Journal of Uncertain Systems
Vol.9, No.4, pp.306-320, 2015
Online at: www.jus.org.uk

Cost Varying Multi-objective Interval Transportation Problem under N-Vehicle

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Received 7 January 2014; Revised 17 June 2015

Abstract

In this paper, we focus on the problem formulation and solution procedure of the multi-objective interval transportation problem(MOITP) where the source and destination parameters are expressed as interval values by the decision maker. This problem is a cost varying multi-objective interval transportation problem(CVMOITP). The intervals cost coefficients of the objective functions of this problem are determined by fixed single trip transportation costs of vehicles. After that, this problem is converted into a classical multi-objective transportation problem(MOTP) where to minimize the interval objective function, the order relations that represent the decision marker's preference between interval is defined by the right limit, centre and half-width of an interval. Finally, the equivalent transformed problem is solved by Zimmerman's fuzzy programming technique.

Keywords: multi-objective interval transportation problem(MOITP), cost varying multi-objective interval transportation problem(CVMOITP), bi-level programming, fuzzy programming

1 Introduction

Transportation problem(TP) is a linear programming problem. TP deals with the distribution of single commodity from various sources to various destination in such a manner that the total transportation cost is minimized. In order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the model must be fixed at crisp values but in real life applications they are interval valued.

A interval transportation problem is such a transportation problem in which the supply, demand and cost parameters are lied in some intervals. This problem is transformed into a classical bi-objective TP where to minimize the interval objective function, the order relations that represent the decision marker's preference between interval profits is defined by the right limit, left limit, centre, and half-width of an interval.

In transportation problem unit transportation cost is constant from each source to each destination. But in reality, it is not constant; it depends on amount of transport quantity and capacity of vehicles. If amount of quantity is small then small(capacity) vehicle is sufficient for deliver. Where as if amount of quantity is large then big(capacity) vehicle is needed. So, depend on amount of transport quantity and the capacity of vehicles, the unit transportation cost is not constant. The cost varying transportation problem is such a transportation problem where unit transportation cost is varied depending on the selection of vehicles and number of vehicles.

The objective of this paper is to develop a transportation problem whose supplies and demands are interval values but varying interval value unit transportation cost. In each cell (i, j), the interval costs are determined by fixed single trip transportation costs of vehicles. Finally, present a solution procedure of this type of problem with a numerical example.

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The basic transportation problem was originally developed by Hitchock [14] and letter by Dantzig [6]. Many researchers [13, 15, 18] did work on fixed charge transportation problem. Gupta and Arora [8] presented a capacitated fixed charge bi-criterion indefinite quadratic transportation problem, giving the same priority to cost as well as time is studied. They developed an algorithm which is based on the concept of solving the indefinite quadratic fixed charge transportation problem. Gupta and Arora [11] discussed on a paradox in a capacitated transportation problem where the objective function is a ratio of two linear functions consisting of variable costs and profits respectively. In another paper, Gupta and Arora [9, 10] discussed on restricted flow in a fixed charge capacitated transportation problem with bounds on total source availabilities and total destination requirements. Dahiya and Verma [5] considered a class of the capacitated transportation problems with bounds on total availabilities at sources and total destination requirements. In this paper, unbalanced capacitated transportation problems have been discussed in the present paper as a particular case of original problem. In addition, they have discussed paradoxical situation in a balanced capacitated transportation problem and have obtained the paradoxical solution by solving one of the unbalanced problems. Arora and Ahuja [1] discussed a paradox in fixed charge transportation problem. Then Arora and Khurana [2] introduced three-dimensional fixed charge transportation problem is an extension of the classical three-dimensional transportation problem in which a fixed cost is incurred for every origin. Basu et al. [3] represented an algorithm for finding the optimum solution of solid fixed charge transportation problem. Then Bit, et. al. developed fuzzy programming technique for multi objective capacitated transportation problem. Singh and Saxena [16] introduced the multiobjective time transportation problem with additional restrictions. Recently, Dutta and Murthy [7] developed fuzzy transportation problem with additional restrictions.

Here we present interval transportation problem. In reality, the interval of the unit cost depending on the interval of sources and demands. In urban region , actually the transportation cost is not depended on the quantities but on the capacity of the transports. So unit cost is vary depended on vehicles. In this paper we determine interval of the parameters of unit cost by our proposed algorithm which develops a multi-level uncertain programming model. Then formulate corresponding multi-objective crisp model. There are various type of methods to solve this type of model, but best one is fuzzy programming technique [20] which is applied here.

2 Mathematical Formulation

2.1 Multi-objective Interval Transportation Problem(MOITP)

The formulation of MOITP is the problem of minimizing interval valued objective function with interval costs, interval sources and interval demands parameters, is given in the following **Model 1.**

Model 1

$$\begin{aligned}
\min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}, \quad k=1,\ldots,K \\
\text{subject to} \quad & c_{ij}^{k} \in [D_{L_{ij}}^{k}, D_{R_{ij}}^{k}], \quad k=1,\ldots,K \\
& \sum_{j=1}^{n} x_{ij} \in [a_{L_i}, a_{R_i}], \quad i=1,\ldots,m \\
& \sum_{i=1}^{m} x_{ij} \in [b_{L_j}, b_{R_j}], \quad j=1,\ldots,n \\
& \sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \\
& \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j} \\
& x_{ij} \geq 0 \quad \forall i, \quad \forall j
\end{aligned}$$

where $c_{ij}^{k} \in [D_{L_{ij}}^{k}, D_{R_{ij}}^{k}], \quad k=1,\ldots,K$ are intervals representing the uncertain cost for transportation problem. The sources parameter lies in $[a_{L_i}, a_{R_i}]$ and destination parameter lies in $[b_{L_j}, b_{R_j}]$.

Depending on $[a_{L_i}, a_{R_i}]$ and $[b_{L_j}, b_{R_j}]$. We determine $c^k_{L_{ij}}$ and $c^k_{R_{ij}}$ which is discussed in the following subsection. Then we define $D_{L^k_{ij}=\min\{c^k_{L_{ij}}, c_{R^k_{ij}}\}}$ and $D_{R^k_{ij}=\max\{c^k_{L_{ij}}, c^k_{R_{ij}}\}}$.

