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First Order Non Homogeneous Ordinary Differential Equation with Initial Value as Triangular Intuitionistic Fuzzy Number

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Abstract

In this paper first order non homogeneous ordinary differential equation is described in intuitionistic fuzzy environment. Here initial condition of the said differential equation is considered as triangular intuitionistic fuzzy number. The method is illustrated by numerical examples. Finally an elementary application on bank account problem is described in intuitionistic fuzzy environment.

Keywords: fuzzy set, intuitionistic fuzzy set, fuzzy differential equation, triangular intuitionistic fuzzy number

1 Introduction

1.1 Fuzzy Set and Intuitionistic Fuzzy Set Theory

Zadeh [41] and Dubois and Parade [15] first introduced the conception on fuzzy number and fuzzy arithmetic. Generalizations of fuzzy sets theory [41] is considered to be one of Intuitionistic fuzzy set (IFS). Out of several higher-order fuzzy sets, IFS was first introduced by Atanassov [2] have been found to be suitable to deal with unexplored areas. The fuzzy set considers only the degree of belongingness and non belongingness. Fuzzy set theory does not incorporate the degree of hesitation (i.e.,degree of non-determinacy defined as, 1 − sum of membership function and non-membership function.To handle such situations, Atanassov [3] explored the concept of fuzzy set theory by intuitionistic fuzzy set (IFS) theory.The degree of acceptance in Fuzzy Sets is only considered, otherwise IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one [3].

Basic arithmetic operations of TIFNs is defined by Li in [25] using membership and non membership values. Basic arithmetic operations of TIFNs such as addition, subtraction and multiplication are defined by Mahapatra and Roy in [26], by considering the six tuple number itself and division by Nagoorgani and Ponnalagu [35].

Now-a-days, IFSs are being studied extensively and being used in different fields of Science and Technology. Amongst the all research works mainly on IFS we can include [3, 4, 5, 40, 10, 6].

1.2 Fuzzy Differential Equation and Its Application

It is seen that in recent years the topic of Fuzzy Differential Equations (FDEs) has been rapidly grown. In the year 1987, the term "fuzzy differential equation" was introduced by Kandel and Byatt [20]. To study FDE there have been many conceptions for the definition of fuzzy derivative. Chang and Zadeh [13] was someone who first introduced the concept of fuzzy derivative, later on it was followed up by Dobois and Prade [16] who used the extension principle in their approach. Other methods have been discussed by Puri and Ralescu [38], Goetschel and Voxman [18], Seikkala [39] and Friedman et al. [17, 21], Cano and Flores [12], Hüllermeier [19], Lan and Nieto [22], Nieto, López and Georgiou [36]. First order linear fuzzy differential equations or systems are researched under various interpretations

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in several papers (see [8, 14]).There are only few papers such as [23, 27] in which intuitionistic fuzzy number are applied in differential equation .

Fuzzy differential equations play an important role in the field of biology, engineering, physics as well as among other field of science. For example, in population models [9], civil engineering [37], Growth model [29], Bacteria culture model [31], bank account and drug concentration problem [33], barometric pressure problem [34]. First order linear fuzzy differential equations are sort of equations which have many applications among all the Fuzzy Differential Equation.

1.3 Intuitionistic Fuzzy Differential Equation

Intutionistic FDE is very rare. Melliani and Chadli [28] solve partial differential equation with intutionistic fuzzy number. Abbasbandy and Allahviranloo [1] discussed numerical Solution of fuzzy differential equations by Runge-Kutta and the Intuitionistic treatment. Lata and Kumar [24] solve time-dependent intuitionistic fuzzy differential equation and its application to analyze the intutionistic fuzzy reliability of industrial system. First order homogeneous ordinary differential equation with initial value as triangular intuitionistic fuzzy number is described by Mondal and Roy [32]. System of differential equation with initial value as triangular intuitionistic fuzzy number and its application is solved by Mondal and Roy [30].

1.4 Motivation

In modeling a real world problem it is need no let every parameter as fixed value. There arise some parameters which value is uncertain or imprecise. When the uncertainty comes in modeling on a problem with differential equation there the concepts of imprecise differential equation is arrise. Many researchers take this impreciseness as fuzzy sense. The fuzzy set considered only degree of belongingness. Fuzzy set theory does not incorporate the degree of hesitation (i.e., degree of non-determinacy) . To handle such situation uses of Intutionistic fuzzy set theory are developed. To the best of our knowledge there are very few works on intutionistc fuzzy differential equation till date after developing intutionistic fuzzy set theory. So, the behavior and solution of a differential equation in intutionistic fuzzy environment is very important.

The theory and models are discussed both fuzzy and intuitionistic fuzzy environments. The coefficients, initial condition of the diferential equation are taken as fuzzy or intuitionistic fuzzy number. In real life situation intutionistic fuzzy differential equation are more acceptable than fuzzy differential equation.

1.5 Novelties

Although some developments are done but some new interest and new work have done by our self which is mentioned bellow:

(i) Differential equation is solved in intuitionistic fuzzy environment.

(ii) Find the exact solution of linear non-homogeneous types of differential equation.

(iii) An application in bank account problem is solved in intuitionistic fuzzy environment.

1.6 Structure of the Paper

The structure of the paper is as follows: In first section we introduce the previous work on fuzzy, intutionistic fuzzy, fuzzy and intutionistic fuzzy differential equation. Second section goes to preliminary concept. We define fuzzy and intutionistic fuzzy number. In third section we solve non-homogeneous first order linear differential equation with triangular intutionistic fuzzy number. Forth section goes for numerical examples. In fifth section we applied this procedure in a bank account problem in intutionistic fuzzy environment. The conclusion is done in sixth section.

2 Preliminary Concepts

2.1 Concept on Fuzzy Set

Definition 1: Fuzzy Set: A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in A, \mu_{\tilde{A}}(x) \in [0,1]\}$. In the pair $(x, \mu_{\tilde{A}}(x))$ the first element x belong to the classical set A, the second element $\mu_{\tilde{A}}(x)$, belong to the interval $[0, 1]$, called membership function.

Definition 2: Height: The height $h(\tilde{A})$, of a fuzzy set $\tilde{A} = (x, \mu_{\tilde{A}}(x): x \in X)$, is the largest membership grade obtained by any element in that set i.e. $h(\tilde{A})$=sup $\mu_{\tilde{A}}(x)$.

