Journal of Uncertain Systems
Vol.9, No.4, pp.251-273, 2015
Online at: www.jus.org.uk

A New Intuitionistic Preference Scale based on Interval Type-2 Fuzzy Set for MCDM Problems

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Received 29 May 2013; Revised 2 August 2015

Abstract

Interval Type-2 Fuzzy TOPSIS (IT2FTOPSIS) is a useful way to handle Fuzzy Multiple Attribute Decision-Making (FMADM) problems in a more flexible and intelligent manner. It is very useful due to the fact that it uses Type-2 Fuzzy Sets (T2FSs) rather than Type-1 Fuzzy Sets (T1FSs) to represent the evaluating values and the weights of attributes. Besides, all the linguistic terms are pointed in Type-1 Fuzzy Numbers (T1FNs) rather than Fuzzy TOPSIS (FTOPSIS), using crisps numbers. However, IT2FTOPSIS only focuses on the membership degree without considering the non-membership degree. In real life situation, evaluation becomes more comprehensive if non-membership degree is considered concurrently. Preference is expected to be more effective when considering both membership and non-membership degree due to the effectiveness of fuzziness taken from the hesitation degree. Therefore, the aim of this paper is to introduce a new preference scale that considers both membership and non-membership degree in IT2FTOPSIS. Both membership and non-membership degree of Intuitionistic Fuzzy Sets (IFSs) are developed under Interval Type-2 FMADM environment. Then, this new method is tested using five illustrative examples. Finally, this new method is applied to a case study on selecting the best of flood control project and the results demonstrate the feasibility. This paper has been proven able to measure human being decision making progress to solve the incomplete information and becomes a new way to deal with the vagueness and uncertainty.

Keywords: intuitionistic fuzzy set, interval type-2 fuzzy sets, interval type-2 fuzzy TOPSIS

1 Introduction

Intuitionistic Fuzzy Set (IFS) was first developed by Atanassov [2]. Atanassov [2] introduced the concept of membership and non-membership degree in IFS, unlike the concept of Fuzzy Set (FS) from Zadeh [47] that considered the membership degree only. IFS has proven to be highly and more useful in dealing with uncertainty and vagueness [44] compared to Fuzzy Set (FS). It is due to the existence of hesitation degree from non-membership degree which proved to be more suitable in dealing with fuzziness and uncertainty. Since that, IFS has been spread vigorously in many areas. For example, Atanassova [4] determined the cardinality of the set of all different types of IFS over a given universe. Besides, Bustince and Burillo [10] recapitulated the definition given by Atanassov [3] of IFSs as well as the definition of vague sets given by Gau and Buehrer [20] and see both definitions coincide. In 1996, Bustince and Burillo [10] defined the distance measure between IFSs and gave an axiom definition of intuitionistic fuzzy entropy. Moreover, in 2005, Duboisa [19] pointed out a clash of terminological with Atanassov's "IFSs" that understood as intuitionistic logic. Furthermore, Cattaneo and Ciucci [11] contributed the terminological debate about Atanassov's use of the term "Intuitionistic" in defining IFS structure based on ortho-pairs of FSs. Besides, Bustince et al. [8] constructed an expression for calculating the total contrast of an image from Atanassov's intuitionistic fuzzy S-implications and from the fuzzy expected values. On the other hand, Chai et al. [12] proposed a new Intuitionistic Fuzzy SIR (IF-SIR) approach and focused on its application to supplier selection which is the important activity in supply chain management.

Owing to the advantage of dealing with uncertain information define from the hesitation degree, many theories and methods on IFS have been put forward and have been used to solve Fuzzy Multi-Criteria Decision Making

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(FMCDM) problems. For example, Boran et al. [6] proposed a Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method combined with IFS to select appropriate supplier in group decision making environment. Next, Yue et al. [46] defined the concepts of supporting, apposing and neutral set of alternative respectively, developed an approach for transform attribute values into Intuitionistic Fuzzy Number (IFN), and determined the order of alternatives based on the score and the degree of accuracy of the IFN. In 2010, Li et al. [28] aimed to develop a new methodology for solving group decision making problems in which preference comparisons between alternatives are expressed with Atanssov's intuitionistic fuzzy (IF) preference relations. Then, also in 2010, Ye [44] extended TOPSIS method for group decision making with Interval-Valued Intuitionistic Fuzzy Numbers (IVIFNs) to solve the partner selection problem under incomplete and uncertain information environment. Moreover, in 2011, Sua et al. [37] investigated the Dynamic Intuitionistic Fuzzy Multi-Attribute Group Decision Making (DIFMAGDM) problems, in which all the attribute values provided by multiple Decision Makers (DMs) at different periods take the form of IFNs, and developed an interactive method to solve the DIFMAGDM problems. Next, Chen [14] conducted a comparative study of score functions in MCDM based on IFSs. Besides, Yue and Jia [45] presented a soft computing model to Multiple Attribute Group Decision-Making (MAGDM) problems and aggregated all individual decisions on an attribute into an IVIFN.

Due to the vagueness and uncertainties in many decision problems, Type-2 Fuzzy Set (T2FS) by Zadeh [48] was introduced in order to offer better solution for the problems. The latter concepts look comprehensive due to the ability of providing more flexibility spaces to represent uncertainties compared to FSs [47]. Then, in the year of 2000, Mendel and Liang [34] upgraded T2FS in the interval concept and became the Interval Type-2 Fuzzy Set (IT2FS). This IT2FS can be viewed as a special case as all values of secondary membership are equal to 1 [24], where the membership functions of IT2FS are three dimensional and include a Footprint of Uncertainty (FOU) which is the third new dimension of T2FS and the footprint of uncertainty provide additional degrees of freedom to directly model and handle uncertainties. Concurrently, the membership function of IT2FS are widely explored and successfully applied in decision-making field. For example, Chen and Lee [16] developed linguistic terms in Type-1 Fuzzy Numbers (T1FNs) rather than Fuzzy TOPSIS (FTOPSIS), using crisps numbers to handle Interval Type-2 Fuzzy Multiple Criteria Group Decision-Making (IT2FMCGDM) problems. Furthermore, Wu and Mendel [42,43] presented a method using the linguistic weighted average and IT2FS for handling Fuzzy Multiple Criteria Group Decision-Making (FMCGDM) problems, Wu and Mendel's FMCGDM method was to make decisions by means of aggregating the opinions of DMs. Moreover, in 2013, Wang et al. [39] investigated the MCGDM problem under IT2 fuzzy environment and developed an approach to handle the situation where the attributes values are characterized by IT2FS and the information about attribute weights is partially known.

However, in real life situation, a membership function express by DMs in IT2MCGDM problems may not be able to accurate his/ her preferences for alternatives due to several reasons. One of the reasons is the DM may not possess a precise or sufficient level of knowledge of the problem due to some hesitation problems in defining the preference. Therefore, the objective of this paper is to introduce a new preference scale that considers membership, non-membership and hesitation degree in interval type-2 fuzzy TOPSIS (IT2FTOPSIS). The IFS with IT2FS is used in order to develop new preference scale. This new preference scale can measure human being decision making progress, solve the incomplete information and to be the new way to deal with the vagueness and uncertainty.

The remaining part of this paper is arranged as follows. In Section 2, some basic concepts on IFS, T2FS, FOU and IT2FS are briefly reviewed. Then, it is followed by the development of a new intuitionistic preference scale with IT2FS (Section 3.1), new IT2FTOPSIS procedure with the new preference scale (Section 3.2) and schematic diagram (Section 3.3). Moreover, a numerical illustration is presented in Section 4 and other examples are validated in Section 4.1. Furthermore, a case study on flood control project selection is performed in Section 5, includes the background of the case study (Section 5.1) and 'analysis and results' (Section 5.2).The other examples are validated in Section 5. Finally, the last section which is Section 6 provides the conclusion.

2 Fundamental Notion

In this section, we shall briefly recall some fundamental notions of IFSs, T2FS, FOU and basic concepts of IT2FS.

2.1. Intuitionistic Fuzzy Sets

IFSs constitute a generalization of the notion of a FS. FSs give the degree of membership of an element in a given set, while IFSs give both a degree of membership and a degree of non-membership, which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than 1 [18].

Definition 2.1 ([2]) An IFS A in X is given by

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle | x\in X\} \tag{2.1}$$

where for all $x\in X$, $\mu_A(x)\in[0,1]$ and $\nu_A(x)\in[0,1]$ are called the membership degree and non-membership degree, respectively, of x in A, and furthermore satisfy $0\le\mu_A(x)+\nu_A(x)\le 1$. The class of intuitionistic fuzzy sets in X is denoted by IFS(X).