2.2 N-Vehicle Cost Varying Multi-objective Transportation Problem

Abbreviations:
$[a_{L_i}, a_{R_i}]$: Availability at Origin/Source $O_i, i = 1, \ldots, m$.
$[b_{L_i}, b_{R_i}]$: Demand at destination $D_j, j = 1, \ldots, n$.
N : Number of vehicles.
V_l : l^{th} vehicle, $l = 1, \ldots, N$.
K : Number of objectives.
$R^r_{ij}(l)$: Transportation cost of single trip of l^{th} vehicle in the $r^{th}, (r = 1, \ldots, K)$ objective in the cell (i, j).
$c^r_{ij} \in [D^r_{L_{ij}}, D^r_{R_{ij}}]$: Unit transportation cost at the cell (i, j) r^{th} objective, where $D_{L^r_{ij}=\min\{c^r_{L_{ij}}, c_{R^r_{ij}}\}}$ and $D_{R^r_{ij}=\max\{c^r_{L_{ij}}, c^r_{R_{ij}}\}}$.
C_l : Capacity(in unit) of the vehicle $V_l, l = 1, \ldots, N$, where $C_1 < C_2 < \cdots < C_N$.

Suppose there are N-types off vehicles $V_l, l = 1, \ldots, N$ from each source to each destination. Let $C_l, l = 1, \ldots, N$ are the capacities (in unit) of the vehicles $V_l, l = 1, \ldots, N$ respectively, where $C_1 < C_2 < \cdots < C_N$. $(R^r_{ij}(1), \ldots, R^r_{ij}(N)), r = 1, \ldots, K$ represent transportation cost for each cell (i, j); where $R^r_{ij}(l), r = 1, \ldots, K; l = 1, \ldots, N$ are the transportation cost from source $O_i, i = 1, \ldots, m$ to the destination $D_j, j = 1, \ldots, n$ by the vehicle V_l. So, for each $r = 1, \ldots, K$ cost varying TP can be represent in the following tabulated form. $c^r_{ij}, i = 1, \ldots, m; j = 1, \ldots, n; r = 1, \ldots, k$ are not constants.

Table 1: N-vehicle multi-objective cost varying transportation problem

	D_1	D_2	...	D_n	*stock*
O_1	$R^r_{11}(1), \ldots, R^r_{11}(N)$	$R^r_{12}(1), \ldots, R^r_{12}(N)$	...	$R^r_{1n}(1), \ldots, R^r_{1n}(N)$	$[a_{L_1}, a_{R_1}]$
O_2	$R^r_{21}(1), \ldots, R^r_{21}(N)$	$R^r_{22}(1), \ldots, R^r_{22}(N)$	...	$R^r_{2n}(1), \ldots, R^r_{2n}(N)$	$[a_{L_2}, a_{R_2}]$
...	...	...	...	...	...
O_m	$R^r_{m1}(1), \ldots, R^r_{m1}(N)$	$R^r_{m2}(1), \ldots, R^r_{m2}(N)$	...	$R^r_{mn}(1), \ldots, R^r_{mn}(N)$	$[a_{L_m}, a_{R_m}]$
Demand	$[b_{L_1}, b_{R_1}]$	$[b_{L_2}, b_{R_2}]$	...	$[b_{L_n}, b_{R_n}]$	

2.2.1 Determination of $c^r_{L_{ij}}$

To solve this problem, apply our proposed **Algorithm** stated as follows:
Algorithm A1:
Step 1. Since lower limit of unit cost is not determined (because it depends on quantity of transport), so North-west corner rule (because it does not depend on unit transportation cost) is applicable to allocate initial B.F.S.
Step 2. After the allocate x_{ij} by North-west corner rule, for basic cell we determine $c^r_{L_{ij}}$ (unit transportation cost from source O_i to destination D_j) as

$$c^r_{L_{ij}} = \begin{cases} \frac{\sum_l L^r_{ij}(l) R^r_{ij}(l)}{x_{ij}} & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0, \end{cases} \qquad r = 1, \ldots, k$$

where $L^r_{ij}(l)$ are integer solutions of

$$\begin{aligned} \min \quad & \sum_l L^r_{ij}(l) R^r_{ij}(l) \\ \text{s.t.} \quad & x_{ij} \leq \sum_l L^r_{ij}(l) C_l. \end{aligned}$$

Step 3. For non-basic cell (i, j) possible allocation is the minimum of allocations in i^{th} row and j^{th} column (for possible loop). If possible allocation be x_{ij}, then for non-basic cell $c^r_{L_{ij}}$ (unit transportation cost from source O_i to destination D_j) as

$$c^r_{L_{ij}} = \begin{cases} \frac{\sum_l L^r_{ij}(l)R^r_{ij}(l)}{x_{ij}} & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0, \end{cases} \qquad r = 1, \ldots, k$$

where $L^r_{ij}(l)$ are integer solutions of

$$\begin{aligned} \min \quad & \sum_l L^r_{ij}(l)R^r_{ij}(l) \\ \text{s.t.} \quad & x_{ij} \leq \sum_l L^r_{ij}(l)C_l. \end{aligned}$$

In this manner we convert cost varying transportation problem to a usual transportation problem but $c^r_{L_{ij}}$ is not fixed, it may be changed (when this allocation will not serve optimal value) during optimality test.
Step 4. During optimality test some basic cell changes to non-basic cell and some non-basic cell changes to basic cell, depends on running basic cell we first fix $c^r_{L_{ij}}$ by **Step 2** and for non-basic we fix $c^r_{L_{ij}}$ by **Step 3**.
Step 5. Repeat **Step 2.** to **Step 4.** until we obtain optimal solution.
Thus to determine $c^r_{L_{ij}}$ we solve the following bi-level programming model**Model 2.L** which is as follows:
Model 2.L

$$\min \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c^r_{L_{ij}} x_{ij}, \tag{1}$$

$$c^r_{L_{ij}} = \begin{cases} \frac{\sum_l L^r_{ij}(l)R^r_{ij}(l)}{x_{ij}} & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases}$$

where $c^r_{L_{ij}}$ is determined by following mathematical programming

$$\begin{aligned} \min \quad & \sum_l L^r_{ij}(l)R^r_{ij}(l) \\ \text{s.t.} \quad & x_{ij} \leq \sum_l L^r_{ij}(l)C_l \\ & \sum_{j=1}^{n} x_{ij} = a_{L_i}, \quad i = 1, \ldots, m \\ & \sum_{i=1}^{m} x_{ij} = b_{L_j}, \quad j = 1, \ldots, n \\ & \sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \\ & 0 \leq x_{ij} \quad \forall i, \quad \forall j \end{aligned}$$

where $L^r_{ij}(l), i = 1, \ldots, m; j = 1, \ldots, n$ are integers.

