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Finding Optimal Decisions in Supply Contracts by Two-stage Bi-objective Stochastic Optimization Method

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Abstract

This paper studies the risk-averse optimal ordering decision problems with random demand in supply contracts. In order to maximize the profit and minimize the risk simultaneously, we present a two-stage bi-objective stochastic model which employs the double evaluation criteria of mean and standard deviation. Constraints are used to guarantee that the optimal decisions are in accordance with the actual situation. In the process of dealing with the model, the main concern is to calculate the mean of random profit and its standard deviation. When the random demand follows common continuous or discrete probability distributions, the proposed optimization model can be turned into single-stage single objective models. Finally, numerical experiments and parameter analyses are given to demonstrate the validity of the proposed model. From the experimental results, we conclude that the distributor's risk preference level has a strong impact on optimal decisions.

Keywords: two-stage stochastic optimization, bi-objective model, options-futures contract, risk-averse, standard deviation

1 Introduction

With the development of business, the diversity of products is increasing, the supply and demand are changing rapidly, and their uncertainties are raising. All of these changes sophisticate the business environment and multiply the risk of supply chain's breakage. The supply chain members will be more concerned with the risk associated with demand uncertainties under random environment. The risk attitude of any firm may affect the decisions of all members. In the 1990s, there was rising a research methodology which made decisions under risk aversion in the inventory and supply chain problems. Finch [10] indicated the importance of undertaking risk assessment. This risk-averse thought was used in numerous fields like finance risk management [20], strategic risk management [2], and project risk management [16].

In the literature, researchers have chosen different risk evaluation criteria. Lau [14] determined the optimum order quantity under two risk management objectives: maximizing expected utility and maximizing the probability of achieving a budgeted profit. Choi et al. [9] took variance as risk evaluation criterion, and discussed the variance of profit associated with the ordering decision. They addressed two fundamental questions in inventory management: when to place the single order and how much to order. In order to find the optimal ordering decisions, Ahiska et al. [1] used a discrete-time Markov decision process to model inventory risk with an unreliable supply chain. Wu et al. [24] researched the risk-averse newsvendor model with a mean-variance objective function, and proposed that stockout cost had a significant impact on the newsvendor's optimal ordering decisions. Ghadge et al. [11] used the method of systematic literature review to summarize literature about supply chain risk management. Wang and Chen [22] adopted the method of mean-standard deviation to averse the risk generated by uncertain cycle time and derived optimal ordering quantity. Ozler et al. [17] utilized value at risk (VaR) as the risk measure in a newsvendor framework and investigated the multi-product newsvendor problem under a VaR constraint. In 2002, Rockafellar and Uryasev [19] proposed conditional value at risk (CVaR) for general loss distributions. It takes the risk of tail distribution that beyond VaR into account. Borgonovo and Peccati [5] discussed the effect of risk measure selection in

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the determination of inventory policies. They derived the optimal ordering decision problems by risk neutral, quadratic utility (variance), mean-absolute deviation and CVaR, respectively. Wu et al. [25] introduced the concept of CVaR as the evaluation criterion in a supply contract model. They derived the manufacturer's optimal decisions and analyzed the impact of risk aversion on them. Despite all of these, Wu et al. [27] deemed that CVaR measure should be used with caution. The main reasons were the CVaR approach did not take the expected profit into consideration, and its result was counter-intuitive that a higher retail price may lead to a smaller order quantity. Wu et al. [26] compared VaR risk criterion with CVaR. They found that the optimal order quantity was not affected by the capacity uncertainty for the risk-neutral newsvendor. However, capacity uncertainty decreased the order quantity with CVaR criterion, and led to an order decrease for low confidence levels but increase for high confidence levels with VaR criterion. They implied that the risk criteria should be carefully selected as it has an important effect on inventory decisions. For a firm in a supply chain with demand risk and supply risk, He [12] and Zhang [28] studied sequential decision problems.

Signing a contract is one of methods to averse risks. Brown and Lee [6] presented an option-based capacity reservation contract, and they coped with the amount of futures and options to maximize the manufacturer's expected profit. Brown and Lee [7] compared the different decisions produced via options-futures contract, options-only contract, backup contract, and quantity-flexibility contract. Cachon [8] researched the impacts of supply chain efficiency with three types of wholesale price contracts (push, pull and advance-purchase discount contracts). The author found that the efficiency of a single wholesale price contract was considerably higher than previous thought as long as firms considered both push and pull contracts. Avinadav et al. [3] employed the revenue sharing contract to circumvent the double marginalization effect that associated with vertical competition, and they showed that risk-seeking might obtain a higher expected profit contrasted with risk-neutral. In recent years, some scholars studied the optimal ordering decisions in supply contract such as Hu et al. [13], Wang and Chen [21], Wang and Luo [23].

In this paper, we are in the perspective of distributor to study problems with random demand. Distributor is a key link of supply chain. Distributors purchase commodities from upstream suppliers then sell them to downstream retailers to benefit from price difference. In this process, distributors are likely to encounter a lot of risks, such as uncertain demand information, uncertain supply, better alternatives, economic recession, natural disaster, and policy change. Some risks like uncertain supply will cause stockout; some risks like better alternatives will lead to inventory backlog; and some risks like uncertain demand information will either bring about stockout or inventory backlog. All these are what we are unwilling to see. The problem is how to make optimal ordering decisions which bring risk-averse distributors the maximum benefit.

In order to averse risks, the distributor signs an options-futures contract with upstream supplier. Standard deviation is used as risk evaluation criterion. We build a two-stage stochastic programming model to find the optimal ordering decisions. The futures purchasing capacity and the maximum options reserve capacity are made in the first stage. After the realization of random demand is known, the purchase quantity of options, as a recourse decision, is made in the second stage. The two-stage stochastic programming is an effective method to solve the decision-making problem with recourse action, and has a wide range of applications. For example, Li et al. [15] utilized two-stage stochastic programming to model insuring critical path problems, and Qin et al. [18] developed a new decomposition method for two-stage birandom programming and gave its application in production planning problem. The interested readers may refer to Birge and Louveaux [4] for detailed discussions.

Compared with the related literature, the main contributions of this paper consist of the following four aspects. Firstly, by means of options-futures contract, distributors' ordering decisions are made twice in the whole time horizon. In this way, according to the realized exact demand, decision makers can make a recourse decision at the second time. Thus this contract reduces the risks under uncertain data, and distributors' ordering decisions problem is built as a two-stage optimization model. Secondly, considering the most concerned two factors of decision makers synthetically, there are two objectives in the proposed model: maximizing the profit and minimizing the risk. Using the weight coefficient method to deal with the bi-objective, decision makers can adjust the share of profit and risk on the basis of their risk preference level. Thirdly, by analyzing the second-stage programming problem, the two-stage bi-objective model is simplified into a tractable single-stage single objective model. Fourthly, from the computational results of numerical experiments, we can get the impacts of the risk preference level on decisions, mean profit and risk.

The structure of this paper is organized as follows. Section 2 builds a two-stage bi-objective model for the ordering decision in a supply chain which includes a supplier, a distributor and some retailers. Section 3 derives the optimal value expression of the second-stage programming problem. Then the proposed two-stage model is transformed into its equivalent single-stage optimization model. This section also deals with the calculations about the expected value and standard deviation of the discrete random demand and the common continuous random demand. Section 4 provides some numerical examples to illustrate the impacts of the risk preference level on the distributor's futures purchasing capacity, maximum reserve capacity, mean profit and risk. Section 5 gives the conclusions of this paper.

2 Formulation of a New Two-stage Bi-objective Options-Futures Contract Model

In this paper, we consider the ordering decision in a supply chain which includes a supplier, a distributor and several retailers. Before describing the research problem, we make the following assumptions: Firstly, there is an options-futures contract allowed to sign between the distributor and the supplier. The options-futures contract lists some agreements that ensure vendees to buy a certain number of commodities at a certain price on a stated future date. Futures are commodities that vendees promise to buy. Options are commodities that vendees book from vendors; the options are flexible, that means vendees can buy discretionary quantity within the booking volume. Secondly, before the sales season, the demand from retailers is undetermined and no one knows how much it will be. Since the production of commodities need time, the distributor has to sign a contract at that uninformed moment, scilicet, decide how much the futures and how much the options are. In the sales season, the exact demand is known, the distributor can decide whether or not to exercise the options and how much to purchase. Thirdly, the distributor is risk-averse.

2.1 Notations

Decision variables

y: futures purchasing capacity;
z: maximum commodities reserve capacity;
q: the distributor's purchase quantity of options;

Parameters

ξ : random demand;
c_f: unit cost of futures;
c_o: unit cost of reserving options;
c_b: unit cost of purchasing options;
r: unit revenue for each commodity.