An IFS A is said to be contained in an IFS B (notation $A\subseteq B$) if and only if $\mu_A(x)\le\mu_B(x)$ and $\nu_A(x)\le\nu_B(x)$, for all $x\in X$. The union of the intersection of two IFSs A and B in X is given by:

$$(A\cap B)(x)=\{(x,\min(\mu_A(x),\mu_B(x)),\max(\nu_A(x),\nu_B(x)))| x\in X\}, \tag{2.2}$$

$$(A\cap B)(x)=\{(x,\max(\mu_A(x),\mu_B(x)),\min(\nu_A(x),\nu_B(x)))| x\in X\}. \tag{2.3}$$

For each IFS in X, we will call $\pi_A(x)=1-\mu_A(x)-\nu_A(x)$ as the intuitionistic index of x in A. It is obvious that $0\le\pi_A(x)\le 1,\ \forall x\in X$. Obviously, every FS A corresponds to the following IFS:

$$A=\{\langle x,\mu_A(x),1-\mu_A(x)\rangle | x\in X\}. \tag{2.4}$$

2.2. Type-2 Fuzzy Sets

Definition 2.2 ([30, 32]) A T2FSs $\tilde{A}$ is characterized by a three-dimensional membership function (type-2 membership function) which itself is fuzzy. This is shown in Eq. (2.5), where $0\le\mu_{\tilde{A}}(x,u)\le 1$. Alternatively, $\tilde{A}$ can be expressed on Eq. (2.6), where the symbol $\int\int$ denotes union over all admissible x and u.

$$\tilde{A}=\{((x,u),\mu_{\tilde{A}}(x,u)) | \forall x\in X,\forall u\in J_x\subseteq[0,1]\} \tag{2.5}$$

$$\tilde{A}=\int_{x\in X}\int_{u\in J_x}\mu_{\tilde{A}}(x,u)/(x,u) \tag{2.6}$$

A very convenient notation of T2FSs is the vertical slice representation [32]. This is shown in Eq. (2.7)-(2.9), where $\mu_{\tilde{A}}(x)$ is the secondary membership function (a T1FS) for a generic element $x\in X$. The domain (J_x) and amplitude $(f_x(u))$ of $\mu_{\tilde{A}}(x)$ are the primary membership of x and secondary grade, respectively.

$$\tilde{A}=\{(x,\mu_{\tilde{A}}(x)) | \forall x\in X\} \tag{2.7}$$

$$\tilde{A}=\int_{x\in X}\mu_{\tilde{A}}(x)/x \tag{2.8}$$

$$\mu_{\tilde{A}}(x)=\int_{u\in J_x} f_x(u)/u,\quad J_x\subseteq[0,1] \tag{2.9}$$

2.3. Fingerprint of Uncertainty (FOU)

Definition 2.3 ([30, 32]) The union $(\bigcup)$ of all primary memberships of $\tilde{A}$ is a bounded region called FOU as showed in Figure 1 and Eq. (2.10). The upper (Eq. (2.11)) and lower (Eq. (2.12)) bounds of the FOU are two type-1 membership functions called the upper and lower memberships of $\tilde{A}$, respectively.

$$FOU(\tilde{A})=\bigcup_{x\in X} J_x \tag{2.10}$$

$$\bar{\mu}_{\tilde{A}}(x)=\overline{FOU(\tilde{A})}=\bigcup_{x\in X}\overline{J_x},\quad \forall x\in X \tag{2.11}$$

$$\underline{\mu}_{\tilde{A}}(x) = \underline{FOU}(\tilde{A}) = \bigcup_{x\in X} \underline{J}_x, \quad \forall x \in X \tag{2.12}$$

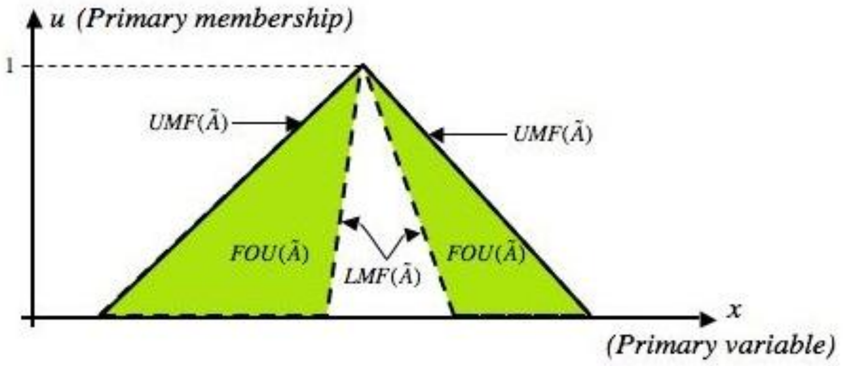

Figure 1: Fingerprint of uncertainty (FOU)

If X and J_x are both discrete, either by problem formulation or by discretization of continuous universes of discourse, then $\tilde{A}$ can be expressed on Eq. (2.13), where the symbols $\sum$ and $+$ also denote union over all admissible x and u.

$$\tilde{A} = \sum_{x\in X}\left[\sum_{u\in J_x} f_x(u)/u\right]\Big/x = \sum_{i=1}^{N}\left[\sum_{u\in J_{x_i}} f_{x_i}(u)/u\right]\Big/x_i = \left[\sum_{k=1}^{M_1} f_{x_i}(u_{1k})/u_{1k}\right]\Big/x_1 + \cdots + \left[\sum_{k=1}^{M_N} f_{x_N}(u_{Nk})/u_{Nk}\right]\Big/x_N \tag{2.13}$$

This last expression (Eq. (2.13)) allows the introduction of the useful concepts of embedded type-2 and embedded T1FSs. For discrete universes of discourse X and U, and embedded type-2 fuzzy set $\tilde{A}_e$ has N elements, where $\tilde{A}_e$ contains exactly one element from $J_{x_1}, J_{x_2}, \ldots, J_{x_N}$, each with its associated secondary grade (Eq. (2.14)). The union of all the primary memberships of $\tilde{A}_e$ is called an embedded T1FS A_e (Eq. (2.15)). Set $\tilde{A}_e$ is embedded in $\tilde{A}$, there are a total of $\prod_{i=1}^{N} M_i$ embedded type-2 and embedded T1FSs in $\tilde{A}$.

$$\tilde{A}_e = \sum_{i=1}^{N}\left[f_x(u)/u\right]/x_i, \quad u_i \in J_{x_i} \subseteq U = [0,1] \tag{2.14}$$

$$A_e = \sum_{i=1}^{N} u_i/x_i, \quad u_i \in J_{x_i} \subseteq U = [0,1] \tag{2.15}$$

Definition 2.4 ([32]) Let $\tilde{A}_e^j$ denote the j-th embedded type-2 fuzzy set for the type-2 fuzzy set $\tilde{A}$ (Eq. (2.16)), then $\tilde{A}$ can be represented as the union of its embedded type-2 fuzzy sets (Eq. (2.17)). This is called wavy slice representation of $\tilde{A}$.

$$\tilde{A}_e^j \equiv \left\{\left(u_i^j, f_{x_i}\left(u_i^j\right)\right), i = 1, \ldots, N\right\} \quad u_i^j \in \left\{u_{ik}, k = 1, \ldots, M_i\right\} \tag{2.16}$$

$$\tilde{A} = \sum_{j=1}^{n} \tilde{A}_e^j, \quad n \equiv \prod_{i=1}^{N} M_i \tag{2.17}$$

2.4. Interval Type-2 Fuzzy Sets

Definition 2.5 ([33]) Let $\tilde{\tilde{A}}$ be a T2FS in the universe of discourse X represented by the type-2 membership function $\mu_{\tilde{\tilde{A}}}$. If all $\mu_{\tilde{\tilde{A}}} = 1$, then A is called an IT2FSs. An IT2FS $\tilde{\tilde{A}}$ can be regarded as a special case of a T2FS, represented as follows:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u\in J_x} 1/(x,u), \tag{2.18}$$

where $J_x \subseteq [0,1]$.

Our purpose is to develop a new preference scale for IT2FTOPSIS. Thus, IFSs and the concept of IT2FSs are used as a stepping-stone in achieving the objectives. Therefore, the development of a new intuitionistic preference scale based on IT2FS for MCDM problems is shown in Section 3.

3 Development of a New IT2FTOPSIS

This section establishes a new preference scale based on IFS and IT2FS which the attributes and alternatives values take the form of uncertain preference scale. Before establishing the new preference scale, this paper introduces some of the new theorem of IFS in IT2FS. Based on the new theorems, the Interval Type-2 Intuitionistic Fuzzy Set (IT2-IFS) preference scale is developed (shown in Section 3.1). Later, this new preference scale is applied in IT2FTOPSIS to handle multi-criteria problems (shown in Section 3.2). Lastly, the schematic diagram of implementing this new preference scale for IT2FTOPSIS is shown in Section 3.3.

3.1. Development of a New Intuitionistic Preference Scale with IT2FS

According to Chen and Tsao [17], sometimes available information is not sufficient for the exact definition of a degree of membership for certain elements. There may be some hesitation degree between membership and non-membership. In view that there are many real life situations where due to insufficiency in information availability, IFS with IT2FS are used to overcome this problems. Therefore, a new preference scale is developed where the hesitation degree is introduced in IT2FTOPSIS. Before we develop a new preference scale, we introduce some of the propositions on IFS with IT2FS. Based on the IFS definition (refer Section 2.1) and IT2FS definition (refer Section 2.4), therefore, an IT2-IFS is developed as follows:

Proposition 3.1: Let

$$\tilde{\tilde{A}} = \{\langle((x,u), \mu_A(x,u), v_A(x,u))\rangle\} \tag{3.1}$$

where for all $\forall x \in X$, and $\forall u \in J_x \subseteq [0,1]$. Then, $\mu_A(x,u) \in [0,1]$ is a type-1 membership degree and $v_A(x,u) \in [0,1]$ is a type-1 non-membership degree, respectively, of *x* and *u* in *A*, and furthermore satisfy $0 \le \mu_A(x,u) + v_A(x,u) \le 1$.