2.2.2 Determination of $c^r_{R_{ij}}$

To solve this problem, apply our proposed algorithm stated as follows:
Algorithm A2:
Step 1. Since lower limit of unit cost is not determined (because it depends on quantity of transport), so North-west corner rule (because it does not depend on unit transportation cost) is applicable to allocate initial B.F.S.
Step 2. After the allocate x_{ij} by North-west corner rule, for basic cell we determine $c^r_{R_{ij}}$ (unit transportation

cost from source O_i to destination D_j) as

$$c^r_{R_{ij}} = \begin{cases} \frac{\sum_l R^r_{ij}(l)R^r_{ij}(l)}{x_{ij}} & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0, \end{cases} \quad r = 1, \ldots, k$$

where $R^r_{ij}(l)$ are integer solutions of

$$\min \quad \sum_l R^r_{ij}(l)R^r_{ij}(l)$$

$$\text{s.t.} \quad x_{ij} \leq \sum_l R^r_{ij}(l)C_l.$$

Step 3. For non-basic cell (i,j) possible allocation is the minimum of allocations in i^{th} row and j^{th} column (for possible loop). If possible allocation be x_{ij}, then for non-basic cell $c^r_{R_{ij}}$ (unit transportation cost from source O_i to destination D_j) as

$$c^r_{R_{ij}} = \begin{cases} \frac{\sum_l R^r_{ij}(l)R^r_{ij}(l)}{x_{ij}} & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0, \end{cases} \quad r = 1, \ldots, k$$

where $R^r_{ij}(l)$ are integer solutions of

$$\min \quad \sum_l R^r_{ij}(l)R^r_{ij}(l)$$

$$\text{s.t.} \quad x_{ij} \leq \sum_l R^r_{ij}(l)C_l.$$

In this manner we convert cost varying transportation problem to a usual transportation problem but $c^r_{R_{ij}}$ is not fixed, it may be changed (when this allocation will not serve optimal value) during optimality test.
Step 4. During optimality test some basic cell changes to non-basic cell and some non-basic cell changes to basic cell, depends on running basic cell we first fix $c^r_{R_{ij}}$ by **Step 2** and for non-basic we fix $c^r_{R_{ij}}$ by **Step 3**.
Step 5. Repeat **Step 2.** to **Step 4.** until we obtain optimal solution.

Thus to determine $c^r_{R_{ij}}$ we solve the following bi-level programming model**Model 2.R** which is as follows:
Model 2.R

$$\min \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c^r_{R_{ij}} x_{ij}, \tag{2}$$

$$c^r_{R_{ij}} = \begin{cases} \frac{\sum_l R^r_{ij}(l)R^r_{ij}(l)}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases}$$

where $c^r_{R_{ij}}$ is determined by following mathematical programming

$$\min \quad \sum_l R^r_{ij}(l)R^r_{ij}(l)$$

$$\text{s.t.} \quad x_{ij} \leq \sum_l R^r_{ij}(l)C_l$$

$$\sum_{j=1}^{n} x_{ij} = a_{R_i}, \quad i = 1, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_{R_j}, \quad j = 1, \ldots, n$$

$$\sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}$$

$$0 \leq x_{ij} \quad \forall i, \quad \forall j$$

where $R^r_{ij}(l), i = 1, \ldots, m; j = 1, \ldots, n$ are integers.

2.2.3 Multi-level Mathematical Programming for Cost Varying Multi-objective Interval Transportation Problem under N-Vehicle (CVMOITPNV)

The Multi-level Mathematical Programming for Cost Varying Interval Transportation Problem under N-Vehicle is formulated in **Model 3** as follows:
Model 3

$$\begin{aligned}
\min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r} x_{ij} \\
\text{subject to} \quad & c_{ij}^{r} \in [D_{L_{ij}}^{r}, D_{R_{ij}}^{r}] \\
& \sum_{j=1}^{n} x_{ij} \in [a_{L_i}, a_{R_i}], \quad i = 1, \ldots, m \\
& \sum_{i=1}^{m} x_{ij} \in [b_{L_j}, b_{R_j}], \quad j = 1, \ldots, n \\
& \sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \\
& \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j} \\
& x_{ij} \geq 0 \ \ \forall i, \ \ \forall j \\
& \textbf{Model 2.L} && (3) \\
& \textbf{Model 2.R} && (4)
\end{aligned}$$

i.e., **Model 3**

$$\begin{aligned}
\min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r} x_{ij} \\
\text{subject to} \quad & c_{ij}^{r} \in [D_{L_{ij}}^{r}, D_{R_{ij}}^{r}] \\
& \sum_{j=1}^{n} x_{ij} \in [a_{L_i}, a_{R_i}], \quad i = 1, \ldots, m \\
& \sum_{i=1}^{m} x_{ij} \in [b_{L_j}, b_{R_j}], \quad j = 1, \ldots, n \\
& \sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \\
& \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j} \\
& x_{ij} \geq 0 \ \ \forall i, \ \ \forall j
\end{aligned}$$

where $D^{r}_{L_{ij}=\min\{c^{r}_{L_{ij}}, c^{r}_{R_{ij}}\}}$ and $D^{r}_{R_{ij}=\max\{c^{r}_{L_{ij}}, c^{r}_{R_{ij}}\}}$. And $[c^{r}_{L_{ij}}, c^{r}_{R_{ij}}]$ is determined by following mathematical programming

$$\min \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_{L_{ij}}^{r} x_{ij}, \tag{5}$$

where $c^{r}_{L_{ij}}$ is determined by following mathematical programming

$$c_{L_{ij}}^{r} = \begin{cases} \frac{\sum_{l} L_{ij}^{r}(l) R_{ij}^{r}(l)}{x_{ij}}, & \text{if} \ \ x_{ij} \neq 0 \\ 0 & \text{if} \ \ x_{ij} = 0 \end{cases}$$

$$
\begin{aligned}
\min \quad & \sum_l L^r_{ij}(l) R^r_{ij}(l) \\
\text{s.t.} \quad & x_{ij} \le \sum_l L^r_{ij}(l) C_l \\
& \sum_{j=1}^{n} x_{ij} = a_{L_i}, \quad i = 1, \dots, m \\
& \sum_{i=1}^{m} x_{ij} = b_{L_j}, \quad j = 1, \dots, n \\
& \sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \\
& 0 \le x_{ij} \quad \forall i, \quad \forall j \\
& \text{where } L^r_{ij}(l), i = 1, \dots, m; j = 1, \dots, n \text{ are integers}
\end{aligned}
$$

