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The Cross Selections of Parametric Interval-Valued Fuzzy Variables*

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Abstract

In fuzzy possibility theory, a parametric interval-valued fuzzy variable is defined as a measurable map from the universe to the set of real numbers, and the possibility of a parametric interval-valued fuzzy variable takes on a real number is a closed subinterval of [0,1]. In this paper, we first define the pessimistic cross selection, the optimistic cross selection, and the lambda cross selection for a parametric interval-valued fuzzy variable. Then, for the parametric interval-valued trapezoidal, normal and Erlang fuzzy variables, we derive the parametric possibility distribution functions of their lambda cross selections.

Keywords: parametric possibility distribution, parametric interval-valued fuzzy variable, cross selection

1 Introduction

The concept of a type-2 (T2) fuzzy set was introduced by Zadeh [15] in 1975, which is an extension of an ordinary fuzzy set. A T2 fuzzy set is characterized by a fuzzy membership function. A T2 fuzzy set represents the uncertainty in terms of the secondary membership function and footprint of uncertainty [8]. As a particular T2 fuzzy set where the secondary membership degree is equal to one, the interval T2 fuzzy set is a generalization of the interval-valued fuzzy set [2].

In 2010, Liu and Liu [6] generalized the usual possibility theory [3, 9, 12, 16] and presented fuzzy possibility theory. They defined a T2 fuzzy variable as a measurable map from the universe to the set of real numbers, and the possibility of a T2 fuzzy variable takes on a real number is a regular fuzzy variable. So we can say that a T2 fuzzy variable is an extension of an ordinary fuzzy variable, and fuzzy possibility theory is a variable-based approach to dealing with T2 fuzziness. Based on [6], Liu and Bai [4] discussed the linear combinations of T2 fuzzy variables. To simplify the T2 fuzzy variables, the literature [10, 11] and [13] gave the reduction methods for the T2 fuzzy variables. In a number of practical optimization and decision-making problems, T2 fuzzy variable was used largely as a result of these reduction methods. In [5], the authors introduced the further developed results of fuzzy possibility theory. Also many interesting applications of the T2 fuzzy variables can be found in this literature. By fuzzy possibility theory, Bai and Liu [1] studied fuzzy supply chain network design in 2014. Yang et al. [14] optimized fuzzy p-hub center problem with generalized value-at-risk criterion. Recently, Liu and Liu [7] defined the concept of a parametric interval-valued fuzzy variable and gave five kinds of common parametric interval-valued fuzzy variables. The parametric interval-valued fuzzy variable is a particular T2 fuzzy variable. In this paper, we present three cross selections for a parametric interval-valued fuzzy variable. We focus our attention on these cross selections of the parametric interval-valued trapezoidal, normal and Erlang fuzzy variables.

The rest of the paper is organized as follows. In Section 2, we first define the pessimistic cross selection, the optimistic cross selection, and the lambda cross selection for a parametric interval-valued fuzzy variable. Then, for five kinds of common parametric interval-valued fuzzy variables, we deduce the parametric possibility distribution functions of their lambda cross selections. We conclude the main results in Section 3.

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2 The Cross Selections of Parametric Interval-Valued Fuzzy Variables

Let Γ be the universe of discourse, $\mathcal{A}$ an ample field on Γ, and $\tilde{\text{Pos}} : \mathcal{P}(\Gamma) \mapsto \mathcal{R}([0,1])$ a fuzzy possibility measure [6]. A T2 fuzzy vector [6] is a map $\xi = (\xi_1, \xi_2, \ldots, \xi_m) : \Gamma \mapsto \Re^m$ with the secondary possibility distribution function $\tilde{\mu}_\xi(x) = \tilde{\text{Pos}}\{\gamma \in \Gamma \mid \xi(\gamma) = x\}$, $x \in \Re^m$, and T2 possibility distribution function $\mu_\xi(x,u) = \text{Pos}\{\tilde{\mu}_\xi(x) = u\}$, $(x,u) \in \Re^m \times J_x$. As $m = 1$, the map ξ is usually called a T2 fuzzy variable. The support [6] of a T2 fuzzy vector ξ is defined as $\text{supp}\,\xi = \{(x,u) \in \Re^m \times [0,1] \mid \mu_\xi(x,u) > 0\}$.