Definition 3: Support of Fuzzy Set: The support of fuzzy set $\tilde{A}$ is the set of all points x in X such that $\mu_{\tilde{A}}(x) > 0$ i.e., $support\ (\tilde{A}) = \{x \mid \mu_{\tilde{A}}(x) > 0\}$.

Definition 4: Convex Fuzzy sets: A fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\} \subseteq X$ is called convex fuzzy set if all A_α for every $\alpha \in [0,1]$ are convex sets i.e. for every element $x_1 \in A_\alpha$ and $x_2 \in A_\alpha$ and $\lambda x_1 + (1-\lambda)x_2 \in A_\alpha, \forall\ \lambda \in [0,1]$. Otherwise the fuzzy set is called non-convex fuzzy set.

Definition 5:α-cut of a fuzzy set: The α-level set (or interval of confidence at level α or α-cut) of the fuzzy set $\tilde{A}$ of universe X is a crisp set A_α that contains all the elements of X that have membership values in A greater than or equal to α i.e. $A_\alpha = \{x: \mu_{\tilde{A}}(x) \geq \alpha\}\ \forall\ \alpha \in [0,1]$.

Definition 6: **Strong α-cut of a fuzzy set:** Strong α-cut is denoted by $A_\alpha^{strong} = \{x: \mu_{\tilde{A}}(x) > \alpha\}\ \forall\ \alpha \in [0,1]$.

Definition 7: Fuzzy Number: A fuzzy set $\tilde{A}$, defined on the universal set of real number R , is said to be a fuzzy number if its possess at least the following properties:

(i) $\tilde{A}$ is convex.

(ii) $\tilde{A}$ is normal i.e., $\exists\ x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$.

(iii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

(iv) A_α must be closed interval for every α [0,1].

(v) The support of $\tilde{A}$, i.e., $support\ (\tilde{A})$ must be bounded.

Definition 8: Positive and Negative fuzzy number: A fuzzy number $\tilde{A}$ is called positive (or negative), denoted by $\tilde{A} > 0$ (or $\tilde{A} < 0$), if its membership function $\mu_{\tilde{A}}(x)$ satisfies $\mu_{\tilde{A}}(x) = 0, \forall\ x < 0\ (x > 0)$.

2.2 Concept on Intuitionistic Fuzzy Set

Definition 9: Intuitionistic Fuzzy Set: Let a set X be fixed. An IFS $\tilde{A}^i$ in X is an object having the form $\tilde{A}^i = \{< x, \mu_{\tilde{A}^i}(x), \vartheta_{\tilde{A}^i}(x) >: x \in X\}$, where the $\mu_{\tilde{A}^i}(x): X \to [0,1]$ and $\vartheta_{\tilde{A}^i}(x): X \to [0,1]$ define the degree of membership and degree of non-membership respectively, of the element $x \in X$ to the set $\tilde{A}^i$, which is a subset of X, for every element of $x \in X$, $0 \leq \mu_{\tilde{A}^i}(x) + \vartheta_{\tilde{A}^i}(x) \leq 1$.

Definition 10: Intuitionistic Fuzzy Number: An IFN $\tilde{A}^i$ is defined as follows:
(i) an intuitionistic fuzzy subject of real line,
(ii) normal,i.e., there is any $x_0 \in R$ such that $\mu_{\tilde{A}^i}(x_0) = 1$(so $\vartheta_{\tilde{A}^i}(x_0) = 0$),
(iii) a convex set for the membership function $\mu_{\tilde{A}^i}(x)$, i.e.,

$$\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \geq min\big(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)\big) \quad \forall\ x_1, x_2 \in R, \lambda \in [0,1],$$

(iv) a concave set for the non-membership function $\vartheta_{\tilde{A}^i}(x)$, i.e.,

$$\vartheta_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \geq max\big(\vartheta_{\tilde{A}^i}(x_1), \vartheta_{\tilde{A}^i}(x_2)\big) \quad \forall\ x_1, x_2 \in R, \lambda \in [0,1].$$

Definition 11: Triangular Intuitionistic Fuzzy Number: A TIFN $\tilde{A}^i$ is a subset of IFN in R with following membership function and non membership function as follows:

$$\mu_{\tilde{A}^i}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & for\ a_1 \leq x < a_2, \\ \frac{a_3-x}{a_3-a_2} & for\ \ a_2 < x\ \leq a_3 \\ 0 & otherwise \end{cases} \quad \text{and} \quad \vartheta_{\tilde{A}^i}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'} & for\ a_1' \leq x < a_2 \\ \frac{x-a_2}{a_3'-a_2} & for\ \ a_2 < x\ \leq a_3' \\ 1 & otherwise \end{cases}$$

where $a_1' \leq a_2 \leq a_3'$ and $a_1 \leq a_2 \leq a_3$.

The TIFN is denoted by $\tilde{A}^i{}_{TIFN} = (a_1, a_2, a_3; a_1', a_2, a_3')$.

Definition 13: α-cut set: A α-cut set of $\tilde{A}^i{}_{TIFN} = (a_1, a_2, a_3; a_1', a_2, a_3')$ is a crisp subset of R which is defined as follows

$$A_\alpha = \{x: \mu_{\tilde{A}^i}(x) \geq \alpha, \forall \alpha \in [0,1]\} = [A_1(\alpha), A_2(\alpha)] = [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)].$$

Definition 14: β-cut set: A β-cut set of $\tilde{A}^{ni}{}_{TIFN} = (a_1, a_2, a_3; a_1', a_2, a_3')$ is a crisp subset of R which is defined as follows

$$A_\beta = \{x: \vartheta_{\tilde{A}^i}(x) \leq \beta, \forall \beta \in [0,1]\} = [A_1'(\beta), A_2'(\beta)] = [a_2 - \beta(a_2 - a_1'), a_2 + \beta(a_3' - a_2)].$$

Definition 15: (α, β)-cut set: A (α, β)-cut set of $\tilde{A}^i{}_{TIFN} = (a_1, a_2, a_3; a_1', a_2, a_3')$ is a crisp subset of R which is defined as follows

$$A_{\alpha,\beta} = \{[A_1(\alpha), A_2(\alpha)]; [A_1'(\beta), A_2'(\beta)]\}, \alpha + \beta \leq 1, \alpha \in [0,1], \beta \in [0,1].$$