Proposition 3.2: Let $\tilde{\tilde{A}}$ be an IT2FSs. Then

$$\tilde{\tilde{A}} = \left(\tilde{A}^l, \tilde{A}^u\right), \quad \tilde{\tilde{A}} = \left(\left((x,u)^l, \mu_A(x,u)^l, v_A(x,u)^l\right), \left((x,u)^u, \mu_A(x,u)^u, v_A(x,u)^u\right)\right)$$

where

$$(x,u)^l = a_{i1}^l,\ \mu_A(x,u)^l = a_{i2}^l,\ v_A(x,u)^l = 1 - a_{i1}^l - a_{i2}^l,$$
$$(x,u)^u = a_{i1}^u,\ \mu_A(x,u)^u = a_{i2}^u,\ v_A(x,u)^u = 1 - a_{i1}^u - a_{i2}^u. \tag{3.2}$$

Thus,

$$\tilde{\tilde{A}}_i = \left(\left(a_{i1}^l, a_{i2}^l, 1 - a_{i1}^l - a_{i2}^l; H\left(\tilde{A}_i^l\right)\right), \left(a_{i1}^u, a_{i2}^u, 1 - a_{i1}^u - a_{i2}^u; H\left(\tilde{A}_i^u\right)\right)\right) \tag{3.3}$$

where a_{i1}^l is a lower membership function and a_{i2}^l is a lower non-membership function, $1 - a_{i1}^l - a_{i2}^l$ is a lower hesitation degree and $H\left(\tilde{A}_i^l\right)$ denotes the lower membership value of the element a_{i1}^l, a_{i2}^l and $1 - a_{i1}^l - a_{i2}^l$, and a_{i1}^u is a upper membership function and a_{i2}^u is a upper non-membership function, $1 - a_{i1}^u - a_{i2}^u$ is a upper hesitation degree and $H\left(\tilde{A}_i^u\right)$ denotes the upper membership value of the element a_{i1}^u, a_{i2}^u and $1 - a_{i1}^u - a_{i2}^u$.
Then, can also be denoted as

$$\tilde{\tilde{A}}_i = \left(\left(a_{i1}^l, a_{i2}^l; H\left(\tilde{A}_i^l\right)\right), \left(a_{i1}^u, a_{i2}^u; H\left(\tilde{A}_i^u\right)\right)\right) \tag{3.4}$$

Therefore, Proposition 3.1 and Proposition 3.2 can be referred as shown in Figure 2.

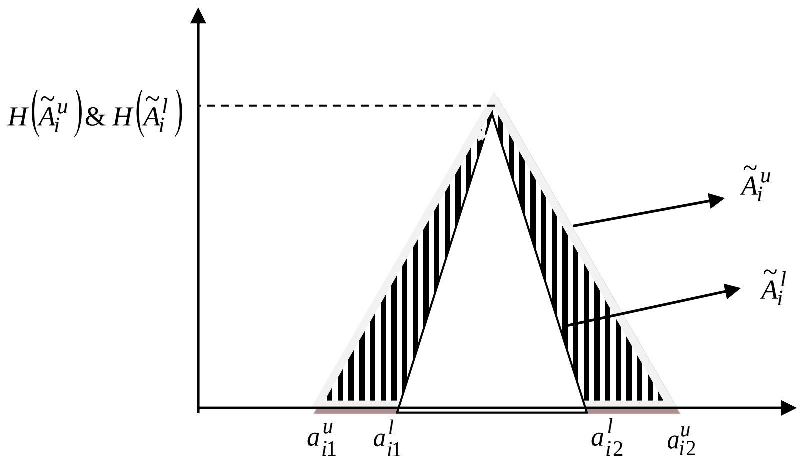

Figure 2: The upper and lower IT2FS

Further explanations regarding this new IT2-IFS are explained in Proposition 3.3 till Proposition 3.6.

Proposition 3.3: The addition operation between two intuitionistic T2FSs

$$\tilde{\tilde{A}}_1 = \left(\left(a_{11}^l, a_{12}^l; H\left(\tilde{A}_1^l\right)\right), \left(a_{11}^u, a_{12}^u; H\left(\tilde{A}_1^u\right)\right)\right) \text{ and } \tilde{\tilde{A}}_2 = \left(\left(a_{21}^l, a_{22}^l; H\left(\tilde{A}_2^l\right)\right), \left(a_{21}^u, a_{22}^u; H\left(\tilde{A}_2^u\right)\right)\right)$$

is defined as follows:

$$\tilde{\tilde{A}}_1 \oplus \tilde{\tilde{A}}_2 = \left(\tilde{A}_1^l, \tilde{A}_1^u\right) \oplus \left(\tilde{A}_2^l, \tilde{A}_2^u\right) = \left(\left(a_{11}^l + a_{21}^l; \min\left(H_1\left(\tilde{A}_1^l\right), H_1\left(\tilde{A}_2^l\right)\right)\right), \left(a_{11}^u + a_{21}^u; \min\left(H_1\left(\tilde{A}_1^u\right), H_1\left(\tilde{A}_2^u\right)\right)\right)\right). \quad (3.5)$$

Proposition 3.4: The substraction operation between two intuitionistic T2FSs

$$\tilde{\tilde{A}}_1 = \left(\left(a_{11}^l, a_{12}^l; H\left(\tilde{A}_1^l\right)\right), \left(a_{11}^u, a_{12}^u; H\left(\tilde{A}_1^u\right)\right)\right) \text{ and } \tilde{\tilde{A}}_2 = \left(\left(a_{21}^l, a_{22}^l; H\left(\tilde{A}_2^l\right)\right), \left(a_{21}^u, a_{22}^u; H\left(\tilde{A}_2^u\right)\right)\right)$$

is defined as follows:

$$\tilde{\tilde{A}}_1 - \tilde{\tilde{A}}_2 = \left(\tilde{A}_1^l, \tilde{A}_1^u\right) - \left(\tilde{A}_2^l, \tilde{A}_2^u\right) = \left(\left(a_{11}^l + a_{21}^l; \min\left(H_1\left(\tilde{A}_1^l\right), H_1\left(\tilde{A}_2^l\right)\right)\right), \left(a_{11}^u + a_{21}^u; \min\left(H_1\left(\tilde{A}_1^u\right), H_1\left(\tilde{A}_2^u\right)\right)\right)\right). \quad (3.6)$$

Proposition 3.5: The multiplication operation between two intuitionistic T2FSs

$$\tilde{\tilde{A}}_1 = \left(\left(a_{11}^l, a_{12}^l; H\left(\tilde{A}_1^l\right)\right), \left(a_{11}^u, a_{12}^u; H\left(\tilde{A}_1^u\right)\right)\right) \text{ and } \tilde{\tilde{A}}_2 = \left(\left(a_{21}^l, a_{22}^l; H\left(\tilde{A}_2^l\right)\right), \left(a_{21}^u, a_{22}^u; H\left(\tilde{A}_2^u\right)\right)\right)$$

is defined as follows:

$$\tilde{\tilde{A}}_1 \otimes \tilde{\tilde{A}}_2 = \left(\tilde{A}_1^l, \tilde{A}_1^u\right) \otimes \left(\tilde{A}_2^l, \tilde{A}_2^u\right) = \left(\left(a_{11}^l \times a_{21}^l; \min\left(H_1\left(\tilde{A}_1^l\right), H_1\left(\tilde{A}_2^l\right)\right)\right), \left(a_{11}^u \times a_{21}^u; \min\left(H_1\left(\tilde{A}_1^u\right), H_1\left(\tilde{A}_2^u\right)\right)\right)\right). \quad (3.7)$$

Proposition 3.6: The arithmetic operation between the intuitionistic T2FSs $\tilde{\tilde{A}}_1 = \left(\left(a_{11}^l, a_{12}^l; H\left(\tilde{A}_1^l\right)\right), \left(a_{11}^u, a_{12}^u; H\left(\tilde{A}_1^u\right)\right)\right)$ and the crisp value k is defined as follows:

$$k\tilde{\tilde{A}}_i = \left(\left(k \times a_{i1}^l, k \times a_{i2}^l, 1 - a_{i1}^l - a_{i2}^l; H\left(\tilde{A}_i^l\right)\right), \left(k \times a_{i1}^u, k \times a_{i2}^u, 1 - a_{i1}^u - a_{i2}^u; H\left(\tilde{A}_i^u\right)\right)\right), \quad (3.8)$$

$$\frac{\tilde{\tilde{A}}_i}{k} = \left(\left(\frac{1}{k} \times a_{i1}^l, \frac{1}{k} \times a_{i2}^l, 1 - a_{i1}^l - a_{i2}^l; H\left(\tilde{A}_i^l\right)\right), \left(\frac{1}{k} \times a_{i1}^u, \frac{1}{k} \times a_{i2}^u, 1 - a_{i1}^u - a_{i2}^u; H\left(\tilde{A}_i^u\right)\right)\right). \quad (3.9)$$

Based on Proposition 3.3 till Proposition 3.6, the computational technique of the new preference scale is defined as follows:

Table 1 shows the current preference scale of IFS by Boran et al. [6], where Table 2 shows the preference scale of IT2FS by Chen and Lee [15]. Information in Table 1 and Table 2 were set as prerequisites to formulate the conversion steps of a new preference scale.

Table 1: Linguistic variables of IFS

Linguistic Variable	IFS [6]
Very very bad (VVB)/ very very low (VVL)	(0.1, 0.9)
Very bad (VB)/very low (VL)	(0.1, 0.75)
Bad (B)/low (L)	(0.25, 0.6)
Medium bad (MB)/ medium low (ML)	(0.4, 0.5)
Fair (F)/medium (M)	(0.5, 0.4)
Medium good (MG)/ medium high (MH)	(0.6, 0.3)
Good (G)/high (H)	(0.7, 0.2)
Very good (VG)/very high (VH)	(0.8, 0.1)
Very very good (VVG)/ very very high (VVH)	(0.9, 0.10)
Extremely good (EG)/ extremely high (EH)	(1.0, 0.0)

Table 2: Linguistic variables of IT2FS

Linguistic Variable	IT2FS [15]
Very bad (VB)/very low (VL)	((0, 0, 0, 0.1; 1, 1), (0, 0, 0, 0.1; 1, 1))
Bad (B)/low (L)	((0, 0.1, 0.1, 0.3; 1, 1), (0, 0.1, 0.1, 0.3; 1, 1))
Medium bad (MB)/ medium low (ML)	((0.1, 0.3, 0.3, 0.5; 1, 1), (0.1, 0.3, 0.3, 0.5; 1, 1))
Fair (F)/medium (M)	((0.3, 0.5, 0.5, 0.7; 1, 1), (0.3, 0.5, 0.5, 0.7; 1, 1))
Medium good (MG)/ medium high (MH)	((0.5, 0.7, 0.7, 0.9; 1, 1), (0.5, 0.7, 0.7, 0.9; 1, 1))
Good (G)/high (H)	((0.7, 0.9, 0.9, 1; 1, 1), (0.7, 0.9, 0.9, 1; 1, 1))
Very good (VG)/very high (VH)	((0.9, 1, 1, 1; 1, 1), (0.9, 1, 1, 1; 1, 1))