AND

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{m} \sum_{j=1}^{n} c^r_{R_{ij}} x_{ij}, \qquad (6) \\
& \text{where } c^r_{R_{ij}} \text{ is determined by following mathematical programming} \\
& c^r_{R_{ij}} = \begin{cases} \frac{\sum_l R^r_{ij}(l) R^r_{ij}(l)}{x_{ij}}, & \text{if } \; x_{ij} \ne 0 \\ 0 & \text{if } \; x_{ij} = 0 \end{cases} \\
\min \quad & \sum_l R^r_{ij}(l) R^r_{ij}(l) \\
\text{s.t.} \quad & x_{ij} \le \sum_l R^r_{ij}(l) C_r \\
& \sum_{j=1}^{n} x_{ij} = a_{R_i}, \quad i = 1, \dots, m \\
& \sum_{i=1}^{m} x_{ij} = b_{R_j}, \quad j = 1, \dots, n \\
& \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j} \\
& 0 \le x_{ij} \quad \forall i, \quad \forall j \\
& \text{where } R^r_{ij}(l), i = 1, \dots, m; j = 1, \dots, n \text{ are integers.}
\end{aligned}
$$

3 Solution Procedure of CVMOITPNV

3.1 Some Definitions

Definition 1. Interval: A closed interval is defined by an order pair of brackets as:

$$A = [a_L, a_R] = \{a : a_L \le a \le a_R, a \in R\}$$

where a_L and a_R are, respectively, the left and right limits of A.

The interval is also denoted by its centre and half width as

$$A = \langle a_c, a_w \rangle = \{a : a_c - a_w \le a \le a_c + a_w, a \in R\}$$

where $a_c = \frac{a_R + a_L}{2}$ and $a_w = \frac{a_R - a_L}{2}$ are respectively, the centre and half width of A.

Definition 2. Operators: If A and B are two closed intervals, and $*$ be a binary operation on the set of real number, then $A * B = \{a * b : a \in A, b \in B\}$ is defined a binary operation.

According to the above definition interval operations are defined as:

$$A + B = [a_L, a_R] + [b_L, b_R] = [a_L + b_L, a_R + b_R],$$

$$A + B = \langle a_c, a_w \rangle + \langle b_c, b_w \rangle = \langle a_c + b_c, a_w + b_w \rangle,$$

$$kA = k[a_L, a_R] = [ka_L, ka_R] \quad if \quad k \geq 0,$$

$$kA = k[a_L, a_R] = [ka_R, ka_L] \quad if \quad k \leq 0$$

where k is a real number.

Definition 3. Order relation $\leq_{LR}$: The order relation $\leq_{LR}$ between $A = [a_L, a_R]$ and $B = [b_L, b_R]$ is defined as $A \leq_{LR} B$ iff $a_L \leq b_L$ and $a_R \leq b_R$, $A <_{LR} B$ iff $A \leq_{LR} B$ and $A \neq B$.

Definition 4. Order relation $\leq_{cw}$: The order relation $\leq_{cw}$ between $A = \langle a_c, a_w \rangle$ and $B = \langle b_c, b_w \rangle$ is defined as $A \leq_{cw} B$ iff $a_c \leq b_c$ and $a_w \leq b_w$, $A <_{cw} B$ iff $A \leq_{cw} B$ and $A \neq B$.

3.2 Formulation of the Crisp Objective Function

Let S be the set of all feasible solution of **Model 3**.

Definition 5. Optimal Solution: For each $r = 1, \ldots, K$, $x^0 \in S$ is an optimal solution of the **Model 3** iff there is no other solution $x \in S$ which satisfies $Z^r(x) <_{LR} Z^r(x^0)$ or $Z^r(x) <_{cw} Z^r(x^0)$.

Definition 6. Order relation $\leq_{Rc}$: The order relation $\leq_{Rc}$ between A and B is defined as $A \leq_{Rc} B$ iff $A \leq_{LR} B$ and $A \leq_{cw} B$, $A <_{Rc} B$ iff $A <_{LR} B$ and $A <_{cw} B$.

Definition 7. Optimal Solution:$r = 1, \ldots, K$, $x^0 \in S$ $x^0 \in S$ is an optimal solution of the **Model 3** iff there is no other solution $x \in S$ which satisfies $Z^r(x) <_{Rc} Z^r(x^0)$ or $Z^r(x) <_{cw} Z^r(x^0)$.

For each $r = 1, \ldots, K$, the right limit $Z_R^r(x)$ of the interval objective function $Z^r(x)$ in given problem may be elicited as $Z_R^r(x) = \sum_{i=1}^m \sum_{j=1}^n D_{c_{ij}}^r x_{ij} + \sum_{i=1}^m \sum_{j=1}^n D_{w_{ij}}^r x_{ij}$. And the centre of the objective function $Z_c^r(x)$ can be elicited as $Z_c^r(x) = \sum_{i=1}^m \sum_{j=1}^n D_{c_{ij}}^r x_{ij}$.

The solution of the **Model 3** by Definition 7 can be taken as the optimal solution of following model **Model 4.**

Model 4

$$\min \quad Z_R^r(x) = \sum_{i=1}^m \sum_{j=1}^n D_{c_{ij}}^r x_{ij} + \sum_{i=1}^m \sum_{j=1}^n D_{w_{ij}}^r x_{ij}, \ r = 1, \ldots, K \tag{7}$$

$$\min \quad Z_c^r(x) = \sum_{i=1}^m \sum_{j=1}^n D_{c_{ij}}^r x_{ij}, \ r = 1, \ldots, K \tag{8}$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{j=1}^n x_{ij} \in [a_{L_i}, a_{R_i}], \quad i = 1, \ldots, m \\ & \sum_{i=1}^m x_{ij} \in [b_{L_j}, b_{R_j}], \quad j = 1, \ldots, n \\ & \sum_{i=1}^m a_{L_i} = \sum_{j=1}^n b_{L_j} \\ & \sum_{i=1}^m a_{R_i} = \sum_{j=1}^n b_{R_j} \\ & x_{ij} \geq 0 \ \ \forall i, \ \ \forall j \end{aligned}$$

$$\textbf{Model 2.L} \tag{9}$$

$$\textbf{Model 2.R} \tag{10}$$

Model 4 is a multi-objective model. Therefore it is not easy to solve by interval optimization method or stochastic optimization method. We can solved it easily by transform it into a single objective model by many methods like scaler method, weighting method, rank method etc. One of the better method is fuzzy optimization technique which is discussed in following subsection.