Definition 16: Intuitionistic fuzzy function: Let $F_1(\alpha)$, $F_2(\alpha)$, $F_1'(\beta)$ and $F_2'(\beta)$ be a continuous functions on an interval I. The set $\tilde{F}$, determined by the membership and non membership function

$$\mu_{\tilde{F}(t)}(m(\alpha)) = \begin{cases} \alpha, m(\alpha) = F_1(\alpha) \; and \; 0 \le \alpha \le 1 \\ \alpha, m(\alpha) = F_2(\alpha) \; and \; 0 \le \alpha \le 1 \\ 0, otherwise \end{cases}$$

and

$$\vartheta_{\tilde{F}(t)}(n(\beta)) = \begin{cases} \beta, n(\beta) = F_1'(\beta) \; and \; 0 \le \beta \le 1 \\ \beta, n(\beta) = F_2'(\beta) \; and \; 0 \le \beta \le 1 \\ 1, otherwise \end{cases}$$

where $0 \le \alpha + \beta \le 1$, is called a intuitionistic fuzzy function and is denoted as $\tilde{F}(t)$, where the (α, β)-cut of $\tilde{F}(t)$ is

$$[F_1(\alpha), F_2(\alpha); F_1'(\beta), F_2'(\beta)].$$

According to the definition, a intuitionistic fuzzy function is a intuitionistic fuzzy set of real function. Two of these functions have the membership degree α: $F_1(\alpha)$ and $F_2(\alpha)$ and two of these functions have the non membership degree β: $F_1'(\beta)$ and $F_2'(\beta)$.

2.3 Concept on Fuzzy Derivative

Theorem 2.1: ([7]) Let $f: (a, b) \to E$ and $x_0 \in (a, b)$. We say that f is strongly generalized differential at x_0 (Bede-Gal differential) if there exists an element $f'(x_0) \in E$, such that

(i) for all $h > 0$ sufficiently small, $\exists \, f(x_0 + h) -^h f(x_0)$, $\exists \, f(x_0) -^h f(x_0 - h)$ and the limits(in the metric D)

$$\lim_{h \searrow 0} \frac{f(x_0 + h) -^h f(x_0)}{h} = \lim_{h \searrow 0} \frac{f(x_0) -^h f(x_0 - h)}{h} = f'(x_0)$$

or

(ii) for all $h > 0$ sufficiently small, $\exists \, f(x_0) -^h f(x_0 + h)$, $\exists \, f(x_0 - h) -^h f(x_0)$ and the limits(in the metric D)

$$\lim_{h \searrow 0} \frac{f(x_0) -^h f(x_0 + h)}{-h} = \lim_{h \searrow 0} \frac{f(x_0 - h) -^h f(x_0)}{-h} = f'(x_0)$$

or

(iii) for all $h > 0$ sufficiently small, $\exists \, f(x_0 + h) -^h f(x_0)$, $\exists \, f(x_0 - h) -^h f(x_0)$ and the limits(in the metric D)

$$\lim_{h \searrow 0} \frac{f(x_0 + h) -^h f(x_0)}{h} = \lim_{h \searrow 0} \frac{f(x_0 - h) -^h f(x_0)}{-h} = f'(x_0)$$

or

(iv) for all $h > 0$ sufficiently small, $\exists \, f(x_0) -^h f(x_0 + h)$, $\exists \, f(x_0) -^h f(x_0 - h)$ and the limits(in the metric D)

$$\lim_{h \searrow 0} \frac{f(x_0) -^h f(x_0 + h)}{-h} = \lim_{h \searrow 0} \frac{f(x_0) -^h f(x_0 - h)}{-h} = f'(x_0)$$

(h and $-h$ at denominators mean $\frac{1}{h}$ and $\frac{-1}{h}$, respectively).

Theorem 2.2: ([11]) Let $f: R \to E$ be a function and denote $f(t) = (f_1(t, r), f_2(t, r))$, for each $r \in [0,1]$. Then,

(1) If f is (i)-differentiable, then $f_1(t, r)$ and $f_2(t, r)$ are differentiable function and $f'(t) = (f_1'(t, r), f_2'(t, r))$.

(2) If f is (ii)-differentiable, then $f_1(t, r)$ and $f_2(t, r)$ are differentiable function and $f'(t) = (f_1'(t, r), f_2'(t, r))$.

Let $f: [0, T] \to \Re_{\mathcal{F}}$. The integral of f in $[0, T]$, (denoted by $\int_{[0,T]} f(t)dt$ or, $\int_0^T f(t)dt$) is defined levelwise as the set if integrals of the (real) measurable selections for $[f]^r = [f_1(t, r) \, , f_2(t, r) \,]$, for each $r \in [0,1]$. We say that f is integrable over $[0, T]$ if $\int_{[0,T]} f(t)dt \in \Re_{\mathcal{F}}$ and we have

$$\left[\int_0^T f(t)dt\right]^r = \left[\int_0^T f_1(t, r)dt \, , \int_0^T f_2(t, r)dt\right] \text{ for each } r \in [0,1].$$

2.4 Concept on Intuitionistic Fuzzy Derivative

Definition 17: The Hausdorff distance between intutionistic fuzzy number is given by $D: \Re_{\mathcal{F}} \times \Re_{\mathcal{F}} \to \mathbb{R}^+ \cup \{0\}$ as

$$D(u, v; p, q) = \sup_{\alpha, \beta \in [0,1]} d\big([u]_\alpha, [v]_\alpha; [p]_\beta, [q]_\beta\big)$$
$$= \sup_{\alpha, \beta \in [0,1]} \max\{|u_1(\alpha) - v_1(\alpha)|, |u_2(\alpha) - v_2(\alpha)|, |p_1(\beta) - q_1(\beta)|, |p_2(\beta) - q_2(\beta)|\}$$

where d is Hausdorff metric. The metric space $(\Re_{\mathcal{F}}, D)$ is complete, separable and locally compact and the following properties from [] for metric D are valid:

1. $D(u\oplus w, v\oplus w; p\oplus z, q\oplus z) = D(u, v; p, q), \forall\, u, v, p, q, w, z \in \Re_{\mathcal{F}}$;
2. $D(\lambda u, \lambda v; \lambda p, \lambda q) = |\lambda| D(u, v; p, q), \lambda \in \mathbb{R}, u, v, p, q \in \Re_{\mathcal{F}}$;
3. $D(u_1\oplus u_2, v_1\oplus v_2; p_1\oplus p_2, q_1\oplus q_1) \leq D(u_1, v_1; p_1, q_1) + D(u_2, v_2; p_2, q_1)\ \forall\, u_1, u_2, v_1, v_2, p_1, p_2, q_1, q_1 \in \Re_{\mathcal{F}}$;
4. $D(u_1 \ominus u_2, v_1 \ominus v_2; p_1 \ominus p_2, q_1 \ominus q_1) \leq D(u_1, v_1; p_1, q_1) + D(u_2, v_2; p_2, q_1)$, as long as $u_1 \ominus u_2, v_1 \ominus v_2; p_1 \ominus p_2, q_1 \ominus q_1$ exists and $u_1, u_2, v_1, v_2, p_1, p_2, q_1, q_1 \in \Re_{\mathcal{F}}$.