In order to make this new preference scale possible, this study uses ten basic linguistic terms as "Very Very Low"(VVL), "Very Low" (VL), "Low" (L), "Medium Low" (ML), "Fair" (F)/"Medium" (M), "Medium High" (MH), "High" (H),"Very High" (VH), "Very Very High" (VVH), "Extremely High" (EH) and linguistic terms for the ratings of the criteria are "Very Very Bad" (VVB)/"Very Very Poor" (VVP), "Very Bad" (VB)/"Very Poor" (VP), "Bad" (B)/"Poor" (P), "Medium Bad" (MB)/"Medium Poor" (MP), "Fair" (F)/"Medium" (M), "Medium Good" (MG), "Good" (G) and "Very Good" (VG). Therefore for the purpose of clarity, preference scale of 'Very good (VG)/very high (VH)' is considered as an example. Below are the ways of converting IFS preference scale [6] and IT2FS preference scale [15] into the new preference scale as follows:

A: We know that intuitionistic fuzzy set *A* in *X* is given by $A = \langle x, \mu_A(x), v_A(x) \rangle x \in X)$, then the preference scale for 'Very good (VG)/very high (VH)' is (0.8, 0.1) where 0.8 is the value of x and 0.1 is the value of $v_A(x)$.

B: We know that T2FS $\tilde{A}$ in X is given by $\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\}$, and IT2FS by $\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} 1/(x,u)$, where

$$\tilde{\tilde{A}}_i = \left(\tilde{A}_i^U, \tilde{A}_i^L\right) = \left(\left(a_{i1}^u, a_{i2}^u, a_{i3}^u, a_{i4}^u; H_1\left(\tilde{A}_i^u\right), H_2\left(\tilde{A}_i^u\right)\right), \left(a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1\left(\tilde{A}_i^L\right), H_2\left(\tilde{A}_i^L\right)\right)\right), \quad [15]$$

then the preference scale for 'Very good (VG)/very high (VH)' is ((0.9, 1, 1, 1; 1, 1), (0.9, 1, 1, 1; 1, 1)).

C: Therefore, based on the idea from **A** and **B**, we develop a new preference scale as:

We know that IT2-IFS $\tilde{\tilde{A}}$ in *X* is $\tilde{\tilde{A}} = \{\langle((x,u), \mu_A(x,u), v_A(x,u))\rangle\}$, then $\tilde{\tilde{A}} = \left(\tilde{A}^l, \tilde{A}^u\right)$,

$$\tilde{\tilde{A}}=\left(\left(\left(x,u\right)^{l},\mu_{A}\left(x,u\right)^{l},\nu_{A}\left(x,u\right)^{l}\right),\left(\left(x,u\right)^{u},\mu_{A}\left(x,u\right)^{u},\nu_{A}\left(x,u\right)^{u}\right)\right).$$

Therefore, the preference scale for ‘Very good (VG)/very high (VH)’ is ((0.1, 0.9;1), (0.1, 0.9;1)). The value for hesitation degree are $1-(x,u)^{l}-\mu_{A}(x,u)^{l}=1-0.1-0.9=0$ and $1-(x,u)^{u}-\mu_{A}(x,u)^{u}=1-0.1-0.9=0$.

Thus, the new preference scale shows as follows:

Table 3: The new preference scale for the relative importance weights of attributes

Linguistic Variable	IT2-IFS
Very very low (VVL)	((0.1, 0.9;1), (0.1, 0.9;1))
Very low (VL)	((0.2, 0.8;1), (0.2, 0.8;1)
Low (L)	((0.3, 0.7;1), (0.3,0.7;1))
Medium low (ML)	((0.4, 0.6;1), (0.4,0.6;1))
Fair (F)/medium (M)	((0.5, 0.5;1), (0.5,0.5;1))
Medium high (MH)	((0.6, 0.4;1), (0.6,0.4;1))
High (H)	((0.7, 0.3;1), (0.7,0.3;1))
Very high (VH)	((0.8, 0.2;1), (0.8,0.2;1))
Very very high (VVH)	((0.9, 0.1;1), (0.9,0.1;1))
Extremely high (EH)	((1.0, 0.0;1), (1.0,0.0;1)

Table 4: The new preference scale for the ratings of attributes

Linguistic Variable	IT2-IFS
Very very bad (VVB)/Very Very Poor (VP)	((0.1, 0.9;1), (0.1, 0.9;1))
Very bad (VB)/ Very Poor (VP)	((0.2, 0.8;1), (0.2, 0.8;1)
Bad (B)/ Poor (P)	((0.3, 0.7;1), (0.3,0.7;1))
Medium bad (MB)/ Medium Poor (MP)	((0.4, 0.6;1), (0.4,0.6;1))
Fair (F)/medium (M)	((0.5, 0.5;1), (0.5,0.5;1))
Medium good (MG)	((0.6, 0.4;1), (0.6,0.4;1))
Good (G)	((0.7, 0.3;1), (0.7,0.3;1))
Very good (VG)	((0.8, 0.2;1), (0.8,0.2;1))
Very very good (VVG)	((0.9, 0.1;1), (0.9,0.1;1))
Extremely good (EG)	((1.0, 0.0;1), (1.0,0.0;1)

This new linguistic variables is still new. Therefore, we have tested this new linguistic variable by using five numerical examples which are [13], [15], [35], [39] and [50], shown in Section 4. Furthermore, a case study on flood control project selection is performed in Section 5.

3.2 A New IT2FTOPSIS Procedure with the New Preference Scale

The new intuitionistic preference scale with IT2FS is applied to the IT2FTOPSIS. Here, we used the IT2FTOPSIS from Chen and Lee [15]. Thus, similar to the existed IT2FTOPSIS, this interval type-2 intuitionistic fuzzy TOPSIS (IT2-IFTOPSIS) is also dealt with the relative between attributes and alternatives in finding the best rank for MCDM problems. The new preference scale of IT2FTOPSIS is used as measurement in judgement matrices. The proposed IT2FTOPSIS method is described as below:

Suppose an IT2FTOPSIS has n alternatives $(A_1,\ldots,A_n)$ and m decision criteria/ attributes $(C_1,\ldots,C_m)$. Each alternative is evaluated with respect to the m criteria/ attributes. All the values/ ratings assigned to the alternatives with respect to each criterion from a decision matrix, denoted by $S=(y_{ij})_{n\times m}$, and the relative weight vector about the criteria, denoted by $W=(w_1,\ldots,w_m)$, that satisfying $\sum_{j=1}^{m} w_j=1$. Therefore, the further explanations of the proposed eight step method are described as follows:

Step 1: Construct hierarchy structure of MCDM problems

Data for attributes and alternatives are identified using the IT2-IFS preference scale as the main part of MCDM problem. Therefore, the decision matrix and fuzzy weight matrix are defined as follows:

$$D = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_i \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_j \end{array} \\ \begin{bmatrix} \tilde{\tilde{A}}_{11} & \tilde{\tilde{A}}_{12} & \cdots & \tilde{\tilde{A}}_{1j} \\ \tilde{\tilde{A}}_{21} & \tilde{\tilde{A}}_{22} & \cdots & \tilde{\tilde{A}}_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{A}}_{i1} & \tilde{\tilde{A}}_{i2} & \cdots & \tilde{\tilde{A}}_{ij} \end{bmatrix} \end{array}, \tag{3.10}$$

$$\tilde{\tilde{W}} = \left(\tilde{\tilde{w}}_1, \ \tilde{\tilde{w}}_2, \ \cdots, \ \tilde{\tilde{w}}_j\right)$$

where $x_1, x_2, \ldots, x_i$ represents the alternative, $C_1, C_2, \ldots, C_j$ represents the attribute and $\tilde{\tilde{W}}$ represents the weight.

Step 2: Determine the weights for MCDM Problems

Determine the weighting matrix W_p of the attributes of the DM and construct the *p*th average weighting matrix $\overline{W}$, respectively, shown as follows:

$$\tilde{\tilde{w}}_j = 1/FOU\left(\tilde{\tilde{w}}_j\right) = \left[\tilde{w}_j{}^l, \tilde{w}_j{}^u\right],$$

$$\overline{W}_p = \left(\tilde{\tilde{w}}_j^p\right)_{1\times m} = \begin{array}{c} \begin{array}{cccc} f_1 & f_2 & \cdots & f_n \end{array} \\ \left[\tilde{\tilde{w}}_1^p \quad \tilde{\tilde{w}}_2^p \quad \cdots \quad \tilde{\tilde{w}}_j^p\right] \end{array} \tag{3.11}$$

$$\overline{W} = \left(\tilde{\tilde{w}}\right)_{1\times m} \tag{3.12}$$

where

$$\tilde{\tilde{w}}_j = \left(\frac{\tilde{\tilde{w}}_j^{1\,l} \oplus \tilde{\tilde{w}}_j^{2\,l} \oplus \ldots \oplus \tilde{\tilde{w}}_j^{k\,l}}{k}, \frac{\tilde{\tilde{w}}_j^{1\,u} \oplus \tilde{\tilde{w}}_j^{2\,u} \oplus \ldots \oplus \tilde{\tilde{w}}_j^{k\,u}}{k}\right),$$

$\tilde{\tilde{w}}_j$ is in IT2-IFS preference scale, $1 \le j \le m$, $1 \le p \le k$ and denotes the number of DMs.