Definition 1. ([7]) *Assume that ξ is a T2 fuzzy variable with the secondary possibility distribution function $\tilde{\mu}_\xi(x)$. If for any $x \in \Re$, $\tilde{\mu}_\xi(x)$ is a subinterval $[\mu_{\xi^L}(x;\theta_l), \mu_{\xi^U}(x;\theta_r)]$ of $[0,1]$ with parameters $\theta_l, \theta_r \in [0,1]$, then ξ is called a parametric interval-valued fuzzy variable.*

Definition 2. *Assume that ξ is a parametric interval-valued fuzzy variable with the secondary possibility distribution function $\tilde{\mu}_\xi(x) = [\mu_{\xi^L}(x;\theta_l), \mu_{\xi^U}(x;\theta_r)]$. Let x^* be the minimum value of x such that $\mu_{\xi^L}(x;\theta_l) = 1$. The fuzzy variable ξ_P with the following parametric possibility distribution function*

$$\mu_{\xi_P}(x;\theta_l,\theta_r) = \begin{cases} \mu_{\xi^U}(x;\theta_r), & \text{if } \ x \le x^* \\ \mu_{\xi^L}(x;\theta_l), & \text{if } \ x > x^* \end{cases} \tag{1}$$

is called the pessimistic cross selection of ξ. The fuzzy variable ξ_O with the following parametric possibility distribution function

$$\mu_{\xi_O}(x;\theta_l,\theta_r) = \begin{cases} \mu_{\xi^L}(x;\theta_l), & \text{if } \ x \le x^* \\ \mu_{\xi^U}(x;\theta_r), & \text{if } \ x > x^* \end{cases} \tag{2}$$

is called the optimistic cross selection of ξ. For any $\lambda \in [0,1]$, a fuzzy variable ξ_λ with the following parametric possibility distribution function

$$\mu_{\xi_\lambda}(x;\theta_l,\theta_r) = \lambda\mu_{\xi_P}(x;\theta_l,\theta_r) + (1-\lambda)\mu_{\xi_O}(x;\theta_l,\theta_r) \tag{3}$$

is called a λ cross selection of ξ.

In the following, for parametric interval-valued trapezoidal, normal and Erlang fuzzy variables, we give the parametric possibility distribution functions of their λ cross selections.

2.1 The Cross Selection of Parametric Interval-Valued Trapezoidal Fuzzy Variable

According to Liu and Liu [7], $\eta = [r_1, r_2, r_3, r_4; \theta_l, \theta_r]$, $r_1 < r_2 \le r_3 < r_4$ is a parametric interval-valued trapezoidal fuzzy variable with the secondary possibility distribution $\tilde{\mu}_\eta(x)$, which is the following subinterval

$$\left[\tfrac{x-r_1}{r_2-r_1} - \theta_l \min\{\tfrac{x-r_1}{r_2-r_1}, \tfrac{r_2-x}{r_2-r_1}\}, \tfrac{x-r_1}{r_2-r_1} + \theta_r \min\{\tfrac{x-r_1}{r_2-r_1}, \tfrac{r_2-x}{r_2-r_1}\}\right]$$

of $[0,1]$ for $x \in [r_1, r_2]$, the interval $[1,1]$ for $x \in [r_2, r_3]$, and the following subinterval

$$\left[\tfrac{r_4-x}{r_4-r_3} - \theta_l \min\{\tfrac{r_4-x}{r_4-r_3}, \tfrac{x-r_3}{r_4-r_3}\}, \tfrac{r_4-x}{r_4-r_3} + \theta_r \min\{\tfrac{r_4-x}{r_4-r_3}, \tfrac{x-r_3}{r_4-r_3}\}\right]$$

of $[0,1]$ for $x \in [r_3, r_4]$. Particularly, η is called a parametric interval-valued triangular fuzzy variable if $r_2 = r_3$, and denoted by $[r_1, r_2, r_3; \theta_l, \theta_r]$ where $r_1 < r_2 < r_3$.

