7.23
	6	4.76	4.83	4.89	5.14	5.48	5.25	6.57	6.26
	7	4.32	4.33	4.5	4.41	4.76	5.5	5.78	5.57
	8	4	4.17	4.21	4.23	4.23	4.91	5.21	5.07
	9	3.86	3.86	3.98	4.08	4.3	4.47	4.76	4.67
	10	3.65	3.78	3.79	3.96	3.93	4.11	4.41	4.36
0.95	0	85.71	91.83	98.03	91.89	91.87	122.48	153.1	91.87
	1	20.31	19.84	22.92	19.76	22.94	21.6	27.01	32.41
	2	11.5	11.48	12.23	11.4	14.18	14.98	13.74	16.49
	3	8.49	8.58	8.67	8.5	9.23	9.93	9.37	11.24
	4	6.68	7.13	6.95	7.06	8.14	7.53	9.41	8.7
	5	5.87	5.91	5.94	6.2	6.52	7.43	7.68	7.23
	6	5.34	5.41	5.28	5.63	6.23	6.29	6.57	6.26
	7	4.8	4.81	5.14	5.22	5.38	5.5	5.78	5.57
	8	4.53	4.58	4.75	4.57	4.76	4.91	5.21	6.25
	9	4.21	4.21	4.45	4.38	4.76	5.1	4.76	5.71
	10	3.96	4.08	4.21	4.22	4.33	4.68	5.14	5.29
0.99	0	134.68	146.92	147.04	153.14	137.8	183.71	153.1	183.73
	1	27.38	27.8	30.01	28.65	29.63	30.59	38.24	32.41
	2	15.35	14.95	15.32	15.28	17.1	18.9	18.72	16.49
	3	10.79	10.66	11.45	10.83	12.75	12.31	12.41	14.89
	4	8.6	8.58	8.88	9.48	9.37	9.2	9.41	11.29
	5	7.32	7.36	7.4	7.42	8.38	8.69	9.29	9.22
	6	6.49	6.56	6.83	6.6	6.97	7.3	7.86	7.88
	7	5.9	6	6.1	6.02	6.61	6.34	6.87	6.94
	8	5.34	5.39	5.56	5.6	5.81	6.35	6.14	6.25
	9	5.02	5.09	5.15	5.26	5.66	5.72	5.58	6.7
	10	4.77	4.85	4.83	5	5.14	5.23	5.85	6.17

About the Tables

Tables 3 through 6, present the results of time truncated acceptance sampling plan when it is assumed that the life time of test items follows the hybrid exponential distribution where the scale parameter is treated as triangular fuzzy variable. Now, we shall discuss the utility of these tables. Table 3 provides minimum sample size as well as the acceptance number c required to ensure that the median life time exceeds a given pre-specified median value θ_m^0 with consumer's confidence level P^*. The calculations were performed on using binomial approximation by assuming that the lot is large enough and p is not very small. Table 4 provides similar results under the poisson approximation to binomial. Operating characteristic function values are shown in Table 5 for different combinations of the median ratio θ_m/θ_m^0, probability P^*, and the experimental time ratio t/θ_m^0. Table 6 shows the minimum ratios of the actual median life to the specified median life for the acceptance of the lot with producer's risk of 0.05.

Assuming that the life time distribution follows hybrid exponential distribution and it is decided to establish a minimum median life time of θ_m^0 =1000 hours with probability P^*=0.95 given that the life test gets terminated at t =628 hours. For this situation, from Table 3, we get the minimum sample size 16 and acceptance number 2. This means that, out of 16 items, if no more than 2 items fail during 628 hours, then the experimenter assures that the actual median life time θ_m of the items is at least 1000 hours with confidence level of 0.95. Table 4 can be used in the same manner when poisson approximation to binomial probabilities is justified.

Table 5 gives the values of the operating characteristic function for the acceptance sampling plan adopted from Table 3, for different values of θ_m/θ_m^0 and P^*. For example, when P^*=0.95, t/θ_m^0 =0.628, c =2, θ_m/θ_m^0 =4, the probability of accepting the lot is 0.58297. It implies that, the lot is accepted with probability 0.58297 when time truncated sampling plan with samples size 16 and acceptance number 2 is used with $\theta_m \geq 4\times t/0.628$ =4000 hours. For the acceptance of a lot, Table 6 provides the minimum ratio of the true median life to the specified median life when the producer's risk is 0.05. For example, if P^*=0.95, t/θ_m^0 =0.628, and c =2, then the table value of θ_m/θ_m^0 is 11.5. This implies when $\theta_m \geq 11.5\times t/0.628$, the lot will be rejected with probability less than or equal to 0.05

5 Conclusion

Thus in this paper, a new hybrid distribution called hybrid exponential distribution is considered based on the Liu's chance theory [25]. Theoretical and numerical studies have been carried out to study its properties. The distribution is developed using the exponential distribution where the mean is treated as a triangular fuzzy variable. In this work, the minimum sample size required to decide to accept/reject a lot based on its specified median life time of the experimental units have been tabulated assuming the life time distribution follows hybrid exponential distribution under time truncated acceptance sampling plan. The optimal sample size provides the desired level of protection for both the customers as well as manufacturers. Apart from this, Operating Characteristic function has been evaluated for different choices of the parameters involved in the hybrid exponential distribution. Finally, values of the minimum ratio of the true median life to the specified median life are also tabulated when the producer's risk is 0.05. The authors are investigating designing such plans under other distributions as well.

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