3.2.1 Fuzzy Programming Technique to Solve Model 4

In fuzzy programming technique, we first find the lower bound as LZ_R and the upper bound as UZ_R for the r^{th} objective function $Z_R(x)$. Similarly the lower bound as LZ_c and the upper bound as UZ_c for the r^{th} objective function $Z_c(x)$. dZ_R =UZ_R - LZ_R the degradation allowance for objective $Z_R(x)$. dZ_c =UZ_c - LZ_{Rc} the degradation allowance for objective $Z_c(x)$

When the aspiration levels for each of the objective have been specified, a fuzzy model is formed and then the fuzzy model is converted into a crisp model. The solution of **Model 4** can be obtained by the following steps:

step 1. Solve the **Model 4** as a single-objective transportation problem 2 times by taking one of the objective at a time.

step 2. From the above results, determine the corresponding values for objective at each solution derived. According to each solution and value for every objective, we can find a pay-off matrix as follows:

Table 2: Objective values

	Z_R	Z_c
x^1	Z_{1R}	Z_{1c}
x^2	Z_{2R}	Z_{2c}

where x^1, x^2 are the isolated optimal solutions of the k different transportation problems for 2 different objective functions. $Z_{1R}, Z_{1c}, Z_{2R}, Z_{2c}$ are the values of objective functions.

step 3. From **Step 2,** find for each objective the U_r and the L_r corresponding to the set of solutions, where $UZ_R = \max\{Z_{1R}, Z_{2R}\}$, $LZ_R = \min\{Z_{1R}, Z_{2R}\}$, $UZ_c = \max\{Z_{1c}, Z_{2c}\}$, $LZ_c = \min\{Z_{1c}, Z_{2c}\}$.

An initial fuzzy model of the problem can be:

Find $x_{ij},\ i = 1, \ldots, m;\ j = 1, \ldots, n,$

$$
\begin{aligned}
& Z_R \preceq LZ_R, && (11)\\
& Z_c \preceq LZ_c && (12)\\
\text{s.t.}\quad & \sum_{j=1}^{n} x_{ij} \in [a_{L_i}, a_{R_i}], \quad i = 1, \ldots, m \\
& \sum_{i=1}^{m} x_{ij} \in [b_{L_j}, b_{R_j}], \quad j = 1, \ldots, n \\
& \sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \\
& \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j} \\
& x_{ij} \geq 0 \ \ \forall i, \ \ \forall j \\
& \textbf{Model 2.L} \\
& \textbf{Model 2.R}
\end{aligned}
$$

step 4. Define a membership function $\mu(Z_R), \mu(Z_c)$, for the Z_R, Z_c respectively.

step 5. Convert the fuzzy model of the problem, obtained in **Step 3,** into the following crisp model, namely, **Model 5.**

Model 5

$$
\begin{aligned}
\max \quad & \lambda && (13)\\
\text{subject to} \quad & \lambda \le \mu(Z_R) && (14)\\
& \lambda \le \mu(Z_c) && (15)\\
& \sum_{j=1}^{n} x_{ij} \in [a_{L_i}, a_{R_i}], \quad i = 1, \ldots, m\\
& \sum_{i=1}^{m} x_{ij} \in [b_{L_j}, b_{R_j}], \quad j = 1, \ldots, n\\
& \sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j}\\
& \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}\\
& x_{ij} \ge 0 \quad \forall i, \quad \forall j\\
& \textbf{Model 2.L}\\
& \textbf{Model 2.R}\\
& \lambda \ge 0
\end{aligned}
$$

step 6. Solve the crisp model by an appropriate mathematical programming algorithm.
step 7. The solution obtained in **step 6** will be optimal compromise solution of the **Model 4**.

3.2.2 Fuzzy Programming Technique with Linear Membership Function

A linear membership function is defined as

$$
\mu_1(Z_R) = \begin{cases} 1 & \text{if } Z_R \le LZ_R \\ 1 - \frac{Z_R - LZ_R}{UZ_R - LZ_R} & \text{if } LZ_R \le Z_R \le UZ_R \\ 0 & \text{if } UZ_R \le Z_R, \end{cases} \qquad (16)
$$

and

$$
\mu_2(Z_c) = \begin{cases} 1 & \text{if } Z_c \le LZ_c \\ 1 - \frac{Z_c - LZ_c}{UZ_c - LZ_c} & \text{if } LZ_c \le Z_c \le UZ_c \\ 0 & \text{if } UZ_c \le Z_c. \end{cases} \qquad (17)
$$

If we use a linear membership function, the crisp model can be simplified in **Model 6** as follows:
Model 6

$$
\begin{aligned}
\max \quad & \lambda && (18)\\
\text{subject to} \quad & Z_R + \lambda(UZ_R - LZ_R) \le UZ_R,\\
& Z_c + \lambda(UZ_c - LZ_c) \le UZ_c,\\
& \sum_{j=1}^{n} x_{ij} \in [a_{L_i}, a_{R_i}], \quad i = 1, \ldots, m\\
& \sum_{i=1}^{m} x_{ij} \in [b_{L_j}, b_{R_j}], \quad j = 1, \ldots, n\\
& \sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j}\\
& \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}
\end{aligned}
$$

$$x_{ij} \geq 0 \quad \forall i, \quad \forall j$$
Model 2.L
Model 2.R
$$\lambda \geq 0.$$

3.2.3 Fuzzy Programming Technique with Exponential Membership Function

Considering $Z_1 = Z_R, Z_2 = Z_c, U_1 = UZ_R, U_2 = UZ_c, L_1 = LZ_R, L_2 = LZ_c$, a exponential membership function is defined as

$$\mu_l(Z_r) = \begin{cases} 1 & \text{if } Z_r \leq L_r \\ 1 - \frac{e^{-s\frac{Z_r - L_r}{U_r - L_r}} - e^{-s}}{1 - e^{-s}} & \text{if } L_r \leq Z_r \leq U_r \quad r = 1, 2. \\ 0 & \text{if } U_r \leq Z_r, \end{cases} \tag{19}$$