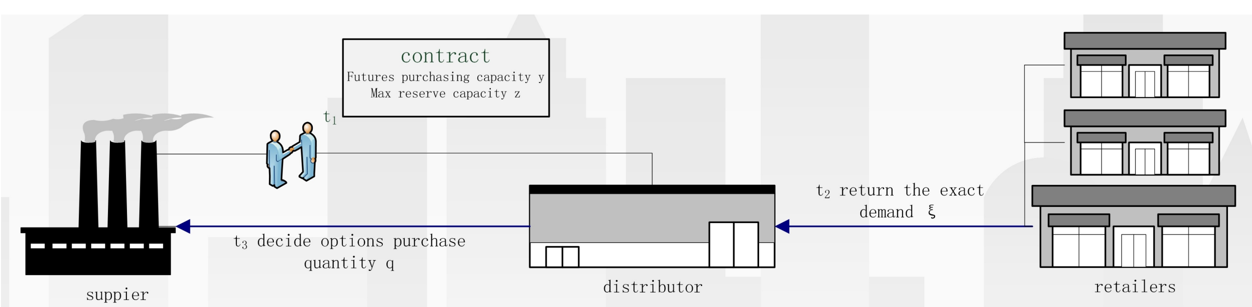


Figure 1: Flow chart of the supply chain about time t

2.2 Problem Description

Now we are in the perspective of distributor to deal with the research problem. As shown in Figure 1, the distributor purchases commodities from an upstream supplier then sells them to downstream retailers. In order to meet customers' demand and achieve zero inventory, the distributor will sign an options-futures contract with the supplier to obtain the maximum profit and the minimum risk before the sales season. The contract ensures the distributor can make twice decisions in the whole time horizon. In the moment of signing the contract, the distributor has to make two initial decisions, which are the futures purchasing capacity y

and the maximum commodities reserve capacity z. In fact, z is not less than y and $z-y$ is the options reserve capacity. They also need to sign the unit cost c_f of the futures, the unit cost c_o of the reserve options and the unit cost c_b of the buying options on the contract. When the exact demand is determined, the distributor can use the second decision-making chance to make a reasonable recourse decision; let q be the purchase quantity of options, where $0 \leqslant q \leqslant z-y$. At the end of the sales season, the distributor obtains revenue r for each commodity which has been sold. According to the actual situation, it is obvious that $c_f < c_o + c_b < r$ and $c_o < c_f$. The initial decision vector (y, z), called the first-stage decision, must be made before knowing the realization of the random demand ξ, while the recourse decision, called the second-stage decision, is made after the realization of the random demand ξ is known.

Distributor, who pursues interests, hopes to get the maximum profit. The profit is determined by the total revenue and costs. The total revenue is the product of the unit revenue and the quantity of the sold commodity. Obviously the latter is the smaller one between demand and the final purchase quantity of the distributor. So the total revenue is $r\min\{\xi, (y+q)\}$. The costs consist of initial costs and recourse costs. The initial costs include the future costs $c_f y$ and the costs of reserving options $c_o(z-y)$. The recourse costs refer to the costs of the exercised options after the demand ξ is known. The recourse costs can be expressed as $c_b q$. In conclusion, the profit function is as follows:

$$r\min\{\xi, (y+q)\} - c_f y - c_o(z-y) - c_b q.$$

2.3 Two-stage Bi-objective Stochastic Model

The maximum profit is

$$\pi(y, z; \xi) = \max_q \quad r\min\{\xi, (y+q)\} - c_f y - c_o(z-y) - c_b q.$$

Since $\pi(y, z; \xi)$ is related to the random demand, it is also a random variable. We measure the profit level by its mean. Considering the distributor is risk-averse, the standard deviation is used to measure the risk. A small standard deviation indicates the small risk, on the contrary, a larger standard deviation indicates the larger risk. We want to obtain the maximum profit as well as bear the minimum risk. Then the problem can be constructed as the following bi-objective model

$$\begin{array}{ll} \max & \mathrm{E}_\xi[\pi(y, z; \xi)] \\ \min & \sigma_\xi[\pi(y, z; \xi)] \\ \text{s.t.} & 0 \leqslant y \leqslant z, \end{array} \tag{1}$$

where

$$\begin{aligned} \pi(y, z; \xi) = & \max_q \quad r\min\{\xi, (y+q)\} - c_f y - c_o(z-y) - c_b q \\ & \text{s.t.} \quad 0 \leqslant q \leqslant z-y. \end{aligned} \tag{2}$$

3 Model Analysis

In this section, we analyze the proposed two-stage bi-objective model (1)-(2). Firstly the second-stage programming is solved for the given first-stage decision vector (y, z) and known realization of random demand ξ. Then the weight coefficient method is used to transform the equivalent single-stage bi-objective model into a single-stage single objective model.

3.1 The Optimal Value of the Second-Stage Model

After the options-futures contract is signed, the futures purchasing capacity y and the maximum reserve capacity z are fixed. Now suppose the retailers' demand is known. Then we solve the second-stage model (2)

$$\begin{aligned} \pi(y, z; \xi) &= \max_q \quad r\min\{\xi, (y+q)\} - c_f y - c_o(z-y) - c_b q \\ &= r\min\{\xi, (y+q^*)\} - c_f y - c_o(z-y) - c_b q^*, \end{aligned} \tag{3}$$

where q^* is the optimal quantity of the exercised options. The value of q^* is

$$q^*(y,z;\xi)=\begin{cases}0, & \text{if } \xi\leqslant y\\ \xi-y, & \text{if } y\leqslant\xi\leqslant z\\ z-y, & \text{if } z\leqslant\xi.\end{cases}\tag{4}$$

It indicates that q^* is dependent on the random demand ξ and decision vector (y,z).

Eq.(4) presents three different relationships between the determined demand and the first-stage decisions. In the first case, the actual demand is less than the futures purchasing capacity, we call it overrated demand. In this case, the amount of futures can meet the demand and the distributor does not need to exercise the options. In the second case, the actual demand is less than the maximum reserve capacity and more than the futures purchasing quantity, we call it anticipated demand. In this case, the amount of futures can not meet the demand and the distributor needs to exercise the options to cover the shortfall. In the third case, the actual demand is more than the maximum reserve capacity, we call it underrated demand. In this case, the maximum reserve capacity can not meet demand. Hence the distributor exercises entire options. The reason we name the above three cases of demands is that: the distributor wishes the actual demand falls in the interval $[y,z]$ when making the first-stage decisions.

These results show that the final purchase quantity of options decided by distributor is based on the retailers' demand, and the distributor will not purchase any commodities beyond the demand.

Substituting Eq.(4) into Eq.(3), we have

$$\pi(y,z;\xi)=\begin{cases}r\xi-c_fy-c_o(z-y), & \text{if } \xi\leqslant y\\ r\xi-c_fy-c_o(z-y)-c_b(\xi-y), & \text{if } y\leqslant\xi\leqslant z\\ rz-c_fy-c_o(z-y)-c_b(z-y), & \text{if } z\leqslant\xi.\end{cases}\tag{5}$$

The above three cases correspond to overrated demand, anticipated demand and underrated demand, respectively. We denote them by π_o, π_a, π_u, respectively. Obviously, π_u is a constant function. According to the actual situation, the realizations of random demand ξ are in the interval $[0,+\infty)$. We draw an approximate image of $\pi(y,z;\xi)$ as Figure 2.

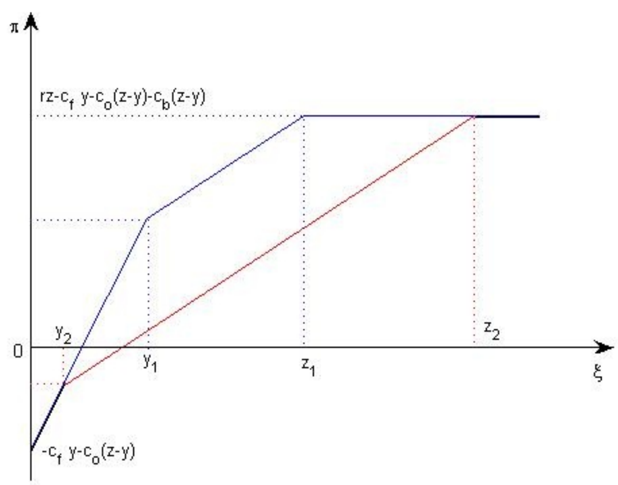


Figure 2: Random profit $\pi(y,z;\xi)$

From Figure 2, it is obvious that $\pi(y,z;\xi)$ is a continuous, piecewise differentiable function with respect to $\xi\in[0,+\infty)$. Also $\pi(y,z;\xi)$ is increasing with respect to the random remand ξ, where π_o, π_a are both strictly increasing, and have a lower bound $-c_fy-c_o(z-y)$ (which is less than 0) and an upper bound $rz-c_fy-c_o(z-y)-c_b(z-y)$ (which is more than 0). The slopes of π_o, π_a and π_u are r, $r-c_b$ and 0, respectively. All these imply that each decision vector (y,z) determines a unique profit function $\pi(y,z;\xi)$.