Theorem 1. *Let $\eta = [r_1, r_2, r_3, r_4; \theta_l, \theta_r]$ be a parametric interval-valued trapezoidal fuzzy variable. Then the λ cross selection variable η_λ of η has the following parametric possibility distribution function*

$$\mu_{\eta_\lambda}(x;\theta_l,\theta_r) = \begin{cases} \frac{x-r_1}{r_2-r_1}[1-\theta_l+\lambda(\theta_r+\theta_l)], & \text{if } \ x \in [r_1, \frac{r_1+r_2}{2}) \\ \frac{x-r_1}{r_2-r_1} + [\lambda(\theta_r+\theta_l)-\theta_l]\frac{r_2-x}{r_2-r_1}, & \text{if } \ x \in [\frac{r_1+r_2}{2}, r_2) \\ 1, & \text{if } \ x \in [r_2, r_3] \\ \frac{r_4-x}{r_4-r_3} + [\theta_r-\lambda(\theta_r+\theta_l)]\frac{x-r_3}{r_4-r_3}, & \text{if } \ x \in (r_3, \frac{r_3+r_4}{2}] \\ \frac{r_4-x}{r_4-r_3}[1+\theta_r-\lambda(\theta_r+\theta_l)], & \text{if } \ x \in (\frac{r_3+r_4}{2}, r_4]. \end{cases} \tag{4}$$

Proof. It is easy to know that $x^* = r_2$. Therefore, the pessimistic cross selection η_P of η has the following parametric possibility distribution function

$$\mu_{\eta_P}(x;\theta_l,\theta_r) = \begin{cases} \frac{x-r_1}{r_2-r_1} + \theta_r \min\{\frac{x-r_1}{r_2-r_1}, \frac{r_2-x}{r_2-r_1}\}, & \text{if } x \in [r_1, r_2] \\ 1, & \text{if } x \in [r_2, r_3] \\ \frac{r_4-x}{r_4-r_3} - \theta_l \min\{\frac{r_4-x}{r_4-r_3}, \frac{x-r_3}{r_4-r_3}\}, & \text{if } x \in [r_3, r_4], \end{cases}$$

the optimistic cross selection η_O of η has the following parametric possibility distribution function

$$\mu_{\eta_O}(x;\theta_l,\theta_r) = \begin{cases} \frac{x-r_1}{r_2-r_1} - \theta_l \min\{\frac{x-r_1}{r_2-r_1}, \frac{r_2-x}{r_2-r_1}\}, & \text{if } x \in [r_1, r_2] \\ 1, & \text{if } x \in [r_2, r_3] \\ \frac{r_4-x}{r_4-r_3} + \theta_r \min\{\frac{r_4-x}{r_4-r_3}, \frac{x-r_3}{r_4-r_3}\}, & \text{if } x \in [r_3, r_4]. \end{cases}$$

So the λ cross selection η_λ of η has the following parametric possibility distribution function

$$\mu_{\eta_\lambda}(x;\theta_l,\theta_r) = \begin{cases} \frac{x-r_1}{r_2-r_1} + [\lambda(\theta_l+\theta_r)-\theta_l] \min\{\frac{x-r_1}{r_2-r_1}, \frac{r_2-x}{r_2-r_1}\}, & \text{if } x \in [r_1, r_2] \\ 1, & \text{if } x \in [r_2, r_3] \\ \frac{r_4-x}{r_4-r_3} + [\theta_r - \lambda(\theta_l+\theta_r)] \min\{\frac{r_4-x}{r_4-r_3}, \frac{x-r_3}{r_4-r_3}\}, & \text{if } x \in [r_3, r_4]. \end{cases}$$

It is equivalent to Equation (4). The proof of theorem is complete. □

Corollary 1. *Let $\eta = [r_1, r_2, r_3; \theta_l, \theta_r]$ be a parametric interval-valued triangular fuzzy variable. Then the λ cross selection variable η_λ of η has the following parametric possibility distribution function*

$$\mu_{\eta_\lambda}(x;\theta_l,\theta_r) = \begin{cases} \frac{x-r_1}{r_2-r_1}[1-\theta_l+\lambda(\theta_r+\theta_l)], & \text{if } x \in [r_1, \frac{r_1+r_2}{2}) \\ \frac{x-r_1}{r_2-r_1} + [\lambda(\theta_r+\theta_l)-\theta_l]\frac{r_2-x}{r_2-r_1}, & \text{if } x \in [\frac{r_1+r_2}{2}, r_2) \\ \frac{r_3-x}{r_3-r_2} + [\theta_r - \lambda(\theta_r+\theta_l)]\frac{x-r_2}{r_3-r_2}, & \text{if } x \in (r_2, \frac{r_2+r_3}{2}] \\ \frac{r_3-x}{r_3-r_2}[1+\theta_r-\lambda(\theta_r+\theta_l)], & \text{if } x \in (\frac{r_2+r_3}{2}, r_3]. \end{cases} \tag{5}$$