Lemma 1: Let $u_n, v_n, p_n, q_n, u, v, p, q \in \Re_{\mathcal{F}}, n \in \mathbb{N}$. Let $u_n \to u, v_n \to v, p_n \to p, q_n \to q$, as $n \to \infty$. Then $D(u_n, v_n; p_n, q_n) \to D(u, v; p, q)$, as $n \to \infty$. In particular we can write

$$\lim_{n\to+\infty} D(u_n, v_n; p_n, q_n) = D\left(\lim_{n\to+\infty} u_n, \lim_{n\to+\infty} v_n; \lim_{n\to+\infty} p_n, \lim_{n\to+\infty} q_n\right) = D(u, v; p, q).$$

Lemma 2: Let $u_n, u \in \Re_{\mathcal{F}}$ and $c_n, c \in \mathbb{R}$, such that $u_n \to u$ and $c_n \to c$ as $n \to +\infty$. Then in D-metric

$$\lim_{n\to+\infty} (u_n \odot c_n) = \left(\lim_{n\to+\infty} u_n\right) \odot \left(\lim_{n\to+\infty} c_n\right) = u \odot c.$$

Definition 18: The generalized Hukuhara difference of two intutionistic fuzzy numbers $a, b \in \Re_{\mathcal{F}}$ is defined as follows

$$a \ominus_{gH} b = k \Leftrightarrow \begin{cases} a = b + k \\ b = a + (-1)k. \end{cases}$$

In terms of (α, β)-cut set we have

$$[a \ominus_{gH} b]_{\alpha,\beta} = [\min\{r_1(\alpha), r_2(\alpha)\}, \max\{r_1(\alpha), r_2(\alpha)\}; \min\{s_1(\beta), s_2(\beta)\}, \max\{s_1(\beta), s_2(\beta)\}]$$

where $r_1(\alpha) = a_1(\alpha) - b_2(\alpha)$, $r_2(\alpha) = a_2(\alpha) - b_1(\alpha)$, $s_1(\beta) = a_1'(\beta) - b_2'(\beta)$ and $s_2(\beta) = a_2'(\beta) - b_1'(\beta)$.

Let $e = a \ominus_{gH} b$. The conditions for which the existence of $a \ominus_{gH} b$ exists

(1) $e_1(\alpha) = a_1(\alpha) - b_2(\alpha)$, $e_2(\alpha) = a_2(\alpha) - b_1(\alpha)$, $e_1'(\beta) = a_1'(\beta) - b_2'(\beta)$ and $e_2'(\beta) = a_2'(\beta) - b_1'(\beta)$ with $e_1(\alpha)$, $e_2'(\beta)$ are increasing and $e_2(\alpha)$, $e_1'(\beta)$ are decreasing function for all $\alpha, \beta \in [0,1]$ and $e_1(\alpha) \leq e_2(\alpha)$, $e_2'(\beta) \leq e_1'(\beta)$.

(2) $e_2(\alpha) = a_1(\alpha) - b_2(\alpha)$, $e_1(\alpha) = a_2(\alpha) - b_1(\alpha)$, $e_2'(\beta) = a_1'(\beta) - b_2'(\beta)$ and $e_1'(\beta) = a_2'(\beta) - b_1'(\beta)$ with $e_1(\alpha)$, $e_2'(\beta)$ are increasing and $e_2(\alpha)$, $e_1'(\beta)$ are decreasing function for all $\alpha, \beta \in [0,1]$ and $e_1(\alpha) \leq e_2(\alpha)$, $e_2'(\beta) \leq e_1'(\beta)$.

Remark: Throughout the paper, we assume that $a \ominus_{gH} b \in \Re_{\mathcal{F}}$.

Definition 19: Generalized Hukuhara derivative: The generalized Hukuhara derivative of a intuitionistic fuzzy valued function $f: (a, b) \to \Re_{\mathcal{F}}$ at t_0 is defined as

$$f'(t_0) = \lim_{h\to 0} \frac{f(t_0+h) \ominus_{gH} f(t_0)}{h}. \tag{2.1}$$

If $f'(t_0) \in \Re_{\mathcal{F}}$ satisfying (2.1) exists, we say that f is generalized Hukuhara differentiable at t_0.
Also we say that $f(t)$ is (i)-gH differentiable at t_0 if

$$[f'(t_0)]_\alpha = [f_1'(t_0, \alpha), f_2'(t_0, \alpha); g_1'(t_0, \alpha), g_2'(t_0, \alpha)] \tag{2.2}$$

and $f(t)$ is (i)-gH differentiable at t_0 if

$$[f'(t_0)]_\alpha = [f_2'(t_0, \alpha), f_1'(t_0, \alpha); g_2'(t_0, \alpha), g_1'(t_0, \alpha)] \tag{2.3}$$

where (α, β)-cut of $f(t)$ is $[f_1(\alpha), f_2(\alpha); g_1(\beta), g_2(\beta)]$.

Definition 20: Strong and Weak solution of Intuitionistic Fuzzy ordinary differential equation: Consider the 1st order linear homogeneous intuitionistic fuzzy ordinary differential equation

$$\frac{dx}{dt} = kx \ \text{ with } (t_0) = x_0 .$$

Here k or (and) x_0 be triangular intuitionistic fuzzy number(s). Let the solution of the above FODE be $\tilde{x}(t)$ and its (α, β)-cut be $x(t, \alpha, \beta) = \left[[x_1(t, \alpha), x_2(t, \alpha); x_1'(t, \beta), x_2'(t, \beta)]\right]$. The solution is a strong solution if

(i) $\frac{dx_1(t,\alpha)}{d\alpha} > 0, \frac{dx_2(t,\alpha)}{d\alpha} < 0\ \forall\, \alpha \in [0,1]$, $x_1(t, 1) \leq x_2(t, 1)$,

(ii) $\frac{dx_1'(t,\beta)}{d\beta} < 0, \frac{dx_2'(t,\beta)}{d\beta} > 0\ \forall\, \beta \in [0,1]$, $x_1'(t, 0) \leq x_2'(t, 0)$.