Nevertheless, for different values of (y, z), we are not sure which is the larger one in $\pi(y, z; \xi)$ and 0 when the realization of random demand ξ is y.

3.2 The Objectives in the First-Stage Model

Substituting the optimal value of model (2) into model (1), the two-stage model (1)-(2) is equivalent to a single-stage model. There are two objectives in the equivalent model, which are maximizing the mean profit and minimizing the profit's standard deviation. We calculate them respectively.

3.2.1 The Expected Value of Random Profit

Note the structure of $\pi(y, z; \xi)$, there is a same term $-[c_f y + c_o(z-y)]$ without random variable ξ in each piecewise expression. Hence we have

$$\pi(y, z; \xi) = -[c_f y + c_o(z-y)] + \pi'(y, z; \xi),$$

where

$$\pi'(y, z; \xi) = \begin{cases} r\xi, & \text{if } \xi \leqslant y \\ r\xi - c_b(\xi - y), & \text{if } y \leqslant \xi \leqslant z \\ rz - c_b(z-y), & \text{if } z \leqslant \xi. \end{cases} \tag{6}$$

Therefore,

$$\mathrm{E}_\xi[\pi(y, z; \xi)] = -[c_f y + c_o(z-y)] + \mathrm{E}_\xi[\pi'(y, z; \xi)].$$

We only need to calculate $\mathrm{E}_\xi[\pi'(y, z; \xi)]$. Next, we deal with the calculation of $\mathrm{E}_\xi[\pi'(y, z; \xi)]$ under common demand distributions. In real life, some commodities are sold by an arbitrary number, others are sold by piece. We divide our discussion into two cases. One is the case that the demand ξ follows discrete distribution, and the second case is that demand follows common continuous distributions.

Case I. Discrete demand distribution

Suppose ξ has the following discrete probability distribution

$$\xi \sim \begin{pmatrix} \hat{\xi}_1 & \hat{\xi}_2 & \cdots & \hat{\xi}_N \\ p_1 & p_2 & \cdots & p_N \end{pmatrix}, \tag{7}$$

with $\hat{\xi}_1 < \hat{\xi}_2 < \cdots < \hat{\xi}_N$. Then we have

$$\begin{aligned} \mathrm{E}_\xi[\pi'(y, z; \xi)] =& \sum_{i=1}^{N} \pi'_i(y, z; \hat{\xi}_i) p_i \\ =& \sum_{i \in I_1} \pi'_i(y, z; \hat{\xi}_i) p_i + \sum_{i \in I_2} \pi'_i(y, z; \hat{\xi}_i) p_i + \sum_{i \in I_3} \pi'_i(y, z; \hat{\xi}_i) p_i \\ =& r \sum_{i \in I_1} \hat{\xi}_i p_i + (r - c_b) \sum_{i \in I_2} \hat{\xi}_i p_i + c_b y \sum_{i \in I_2 \cup I_3} p_i + (r - c_b) z \sum_{i \in I_3} p_i \\ =& (1 - F(y)) c_b y + (1 - F(z))(r - c_b) z + r \sum_{i \in I_1} \hat{\xi}_i p_i + (r - c_b) \sum_{i \in I_2} \hat{\xi}_i p_i, \end{aligned} \tag{8}$$

where $I_1 = \{i \mid 0 \leqslant \hat{\xi}_i \leqslant y\}, I_2 = \{i \mid y < \hat{\xi}_i \leqslant z\}$, and $I_3 = \{i \mid \hat{\xi}_i > z\}$. $F(\cdot)$ is the probability distribution function.

As a result, the mean profit is

$$\mathrm{E}_\xi[\pi(y, z; \xi)] = [c_o + (1 - F(y)) c_b - c_f] y + [(1 - F(z))(r - c_b) - c_o] z + r \sum_{i \in I_1} \hat{\xi}_i p_i + (r - c_b) \sum_{i \in I_2} \hat{\xi}_i p_i. \tag{9}$$

If ξ follows the following equiprobable distribution

$$\xi \sim \begin{pmatrix} \hat{\xi}_1 & \hat{\xi}_2 & \cdots & \hat{\xi}_N \\ \frac{1}{N} & \frac{1}{N} & \cdots & \frac{1}{N} \end{pmatrix} \tag{10}$$

with $\hat{\xi}_1 < \hat{\xi}_2 < \cdots < \hat{\xi}_N$. Then, according to Eq.(9), the mean profit is

$$\mathrm{E}_\xi[\pi(y,z;\xi)] = [c_o + (1-F(y))c_b - c_f]y + [(1-F(z))(r-c_b) - c_o]z + \frac{r}{N}\sum_{i\in I_1}\hat{\xi}_i + \frac{(r-c_b)}{N}\sum_{i\in I_2}\hat{\xi}_i,$$

where

$$F(y) = \frac{i_y}{N}, i_y = \max\{i \mid \hat{\xi}_i \leqslant y\} \text{ and } F(z) = \frac{i_z}{N}, i_z = \max\{i \mid \hat{\xi}_i \leqslant z\}.$$

Case II. Continuous demand distribution

Suppose ξ is a continuous random variable. Then

$$\begin{aligned} \mathrm{E}_\xi[\pi'(y,z;\xi)] &= \int_0^{+\infty} \pi'(y,z;x)\mathrm{d}F_\xi(x) \\ &= \int_0^y rx\mathrm{d}F_\xi(x) + \int_y^z rx - c_b(x-y)\mathrm{d}F_\xi(x) + \int_z^{+\infty} rz - c_b(z-y)\mathrm{d}F_\xi(x) \\ &= r\int_0^y x\mathrm{d}F_\xi(x) + (r-c_b)\int_y^z x\mathrm{d}F_\xi(x) + c_b y\int_y^z \mathrm{d}F_\xi(x) + [rz - c_b(z-y)]\int_z^{+\infty}\mathrm{d}F_\xi(x) \\ &= c_b y + (r-c_b)z - r\int_0^y F_\xi(x)\mathrm{d}x - (r-c_b)\int_y^z F_\xi(x)\mathrm{d}x. \end{aligned} \tag{11}$$

As a result, one has

$$\mathrm{E}_\xi[\pi(y,z;\xi)] = (c_o + c_b - c_f)y + (r - c_o - c_b)z - r\int_0^y F_\xi(x)\mathrm{d}x - (r-c_b)\int_y^z F_\xi(x)\mathrm{d}x. \tag{12}$$

When ξ follows uniform distribution on the interval $[a,b]$, the minimum demand is a, the maximum demand is b. The distributor will determine the appropriate number of futures within this range. Therefore distributor's optimal decisions y and z are certainly such that $a \leqslant y \leqslant z \leqslant b$. So, based on Eq.(12), the mean profit is

$$\mathrm{E}_\xi[\pi(y,z;\xi)] = (c_o + c_b - c_f)y + (r - c_o - c_b)z - \frac{1}{2(b-a)}[c_b(y-a)^2 + (r-c_b)(z-a)^2].$$

When ξ follows exponential distribution with parameter k, based on Eq.(12), the mean profit is

$$\mathrm{E}_\xi[\pi(y,z;\xi)] = (c_o - c_f)y - c_o z + \frac{1}{k}[r - c_b e^{-ky} - (r-c_b)e^{-kz}].$$

When ξ follows normal distribution $N(\mu,\sigma^2)$, based on Eq.(12), the mean profit is

$$\mathrm{E}_\xi[\pi(y,z;\xi)] = (c_o + \frac{c_b}{2} - c_f)y + (\frac{(r-c_b)}{2} - c_o)z - \frac{r}{2}\int_0^y erf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x - \frac{(r-c_b)}{2}\int_y^z erf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x,$$

where

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\mathrm{d}t.$$

The aforementioned integral $erf(x)$ can not be evaluated in closed form in accordance with elementary functions. The integral can be calculated by using Maclaurin series

$$erf(x) = \frac{2}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{n!(2n+1)},$$

which is derived by expanding the integrand e^{-t^2} into its Maclaurin series, then it can be integrated term by term.