Step 3: Weighted decision matrix

Construct the weighted decision matrix $\overline{Y}_w$,

$$\tilde{\tilde{v}}_{ij} = 1/FOU\left(\tilde{\tilde{v}}_{ij}\right) = \left[\tilde{v}_{ij}{}^l, \tilde{v}_{ij}{}^u\right] \tag{3.13}$$

$$\left[\tilde{v}_{ij}{}^l, \tilde{v}_{ij}{}^u\right] = \left(\left[\tilde{w}_{ij}{}^l \otimes \tilde{r}_{ij}{}^l\right], \left[\tilde{w}_{ij}{}^u \otimes \tilde{r}_{ij}{}^u\right]\right) \tag{3.14}$$

where

$$\tilde{r}_{ij} = \left(\frac{\tilde{a}_{ij}^{1l} \oplus \tilde{a}_{ij}^{2l} \oplus \ldots \oplus \tilde{a}_{ij}^{kl}}{k}, \frac{\tilde{a}_{ij}^{1u} \oplus \tilde{a}_{ij}^{1u} \oplus \ldots \oplus \tilde{a}_{ij}^{1u}}{k}\right).$$

Then the weighted decision matrix can be expressed as follows:

$$\overline{Y}_{w_{ij}} = \begin{array}{c} \\ f_1 \\ f_2 \\ \vdots \\ f_i \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_j \end{array} \\ \begin{bmatrix} \tilde{\tilde{v}}_{11} & \tilde{\tilde{v}}_{12} & \cdots & \tilde{\tilde{v}}_{1j} \\ \tilde{\tilde{v}}_{21} & \tilde{\tilde{v}}_{22} & \cdots & \tilde{\tilde{v}}_{2j} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{v}}_{i1} & \tilde{\tilde{v}}_{i2} & \cdots & \tilde{\tilde{v}}_{ij} \end{bmatrix} \end{array}, \tag{3.15}$$

where $\tilde{\tilde{v}}_{ij} = \left(\left(\tilde{v}_{ij}^l, \tilde{v}_{ij}^l; H\left(\tilde{v}_{ij}^l\right)\right), \left(\tilde{v}_{ij}^u, \tilde{v}_{ij}^u; H\left(\tilde{v}_{ij}^u\right)\right)\right)$ is a weighted decision matrix, where $1 \le i \le m$, and $1 \le j \le n$.

Step 4: Calculate the ranking value

Calculate the ranking value $Rank\left(\tilde{\tilde{v}}_{ij}\right)$ of the IT2FTOPSIS $\tilde{\tilde{v}}_{ij}$, where $1 \le j \le n$. Then, construct the ranking weighted decision matrix $\overline{Y}_w^*$,

$$\overline{Y}_w^* = \left(Rank\left(\tilde{\tilde{v}}_{ij}\right)\right)_{m\times n}, \tag{3.16}$$

where $1 \le i \le m$, and $1 \le j \le n$.

The ranking value $Rank(\tilde{\tilde{v}}_{ij})$ that was proposed by Lee and Chen [26] is modified in term of the IT2-IFS preference scale and defined as follows:

$$\begin{aligned} Rank(\tilde{\tilde{v}}_{ij}) &= M_1(\tilde{v}_{ij}^l) + M_1(\tilde{v}_{ij}^u) + M_2(\tilde{v}_{ij}^l) + M_2(\tilde{v}_{ij}^u) + \left(S_1(\tilde{v}_{ij}^l) + S_1(\tilde{v}_{ij}^u) + S_2(\tilde{v}_{ij}^l) + S_2(\tilde{v}_{ij}^u)\right) \\ &+ H_1(\tilde{v}_{ij}^l) + H_1(\tilde{v}_{ij}^u) + H_2(\tilde{v}_{ij}^l) + H_2(\tilde{v}_{ij}^u) , \end{aligned} \tag{3.17}$$

where $M_p(\tilde{v}_i^j)$ denoted the average of the elements v_{ip}^j and $v_{i(p+1)}^j$, $M_p(\tilde{v}_i^j) = \left(v_{ip}^j + v_{i(p+1)}^j\right)/2$, $1 \le p \le 3$, $S_q(\tilde{v}_i^j)$ denotes the standard deviation of the elements v_{iq}^j and $v_{i(q+1)}^j$, $S_p(\tilde{v}_i^j) = \sqrt{\frac{1}{2}\sum_{k=q}^{q+1}\left(v_{ik}^j - \frac{1}{2}\sum_{k=q}^{q+1} v_{ik}^j\right)^2}$, $1 \le q \le 3$, $H_p(\tilde{v}_i^j)$ denotes the membership value of the element $v_{i(p+1)}^j$ in the IT2-IFS preference scale membership function $\tilde{v}_i^j$, $1 \le p \le 2$, $j \in \{U, L\}$, and $1 \le i \le n$.

Step 5: Calculate the positive and negative ideal solution

Determine the positive ideal solution $x^+ = (v_1^+, v_2^+, \ldots, v_m^+)$ and the negative-ideal solution $x^- = (v_1^-, v_2^-, \ldots, v_m^-)$ where

$$v_i^+ = \begin{cases} \max\limits_{1 \le j \le n} \left\{Rank(\tilde{\tilde{v}}_{ij})\right\}, & if \ f_i \in F_1 \\ \min\limits_{1 \le j \le n} \left\{Rank(\tilde{\tilde{v}}_{ij})\right\}, & if \ f_i \in F_2 \end{cases} \tag{3.18}$$

and

$$v_i^- = \begin{cases} \min\limits_{1 \le j \le n} \left\{Rank(\tilde{\tilde{v}}_{ij})\right\}, & if \ f_i \in F_1 \\ \max\limits_{1 \le j \le n} \left\{Rank(\tilde{\tilde{v}}_{ij})\right\}, & if \ f_i \in F_2 \end{cases} \tag{3.19}$$

where F_1 denotes the set of benefit attributes, F_2 denotes the set of cost attributes, and $1 \le i \le m$.

Step 6: Calculate the distance

Calculate the distance $d^+(x_j)$ between each alternative x_j and the positive ideal solution x^+, as shown as follows:

$$d^+(x_j) = \sqrt{\sum_{i=1}^{m}\left(Rank(\tilde{\tilde{v}}_{ij}) - v_i^+\right)^2}, \tag{3.20}$$

where $1 \le j \le n$. Calculate the distance $d^-(x_j)$ between each alternative x_j and the negative-ideal solution x^-, shown as follows:

$$d^-(x_j) = \sqrt{\sum_{i=1}^{m}\left(Rank(\tilde{\tilde{v}}_{ij}) - v_i^-\right)^2}, \tag{3.21}$$

where $1 \le j \le n$.

Step 7: Calculate the closeness

Calculate the relative degree of closeness $C(x_j)$ of x_j with respect to the positive ideal solution x^+, shown as follow:

$$C(x_j) = \frac{d^-(x_j)}{d^+(x_j) + d^-(x_j)}, \tag{3.22}$$

Step 8: Rank the values

Sort the values of $C(x_j)$ in a descending sequence, where $1 \le j \le n$. The larger the value of $C(x_j)$, the higher the preference of the alternatives x_j, where $1 \le j \le n$.

In this IT2FTOPSIS framework, we introduced a new and standardized preference scale in the term of IT2FS notation instead of using crisp number. This new preference scale which is known as IT2-IFS considers the concept of hesitation. This new IT2-IFS preference scale is hoped to be one of the standard scales while using in IT2 FT process.

3.3 Schematic Diagram of Implementing a New IT2-IFS Preference Scale in IT2FTOPSIS

In this paper, we introduced the new preference scale using the intuitionistic fuzzy set in term of IT2FS. Then, this new IT2-IFS preference scale is implemented in IT2FTOPSIS. The general steps of the proposed IT2FTOPSIS were arranged in well-ordered and shown in Section 3.1 and Section 3.2. The summary of all steps in Section 3.3 is shown as Figure 3.

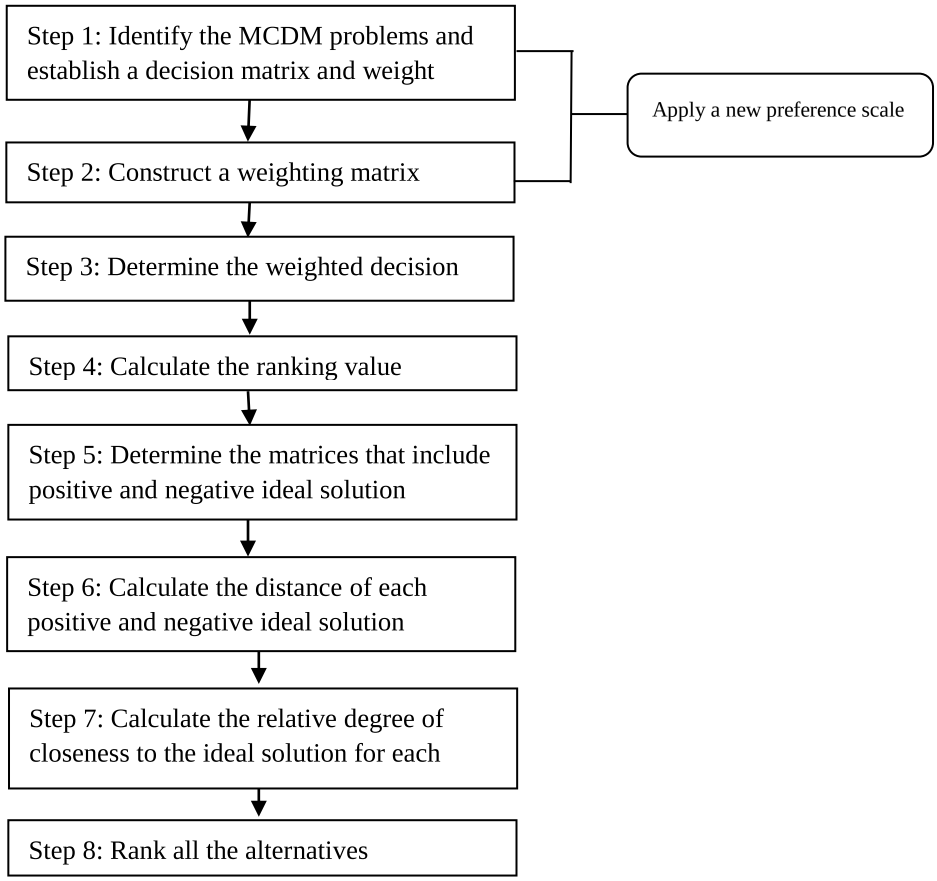

Figure 3: IT2FTOPSIS with new preference scale schematic diagram

4 Numerical Illustration

The following practical example involves determining a suitable location for a oil gas station in Iran city. This example is originally taken from Mokhtarian [35]. However, only half-part of Mokhtarian example is used without modification. This example has pointed three consultants as decision makers $\{D_1, D_2, D_3\}$ to determine a suitable location for the oil gas station. These consultants consider five locations for constructing the oil gas station and evaluate them by six attributes. However, our studies only used three attributes due its suitability to our model. It is because Mokhtarian only used three qualitative attributes and the rest of them were quantitative. The qualitative attributes are $\{C_1, C_3, C_4\}$. Besides, eight alternatives are taken account, which are $\{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8\}$. After a hard effort of consideration, these three consultants describe the information of constructing the oil gas station with respect to three attributes using the new preference scales, which are shown in Table 5 and Table 6. Thus, the computation are executed in the eight steps follows:

Table 5: Linguistic of decision matrix

Criteria	Alternatives	Decision-Makers		
		D_1	D_2	D_3
C_1	A_1	VG	G	VG
	A_2	VG	VG	G
	A_3	MG	F	F
	A_4	G	MG	G
	A_5	MG	G	G
	A_6	F	MG	F
	A_7	F	MG	G
	A_8	MG	F	F
C_3	A_1	VG	VG	VG
	A_2	MP	P	P
	A_3	P	P	P
	A_4	G	MG	G
	A_5	F	F	MG
	A_6	MP	MP	F
	A_7	P	P	MP
	A_8	F	MP	F
C_4	A_1	G	G	MG
	A_2	VG	VG	VG
	A_3	VG	VG	VG
	A_4	G	VG	G
	A_5	G	MG	G
	A_6	MG	G	MG
	A_7	F	F	MG
	A_8	VG	VG	G

Table 6: Linguistic of weighted matrix

Criteria	Decision-Makers		
	D_1	D_2	D_3
C_1	H	VH	VH
C_3	VH	H	VH
C_4	M	MH	H

Step 1: Establish a decision matrix

Establish a IT2-IFS preference scale decision matrix as follows:

The data that considers IT2-IFS preference scale (see Table 4) are referred in order to construct matrix of attributes. Therefore, let's take the example on calculating the $\approx{A}_{11}$.

$$\text{VG} = ((0.8, 0.2; 1), (0.8, 0.2; 1))$$
$$\text{G} = ((0.7, 0.3; 1), (0.7, 0.3; 1))$$
$$\text{VG} = ((0.8, 0.2; 1), (0.8, 0.2; 1))$$

Then, the average for VG, G and VG is

$$((0.7667, 0.2333; 1),(0.7667,0.2333;1)).$$

Apply the same calculations as $\overset{\approx}{A}_{11}$, thus, the whole results for matrix of criteria is summarized as Table 7.

Table 7: The decision matrix and weight matrix

	C_1	C_3	C_4
A_1	((0.7667,0.2333;1), (0.7667,0.2333;1))	((0.8,0.2;1), (0.8,0.2;1))	((0.8,0.2;1), (0.8,0.2;1))
A_2	((0.7667,0.2333;1), (0.7667,0.2333;1))	((0.4,0.6;1), (0.4,0.6;1))	((0.8,0.2;1), (0.8,0.2;1))
A_3	((0.5333,0.4667;1), (0.5333,0.4667;1))	((0.3,0.7;1), (0.3,0.7;1))	((0.7333,0.2667;1), (0.7333,0.2667;1))
A_4	((0.6667,0.3333;1), (0.6667,0.3333;1))	((0.6667,0.3333;1), (0.6667,0.3333;1))	((0.6667,0.3333;1), (0.6667,0.3333;1))
A_5	((0.6667,0.3333;1), (0.6667,0.3333;1))	((0.5333,0.4667;1), (0.5333,0.4667;1))	((0.6333,0.3667;1), (0.6333,0.3667;1))
A_6	((0.5333,0.4667;1), (0.5333,0.4667;1))	((0.4333,0.5667;1), (0.4333,0.5667;1))	((0.5333,0.4667;1), (0.5333,0.4667;1))
A_7	((0.6,0.4;1), (0.6,0.4;1))	((0.3333,0.6667;1), (0.3333,0.6667;1))	((0.7667,0.2333;1), (0.7667,0.2333;1))
A_8	((0.5333,0.4667;1), (0.5333,0.4667;1))	((0.4667,0.5333;1), (0.4667,0.5333;1))	((0.8,0.2;1), (0.8,0.2;1))

Step 2: Weighting matrix

All weights that have given by three DMs are constructed using the weighting matrix W_p in Eq. (3.9). Therefore, let's take the example on calculating $\tilde{\tilde{W}}_1$.

H = ((0.7, 0.3; 1), (0.7, 0.3; 1))

VH = ((0.8, 0.2; 1), (0.8, 0.2; 1))

VH = ((0.8, 0.2; 1), (0.8, 0.2; 1))

Then, the average for H, VH and VH is

((0.7667, 0.2333; 1), (0.7667, 0.2333; 1)).

Apply the same calculations as $\tilde{\tilde{W}}_1$, thus, the whole results for weighting matrix is stated in Table 8.

Table 8: The weighting matrix

	weights
C_1	((0.7667,0.2333;1),(0.7667,0.2333;1))
C_3	((0.7667,0.2333;1),(0.7667,0.2333;1))
C_4	((0.6,0.4;1),(0.6,0.4;1))

Step 3: Weighted decision matrix

After obtaining the decision and weighting matrices, then, the weighted decision matrix is constructed using the formula given by Eq. (3.11) and achieved the decision matrix as Matrix 3.12. Thus, the weighted decision matrix can be expressed as follows Table 9.

Step 4: Calculate the ranking value

Calculate the ranking value $Rank\left(\tilde{\tilde{v}}_{ij}\right)$ of the interval type-2 fuzzy set $\tilde{\tilde{v}}_{ij}$, where $1 \le j \le n$. Construct the ranking weighted decision matrix $\overline{Y}_w^*$,

$$Rank\left(\tilde{\tilde{v}}_{11}\right) = M_1\left(\tilde{A}_i^U\right) + M_1\left(\tilde{A}_i^L\right) - \frac{1}{4}\left(S_1\left(\tilde{A}_i^U\right) + S_1\left(\tilde{A}_i^L\right) + S_2\left(\tilde{A}_i^U\right) + S_2\left(\tilde{A}_i^L\right)\right) + H_1\left(\tilde{A}_i^U\right) + H_1\left(\tilde{A}_i^L\right),$$

$$Rank\left(\tilde{\tilde{v}}_{11}\right) = 0.3211 + 0.3211 - \frac{1}{4}\left(0.1886 + 0.1886 + 0.1886 + 0.1886\right) + 1 + 1 = 2.4537.$$

In the same way, we can construct the ranking weighted decision matrix $\tilde{Y}_w^*$ as Table 10.

Table 9: Weighted decision matrix

	C_1	C_3	C_4
1	((0.5878,0.0544;1), (0.5878,0.0544;l))	((0.6133,0.0467;1), (0.6133,0.0467;1))	((0.4,0.1333;1), (0.4,0.1333;1))
2	((0.5878,0.0544;1), (0.5878,0.0544;l))	((0.3067,0.14;1), (0.3067,0.14;1))	((0.48,0.08;1), (0.48,0.08;1))
3	((0.4089,0.1089;1), (0.4089,0.1089;1))	((0.23,0.1633;1), (0.23,0.1633;1))	((0.48,0.08;1), (0.48,0.08;1))
4	((0.5111,0.0778;1), (0.5111,0.0778;1))	((0.5111,0.0778;1), (0.5111,0.0778;1))	((0.44,0.1067;1), (0.44,0.1067;1))
5	((0.5111,0.0778;1), (0.5111,0.0778;1))	((0.4089,0.1089;1), (0.4089,0.1089;1))	((0.4,0.1333;1), (0.4,0.1333;1))
6	((0.4089,0.1089;1), (0.4089,0.1089;1))	((0.3322,0.1322;1), (0.3322,0.1322;1))	((0.38,0.1467;1), (0.38,0.1467;1))
7	((0.46,0.0933;1), (0.46,0.0933;1))	((0.2556,0.1556;1), (0.2556,0.1556;1))	((0.32,0.1867;1), (0.32,0.1867;1)))
8	((0.4089,0.1089;1), (0.4089,0.1089;1))	((0.3578,0.1244;1), (0.3578,0.1244;1))	((0.46,0.0933;1), (0.46,0.0933;1))

Table 10: Ranking value

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
C_1	2.45366	2.45366	2.411712	2.435682	2.435682	2.411712	2.423697	2.411712
C_3	2.459653	2.387741	2.369763	2.435682	2.411712	2.393734	2.375756	2.399726
C_4	2.439052	2.418579	2.418579	2.428816	2.439052	2.444171	2.459526	2.423697

Step 5: Calculate the positive and negative ideal solution

Determine the positive ideal solution $x^+ = \left(v_1^+, v_2^+, \ldots, v_m^+\right)$ and the negative-ideal solution $x^- = \left(v_1^-, v_2^-, \ldots, v_m^-\right)$ where

$$x^+ = \left(v_1^+, v_2^+, \ldots, v_m^+\right) = \left(Rank\left(\tilde{\tilde{v}}_{11}\right) \quad Rank\left(\tilde{\tilde{v}}_{21}\right) \quad Rank\left(\tilde{\tilde{v}}_{32}\right)\right) = (2.4537,\ 2.4597,\ 2.4596), \tag{3.23}$$

$$x^- = \left(v_1^-, v_2^-, \ldots, v_m^-\right) = \left(Rank\left(\tilde{\tilde{v}}_{13}\right) \quad Rank\left(\tilde{\tilde{v}}_{23}\right) \quad Rank\left(\tilde{\tilde{v}}_{37}\right)\right) = (2.4117,\ 2.3698,\ 2.4186). \tag{3.24}$$

Step 6: Calculate the distance

The distance $d^+\left(x_j\right)$ is calculated (using Eq. (3.20)) between each alternative x_j and the positive-ideal solution x^+, as shown as Table 11.