Otherwise the solution is week solution.

3 Fuzzy Differential Equations with Intuitionistic Fuzzy number

Consider the differential equation

$$\frac{dx(t)}{dt} = kx(t) + \epsilon \text{ with } x(0) = \tilde{\lambda}^i = (a_1, a_2, a_3; a_1', a_2, a_3'). \tag{3.1}$$

3.1 Solution when $x(t)$ is (i)-gH Differentiable

3.1.1 When the Coefficients is Positive i.e., $k > 0$

Taking α-cut of equation (3.1) we get

$$\frac{d}{dt}([x_1(t,\alpha),x_2(t,\alpha)];[x_1'(t,\beta),x_2'(t,\beta)]) = k([x_1(t,\alpha),x_2(t,\alpha)];[x_1'(t,\beta),x_2'(t,\beta)]) \tag{3.2}$$

with initial condition

$$x(t_0;\alpha,\beta) = ([a_1(\alpha),a_2(\alpha)];[a_1'(\beta),a_2'(\beta)],\alpha+\beta\le 1,\alpha,\beta\in[0,1]),$$

i.e.,

$$\begin{gathered}\dot{x}_1(t,\alpha) = kx_1(t,\alpha)+\epsilon\\ \dot{x}_2(t,\alpha) = kx_2(t,\alpha)+\epsilon\\ \dot{x}_1'(t,\beta) = kx_1'(t,\beta)+\epsilon\\ \dot{x}_2'(t,\beta) = kx_2'(t,\beta)+\epsilon\end{gathered}$$

with initial condition

$$\begin{gathered}x_1(t_0,\alpha) = a_1(\alpha)\\ x_2(t_0,\alpha) = a_2(\alpha)\\ x_1'(t_0,\beta) = a_1'(\beta)\\ x_2'(t_0,\beta) = a_2'(\beta).\end{gathered}$$

The solution is given by

$$\begin{gathered}x_1(t,\alpha) = -\frac{\epsilon}{k}+\left\{\frac{\epsilon}{k}+a_1(\alpha)\right\}e^{k(t-t_0)}\\ x_2(t,\alpha) = -\frac{\epsilon}{k}+\left\{\frac{\epsilon}{k}+a_2(\alpha)\right\}e^{k(t-t_0)}\\ x_1'(t,\beta) = -\frac{\epsilon}{k}+\left\{\frac{\epsilon}{k}+a_1'(\beta)\right\}e^{k(t-t_0)}\\ x_2'(t,\beta) = -\frac{\epsilon}{k}+\left\{\frac{\epsilon}{k}+a_2'(\beta)\right\}e^{k(t-t_0)}.\end{gathered}$$

The solution is a strong solution if

(i) $\frac{dx_1(t,\alpha)}{d\alpha} > 0, \frac{dx_2(t,\alpha)}{d\alpha} < 0 \;\forall\, \alpha\in[0,1],\; x_1(t,1)\le x_2(t,1),$

(ii) $\frac{dx_1'(t,\beta)}{d\beta} < 0, \frac{dx_2'(t,\beta)}{d\beta} > 0 \;\forall\, \beta\in[0,1],\; x_1'(t,0)\le x_2'(t,0).$

Otherwise the solution is week solution.

3.1.2 When the Coefficient is Negative i.e., $k < 0$

Let $k=-m, m>0$. Taking α-cut of equation (3.1) we get

$$\frac{d}{dt}([x_1(t,\alpha),x_2(t,\alpha)];[x_1'(t,\beta),x_2'(t,\beta)]) = -m([x_1(t,\alpha),x_2(t,\alpha)];[x_1'(t,\beta),x_2'(t,\beta)])+\epsilon \tag{3.3}$$

with initial condition

$$x(t_0;\alpha,\beta) = ([a_1(\alpha),a_2(\alpha)];[a_1'(\beta),a_2'(\beta)]),\alpha+\beta\le 1,\alpha,\beta\in[0,1],$$

i.e.,

$$\begin{gathered}\dot{x}_1(t,\alpha) = -mx_2(t,\alpha)+\epsilon\\ \dot{x}_2(t,\alpha) = -mx_1(t,\alpha)+\epsilon\\ \dot{x}_1'(t,\beta) = -mx_2'(t,\beta)+\epsilon\\ \dot{x}_2'(t,\beta) = -mx_1'(t,\beta)+\epsilon\end{gathered}$$

with initial condition

$$\begin{gathered}x_1(t_0,\alpha) = a_1(\alpha)\\ x_2(t_0,\alpha) = a_2(\alpha)\\ x_1'(t_0,\beta) = a_1'(\beta)\\ x_2'(t_0,\beta) = a_2'(\beta).\end{gathered}$$

The solution is given by

$$\begin{gathered}x_1(t,\alpha) = \frac{\epsilon}{m}+\frac{1}{2}\left\{-\frac{2\epsilon}{m}+a_1(\alpha)+a_2(\alpha)\right\}e^{-m(t-t_0)}+\frac{1}{2}\{a_1(\alpha)-a_2(\alpha)\}e^{m(t-t_0)}\\ x_2(t,\alpha) = \frac{\epsilon}{m}+\frac{1}{2}\left\{-\frac{2\epsilon}{m}+a_1(\alpha)+a_2(\alpha)\right\}e^{-m(t-t_0)}-\frac{1}{2}\{a_1(\alpha)-a_2(\alpha)\}e^{m(t-t_0)}\end{gathered}$$

$$x_1'(t,\beta)=\frac{\epsilon}{m}+\frac{1}{2}\left\{-\frac{2\epsilon}{m}+a_1'(\beta)+a_2'(\beta)\right\}e^{-m(t-t_0)}+\frac{1}{2}\{a_1'(\beta)-a_2'(\beta)\}e^{m(t-t_0)}$$
$$x_2'(t,\beta)=\frac{\epsilon}{m}+\frac{1}{2}\left\{-\frac{2\epsilon}{m}+a_1'(\beta)+a_2'(\beta)\right\}e^{-m(t-t_0)}-\frac{1}{2}\{a_1'(\beta)-a_2'(\beta)\}e^{m(t-t_0)}.$$

The solution is a strong solution if $\frac{dx_1(t,\alpha)}{d\alpha}>0,\frac{dx_2(t,\alpha)}{d\alpha}<0\ \forall\ \alpha\in[0,1]$, $x_1(t,1)\le x_2(t,1)$ and $\frac{dx_1'(t,\beta)}{d\beta}<0,\frac{dx_2'(t,\beta)}{d\beta}>0\ \forall\ \beta\in[0,1]$, $x_1'(t,0)\le x_2'(t,0)$. Otherwise the solution is week solution.