3.2.2 The Standard Deviation of Random Profit

We next derive the analytic expressions of $\mathrm{E}_\xi[\pi^2(y,z;\xi)]$ and $(\mathrm{E}_\xi[\pi(y,z;\xi)])^2$, respectively. Then, on the basis of

$$\mathrm{Var}[x] = \mathrm{E}[x^2] - (\mathrm{E}[x])^2,$$

we can obtain the standard deviation.

We denote

$$A = [c_f y + c_o(z-y)].$$

It is convenient to derive the above expressions. Eq.(5) can be changed equivalently into

$$\pi(y,z;\xi) = \begin{cases} r\xi - A, & \text{if } \xi \leqslant y \\ r\xi - c_b(\xi - y) - A, & \text{if } y \leqslant \xi \leqslant z \\ rz - c_b(z-y) - A, & \text{if } z \leqslant \xi. \end{cases} \tag{13}$$

Then

$$\pi^2(y,z;\xi) = \begin{cases} r^2\xi^2 - 2Ar\xi + A^2, & \text{if } \xi \leqslant y \\ (r-c_b)^2\xi^2 + 2(r-c_b)(c_b y - A)\xi + (c_b y - A)^2, & \text{if } y \leqslant \xi \leqslant z \\ ([rz - c_b(z-y)] - A)^2, & \text{if } z \leqslant \xi. \end{cases} \tag{14}$$

We denote $\pi^2(y,z;\xi)$ in the above three cases as π_o^2, π_a^2, π_u^2, respectively.

Case I. Discrete demand distribution

Suppose ξ has the discrete probability distribution which is given in Eq. (7) with $\hat{\xi}_1 < \hat{\xi}_2 < \cdots < \hat{\xi}_N$. Then

$$\begin{aligned} \mathrm{E}_\xi[\pi^2(y,z;\xi)] =& \sum_{i=1}^{n} \pi_i^2(y,z;\hat{\xi}_i)p_i \\ =& \sum_{i\in I_1} \pi_{o,i}^2 p_i + \sum_{i\in I_2} \pi_{a,i}^2 p_i + \sum_{i\in I_3} \pi_{u,i}^2 p_i \\ =& A^2 + (1-F(y))(c_b y - 2A)c_b y + (1-F(z))(r-c_b)^2 z^2 \\ &+ 2(1-F(z))(r-c_b)z(c_b y - A) - 2Ar\sum_{i\in I_1}\hat{\xi}_i p_i + 2(r-c_b)(c_b y - A)\sum_{i\in I_2}\hat{\xi}_i p_i \\ &+ r^2\sum_{i\in I_1}\hat{\xi}_i^2 p_i + (r-c_b)^2\sum_{i\in I_2}\hat{\xi}_i^2 p_i \\ =& [(c_f - c_o)^2 + 2(1-F(y))c_b(c_o - c_f) + (1-F(y))c_b^2]y^2 \\ &+ 2[-c_o(c_o + (1-F(y))c_b - c_f) + (1-F(z))(r-c_b)(c_o + c_b - c_f)]yz \\ &+ [c_o^2 - 2(1-F(z))(r-c_b)c_o + (1-F(z))(r-c_b)^2]z^2 \\ &- 2[c_f y + c_o(z-y)]r\sum_{i\in I_1}\hat{\xi}_i p_i + 2(r-c_b)(c_b y - [c_f y + c_o(z-y)])\sum_{i\in I_2}\hat{\xi}_i p_i \\ &+ r^2\sum_{i\in I_1}\hat{\xi}_i^2 p_i + (r-c_b)^2\sum_{i\in I_2}\hat{\xi}_i^2 p_i, \end{aligned} \tag{15}$$

and

$$
\begin{aligned}
(\mathrm{E}_\xi[\pi(y,z;\xi)])^2 =& \Bigg\{[c_o+(1-F(y))c_b-c_f]y+[(1-F(z))(r-c_b)-c_o]z+r\sum_{i\in I_1}\hat{\xi}_i p_i \\
&+(r-c_b)\sum_{i\in I_2}\hat{\xi}_i p_i\Bigg\}^2 \\
=&[(c_o-c_f)^2+2(1-F(y))c_b(c_o-c_f)+(1-F(y))^2c_b^2]y^2 \\
&+2[-c_o(c_o+(1-F(y))c_b-c_f)+(1-F(z))(r-c_b)(c_o+(1-F(y))c_b-c_f)]yz \\
&+[c_o^2-2(1-F(z))(r-c_b)c_o+(1-F(z))^2(r-c_b)^2]z^2 \\
&-2[[c_o+(1-F(y))c_b-c_f]y \\
&+[(1-F(z))(r-c_b)-c_o]z][r\sum_{i\in I_1}\hat{\xi}_i p_i+(r-c_b)\sum_{i\in I_2}\hat{\xi}_i p_i] \\
&+[r\sum_{i\in I_1}\hat{\xi}_i p_i+(r-c_b)\sum_{i\in I_2}\hat{\xi}_i p_i]^2.
\end{aligned} \tag{16}
$$

It follows from Eq.(15) and Eq.(16) that

$$
\begin{aligned}
\mathrm{Var}_\xi[\pi(y,z;\xi)] =&\mathrm{E}_\xi[\pi^2(y,z;\xi)]-(\mathrm{E}_\xi[\pi(y,z;\xi)])^2 \\
=&F(y)(1-F(y))c_b^2y^2+2(1-F(z))(r-c_b)F(y)c_byz+F(z)(1-F(z))(r-c_b)^2z^2 \\
&-2r[[2c_f-(1-F(y))c_b-2c_o]y-[(1-F(z))(r-c_b)-2c_o]z]\sum_{i\in I_1}\hat{\xi}_i p_i \\
&+2(r-c_b)[[2c_o-(2-F(y))c_b-2c_f]y+[(1-F(z))(r-c_b)-2c_o]z]\sum_{i\in I_2}\hat{\xi}_i p_i \\
&+r^2\sum_{i\in I_1}\hat{\xi}_i^2 p_i+(r-c_b)^2\sum_{i\in I_2}\hat{\xi}_i^2 p_i-[r\sum_{i\in I_1}\hat{\xi}_i p_i+(r-c_b)\sum_{i\in I_2}\hat{\xi}_i p_i]^2.
\end{aligned} \tag{17}
$$

Therefore, we have

$$
\begin{aligned}
\sigma_\xi[\pi(y,z;\xi)] =&\{\mathrm{Var}_\xi[\pi(y,z;\xi)]\}^{\frac{1}{2}} \\
=&\big\{F(y)(1-F(y))c_b^2y^2+2(1-F(z))(r-c_b)F(y)c_byz+F(z)(1-F(z))(r-c_b)^2z^2 \\
&-2r[[2c_f-(1-F(y))c_b-2c_o]y-[(1-F(z))(r-c_b)-2c_o]z]\sum_{i\in I_1}\hat{\xi}_i p_i \\
&+2(r-c_b)[[2c_o-(2-F(y))c_b-2c_f]y+[(1-F(z))(r-c_b)-2c_o]z]\sum_{i\in I_2}\hat{\xi}_i p_i \\
&+r^2\sum_{i\in I_1}\hat{\xi}_i^2 p_i+(r-c_b)^2\sum_{i\in I_2}\hat{\xi}_i^2 p_i-[r\sum_{i\in I_1}\hat{\xi}_i p_i+(r-c_b)\sum_{i\in I_2}\hat{\xi}_i p_i]^2\big\}^{\frac{1}{2}}.
\end{aligned} \tag{18}
$$

If ξ follows the equiprobable distribution given in Eq. (10), then the standard deviation is as follows

$$
\begin{aligned}
\sigma_\xi[\pi(y,z;\xi)] =&\big\{F(y)(1-F(y))c_b^2y^2+2(1-F(z))(r-c_b)F(y)c_byz+F(z)(1-F(z))(r-c_b)^2z^2 \\
&-2\frac{r}{n}[[2c_f-(1-F(y))c_b-2c_o]y-[(1-F(z))(r-c_b)-2c_o]z]\sum_{i\in I_1}\hat{\xi}_i \\
&+2\frac{(r-c_b)}{n}[[2c_o-(2-F(y))c_b-2c_f]y+[(1-F(z))(r-c_b)-2c_o]z]\sum_{i\in I_2}\hat{\xi}_i \\
&+\frac{r^2}{n}\sum_{i\in I_1}\hat{\xi}_i^2+\frac{(r-c_b)^2}{n}\sum_{i\in I_2}\hat{\xi}_i^2-\frac{1}{n^2}[r\sum_{i\in I_1}\hat{\xi}_i+(r-c_b)\sum_{i\in I_2}\hat{\xi}_i]^2\big\}^{\frac{1}{2}}.
\end{aligned} \tag{19}
$$