Table 11: Distance for positive-ideal solution

	A_1^+	A_2^+	A_3^+	A_4^+	A_5^+	A_6^+	A_7^+	A_8^+
C_1	0	0	0.00176	0.000323	0.000323	0.00176	0.000898	0.00176
C_3	0	0.005171	0.00808	0.000575	0.002298	0.004345	0.007039	0.003591
C_4	0.000419	0.001677	0.001677	0.000943	0.000419	0.000236	0	0.001284

Next, the distance $d^-\left(x_j\right)$ is calculated (using Eq. (3.21)) between each alternative x_j and the negative-ideal solution x^-, shown as Table 12.

Table 12: Distance for negative-ideal solution

	A_1^-	A_2^-	A_3^-	A_4^-	A_5^-	A_6^-	A_7^-	A_8^-
C_1	0.00176	0.00176	0	0.000575	0.000575	0	0.000144	0
C_3	0.00808	0.000323	0	0.004345	0.00176	0.000575	3.59E-05	0.000898
C_4	0.000419	0	0	0.000105	0.000419	0.000655	0.001677	2.62E-05

Step 7: Calculate the closeness

The relative degree of closeness $C(x_j)$ of x_j with respect to the positive ideal solution x^+, are obtained as Table 13.

Table 13: Positive and negative-ideal solution

	$C(x_j)^+$	$C(x_j)^-$
A_1	0.101287	0.020474
A_2	0.045639	0.082753
A_3	0	0.107315
A_4	0.070885	0.042906
A_5	0.052473	0.055143
A_6	0.035065	0.079629
A_7	0.043084	0.089087
A_8	0.030397	0.081453

Step 8: Rank the values

Then, the values of $C(x_j)$ are sorted in a decending sequence, where the larger the value of $C(x_j)$, the higher the preference of the alternatives x_j. The relative closeness of the alternative x_j with respect to f^* for A_1 is defined as

$$C_j^* = \frac{0.1013}{0.0205 + 0.1013} = 0.8317.$$

Thus, the whole results for the relative closeness coefficients is stated as in Table 14.

Table 14: Final ranking order

	Closeness Coefficient, $(CC)_i$	Ranking
A_1	0.831852496	1
A_2	0.355465328	4
A_3	0	8
A_4	0.622940298	2
A_5	0.487596543	3
A_6	0.305725689	6
A_7	0.325972687	5
A_8	0.271767666	7

Lastly, the results comparison between our proposed method and Mokhtarian (2011) are stated as in Table 15.

Table 15: Results comparison

Result	Ranking
Mokhtarian (2011)	$A_1 > A_4 > A_5 > A_2 > A_8 > A_3 > A_6 > A_7$
Proposed Method	$A_1 > A_4 > A_5 > A_2 > A_7 > A_6 > A_8 > A_3$

As a conclusion, the best alternative selection is A_1 and the ranking order of the alternatives in selecting a set of six robots is given by $A_1 > A_4 > A_5 > A_2 > A_7 > A_6 > A_8 > A_3$. Where the results from Mokhtarian [35] are $A_1 > A_4 > A_5 > A_2 > A_8 > A_3 > A_6 > A_7$ as shown in Table 15. It is better to note that; the ranking of all eight oil gas location is slightly differ from the Mokhtarian's result. Here, the *"A_7 , A_6 , A_8"* , "A_3" are locality changed, where "A_1, A_4, A_5, A_2" and "A_6" are stated at the same place. From the results above, we can see that the proposed method can be used to rank the alternatives, but the ranking results may be different due to the difference of new preference scale information assumptions. Besides, this can avoid the unreasonably high or low evaluation values caused by the DMs' preferences because this proposed method considers the hesitation degree in determining the scales. This proposed method is seen to get more reasonable results.

The validation process is a need in this study. This validation process used different illustrative examples ([13,15,39,50]) and showed how valid each example towards the proposed method. The ranking of the proposed method with the examples from [13, 15, 39] is seen similar with all the examples' results. Whereas, only two place of alternatives are locality changed in [50]. It is better to note that all the ranking order from the proposed method is consistent with the existed examples.

The summary of the new ranking order of the alternatives of MCDM problems using the proposed IT2FTOPSIS with new preference scale is given in Table 16.

Table 16: Ranking of the problems under different methods

Examples	Result for the Example	Proposed Method
Chen [13]	$x_1 = 0.19$ $x_2 = 1$ $x_3 = 0.56$ $x_2 > x_3 > x_1$	$x_1 = 0.5524$ $x_2 = 0.3726$ $x_3 = 0.7842$ $x_2 > x_3 > x_1$
Chen and Lee [15]	$x_1 = 0.61$ $x_2 = 0.87$ $x_3 = 0.31$ $x_2 > x_1 > x_3$	$x_1 = 0.7189$ $x_2 = 0.8$ $x_3 = 0.2821$ $x_2 > x_1 > x_3$
Wang et al. [39]	$x_1 = 0.3323$ $x_2 = 0.3399$ $x_3 = 0.3278$ $x_2 > x_1 > x_3$	$x_1 = 0.5478$ $x_2 = 0.5481$ $x_3 = 0.5368$ $x_2 > x_1 > x_3$
Zhang and Zhang [50]	$x_1 = 0.3430$ $x_2 = 0.7250$ $x_3 = 0.7125$ $x_4 = 0.5511$ $x_5 = 0.7552$ $x_6 = 0.3412$ $x_5 > x_1 > x_3 > x_4 > x_2 > x_6$	$x_1 = 0.2411$ $x_2 = 0.9783$ $x_3 = 0.7476$ $x_4 = 0.6772$ $x_5 = 0.7482$ $x_6 = 0.1940$ $x_5 > x_2 > x_3 > x_4 > x_1 > x_6$

5 Case Study

Besides the numerical examples, we offer a real case study of flood control project selection. This flood control project selection is offered in this paper to confirm the effectiveness of the new preference scale with IT2FTOPSIS towards the real case study. Therefore, this section discusses on the backgrounds off the flood control and flow of analysis' algorithm.

5.1 Backgrounds

Flood control is a measure such as to reduce the flood damages in a minimum constant with the cost involved. The aims of flood control measures was to achieve at least one of the objectives by reducing the area flooded, a reduction in depth of flood waters or a reduction in flood discharge [40]. Structural measures represent traditional flood damage reduction by physical means [38]. In other words, the construction of flood control facilities can be referred as structural measures. Four basic schemes of flood control structures recently used to reduce flood damages are the construction of embankments (dike or levees, floodwalls) to confine the floodwaters, the improvement of river channel to enlarge their discharge capacity, the construction of bypass and diversion channels to carry some of the excess floodwater away from the area to be protected and the construction of reservoir for the temporary storage of floodwaters. The details of these alternative flood control schemes are discussed in Table 17.

Since flood control options has pros and cons of each criterion, the selection process must be evaluated carefully because each implementation of course involve huge investment. The impact on environment and social also maybe as huge as cost for the implementation project. Hence, the selection of flood control projects must be considered from different angles to achieve an optimum solution or holistic approach. Thus, the evaluation of attributes and sub-attributes (Table 18) are introduced in line with the alternative flood control schemes (Table 17) in order to find the best flood control project.

Table 17: Elaboration of the alternatives

Alternatives	Description
Reservoir	The function of a reservoir is to store a portion of the flood flow in such a way to minimize the flood peak at the point to be protected [22].
Dikes (levees/ embankment and flood walls)	Kamarouz et al. [25] mentioned that the levees or dikes are constructed parallel to the rivers to prevent overflow of floodwater to the floodplain.
	Levees and floodwalls can increase the peak discharge of floodwater downstream because natural storage in the floodplain is decreased. Levees are most frequently used for flood control works because they can be built at a relatively low cost with materials available at side.
Channel Improvement	A method adopted for flood control measures in order to increase the flood carrying capacity of a stream. This approach enables the water to flow off faster and thus decrease the height and duration of floods and reduce the frequency of flood damage.
Diversion Scheme	A flood diversion is an artificial channel that is used to divert all or part of the river flow.

Table 18: Elaboration of the evaluation attributes and sub-attributes

Attribute/ Sub-attribute	Description
Economic factors (C_1) Project costs (C_{11}) Operation and maintenance (C_{12}) Project benefit (C_{13}) Reliability economic parameter (C_{14})	• The estimated total cost for project • The estimated operation and maintenance cost per year • It concerns the long-term benefit of the project such as flood damage reduction, sosio-economic benefit, national/ regional economic development etc.
Social factors (C_2) Social acceptability (C_{21}) Demographic changes (C_{22}) Effects on infrastructure (C_{23}) Recreation activity (C_{24})	• The public perception about risk to life and health, community vitality, fiscal health and displacement • Effects on social fabric, geographic and demographic distributions of income and employment • The general effects to the surrounding infrastructure, historical places
Environment factors (C_3) Water quality impact (C_{31}) Nature conservation (C_{32}) Soil impact (C_{33}) Landscape (C_{34}) Sanitary condition (C_{35})	• Effect on hydrological system's surface and groundwater levels • Impacts on flora and fauna, endangered species habitat • Impacts on area of agriculture soil and soil contamination • Urban integration and enhancement of landscape • Long term sustainability development
Technical factors (C_4) Lifetime (C_{41}) Adaptability (C_{42}) Level of protection (C_{43}) Technical complexity (C_{44}) Flexibility of the project (C_{45})	• The estimated lifetime of the alternative • The adaptability to the local condition, related to flood magnitude • The long terms protection of the project at the flood risk area and nearby area

All the attributes and sub-attributes that used in this study referred to several articles as follows: [41, 5, 7, 29, 27] and [51]. Table 18 shows the specific attributes and sub-attributes adopted for this research.