3.2 Solution when $x(t)$ is (ii)-gH Differentiable

3.2.1 When Coefficient is Positive i.e., $k>0$

Taking α-cut of equation (3.1) we get

$$\frac{d}{dt}([x_2(t,\alpha),x_1(t,\alpha)];[x_2'(t,\beta),x_1'(t,\beta)])=k([x_1(t,\alpha),x_2(t,\alpha)];[x_1'(t,\beta),x_2'(t,\beta)])+\epsilon \tag{3.4}$$

with initial condition

$$x(t_0;\alpha,\beta)=([a_1(\alpha),a_2(\alpha)];[a_1'(\beta),a_2'(\beta)],\alpha+\beta\le 1,\alpha,\beta\in[0,1]),$$

i.e.,

$$\dot{x}_2(t,\alpha)=kx_1(t,\alpha)+\epsilon$$
$$\dot{x}_1(t,\alpha)=kx_2(t,\alpha)+\epsilon$$
$$\dot{x}_2'(t,\beta)=kx_1'(t,\beta)+\epsilon$$
$$\dot{x}_1'(t,\beta)=kx_2'(t,\beta)+\epsilon$$

with initial condition

$$x_1(t_0,\alpha)=a_1(\alpha)$$
$$x_2(t_0,\alpha)=a_2(\alpha)$$
$$x_1'(t_0,\beta)=a_1'(\beta)$$
$$x_2'(t_0,\beta)=a_2'(\beta).$$

The solution is given by

$$x_1(t,\alpha)=-\frac{\epsilon}{k}+\frac{1}{2}\left\{\frac{2\epsilon}{k}+a_1(\alpha)+a_2(\alpha)\right\}e^{k(t-t_0)}+\frac{1}{2}\{a_1(\alpha)-a_2(\alpha)\}e^{-k(t-t_0)}$$
$$x_2(t,\alpha)=-\frac{\epsilon}{k}+\frac{1}{2}\left\{\frac{2\epsilon}{k}+a_1(\alpha)+a_2(\alpha)\right\}e^{k(t-t_0)}-\frac{1}{2}\{a_1(\alpha)-a_2(\alpha)\}e^{-k(t-t_0)}$$
$$x_1'(t,\beta)=-\frac{\epsilon}{k}+\frac{1}{2}\left\{\frac{2\epsilon}{k}+a_1'(\beta)+a_2'(\beta)\right\}e^{k(t-t_0)}+\frac{1}{2}\{a_1'(\beta)-a_2'(\beta)\}e^{-k(t-t_0)}$$
$$x_2'(t,\beta)=-\frac{\epsilon}{k}+\frac{1}{2}\left\{\frac{2\epsilon}{k}+a_1'(\beta)+a_2'(\beta)\right\}e^{k(t-t_0)}-\frac{1}{2}\{a_1'(\beta)-a_2'(\beta)\}e^{-k(t-t_0)}.$$

The solution is a strong solution if

(i) $\frac{dx_1(t,\alpha)}{d\alpha}>0,\frac{dx_2(t,\alpha)}{d\alpha}<0\ \forall\ \alpha\in[0,1]$, $x_1(t,1)\le x_2(t,1)$,

(ii) $\frac{dx_1'(t,\beta)}{d\beta}<0,\frac{dx_2'(t,\beta)}{d\beta}>0\ \forall\ \beta\in[0,1]$, $x_1'(t,0)\le x_2'(t,0)$.

Otherwise the solution is week solution.

3.2.2 When Coefficient is Negative i.e., $k<0$

Let $k=-m$. Taking α-cut of equation (3.1) we get

$$\frac{d}{dt}([x_2(t,\alpha),x_2(t,\alpha)];[x_2'(t,\beta),x_1'(t,\beta)])=-m([x_1(t,\alpha),x_2(t,\alpha)];[x_1'(t,\beta),x_2'(t,\beta)])+\epsilon \tag{3.5}$$

with initial condition

$$x(t_0;\alpha,\beta)=([a_1(\alpha),a_2(\alpha)];[a_1'(\beta),a_2'(\beta)],\alpha+\beta\le 1,\alpha,\beta\in[0,1])$$

i.e.,

$$\dot{x}_2(t,\alpha)=-mx_2(t,\alpha)+\epsilon$$
$$\dot{x}_1(t,\alpha)=-mx_1(t,\alpha)+\epsilon$$
$$\dot{x}_2'(t,\beta)=-mx_2'(t,\beta)+\epsilon$$
$$\dot{x}_1'(t,\beta)=-mx_1'(t,\beta)+\epsilon$$

with initial condition

$$x_1(t_0,\alpha) = a_1(\alpha)$$
$$x_2(t_0,\alpha) = a_2(\alpha)$$
$$x_1'(t_0,\beta) = a_1'(\beta)$$
$$x_2'(t_0,\beta) = a_2'(\beta).$$

The solution is given by

$$x_1(t,\alpha) = \frac{\epsilon}{m} + \left\{-\frac{x_0}{m} + a_1(\alpha)\right\} e^{-m(t-t_0)}$$
$$x_2(t,\alpha) = \frac{\epsilon}{m} + \left\{-\frac{x_0}{m} + a_2(\alpha)\right\} e^{-m(t-t_0)}$$
$$x_1'(t,\beta) = \frac{\epsilon}{m} + \left\{-\frac{x_0}{m} + a_1'(\beta)\right\} e^{-m(t-t_0)}$$
$$x_2'(t,\beta) = \frac{\epsilon}{m} + \left\{-\frac{x_0}{m} + a_2'(\beta)\right\} e^{-m(t-t_0)}.$$

The solution is a strong solution if

(i) $\frac{dx_1(t,\alpha)}{d\alpha} > 0, \frac{dx_2(t,\alpha)}{d\alpha} < 0 \ \forall\ \alpha \in [0,1],\ x_1(t,1) \le x_2(t,1),$

(ii) $\frac{dx_1'(t,\beta)}{d\beta} < 0, \frac{dx_2'(t,\beta)}{d\beta} > 0 \ \forall\ \beta \in [0,1],\ x_1'(t,0) \le x_2'(t,0).$

Otherwise the solution is week solution.