If we use a exponential membership function, the crisp model can be simplified as **Model 7**.
Model 7

$$\begin{aligned} \max \quad & \lambda \\ \text{subject to} \quad & \\ & e^{-s\frac{Z_r - L_r}{U_r - L_r}} - \lambda(1 - e^{-s}) \geq e^{-s}, \ r = 1, \ldots, k \\ & \sum_{j=1}^{n} x_{ij} \in [a_{L_i}, a_{R_i}], \quad i = 1, \ldots, m \\ & \sum_{i=1}^{m} x_{ij} \in [b_{L_j}, b_{R_j}], \quad j = 1, \ldots, n \\ & \sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \\ & \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j} \\ & x_{ij} \geq 0 \quad \forall i, \quad \forall j \\ & \textbf{Model 2.L} \\ & \textbf{Model 2.R} \\ & \lambda \geq 0. \end{aligned} \tag{20}$$

4 Numerical Examples

Example 1. The data is collected by a person who supplies products to different companies after tacking different sources. There are three different suppliers(origins) named as O_1, O_2, O_3 and three different destinations namely D_1, D_2, D_3. How much amount of materials be supplied from different sources to all other destinations so that transportation cost(s) are minimum. Data are given below in the following Table 3 and Table 4.

Consider a 2-vehicle cost varying transportation problem as

It is also given that there are three types of vehicle V_1, V_2 and V_3. For each $r = 1, 2$ the cost of V_1 from source 'i' destination 'j' is $R^r_{ij}(1)$ for a single trip. The cost of V_2 from source 'i' destination 'j' is $R^r_{ij}(2)$ and that of V_3 is $R^r_{ij}(3)$ for a single trip. It is also given that the capacity of V_1 is $C_1 = 10$ and that of V_2 and V_3 are $C_2 = 15$ and $C_3 = 20$, respectively.

Then we have by **Model 2.L**

$c^1_{L11} = \frac{119}{145}, c^1_{L12} = \frac{14}{5}, c^1_{L13} = \frac{15}{5}, c^1_{L21} = \frac{103}{145}, c^1_{L22} = \frac{95}{145}, c^1_{L23} = \frac{12}{15}, c^1_{L31} = \frac{114}{120}, c^1_{L32} = \frac{126}{120}, c^1_{L33} = \frac{72}{120},$

and

Table 3: Lower bounds of cost intervals of the vehicles of Example 1

	D_1	D_2	D_3	*stock*
O_1	$10, 13, 16$	$14, 17, 20$	$15, 18, 21$	$[150, 170]$
O_2	$8, 11, 14$	$7, 10, 13$	$9, 12, 15$	$[160, 190]$
O_3	$13, 16, 19$	$15, 18, 21$	$6, 9, 12$	$[120, 130]$
Demand	$[145, 165]$	$[150, 170]$	$[135, 155]$	

Table 4: Upper bounds of cost intervals of the vehicles of Example 1

	D_1	D_2	D_3	*stock*
O_1	$14, 17, 20$	$18, 21, 24$	$20, 23, 26$	$[150, 170]$
O_2	$15, 18, 21$	$8, 11, 14$	$10, 13, 16$	$[160, 190]$
O_3	$16, 19, 22$	$19, 22, 25$	$6, 9, 12$	$[120, 130]$
Demand	$[145, 165]$	$[150, 170]$	$[135, 155]$	

$$c^2_{L11} = \tfrac{151}{145}, c^2_{L12} = \tfrac{18}{5}, c^2_{L13} = \tfrac{20}{5}, c^2_{L21} = \tfrac{159}{145}, c^2_{L22} = \tfrac{103}{145}, c^2_{L23} = \tfrac{13}{15}, c^2_{L31} = \tfrac{132}{120}, c^2_{L32} = \tfrac{150}{120}, c^2_{L33} = \tfrac{72}{120}.$$

We have by **Model 2.R**

$$c^1_{R11} = \tfrac{135}{165}, c^1_{R12} = \tfrac{14}{5}, c^1_{R13} = \tfrac{15}{5}, c^1_{R21} = \tfrac{103}{145}, c^1_{L22} = \tfrac{95}{145}, c^1_{L23} = \tfrac{21}{25}, c^1_{L31} = \tfrac{162}{130}, c^1_{L32} = \tfrac{141}{130}, c^1_{L33} = \tfrac{78}{130},$$

and

$$c^2_{R11} = \tfrac{171}{165}, c^2_{R12} = \tfrac{18}{5}, c^2_{R13} = \tfrac{20}{5}, c^2_{R21} = \tfrac{180}{165}, c^2_{L22} = \tfrac{117}{165}, c^2_{L23} = \tfrac{23}{25}, c^2_{L31} = \tfrac{148}{130}, c^2_{L32} = \tfrac{169}{130}, c^2_{L33} = \tfrac{78}{130}.$$

So, interval c_{ij} of interval TP is determined as $c^r_{ij} \in [\min\{c^r_{Lij}, c^r_{Rij}\}, \max\{c^r_{Lij}, c^r_{Rij}\}]$ i.e., we have

$$c^1_{11} \in [\tfrac{135}{165}, \tfrac{119}{145}], c^1_{12} \in [\tfrac{14}{5}, \tfrac{14}{5}], c^1_{13} \in [\tfrac{15}{5}, \tfrac{15}{5}], c^1_{21} \in [\tfrac{103}{145}, \tfrac{103}{145}], c^1_{22} \in [\tfrac{95}{145}, \tfrac{95}{145}],$$
$$c^1_{23} \in [\tfrac{12}{15}, \tfrac{21}{25}], c^1_{31} \in [\tfrac{114}{120}, \tfrac{162}{130}], c^1_{32} \in [\tfrac{126}{120}, \tfrac{141}{130}], c^1_{33} \in [\tfrac{72}{120}, \tfrac{78}{130}],$$

and

$$c^2_{11} \in [\tfrac{171}{165}, \tfrac{151}{145}], c^2_{12} \in [\tfrac{18}{5}, \tfrac{18}{5}], c^2_{13} \in [\tfrac{20}{5}, \tfrac{20}{5}], c^2_{21} \in [\tfrac{180}{165}, \tfrac{159}{145}], c^2_{22} \in [\tfrac{117}{165}, \tfrac{103}{145}],$$
$$c^2_{23} \in [\tfrac{13}{15}, \tfrac{23}{25}], c^2_{31} \in [\tfrac{132}{120}, \tfrac{148}{130}], c^2_{32} \in [\tfrac{150}{120}, \tfrac{169}{130}], c^2_{33} \in [\tfrac{72}{120}, \tfrac{78}{130}].$$

Then we formulate cost varying interval TP by **Model 4** which is **Model 8**.