5.2 Analysis and Results

Thus, in reviewing the backgrounds, data for alternatives is stated as Table 17 i.e. reservoir (A_1), channel improvement (A_2), diversion scheme (A_3) and dikes (levees/embankment/floodwalls) (A_4) are set as alternatives for four flood control project. Then, based on Table 18, the evaluation attributes and sub-attributes were selected based on the alternatives. There are four main attributes and eighteen sub- attributes considered in this study. Full structure of the flood control project decision-making problem formulated in this study is presented in Figure 4. Furthermore, a committee of three decision-makers or experts has been identified to seek the reliable data over the flood control project. Data in the form of linguistics variables were collected through interviewing of three authorized personnel from three Malaysian Government agencies. The interview was conducted in three separated sessions to elicit the information about factors that regularly lead to flood control project selection. The three decision-makers were:

1. Vice Director of Department of Drainage of Irrigation (DID) (D_1),
2. Vice Director of Department of Environment (DOE) (D_2) ,
3. Officer of Meteorological Department (D_4).

Thus, all the relative importance weights and ratings (i.e. the attributes values) are described using linguistic variables which are defined in Table 3 and Table 4 (refer Section 4).

The hierarchical structure of this experiment can be seen in Figure 4.

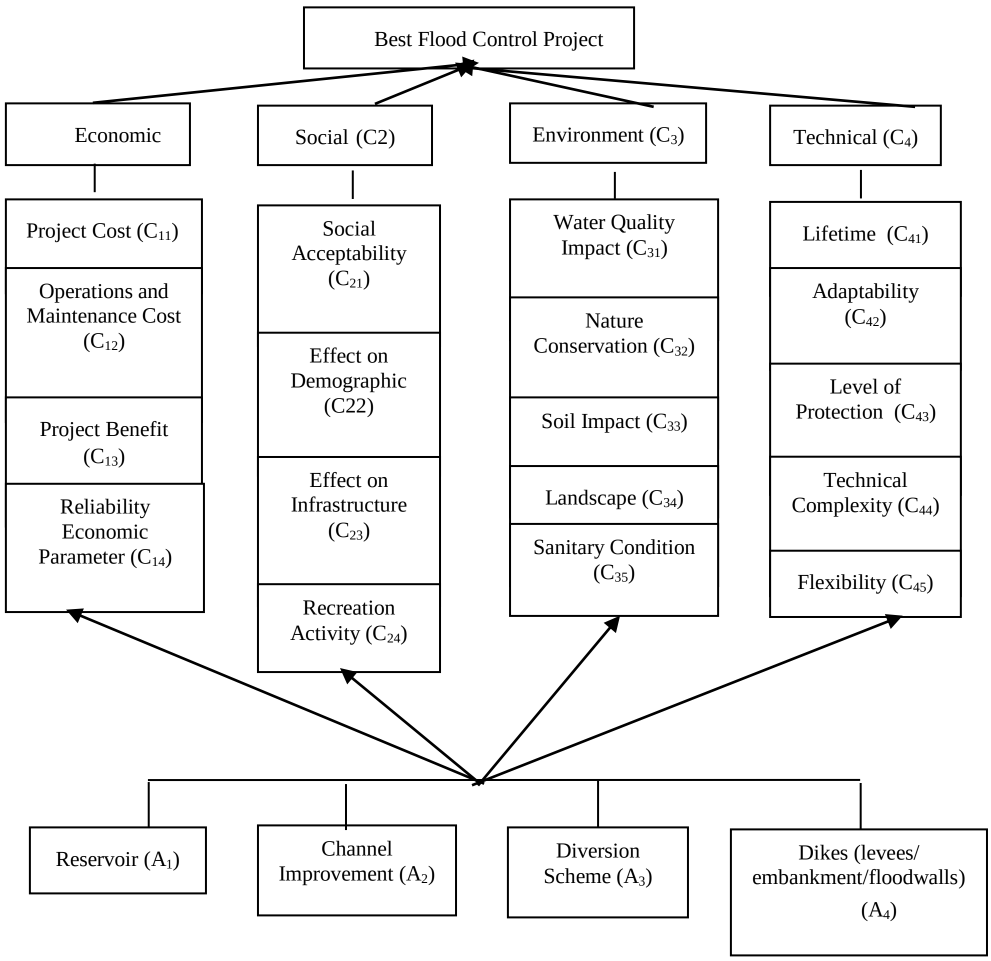

Figure 4: Decision attribute hierarchy

The decision-makers used the linguistic rating variables (see Table 3) to evaluate the rating of alternatives with respect to each criterion in form of decision matrix. The weight for each attribute is also translated into fuzzy weight based on definition in Table 4. These results are presented in Table 19 and Table 20.

Table 19: Weights of the attributes evaluated by decision-makers

Attributes		Decision-makers		
		D_1	D_2	D_3
C_1	C_{11}	H	M	VH
	C_{12}	MH	H	H
C_2	C_{21}	M	H	MH
	C_{22}	H	VH	H
	C_{23}	VH	VH	H
	C_{24}	H	MH	M
C_3	C_{31}	M	M	MH
	C_{32}	VH	H	VH
	C_{33}	H	MH	H
	C_{34}	M	H	VH
C_4	C_{41}	VH	VH	H
	C_{42}	H	VH	H
	C_{43}	H	MH	M
	C_{44}	M	H	MH

Table 20: Linguistic of decision matrix

Attributes		Alternatives	Decision-Makers		
			D_1	D_2	D_3
C_1	C_{11}	A_1	VG	VG	VG
	C_{12}		H	MH	MH
	C_{11}	A_2	M	MH	M
	C_{12}		M	M	M
	C_{11}	A_3	MH	MH	MH
	C_{12}		MH	M	H
	C_{11}	A_4	H	MH	H
	C_{12}		H	MH	MH
C_2	C_{21}	A_1	H	MH	H
	C_{22}		M	ML	ML
	C_{23}		L	M	ML
	C_{24}		VH	H	H
	C_{21}	A_2	VH	MH	MH
	C_{22}		VL	L	L
	C_{23}		VL	L	L
	C_{24}		VL	L	VL
	C_{21}	A_3	MH	H	MH
	C_{22}		M	M	M
	C_{23}		L	ML	ML
	C_{24}		VH	H	H
	C_{21}	A_4	MH	MH	MH
	C_{22}		ML	ML	ML

	C_{23}		L	L	L
	C_{24}		VH	MH	MH
C_3	C_{31}	A_1	L	L	L
	C_{32}		L	ML	L
	C_{33}		VH	VH	H
	C_{34}		MH	MH	H
	C_{31}	A_2	M	ML	M
	C_{32}		M	M	MH
	C_{33}		VL	L	L
	C_{34}		VH	L	VL
	C_{31}	A_3	L	L	L
	C_{32}		L	ML	M
	C_{33}		VH	H	H
	C_{34}		MH	H	MH
	C_{31}	A_4	L	L	L
	C_{32}		L	L	ML
	C_{33}		VH	H	H
	C_{34}		MH	H	MH
C_4	C_{41}	A_1	VH	H	H
	C_{42}		VH	H	H
	C_{43}		L	L	L
	C_{44}		VH	H	VH
	C_{41}	A_2	M	M	M
	C_{42}		ML	M	ML
	C_{43}		ML	L	ML
	C_{44}		M	M	M
	C_{41}	A_3	MH	H	H
	C_{42}		MH	H	H
	C_{43}		MH	M	M
	C_{44}		H	MH	H
	C_{41}	A_4	H	MH	MH
	C_{42}		H	MH	H
	C_{43}		L	ML	ML
	C_{44}		MH	M	M

As shown in Table 21, the results for this investigation show that the best flood control project is A_2, A_4, A_3 and A_1. Channel Improvement is ranked first, followed by dikes and diversion scheme. Reservoir is ranked last. Thus, after taking into account the four attributes, eighteen sub- attributes and the opinion from the three experts, a single measurement for flood control project are obtained and channel improvement recorded the highest closeness coefficient at 1.

Table 21: Results comparison

Alternative	Result	Ranking
A_1	0.00066	4
A_2	1	1
A_3	0.07165	3
A_4	0.23509	2

6 Conclusions

IT2FTOPSIS has more flexibility in capturing uncertainties in the real world due to the fact that it is used T2FSs rather than T1FS. However, IT2FTOPSIS only focuses on the membership degree without considering the non-membership degree. In real life situation, evaluation becomes more comprehensive if non-membership degree were considered concurrently. Preference was expected to be more effective when considering both membership and non-membership degree due to the effectiveness of fuzziness taken from the hesitation degree. In this paper, we have introduced a new preference scale where the non-membership and hesitation degree were developed based on IFS and IT2FS. Later, this new preference scale was successfully applied into IT2FTOPSIS. Then, this new method was applied in five illustrative examples. [13], [15], [35], [39] and [50]s' examples have been chosen to test the proposed method. All the examples helped in searching the validity of the proposed method. Moreover, a case study on selecting the best of flood control project was applied into the new method and the results proved the feasibility of the new method. As a conclusion, this paper can appropriately measure human being decision making progress and can also properly solved the incomplete information and becoming a new way to deals with the vagueness and uncertainty.

Acknowledgement

This research is supported by MyBrain15 scholarship (MyPhD). This support is gratefully acknowledged.

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