Case II. Continuous demand distribution

Suppose ξ is a continuous random variable. Then we can derive the following formulas

$$\begin{aligned}\mathrm{E}_\xi[\pi^2(y,z;\xi)] &= \int_0^{+\infty} \pi^2(y,z;x)\mathrm{d}F_\xi(x)\\ &= \int_0^y \pi_o^2 \mathrm{d}F_\xi(x) + \int_y^z \pi_a^2 \mathrm{d}F_\xi(x) + \int_z^{+\infty} \pi_u^2 \mathrm{d}F_\xi(x)\\ &=(r-c_b)^2z^2 + 2(r-c_b)(c_by-A)z + (c_by-A)^2 - r^2\int_0^y 2xF_\xi(x)\mathrm{d}x\\ &\quad - (r-c_b)^2\int_y^z 2xF_\xi(x)\mathrm{d}x + 2rA\int_0^y F_\xi(x)\mathrm{d}x - 2(r-c_b)(c_by-A)\int_y^z F_\xi(x)\mathrm{d}x\\ &=[(c_o+c_b-c_f)y + (r-c_o-c_b)z]^2 - 2r\int_0^y \pi_oF_\xi(x)\mathrm{d}x - 2(r-c_b)\int_y^z \pi_aF_\xi(x)\mathrm{d}x,\end{aligned} \tag{20}$$

and

$$\begin{aligned}(\mathrm{E}_\xi[\pi(y,z;\xi)])^2 &= \left\{(c_o+c_b-c_f)y + (r-c_o-c_b)z - r\int_0^y F_\xi(x)\mathrm{d}x - (r-c_b)\int_y^z F_\xi(x)\mathrm{d}x\right\}^2\\ &=[(c_o+c_b-c_f)y + (r-c_o-c_b)z]^2\\ &\quad - 2[(c_o+c_b-c_f)y + (r-c_o-c_b)z][r\int_0^y F_\xi(x)\mathrm{d}x + (r-c_b)\int_y^z F_\xi(x)\mathrm{d}x]\\ &\quad + [r\int_0^y F_\xi(x)\mathrm{d}x + (r-c_b)\int_y^z F_\xi(x)\mathrm{d}x]^2.\end{aligned} \tag{21}$$

By (20) and (21), we get

$$\begin{aligned}\mathrm{Var}_\xi[\pi(y,z;\xi)] &= \mathrm{E}_\xi[\pi^2(y,z;\xi)] - (\mathrm{E}_\xi[\pi(y,z;\xi)])^2\\ &=2r[c_by + (r-c_b)z]\int_0^y F_\xi(x)\mathrm{d}x - 2r^2\int_0^y xF_\xi(x)\mathrm{d}x\\ &\quad + 2(r-c_b)^2z\int_y^z F_\xi(x)\mathrm{d}x - 2(r-c_b)^2\int_y^z xF_\xi(x)\mathrm{d}x\\ &\quad - [r\int_0^y F_\xi(x)\mathrm{d}x + (r-c_b)\int_y^z F_\xi(x)\mathrm{d}x]^2.\end{aligned} \tag{22}$$

So, one has

$$\begin{aligned}\sigma_\xi[\pi(y,z;\xi)] = \Big\{&2r[c_by + (r-c_b)z]\int_0^y F_\xi(x)\mathrm{d}x - 2r^2\int_0^y xF_\xi(x)\mathrm{d}x\\ &+ 2(r-c_b)^2z\int_y^z F_\xi(x)\mathrm{d}x - 2(r-c_b)^2\int_y^z xF_\xi(x)\mathrm{d}x\\ &-[r\int_0^y F_\xi(x)\mathrm{d}x + (r-c_b)\int_y^z F_\xi(x)\mathrm{d}x]^2\Big\}^{\frac{1}{2}}.\end{aligned} \tag{23}$$

When ξ follows uniform distribution on the interval $[a,b]$, we deduce

$$\begin{aligned}\sigma_\xi[\pi(y,z;\xi)] = \Big\{&\frac{1}{(b-a)}[c_b((r-c_b)z + ry)(y-a)^2 + (r-c_b)^2z(z-a)^2\\ &- \frac{2}{3}r^2(y^3-a^3) + ar^2(y^2-a^2) - \frac{2}{3}(r-c_b)^2(z^3-y^3) + a(r-c_b)^2(z^2-y^2)]\\ &-[\frac{1}{2(b-a)}[c_b(y-a)^2 + (r-c_b)(z-a)^2]]^2\Big\}^{\frac{1}{2}}.\end{aligned} \tag{24}$$

When ξ follows exponential distribution with parameter k, we obtain

$$\begin{aligned}\sigma_\xi[\pi(y,z;\xi)] = \Big\{ & \frac{1}{k}[-2rc_b y e^{-ky} - 2(r-c_b)[c_b y + (r-c_b)z]e^{-kz}] \\ & + \frac{1}{k^2}[-2c_b(r-c_b)e^{-ky} + 2c_b(r-c_b)e^{-kz} - [c_b e^{-ky} + (r-c_b)e^{-kz}]^2 + r^2]\Big\}^{\frac{1}{2}}.\end{aligned} \tag{25}$$

When ξ follows normal distribution $N(\mu, \sigma^2)$, we have

$$\begin{aligned}\sigma_\xi[\pi(y,z;\xi)] = \Big\{ & \frac{1}{4}[c_b y + (r-c_b)z]^2 + \frac{1}{2}r[c_b y + (r-c_b)z]\int_0^y erf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x \\ & + \frac{1}{2}(r-c_b)[(r-c_b)z - c_b y]\int_y^z erf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x \\ & - r^2\int_0^y x erf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x - (r-c_b)^2\int_y^z x erf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x \\ & - \frac{1}{4}[r\int_0^y erf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x + (r-c_b)\int_y^z erf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x]^2\Big\}^{\frac{1}{2}}.\end{aligned} \tag{26}$$

Now, under some common demand distributions, we have obtained the equivalent forms of the objective functions $\mathrm{E}_\xi[\pi(y,z;\xi)]$ and $\sigma_\xi[\pi(y,z;\xi)]$ in the equivalent single-stage model of the two-stage model (1)-(2). Next, we are going to solve the proposed two-stage model via the equivalent forms.

3.3 The Single Objective Models

Although model (1)-(2) reflects all distributors' ordering expectations, it describes only an ideal situation which is difficult to realize in a real supply chain management. That is to say we can not find a decision vector (y,z) which is suitable for the distributor to minimize the risk and maximize the profit simultaneously. Furthermore, we usually obtain the Pareto-optimal solutions rather than an optimal solution of the above model. Here we use the weight coefficient method to transform the original model into the following single objective model

$$\begin{array}{ll}\max & \lambda \mathrm{E}_\xi[\pi(y,z;\xi)] - (1-\lambda)\sigma_\xi[\pi(y,z;\xi)] \\ \text{s. t.} & 0 \leqslant y \leqslant z,\end{array} \tag{27}$$

where

$$\begin{array}{ll}\pi(y,z;\xi) = \max\limits_q & r\min\{\xi, (y+q)\} - c_f y - c_o(z-y) - c_b q \\ \text{s. t.} & 0 \leqslant q \leqslant z-y,\end{array} \tag{28}$$

where $\lambda \in [0,1]$ is the weight coefficient, which represents the distributor's preference level towards risk. $\lambda = 1$ stands for the distributor is risk-neutrality and aims to maximize the mean profit; $\lambda = 0$ denotes the distributor is risk-averse completely and aims to minimize the risk.

In section 3.2, we have derived the expressions of $\mathrm{E}_\xi[\pi(y,z;\xi)]$ and $\sigma_\xi[\pi(y,z;\xi)]$. The next work is to seek the optimal decisions about the futures purchasing capacity y and the distributor's maximum reserve capacity z. Substituting the expressions of $\mathrm{E}_\xi[\pi(y,z;\xi)]$ and $\sigma_\xi[\pi(y,z;\xi)]$ into model (27), then the equivalent forms of model (27)-(28) are provided in Theorems 1 and 2.