Model 8

$$\begin{aligned}
\min \quad & Z^1_R(x) = (0.002 + 0.819) * x_{11} + (0.0 + 2.8) * x_{12} + (0.0 + 3) * x_{13} \\
& \qquad + (0.0 + 0.71) * x_{21} + (0.0 + 0.655) * x_{22} + (0.4 + 0.82) * x_{23} \\
& \qquad + (0.296 + 1.098) * x_{31} + (0.035 + 1.067) * x_{32} + (0.0 + 0.6) * x_{33} \\
\min \quad & Z^1_C(x) = (0.0 + 0.819) * x_{11} + (0.0 + 2.8) * x_{12} + (0.0 + 3) * x_{13} \\
& \qquad + (0.0 + 0.71) * x_{21} + (0.0 + 0.655) * x_{22} + (0.0 + 0.82) * x_{23} \\
& \qquad + (0.0 + 1.098) * x_{31} + (0.0 + 1.067) * x_{32} + (0.0 + 0.6) * x_{33} \\
\min \quad & Z^2_R(x) = (0.005 + 1.039) * x_{11} + (0.0 + 3.6) * x_{12} + (0.0 + 4) * x_{13} \\
& \qquad + (0.005 + 1.094) * x_{21} + (0.001 + 0.709) * x_{22} + (0.05 + 0.893) * x_{23} \\
& \qquad + (0.038 + 1.12) * x_{31} + (0.05 + 1.275) * x_{32} + (0.0 + 0.6) * x_{33} \\
\min \quad & Z^2_C(x) = (0.0 + 1.039) * x_{11} + (0.0 + 3.6) * x_{12} + (0.0 + 4) * x_{13} \\
& \qquad + (0.0 + 1.094) * x_{21} + (0.0 + 0.709) * x_{22} + (0.0 + 0.893) * x_{23} \\
& \qquad + (0.0 + 1.12) * x_{31} + (0.0 + 1.275) * x_{32} + (0.0 + 0.6) * x_{33} \\
& x_{11} + x_{12} + x_{13} >= 150; x_{11} + x_{12} + x_{13} <= 170 \\
& x_{21} + x_{22} + x_{23} >= 160; x_{21} + x_{22} + x_{23} <= 190
\end{aligned}$$

$$x_{31}+x_{32}+x_{33}>=120; x_{31}+x_{32}+x_{33}<=130$$
$$x_{11}+x_{21}+x_{31}>=145 x_{11}+x_{21}+x_{31}<=165;$$
$$x_{12}+x_{22}+x_{32}>=150; x_{12}+x_{22}+x_{32}<=170$$
$$x_{13}+x_{23}+x_{33}>=135; x_{13}+x_{23}+x_{33}<=155$$
$$x_{ij}\geq 0, i=1,2,3; j=1,2,3. \tag{21}$$

Then solved by **Model 6** by Lingo package we have the following result: $\lambda = 1.0$, $Z_R^1 = 308.6$, $Z_C^1 = 304.3$, $Z_R^2 = 347.53$, $Z_C^2 = 346.13$, $x_{11} = 150$, $x_{22} = 150$, $x_{23} = 10$, $x_{33} = 125$.

Example 2. The data is collected by a person who supplies products to different companies after tacking different sources. There are three different suppliers(origins) named as O_1, O_2, O_3 and three different destinations namely D_1, D_2, D_3. How much amount of materials be supplied from different sources to all other destinations so that transportation cost(s) are minimum. Data are given below in the following Table 5 and Table 6. Consider a 2-vehicle cost varying transportation problem as

Table 5: Lower bounds of cost intervals of the vehicles of Example 2

	D_1	D_2	D_3	*stock*
O_1	$5,7$	$4,6$	$9,11$	$[60,90]$
O_2	$2,3$	$6,8$	$7,9$	$[40,80]$
O_3	$3,4$	$10,12$	$4,6$	$[20,30]$
Demand	$[50,70]$	$[40,50]$	$[30,80]$	

Table 6: Upper bounds of cost intervals of the vehicles of Example 2

	D_1	D_2	D_3	*stock*
O_1	$10,12$	$11,13$	$12,14$	$[60,90]$
O_2	$8,10$	$6,9$	$5,7$	$[40,80]$
O_3	$15,18$	$14,16$	$4,6$	$[20,30]$
Demand	$[50,70]$	$[40,50]$	$[30,80]$	

It is also given that there are two types of vehicle V_1 and V_2. For each $r = 1,\ldots,K$ the cost of V_1 from source 'i' destination 'j' is $R_{ij}^r(1)$ for a single trip. The cost of V_2 from source 'i' destination 'j' is $R_{ij}^r(2)$ for a single trip. It is also given that the capacity of V_1 is $C_1 = 10$ and that of V_2 is $C_2 = 20$.

Then we have by **Model 2.L**

$$c_{L11}^1=\tfrac{7}{20}, c_{L12}^1=\tfrac{12}{40}, c_{L13}^1=\tfrac{9}{10}, c_{L21}^1=\tfrac{6}{30}, c_{L22}^1=\tfrac{15}{30}, c_{L23}^1=\tfrac{7}{10}, c_{L31}^1=\tfrac{7}{20}, c_{L32}^1=\tfrac{12}{40}, c_{L33}^1=\tfrac{17}{20},$$

and

$$c_{L11}^2=\tfrac{34}{50}, c_{L12}^2=\tfrac{11}{10}, c_{L13}^2=\tfrac{12}{10}, c_{L21}^2=\tfrac{20}{30}, c_{L22}^2=\tfrac{15}{30}, c_{L23}^2=\tfrac{5}{10}, c_{L31}^2=\tfrac{18}{20}, c_{L32}^2=\tfrac{16}{20}, c_{L33}^2=\tfrac{6}{20}.$$