4 Example

Example 4.1: Consider the FODE $\frac{dx(t)}{dt} = \frac{1}{5}x(t) + 2$ with $\tilde{x}(t=0) = (4,5,6; 4.2,5,6.2)$.

Solution: The solution when $x(t)$ is (i)-gH differentiable is given by

$$x_1(t,\alpha) = -10 + \{10 + (4+\alpha)\}e^{\frac{1}{5}t}$$
$$x_2(t,\alpha) = -10 + \{10 + (6-\alpha)\}e^{\frac{1}{5}t}$$
$$x_1'(t,\beta) = -10 + \{10 + (5-0.8\beta)\}e^{\frac{1}{5}t}$$
$$x_2'(t,\beta) = -10 + \{10 + (5+1.2\beta)\}e^{\frac{1}{5}t}.$$

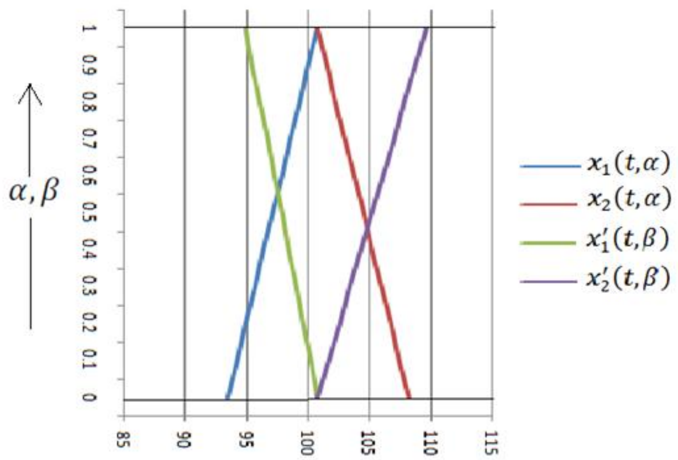

Figure 1: Graph of $x_1(t,\alpha)$, $x_2(t,\alpha)$, $x_1'(t,\beta)$ and $x_2'(t,\beta)$for $t = 20$

Note: From above graph we conclude that the solution is a strong solution.

The solution is a triangular Intuitionistic fuzzy number which is written as

$$\tilde{x}^i = \left(-10 + 14e^{\frac{t}{5}}, -10 + 15e^{\frac{t}{5}}, -10 + 16e^{\frac{t}{5}}; -10 + 14.2e^{\frac{t}{5}}, -10 + 15e^{\frac{t}{5}}, -10 + 16.2e^{\frac{t}{5}}\right).$$

The membership function and non-membership function are as follows

$$\mu_{\tilde{x}^i}(x)=\begin{cases}\dfrac{x-(-10+14e^{\frac{t}{5}})}{e^{\frac{t}{5}}}, & -10+14e^{\frac{t}{5}}\le x\le -10+15e^{\frac{t}{5}}\\ \dfrac{(-10+16e^{\frac{t}{5}})-x}{e^{\frac{t}{5}}}, & -10+15e^{\frac{t}{5}}\le x\le -10+16e^{\frac{t}{5}}\\ 0, & otherwise\end{cases}$$

and

$$\vartheta_{\tilde{x}^i}(x)=\begin{cases}\dfrac{(-10+15e^{\frac{t}{5}})-x}{0.8e^{\frac{t}{5}}}, & -10+14.2e^{\frac{t}{5}}\le x\le -10+15e^{\frac{t}{5}}\\ \dfrac{x-(-10+15e^{\frac{t}{5}})}{1.2e^{\frac{t}{5}}}, & -10+15e^{\frac{t}{5}}\le x\le -10+16.2e^{\frac{t}{5}}\\ 1, & otherwise.\end{cases}$$

Example 3.2: Consider the FODE $\frac{dx(t)}{dt}=-\frac{1}{10}x(t)+2$ with $\tilde{x}(t=0)=(7,10,14;7.8,10,11.6)$.

Solution: The solution when $x(t)$ is (i)-gH differentiable is given by

$$x_1(t,\alpha)=20-(9.5+0.5\alpha)e^{-\frac{1}{10}t}+(-3.5+3.5\alpha)e^{\frac{1}{10}t}$$
$$x_2(t,\alpha)=20-(9.5+0.5\alpha)e^{-\frac{1}{10}t}-(-3.5+3.5\alpha)e^{\frac{1}{10}t}$$
$$x_1'(t,\beta)=20-(10+0.3\beta)e^{-\frac{1}{10}t}-1.9\beta e^{\frac{1}{10}t}$$
$$x_2'(t,\beta)=20-(10+0.3\beta)e^{-\frac{1}{10}t}+1.9\beta e^{\frac{1}{10}t}.$$

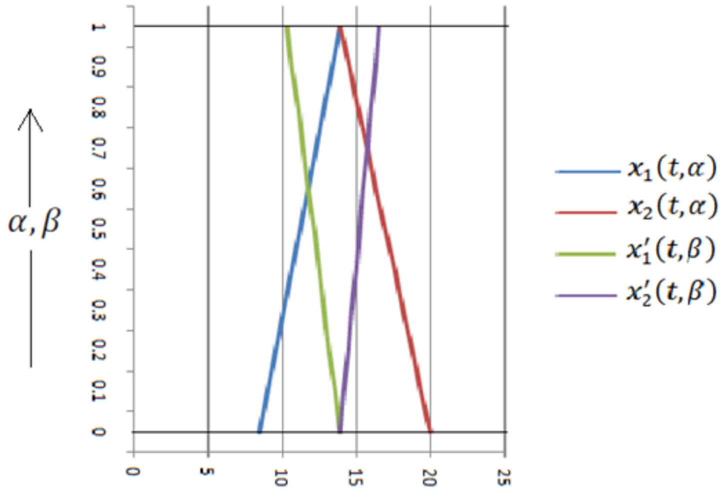

Figure 2: Graph of $x_1(t,\alpha)$, $x_2(t,\alpha)$, $x_1'(t,\beta)$ and $x_2'(t,\beta)$ for $t=5$

Note: From above graph we conclude that the solution is a strong solution.