Theorem 1. *Suppose ξ has the following discrete probability distribution*

$$\xi \sim \begin{pmatrix} \hat{\xi}_1 & \hat{\xi}_2 & \cdots & \hat{\xi}_N \\ p_1 & p_2 & \cdots & p_N \end{pmatrix} \tag{29}$$

with $\hat{\xi}_1 < \hat{\xi}_2 < \cdots < \hat{\xi}_N$. The probability distribution function of ξ is denoted as $F(\cdot)$. Then model (27)-(28) can be transformed equivalently into the following model

$$\begin{aligned}
\max \quad & \lambda\left\{[c_o+(1-F(y))c_b-c_f]y+[(1-F(z))(r-c_b)-c_o]z+r\sum_{i\in I_1}\hat{\xi}_ip_i+(r-c_b)\sum_{i\in I_2}\hat{\xi}_ip_i\right\}\\
& -(1-\lambda)\left\{F(y)(1-F(y))c_b^2y^2+2(1-F(z))(r-c_b)F(y)c_byz+F(z)(1-F(z))(r-c_b)^2z^2\right.\\
& -2r[[2c_f-(1-F(y))c_b-2c_o]y-[(1-F(z))(r-c_b)-2c_o]z]\sum_{i\in I_1}\hat{\xi}_ip_i\\
& +2(r-c_b)[[2c_o-(2-F(y))c_b-2c_f]y+[(1-F(z))(r-c_b)-2c_o]z]\sum_{i\in I_2}\hat{\xi}_ip_i\\
& \left.+r^2\sum_{i\in I_1}\hat{\xi}_i^2p_i+(r-c_b)^2\sum_{i\in I_2}\hat{\xi}_i^2p_i-[r\sum_{i\in I_1}\hat{\xi}_ip_i+(r-c_b)\sum_{i\in I_2}\hat{\xi}_ip_i]^2\right\}^{\frac{1}{2}}\\
\text{s.t.} \quad & 0\leqslant y\leqslant z.
\end{aligned} \tag{30}$$

Theorem 2. *If ξ is a continuous random variable with probability distribution function $F(\cdot)$, then model (27)-(28) can be transformed equivalently into the following model*

$$\begin{aligned}
\max \quad & \lambda\left\{(c_o+c_b-c_f)y+(r-c_o-c_b)z-r\int_0^y F_\xi(x)\mathrm{d}x-(r-c_b)\int_y^z F_\xi(x)\mathrm{d}x\right\}\\
& -(1-\lambda)\left\{2r[c_by+(r-c_b)z]\int_0^y F_\xi(x)\mathrm{d}x-2r^2\int_0^y xF_\xi(x)\mathrm{d}x\right.\\
& \qquad +2(r-c_b)^2z\int_y^z F_\xi(x)\mathrm{d}x-2(r-c_b)^2\int_y^z xF_\xi(x)\mathrm{d}x\\
& \qquad \left.-[r\int_0^y F_\xi(x)\mathrm{d}x+(r-c_b)\int_y^z F_\xi(x)\mathrm{d}x]^2\right\}^{\frac{1}{2}}\\
\text{s.t.} \quad & 0\leqslant y\leqslant z.
\end{aligned} \tag{31}$$

When random demand ξ follows uniform distribution, exponential distribution, normal distribution and equiprobable discrete distribution, we have the following Theorems 3, 4, 5 and 6. Models (32), (33), (34) and (35) can be solved by LINGO software.

Theorem 3. *If ξ follows uniform distribution on the interval $[a,b]$, then model (27)-(28) can be transformed equivalently into the following model*

$$\begin{aligned}
\max \quad & \lambda\left\{(c_o+c_b-c_f)y+(r-c_o-c_b)z-\frac{1}{2(b-a)}[c_b(y-a)^2+(r-c_b)(z-a)^2]\right\}\\
& -(1-\lambda)\{\frac{1}{(b-a)}[c_b((r-c_b)z+ry)(y-a)^2+(r-c_b)^2z(z-a)^2\\
& \qquad -\frac{2}{3}r^2(y^3-a^3)+ar^2(y^2-a^2)-\frac{2}{3}(r-c_b)^2(z^3-y^3)+a(r-c_b)^2(z^2-y^2)]\\
& \qquad -[\frac{1}{2(b-a)}[c_b(y-a)^2+(r-c_b)(z-a)^2]]^2\}^{\frac{1}{2}}\\
\text{s.t.} \quad & a\leqslant y\leqslant z\leqslant b.
\end{aligned} \tag{32}$$

Theorem 4. *If ξ follows exponential distribution with parameter k, then model (27)-(28) can be transformed equivalently into the following model*

$$\begin{aligned}
\max \quad & \lambda\left\{(c_o-c_f)y-c_oz+\frac{1}{k}[r-c_be^{-ky}-(r-c_b)e^{-kz}]\right\}\\
& -(1-\lambda)\left\{\frac{1}{k}[-2rc_bye^{-ky}-2(r-c_b)[c_by+(r-c_b)z]e^{-kz}]\right.\\
& \left.+\frac{1}{k^2}[-2c_b(r-c_b)e^{-ky}+2c_b(r-c_b)e^{-kz}-[c_be^{-ky}+(r-c_b)e^{-kz}]^2+r^2]\right\}^{\frac{1}{2}}\\
\text{s.t.} \quad & 0\leqslant y\leqslant z.
\end{aligned} \tag{33}$$

Theorem 5. *If ξ follows normal distribution $N(\mu, \sigma^2)$, then model (27)-(28) can be transformed equivalently into the following model*

$$\begin{aligned}
\max\quad & \lambda\left\{(c_o+\frac{c_b}{2}-c_f)y+(\frac{(r-c_b)}{2}-c_o)z-\frac{r}{2}\int_0^y erf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x-\frac{(r-c_b)}{2}\int_y^z erf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x\right\}\\
& -(1-\lambda)\left\{\frac{1}{4}[c_by+(r-c_b)z]^2+\frac{1}{2}r[c_by+(r-c_b)z]\int_0^y erf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x\right.\\
& +\frac{1}{2}(r-c_b)[(r-c_b)z-c_by]\int_y^z erf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x\\
& -r^2\int_0^y xerf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x-(r-c_b)^2\int_y^z xerf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x\\
& \left.-\frac{1}{4}[r\int_0^y erf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x+(r-c_b)\int_y^z erf(\frac{x-\mu}{\sqrt{2}\sigma})\mathrm{d}x]^2\right\}^{\frac{1}{2}}\\
\text{s.t.}\quad & 0\leqslant y\leqslant z.
\end{aligned}\tag{34}$$

Theorem 6. *If ξ follows the following equiprobable distribution*

$$\xi\sim\begin{pmatrix}\hat{\xi}_1 & \hat{\xi}_2 & \cdots & \hat{\xi}_N\\ \frac{1}{N} & \frac{1}{N} & \cdots & \frac{1}{N}\end{pmatrix}$$

with $\hat{\xi}_1<\hat{\xi}_2<\cdots<\hat{\xi}_N$, the probability distribution function of ξ is denoted as $F(\cdot)$, then model (27)-(28) can be transformed equivalently into the following model

$$\begin{aligned}
\max\quad & \lambda\left\{[c_o+(1-F(y))c_b-c_f]y+[(1-F(z))(r-c_b)-c_o]z+\frac{r}{n}\sum_{i\in I_1}\hat{\xi}_i+\frac{(r-c_b)}{n}\sum_{i\in I_2}\hat{\xi}_i\right\}\\
& -(1-\lambda)\left\{F(y)(1-F(y))c_b^2y^2+2(1-F(z))(r-c_b)F(y)c_byz+F(z)(1-F(z))(r-c_b)^2z^2\right.\\
& -2\frac{r}{N}[[2c_f-(1-F(y))c_b-2c_o]y-[(1-F(z))(r-c_b)-2c_o]z]\sum_{i\in I_1}\hat{\xi}_i\\
& +2\frac{(r-c_b)}{N}[[2c_o-(2-F(y))c_b-2c_f]y+[(1-F(z))(r-c_b)-2c_o]z]\sum_{i\in I_2}\hat{\xi}_i\\
& \left.+\frac{r^2}{N}\sum_{i\in I_1}\hat{\xi}_i^2+\frac{(r-c_b)^2}{N}\sum_{i\in I_2}\hat{\xi}_i^2-\frac{1}{N^2}[r\sum_{i\in I_1}\hat{\xi}_i+(r-c_b)\sum_{i\in I_2}\hat{\xi}_i]^2\right\}^{\frac{1}{2}}\\
\text{s.t.}\quad & 0\leqslant y\leqslant z.
\end{aligned}\tag{35}$$

3.4 Risk Visualization

The risk expressed by standard deviation is not intuitive. We can only compare the relative sizes of its values, e.g. standard deviation 100000 runs more risk than standard deviation 10. However, how much is the risk that the distributor should burden on earth when the standard deviation is 100000? To overcome this problem, we propose the following two theorems.

The following theorem characterizes risk by using probability that profit is less than 0.