Then we have by **Model 2.R**

$$c_{R11}^1=\tfrac{14}{40}, c_{R12}^1=\tfrac{16}{50}, c_{R13}^1=\tfrac{31}{50}, c_{R21}^1=\tfrac{5}{30}, c_{L22}^1=\tfrac{15}{30}, c_{L23}^1=\tfrac{36}{80}, c_{L31}^1=\tfrac{7}{30}, c_{L32}^1=\tfrac{22}{30}, c_{L33}^1=\tfrac{10}{30},$$

and

$$c_{R11}^2=\tfrac{46}{70}, c_{R12}^2=\tfrac{13}{20}, c_{R13}^2=\tfrac{14}{20}, c_{R21}^2=\tfrac{20}{30}, c_{L22}^2=\tfrac{15}{30}, c_{L23}^2=\tfrac{19}{50}, c_{L31}^2=\tfrac{43}{30}, c_{L32}^2=\tfrac{30}{30}, c_{L33}^2=\tfrac{10}{13}.$$

So, interval c_{ij} of interval TP determined as $c_{ij}^r \in [\min\{c_{Lij}^r, c_{Rij}^r\}, \max\{c_{Lij}^r, c_{Rij}^r\}]$ i.e., we have

$$c_{11}^1 \in [\tfrac{7}{20}, \tfrac{14}{20}], c_{12}^1 \in [\tfrac{12}{40}, \tfrac{16}{50}], c_{13}^1 \in [\tfrac{31}{50}, \tfrac{9}{10}], c_{21}^1 \in [\tfrac{5}{30}, \tfrac{20}{30}], c_{22}^1 \in [\tfrac{15}{30}, \tfrac{15}{30}],$$
$$c_{23}^1 \in [\tfrac{36}{80}, \tfrac{7}{10}], c_{31}^1 \in [\tfrac{7}{30}, \tfrac{7}{10}], c_{32}^1 \in [\tfrac{12}{40}, \tfrac{22}{30}], c_{33}^1 \in [\tfrac{10}{30}, \tfrac{17}{20}],$$

and

$$c_{11}^2 \in [\tfrac{46}{70}, \tfrac{34}{50}], c_{12}^2 \in [\tfrac{13}{20}, \tfrac{11}{10}], c_{13}^2 \in [\tfrac{14}{20}, \tfrac{9}{10}], c_{21}^2 \in [\tfrac{20}{30}, \tfrac{20}{30}], c_{22}^2 \in [\tfrac{15}{30}, \tfrac{15}{30}],$$

$$c_{23}^2 \in [\tfrac{19}{50}, \tfrac{5}{10}], c_{31}^2 \in [\tfrac{18}{20}, \tfrac{43}{30}], c_{32}^2 \in [\tfrac{16}{20}, \tfrac{30}{30}], c_{33}^2 \in [\tfrac{6}{20}, \tfrac{10}{13}].$$

Then we formulate cost varying interval TP by **Model 4** which is **Model 9**.
Model 9

$$\begin{aligned}
\min \quad & Z_R^1(x) = (0.35+0.525)*x_{11} + (0.02+0.31)*x_{12} + (0.28+0.76)*x_{13} \\
& \qquad + (0.5+0.42)*x_{21} + (0.0+0.5)*x_{22} + (0.25+0.575)*x_{23} \\
& \qquad + (0.47+0.47)*x_{31} + (0.43+0.516)*x_{32} + (0.52+0.592)*x_{33} \\
\min \quad & Z_C^1(x) = (0.0+0.525)*x_{11} + (0.0+0.31)*x_{12} + (0.0+0.76)*x_{13} \\
& \qquad + (0.0+0.42)*x_{21} + (0.0+0.5)*x_{22} + (0.0+0.575)*x_{23} \\
& \qquad + (0.0+0.47)*x_{31} + (0.0+0.516)*x_{32} + (0.0+0.592)*x_{33} \\
\min \quad & Z_R^2(x) = (0.023+0.669)*x_{11} + (0.45+0.875)*x_{12} + (0.2+0.8)*x_{13} \\
& \qquad + (0.0+0.667)*x_{21} + (0.0+0.5)*x_{22} + (0.12+0.44)*x_{23} \\
& \qquad + (0.533+1.16)*x_{31} + (0.2+0.9)*x_{32} + (0.469+0.535)*x_{33} \\
\min \quad & Z_C^2(x) = (0.0+0.669)*x_{11} + (0.0+0.875)*x_{12} + (0.0+0.8)*x_{13} \\
& \qquad + (0.0+0.667)*x_{21} + (0.0+0.5)*x_{22} + (0.0+0.44)*x_{23} \\
& \qquad + (0.0+1.16)*x_{31} + (0.0+0.9)*x_{32} + (0.0+0.535)*x_{33} \\
& x_{11}+x_{12}+x_{13} >= 60; x_{11}+x_{12}+x_{13} <= 90 \\
& x_{21}+x_{22}+x_{23} >= 40; x_{21}+x_{22}+x_{23} <= 80 \\
& x_{31}+x_{32}+x_{33} >= 20; x_{31}+x_{32}+x_{33} <= 30 \\
& x_{11}+x_{21}+x_{31} >= 50; x_{11}+x_{21}+x_{31} <= 70 \\
& x_{12}+x_{22}+x_{32} >= 40; x_{12}+x_{22}+x_{32} <= 50 \\
& x_{13}+x_{23}+x_{33} >= 30; x_{13}+x_{23}+x_{33} <= 80 \\
& x_{ij} \geq 0, i=1,2,3; j=1,2,3. \qquad (22)
\end{aligned}$$

Then solved by **Model 6** by Lingo package we have the following result $\lambda = 0.896023$, $Z_R^1 = 96.00$, $Z_C^1 = 64.37989$, $Z_R^2 = 120.288$, $Z_C^2 = 91.00$, $x_{11} = 45.40$, $x_{12} = 36.12$, $x_{22} = 13.89$, $x_{23} = 19.35$, $x_{31} = 4.6$, $x_{33} = 15.4$.

5 Conclusion

In this paper we have presented a solution procedure of cost varying interval transportation problem under two vehicles. Here the source and destination parameters are considered as intervals. Initially, depending on cost of vehicles we determine interval of the parameter of the objective function, and the problem is converted into classical single objective interval transportation problem. Then this model converted to a bi-objective transportation problem, one is the right limit and other is center of the objective which are minimized.

To obtain the solution of this bi-objective model, the fuzzy programming technique is used. Here different types of membership functions may be used (like, linear, hyperbolic, exponential). But we use only linear membership function.

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