2.2 The Cross Selection of Parametric Interval-Valued Normal Fuzzy Variable

According to Liu and Liu [7], ζ is called a parametric interval-valued normal fuzzy variable if its secondary possibility distribution $\tilde{\mu}_\zeta(x)$ is the following subinterval

$$\Big[\exp(-\tfrac{(x-\mu)^2}{2\sigma^2}) - \theta_l \min\{1-\exp(-\tfrac{(x-\mu)^2}{2\sigma^2}), \exp(-\tfrac{(x-\mu)^2}{2\sigma^2})\}, \\ \exp(-\tfrac{(x-\mu)^2}{2\sigma^2}) + \theta_r \min\{1-\exp(-\tfrac{(x-\mu)^2}{2\sigma^2}), \exp(-\tfrac{(x-\mu)^2}{2\sigma^2})\}\Big]$$

of $[0,1]$ for any $x \in \Re$, where $\mu \in \Re$ and $\sigma > 0$. The parametric interval-valued normal fuzzy variable ζ with the above distribution is denoted by $n(\mu, \sigma^2; \theta_l, \theta_r)$.

Theorem 2. *Let $\zeta = n(\mu, \sigma^2; \theta_l, \theta_r)$ be a parametric interval-valued normal fuzzy variable. Then the λ cross selection variable ζ_λ of ζ has the following parametric possibility distribution function*

$$\mu_{\zeta_\lambda}(x;\theta_l,\theta_r) = \begin{cases} \exp(-\frac{(x-\mu)^2}{2\sigma^2})[1+\lambda(\theta_l+\theta_r)-\theta_l], & \text{if } x \le \mu - \sqrt{2\ln 2}\sigma \\ \exp(-\frac{(x-\mu)^2}{2\sigma^2})[1-\lambda(\theta_l+\theta_r)+\theta_l] + [\lambda(\theta_l+\theta_r)-\theta_l], & \text{if } \mu - \sqrt{2\ln 2}\sigma < x \le \mu \\ \exp(-\frac{(x-\mu)^2}{2\sigma^2})[1+\lambda(\theta_l+\theta_r)-\theta_r] + [\theta_r - \lambda(\theta_l+\theta_r)], & \text{if } \mu < x \le \mu + \sqrt{2\ln 2}\sigma \\ \exp(-\frac{(x-\mu)^2}{2\sigma^2})[1-\lambda(\theta_l+\theta_r)+\theta_r], & \text{if } x > \mu + \sqrt{2\ln 2}\sigma. \end{cases} \tag{6}$$

Proof. Based on the definition of the parametric interval-valued normal fuzzy variable, we have $x^* = \mu$. Therefore, the pessimistic cross selection ζ_P of ζ has the following parametric possibility distribution function

$$\mu_{\zeta_P}(x;\theta_l,\theta_r) = \begin{cases} \exp(-\frac{(x-\mu)^2}{2\sigma^2}) + \theta_r \min\{1-\exp(-\frac{(x-\mu)^2}{2\sigma^2}), \exp(-\frac{(x-\mu)^2}{2\sigma^2})\}, & \text{if } x \le \mu \\ \exp(-\frac{(x-\mu)^2}{2\sigma^2}) - \theta_l \min\{1-\exp(-\frac{(x-\mu)^2}{2\sigma^2}), \exp(-\frac{(x-\mu)^2}{2\sigma^2})\}, & \text{if } x > \mu, \end{cases}$$

the optimistic cross selection ζ_O of ζ has the following parametric possibility distribution function

$$\mu_{\zeta_O}(x;\theta_l,\theta_r) = \begin{cases} \exp(-\frac{(x-\mu)^2}{2\sigma^2}) - \theta_l \min\{1-\exp(-\frac{(x-\mu)^2}{2\sigma^2}), \exp(-\frac{(x-\mu)^2}{2\sigma^2})\}, & \text{if } x \le \mu \\ \exp(-\frac{(x-\mu)^2}{2\sigma^2}) + \theta_r \min\{1-\exp(-\frac{(x-\mu)^2}{2\sigma^2}), \exp(-\frac{(x-\mu)^2}{2\sigma^2})\}, & \text{if } x > \mu. \end{cases}$$