Theorem 7 (**Risk visualization theorem**). *For any given decision vector (y, z), we have the following two assertions.*

(*i*) *If $y/z > c_o/(r-c_f+c_o)$, then the critical demand such that the profit is less than zero is*

$$\xi_0=\frac{(c_f-c_o)y+c_oz}{r}.$$

(*ii*) *If $y/z < c_o/(r-c_f+c_o)$, then the critical demand such that the profit is less than zero is*

$$\xi_0=\frac{(c_f-c_o-c_b)y+c_oz}{r-c_b}.$$

In addition, the probability $\Pr\{\xi \leqslant \xi_0\}$ *can be calculated via a given probability distribution function.*

Proof. (i) If the realized value of ξ is y, and $\pi(y,z;\xi)_{\xi=y} = ry - c_f y - c_o(z-y) > 0$, namely

$$\frac{y}{z} > \frac{c_o}{r - c_f + c_o},$$

then the zero point of profit function $\pi(y,z;\xi)$ is on y's left, i.e. $\xi_0 \leqslant y$. So $r\xi_0 - c_f y - c_o(z-y) = 0$ implies

$$\xi_0 = \frac{(c_f - c_o)y + c_o z}{r}.$$

(ii) If the realized value of ξ is y, and $\pi(y,z;\xi)_{\xi=y} = ry - c_f y - c_o(z-y) < 0$, namely

$$\frac{y}{z} < \frac{c_o}{r - c_f + c_o},$$

then the zero point of profit function $\pi(y,z;\xi)$ is on y's right, i.e. $y \leqslant \xi_0 \leqslant z$. Since $r\xi_0 - c_f y - c_o(z-y) - c_b(\xi_0 - y) = 0$, one has

$$\xi_0 = \frac{(c_f - c_o - c_b)y + c_o z}{r - c_b}.$$

□

The following theorem characterizes risk by using probability that profit is less than ω ($0 < \omega < (c_o - c_f + c_b)y + (r - c_o - c_b)z$).

Theorem 8 (**General risk visualization theorem**). *For any given decision vector* (y,z)*, we have the following two assertions.*

(i) If $(r - c_f + c_o)y - c_o z > \omega$*, then the critical demand that the profit is less than* ω *is*

$$\xi_\omega = \frac{(c_f - c_o)y + c_o z + \omega}{r}.$$

(ii) If $(r - c_f + c_o)y - c_o z < \omega$*, then the critical demand that the profit is less than* ω *is*

$$\xi_\omega = \frac{(c_f - c_o - c_b)y + c_o z + \omega}{r - c_b}.$$

In addition, the probability $\Pr\{\xi \leqslant \xi_\omega\}$ *can be calculated via a given probability distribution function.*

Proof. The proof is similar to that of Theorem 7.

(i) If the profit $\pi(y,z;\xi)_{\xi=y} = ry - c_f y - c_o(z-y) > \omega$, namely $(r - c_f + c_o)y - c_o z > \omega$, then $\xi_\omega \leqslant y$. By using $r\xi_\omega - c_f y - c_o(z-y) = \omega$, one has

$$\xi_\omega = \frac{(c_f - c_o)y + c_o z + \omega}{r}.$$

(ii) If $\pi(y,z;\xi)_{\xi=y} = ry - c_f y - c_o(z-y) < \omega$, namely $(r - c_f + c_o)y - c_o z < \omega$, then $y \leqslant \xi_\omega \leqslant z$. By using $r\xi_\omega - c_f y - c_o(z-y) - c_b(\xi_\omega - y) = \omega$, we have

$$\xi_\omega = \frac{(c_f - c_o - c_b)y + c_o z + \omega}{r - c_b}.$$

□

After making the final decisions, we can use Theorem 7 or Theorem 8 to calculate the probability that profit is less than some determined level. The calculated probability is corresponding to the standard deviation that represents the risk in the original model. Based on this probability value, it is easier to judge whether the risk is acceptable and to make appropriate adjustments.

4 Numerical Experiments

In this section, we present a set of numerical experiments to demonstrate the feasibility and effectiveness of the proposed optimization methods and observe the impact of the risk preference level λ on the distributor's optimal decisions about the futures purchasing capacity y and the distributor's maximum reserve capacity z. We first give some descriptions about our natural gas supply problem in the next subsection.

4.1 Natural Gas Supply Problem

A firm, as distributor, first purchases natural gas from some natural gas processing plants, then provides natural gas for the residents to meet the needs of winter heating. The production of natural gas requires purification treatment, such as desulphurization, decarburization, dehydration. That means natural gas needs to be prefabricated. Production of natural gas requires a certain period. Therefore, it is necessary for the distributor to order a certain amount of natural gas from the supply plants.

In a warm winter, a small amount of heat supply can maintain an appropriate indoor temperature. On the other hand, a cold winter requires a lot of heat supply. Hence the distributor cannot know the exact demand of natural gas before the onset of winter. The distributor can sign an options-futures contract with the supplier before the sales season. This contract make natural gas supply meet downstream demand and achieve zero inventory as far as possible. When they sign the contract, the distributor make two initial decisions on the basis of supplier's quotation. The two initial decisions are the futures purchasing capacity and the maximum reserve capacity. When winter comes, the exact demand is known. So the distributor can make the recourse decision, i.e., whether the distributor exercises the options and how much to purchase.

According to the sales data in previous years, we suppose that the random demand ξ follows uniform distribution on the interval $[5000, 15000]$, and kilostere is the corresponding measure unit. The parameters r, c_f, c_o and c_b denote the revenue, the futures' cost, the options' reserve cost and the options' purchase cost, respectively, all these are for per kilostere natural gas. The trade contract stipulates for the settlement of balances in RMB.

This problem completely accords with the conditions of Theorem 3, we can apply Theorem 3 to solve it.

4.2 Computational Results

We adopt the following parameters used in Brown and Lee [7]: $r = 2500$, $c_f = 2000$, $c_o = 400$, $c_b = 1800$. Theorem 7 is used to transform the risk. The distributor's risk preference level λ takes values from 0.0 to 1.0 with the increment 0.1. LINGO provides the computational results which are shown in Table 1.

Table 1: Risk preference level and optimal decisions

λ	0	0.1	0.2	0.3	0.4	0.5
y	5000.000	5003.835	5018.794	5051.355	5108.644	5195.940
z	5000.000	5031.162	5151.403	5405.908	5829.107	6412.674
$E_\xi[\pi(y,z;\xi)]$	2500000.	2510080.	2548346.	2626039.	2745339.	2889687.
$\sigma_\xi[\pi(y,z;\xi)]$	0	744.6276	7947.768	34642.68	99978.32	219097.7
ξ_0	4000.000	4007.440	4036.252	4097.813	4202.189	4351.429
$Pr\{\xi \leqslant \xi_0\}$	0	0	0	0	0	0
λ	0.6	0.7	0.8	0.9	1	
y	5314.910	5465.147	5647.283	5863.126	6111.111	
z	7090.469	7769.353	8377.522	8883.360	9285.714	
$E_\xi[\pi(y,z;\xi)]$	3028245.	3135937.	3205738.	3242767.	3253968.	
$\sigma_\xi[\pi(y,z;\xi)]$	388832.2	588351.8	796599.4	1004802.	1214268.	
ξ_0	4536.018	4740.790	4954.665	5173.738	5396.825	
$Pr\{\xi \leqslant \xi_0\}$	0	0	0	0.017374	0.039683	

According to the data in Table 1, when risk preference level equals 0.8, the standard deviation is 796599.4. The value of standard deviation, which is used to characterize the risk, seems so big that its corresponding decisions cannot be chosen. While the probability that profit is less than zero is still zero. That means risk is not so big at this point. If we choose the more risk-averse attitude only by considering the standard deviation value, we will loss the opportunity to gain greater profit. The above interpretation shows that risk visualization theorem is meaningful.

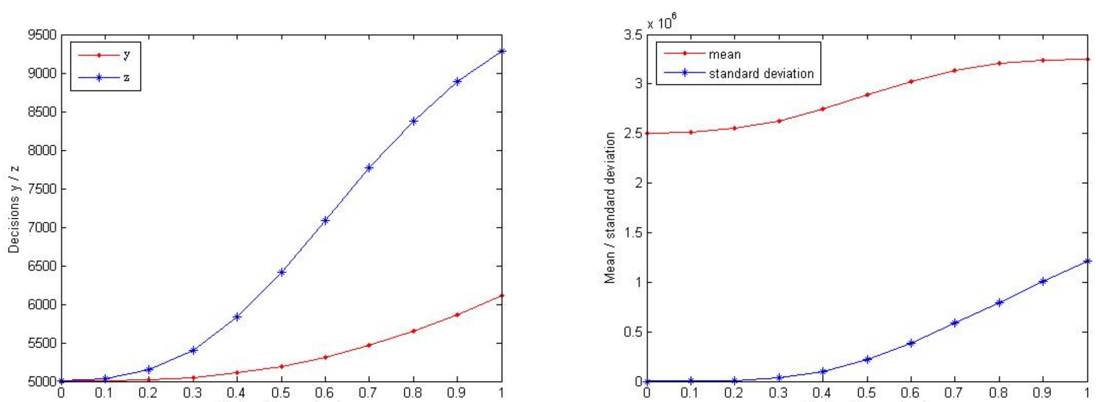
The values of c_o and c_b are changed to 100 and 2100, respectively. It can be seen that the total cost of buying each commodity which exercises options consists with the front. The other assumptions are remain unchanged. The distributor's risk preference level λ takes values from 0.0 to 1.0 with the increment 0.1. LINGO provides the computational results which are shown in Table 2.