The solution is a triangular Intuitionistic fuzzy number which is written as

$$\tilde{x}^i=\left(20-9.5e^{-\frac{t}{10}}-3.5e^{\frac{t}{10}},20-10e^{-\frac{t}{10}},20-9.5e^{-\frac{t}{10}}+3.5e^{\frac{t}{10}};20-10.3e^{-\frac{t}{10}}-1.9e^{\frac{t}{10}},20-10e^{-\frac{t}{10}},20-10.3e^{-\frac{t}{10}}+1.9e^{\frac{t}{10}}\right).$$

The membership function and non-membership function as follows

$$\mu_{\tilde{x}^i}(x) = \begin{cases} \dfrac{x-(20-9.5e^{-\frac{t}{10}}-3.5e^{\frac{t}{10}})}{-0.5e^{-\frac{t}{10}}+3.5e^{\frac{t}{10}}}, & 20-9.5e^{-\frac{t}{10}}-3.5e^{\frac{t}{10}} \le x \le 20-10e^{-\frac{t}{10}} \\ \dfrac{20-9.5e^{-\frac{t}{10}}+3.5e^{\frac{t}{10}}-x}{0.5e^{-\frac{t}{10}}+3.5e^{\frac{t}{10}}}, & 20-10e^{-\frac{t}{10}} \le x \le 20-9.5e^{-\frac{t}{10}}+3.5e^{\frac{t}{10}} \\ 0, & otherwise \end{cases}$$

and

$$\vartheta_{\tilde{x}^i}(x) = \begin{cases} \dfrac{(20-10e^{-\frac{t}{10}})-x}{0.3e^{-\frac{t}{10}}+1.9e^{\frac{t}{10}}}, & 20-10.3e^{-\frac{t}{10}}-1.9e^{\frac{t}{10}} \le x \le 20-10e^{-\frac{t}{10}} \\ \dfrac{x-(20-10e^{-\frac{t}{10}})}{-0.3e^{-\frac{t}{10}}+1.9e^{\frac{t}{10}}}, & 20-10e^{-\frac{t}{10}} \le x \le 20-10.8e^{-\frac{t}{10}}+1.9e^{\frac{t}{10}} \\ 1, & otherwise. \end{cases}$$

5 Applications

Bank account problem: Consider the differential equation $\frac{dM(t)}{dt} = 0.04M(t) - 4000$, where $M(t)$ is the amount of money in a bank account at time$t$, where t is given in years. The differential equation reflects the situation in which interest is being paid at a rate of 4% per year compounded continuously and money is being withdrawn at a constant rate of $4000 per year.

Suppose the initial deposit is $(\$49750, \$50000, \$50350; \$49700, \$50000, \$50250)$. How much money is there after 5 years?

Solution: The solution when $M(t)$ is (i)-gH differentiable is

$$M_1(t,\alpha) = 100000 + \{-50250 + 250\alpha\}e^{0.04t}$$
$$M_2(t,\alpha) = 100000 + \{-49650 - 350\alpha\}e^{0.04t}$$
$$M_1'(t,\beta) = 100000 + \{-50000 - 300\beta\}e^{0.04t}$$
$$M_2'(t,\beta) = 100000 + \{-50000 + 250\beta\}e^{0.04t}.$$

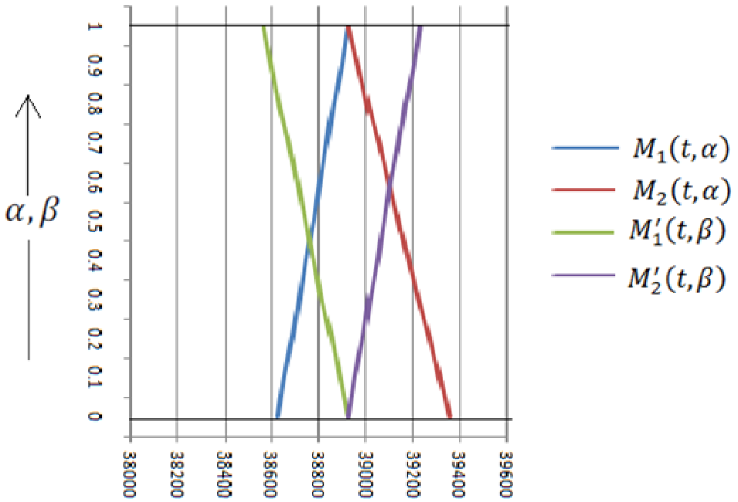

Figure 3: Graph of $M_1(t,\alpha)$, $M_2(t,\alpha)$, $M_1'(t,\beta)$ and $M_2'(t,\beta)$For $t = 5$

Note: From above graph we conclude that the solution is a strong solution.

The solution is a triangular Intuitionistic fuzzy number which is written as

$$\widetilde{M}^i = (100000 - 50250e^{0.04t}, 100000 - 50000e^{0.04t}, 100000 - 49650e^{0.04t}; 100000 - 50300e^{0.04t}, 100000 - 50000e^{0.04t}, 100000 - 49750e^{0.04t}).$$

The membership function and non-membership function as follows

$$\mu_{\tilde{M}^i}(x)=\begin{cases}\dfrac{x-(100000-50250e^{0.04t})}{250e^{0.04t}}, & 100000-50250e^{0.04t}\le x\le 100000-50000e^{0.04t}\\ \dfrac{(100000-49650e^{0.04t})-x}{350e^{0.04t}}, & 100000-50000e^{0.04t}\le x\le 100000-49650e^{0.04t}\\ 0, & otherwise\end{cases}$$

and

$$\vartheta_{\tilde{M}^i}(x)=\begin{cases}\dfrac{(100000-50000e^{0.04t})-x}{300e^{0.04t}}, & 100000-50300e^{0.04t}\le x\le 100000-50000e^{0.04t}\\ \dfrac{x-(100000-50000e^{0.04t})}{250e^{0.04t}}, & 100000-50000e^{0.04t}\le x\le 100000-49750e^{0.04t}\\ 1, & otherwise.\end{cases}$$

6 Conclusion

In this paper we have solved first order non homogeneous ordinary differential equation in intutionistic fuzzy environment. Here we have discussed initial value as triangular intutionistic fuzzy number. The solution procedure is taken both cases positive coefficient and negative coefficients. A bank account problem is discussed in intutionistic fuzzy environment. For further work the same process can be solved by using generalised triangular intutionistic fuzzy number, generalised trapizoidal intutionistic fuzzy number, generalised L-R type intutionistic fuzzy number. Also we can follow the same for first order system of ordinary differential equation. This process can be followed for any economical, bio-mathematical and engineering sciences problem in fuzzy environment.

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