So the λ cross selection ζ_λ of ζ has the following parametric possibility distribution function

$$\mu_{\zeta_\lambda}(x;\theta_l,\theta_r)=\begin{cases}\exp(-\frac{(x-\mu)^2}{2\sigma^2})+[\lambda(\theta_l+\theta_r)-\theta_l]\min\{1-\exp(-\frac{(x-\mu)^2}{2\sigma^2}),\exp(-\frac{(x-\mu)^2}{2\sigma^2})\}, & \text{if} \quad x\le\mu\\ \exp(-\frac{(x-\mu)^2}{2\sigma^2})+[\theta_r-\lambda(\theta_l+\theta_r)]\min\{1-\exp(-\frac{(x-\mu)^2}{2\sigma^2}),\exp(-\frac{(x-\mu)^2}{2\sigma^2})\}, & \text{if} \quad x>\mu,\end{cases}$$

which is equivalent to Equation (6). The proof of theorem is complete. □

2.3 The Cross Selection of Parametric Interval-Valued Erlang Fuzzy Variable

According to Liu and Liu [7], $\xi=\mathrm{Er}(\rho,\kappa;\theta_l,\theta_r)$ is called a parametric interval-valued Erlang fuzzy variable if its secondary possibility distribution $\tilde{\mu}_\xi(x)$ is the following subinterval

$$\Big[\left(\frac{x}{\kappa\rho}\right)^\kappa\exp(\kappa-\frac{x}{\rho})-\theta_l\min\{1-\left(\frac{x}{\kappa\rho}\right)^\kappa\exp(\kappa-\frac{x}{\rho}),\left(\frac{x}{\kappa\rho}\right)^\kappa\exp(\kappa-\frac{x}{\rho})\},$$
$$\left(\frac{x}{\kappa\rho}\right)^\kappa\exp(\kappa-\frac{x}{\rho})+\theta_r\min\{1-\left(\frac{x}{\kappa\rho}\right)^\kappa\exp(\kappa-\frac{x}{\rho}),\left(\frac{x}{\kappa\rho}\right)^\kappa\exp(\kappa-\frac{x}{\rho})\}\Big]$$

of $[0,1]$ for any $x\ge 0$, where $\rho>0$ and $\kappa\in N^+$. Particularly, if $\kappa=1$, then ξ is called a parametric interval-valued exponential fuzzy variable and usually denoted by $\exp(\rho;\theta_l,\theta_r)$.

Theorem 3. *Let $\xi=\mathrm{Er}(\rho,\kappa;\theta_l,\theta_r)$ be a parametric interval-valued Erlang fuzzy variable. Then the λ cross selection variable ξ_λ of ξ has the following parametric possibility distribution function*

$$\mu_{\xi_\lambda}(x;\theta_l,\theta_r)$$
$$=\begin{cases}\left(\frac{x}{\kappa\rho}\right)^\kappa\exp(\kappa-\frac{x}{\rho})[1+\lambda(\theta_l+\theta_r)-\theta_l], & \text{if } x\le\kappa\rho,\left(\frac{x}{\kappa\rho}\right)^\kappa\exp(\kappa-\frac{x}{\rho})\le\frac{1}{2}\\ \left(\frac{x}{\kappa\rho}\right)^\kappa\exp(\kappa-\frac{x}{\rho})[1-\lambda(\theta_l+\theta_r)+\theta_l]+[\lambda(\theta_l+\theta_r)-\theta_l], & \text{if } x\le\kappa\rho,\left(\frac{x}{\kappa\rho}\right)^\kappa\exp(\kappa-\frac{x}{\rho})>\frac{1}{2}\\ \left(\frac{x}{\kappa\rho}\right)^\kappa\exp(\kappa-\frac{x}{\rho})[1+\lambda(\theta_l+\theta_r)-\theta_r]+[\theta_r-\lambda(\theta_l+\theta_r)], & \text{if } x>\kappa\rho,\left(\frac{x}{\kappa\rho}\right)^\kappa\exp(\kappa-\frac{x}{\rho})>\frac{1}{2}\\ \left(\frac{x}{\kappa\rho}\right)^\kappa\exp(\kappa-\frac{x}{\rho})[1-\lambda(\theta_l+\theta_r)+\theta_r], & \text{if } x>\kappa\rho,\left(\frac{x}{\kappa\rho}\right)^\kappa\exp(\kappa-\frac{x}{\rho})\le\frac{1}{2}.\end{cases} \tag{7}$$