Table 2: Risk preference level and optimal decisions

λ	0	0.1	0.2	0.3	0.4	0.5
y	5000.000	5005.377	5026.018	5069.161	5139.415	5234.249
z	5000.000	5093.272	5448.442	6174.455	7299.053	8666.695
$E_\xi[\pi(y,z;\xi)]$	2500000.	2528880.	2635643.	2838080.	3109845.	3372204.
$\sigma_\xi[\pi(y,z;\xi)]$	0.064349	2131.055	22176.81	91444.78	239323.6	453778.6
λ	0.6	0.7	0.8	0.9	1	
y	5345.413	5467.809	5604.440	5763.279	5952.381	
z	9976.380	11001.76	11713.07	12186.05	12500.00	
$E_\xi[\pi(y,z;\xi)]$	3554182.	3650689.	3695141.	3714512.	3720238.	
$\sigma_\xi[\pi(y,z;\xi)]$	673917.7	850224.7	981221.9	1089472.	1198017.	

4.3 Sensitivity Analysis

According to the data in Table 1, Figure 3 is plotted to show the impacts of the risk preference level on the initial decisions, the mean profit and standard deviation.

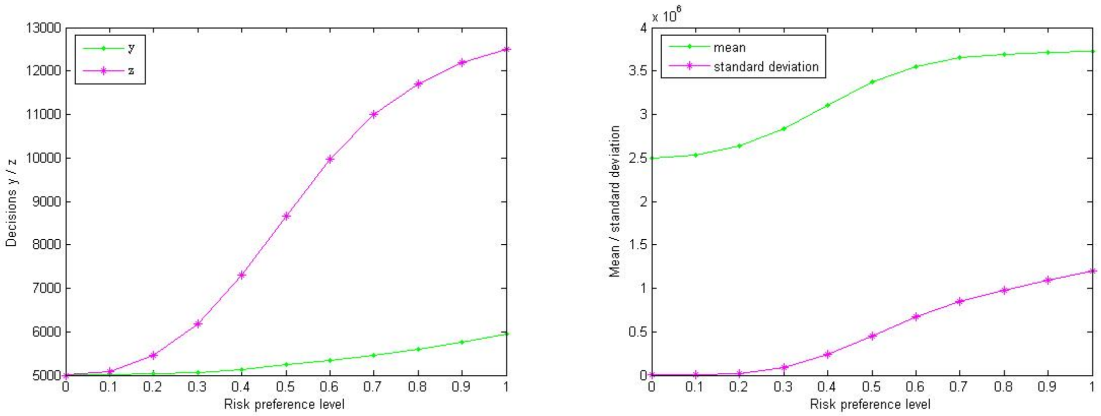


(a) (b)

Figure 3: Impact of the risk preference level λ

Figure 3(a) demonstrates intuitively the impacts of the risk preference level λ on decisions y and z. The distributor's futures purchasing capacity y and maximum reserve capacity z are both increasing with respect to the risk preference level λ. Compared with y, the growth of z is faster. When the distributor is more risk-averse, i.e. λ tends to zero, the amount of futures purchasing capacity y and the maximum reserve capacity z are mighty close. However, when the distributor is risk-neutral, i.e. λ tends to 1, the gap between futures purchasing capacity y and the maximum reserve capacity z is very big. The impacts of the risk preference level λ on the mean profit $\mathrm{E}_\xi[\pi(y,z;\xi)]$ and the standard deviation $\sigma_\xi[\pi(y,z;\xi)]$ are shown in Figure 3(b) intuitively. The distributor's mean profit $\mathrm{E}_\xi[\pi(y,z;\xi)]$ and the standard deviation $\sigma_\xi[\pi(y,z;\xi)]$ are both increasing with respect to the risk preference level λ. To obtain more mean profit, distributor must bear larger risk relatively.

According to the data reported in Table 2, Figure 4 is plotted to show the impacts of the risk preference level on the initial decisions, mean profit and standard deviation.



(a) (b)

Figure 4: Impact of the risk preference level λ

Figure 4(a) describes the impacts of the risk preference level λ on decisions y and z. Figure 4(b) describes the impacts of the risk preference level λ on the mean profit $E_\xi[\pi(y,z;\xi)]$ and the standard deviation $\sigma_\xi[\pi(y,z;\xi)]$. Obviously, Figures 4(a) and 4(b) are similar to Figures 3(a) and 3(b), and leads to the same conclusions. The decisions y and z are both increasing with respect to λ. y grows slowly but z grows fast. When λ tends to zero, the values of y and z are mighty close. While the gap between y and z is very big when λ tends to 1. The distributor's mean profit and the standard deviation are both increasing with respect to the risk preference level λ. To obtain more mean profit, distributor must bear larger risk relatively.

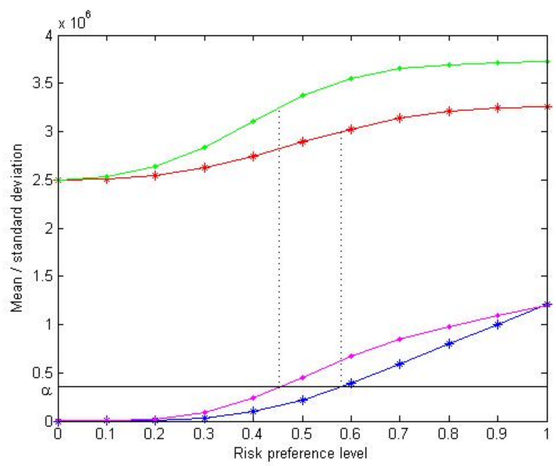


Figure 5: Profit-risk contrast

Together Figures 3(b) and 4(b), we plot Figure 5, in which α is the risk level. Figure 5 shows intuitively that: when the total cost of buying each commodity which exercises options is same, the lower the unit cost of reserve options is, the smaller the risk the distributor will bear. Under the same risk level α, the mean profit corresponding to lower reserve cost is higher than that corresponding to higher reserve cost. Therefore, a risk-averse distributor will choose the supplier with lower unit cost of reserve options, or try to cut the unit cost of reserve options in the contract.

5 Conclusions

In this paper we studied how a risk-averse distributor makes optimal ordering decisions in supply contracts when the retailers' demand is random. The major results are summarized as follows.

(*i*) We established a two-stage bi-objective stochastic model for the problem, in which the mean profit was maximized and the corresponding standard deviation was minimized. Dividing all the decisions into the initial decisions made in the first-stage and the recourse decision made in the second-stage facilitates distributor to meet customers' demand and achieve zero inventory. This decision process controls commodity supply within a certain range by the contract, so it effectively restrains the bullwhip effect.

(*ii*) The optimal value expression of the second-stage programming problem was derived. As a consequence, the proposed two-stage bi-objective model was equivalent to a single-stage bi-objective model. We used the weight coefficient method to deal with the model's objectives, then transformed the equivalent single-stage bi-objective model into a single-stage single objective model.

(*iii*) In the cases that random demand follows discrete distribution and common continuous distributions, we derived the deterministic expressions of the mean profit and the corresponding standard deviation. By solving the equivalent models given in Theorem 3 (uniform distribution), Theorem 4 (exponential distribution), Theorem 5 (normal distribution) and Theorem 6 (discrete distribution), the optimal decisions of the distributor can be obtained.

(*iv*) Based on risk visualization theorems (Theorems 7 and 8), it is easier to judge whether the risk is acceptable and to make appropriate adjustments.

(*v*) From the computational results of the numerical experiments (Figures 3-5), the distributor's futures purchasing capacity and maximum reserve capacity are both increasing with respect to the risk preference level, the former's growth is slower than that of the latter. The distributor's mean profit and the standard deviation are also increasing with respect to the risk preference level. Under the same conditions, the lower unit cost of reserve options is more favorable for distributors.

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