Proof. It is easy to know that $x^*=\kappa\rho$. The proof of theorem is similar to that of Theorem 1. □

Corollary 2. *Let $\xi=\exp(\rho;\theta_l,\theta_r)$ be a parametric interval-valued exponential fuzzy variable. Then the λ cross selection variable ξ_λ of ξ has the following parametric possibility distribution function*

$$\mu_{\xi_\lambda}(x;\theta_l,\theta_r)=\begin{cases}\frac{x}{\rho}\exp(1-\frac{x}{\rho})[1+\lambda(\theta_l+\theta_r)-\theta_l], & \text{if} \quad x\le\rho,\frac{x}{\rho}\exp(1-\frac{x}{\rho})\le\frac{1}{2}\\ \frac{x}{\rho}\exp(1-\frac{x}{\rho})[1-\lambda(\theta_l+\theta_r)+\theta_l]+[\lambda(\theta_l+\theta_r)-\theta_l], & \text{if} \quad x\le\rho,\frac{x}{\rho}\exp(1-\frac{x}{\rho})>\frac{1}{2}\\ \frac{x}{\rho}\exp(1-\frac{x}{\rho})[1+\lambda(\theta_l+\theta_r)-\theta_r]+[\theta_r-\lambda(\theta_l+\theta_r)], & \text{if} \quad x>\rho,\frac{x}{\rho}\exp(1-\frac{x}{\rho})>\frac{1}{2}\\ \frac{x}{\rho}\exp(1-\frac{x}{\rho})[1-\lambda(\theta_l+\theta_r)+\theta_r], & \text{if} \quad x>\rho,\frac{x}{\rho}\exp(1-\frac{x}{\rho})\le\frac{1}{2}.\end{cases} \tag{8}$$

Example 1: Let $\eta=[1,2,3,5;0.6,0.8]$ be a parametric interval-valued trapezoidal fuzzy variable, $\zeta=n(0,4^2;0.5,0.6)$ a parametric interval-valued normal fuzzy variable, and $\xi=\mathrm{Er}(0.2,1;0.3,0.7)$ a parametric interval-valued Erlang fuzzy variable. In Figure 1, the red line, the blue line and the area surrounded by the red line and the blue line characterize the support of η, the red line shows the parametric possibility distribution of the pessimistic cross selection η_P, and the blue line plots the parametric possibility distribution of the optimistic cross selection η_O. Correspondingly, the support, the pessimistic cross selection and the optimistic cross selection of ζ are showed in Figure 2, and the support, the pessimistic cross selection and the optimistic cross selection of ξ are showed in Figure 3.

3 Conclusions

In the current development, we studied the parametric interval-valued fuzzy variable from a new viewpoint. We first defined the pessimistic cross selection and optimistic cross selection for the parametric interval-valued fuzzy variable. Based on these two concepts, we defined the λ cross selection for the parametric interval-valued fuzzy variable. For the parametric interval-valued fuzzy variables which are trapezoidal, triangular, normal, Erlang and exponential, we derived the parametric possibility distributions of their λ cross selections. The major results in this paper will make it more convenient to use the parametric interval-valued fuzzy variables in practical optimization and decision-making problems.

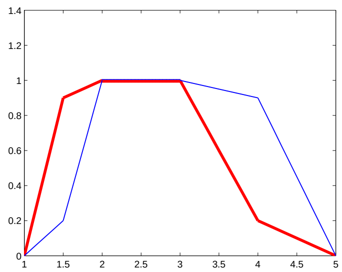

Figure 1: The pessimistic cross selection and optimistic cross selection of η

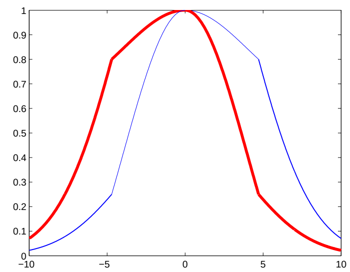

Figure 2: The pessimistic cross selection and optimistic cross selection of ζ

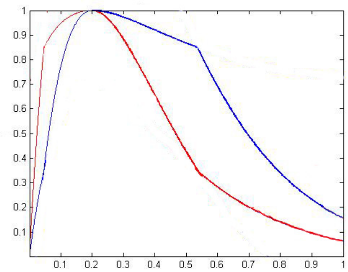

Figure 3: The pessimistic cross selection and optimistic cross selection of ξ

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