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Linear Programming Problems in Fuzzy Environment: The Post Optimal Analyses

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Abstract

This paper proposes a new method of Robust ranking technique, which is used for defuzzifying the trapezoidal fuzzy number into a crisp number to represent the fuzzy set. In practice, there are many problems in which all decision parameters are fuzzy numbers, and such problems are usually solved by either probabilistic programming or multi objective programming methods. Unfortunately all these methods have shortcomings. In this paper, using the concept of comparison of fuzzy numbers, a very effective method is introduced for solving these problems. The model is illustrated with numerical application to generate a good solution and post optimal analyses are obtained. Investigation of the properties of an optimal solution allows developing a simplex algorithm in fuzzy environment. Furthermore, the proposed technique allows the significant ways to help the decision-maker for formulating their decisions and drawing managerial insights efficiently.
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Keywords: linear programming, trapezoidal fuzzy number, robust ranking technique, simplex method, post optimal analyses, decision-making

1 Introduction

In real physical world, there are many problems in which all decision parameters are fuzzy numbers, and such problems are usually solved by either probabilistic programming or multi-objective programming methods. Unfortunately all these methods have shortcomings. In this paper, the concept of comparison using fuzzy numbers is introduced and is very effective method for solving the problems. To handle the fuzzy decision variables can be initially generated and then solved and improved sequentially using the fuzzy decision approach by introducing Robust ranking technique. Linear programming is the optimization technique which is applied most frequently in the real-world problems and it is important to introduce new tools in the approach that allow the model to fit into the real world as much as possible. Any linear programming model representing real-world situations involves a lot of parameters whose values are assigned by experts, and in the conventional approach, it is required to fix an exact value to the aforementioned parameters. However, both experts and the decision maker frequently do not know precisely the value of those parameters. If the exact values are suggested, these are only statistical inference from the past data and their stability is doubtful, so the parameters of the problem are usually defined by the decision maker in an uncertain space. Therefore, it is useful to consider the knowledge of the experts about the parameters as fuzzy data.

Two significant questions may be found in these kinds of problems: how to handle the relationship between the fuzzy parameters, and how to find the optimal values for the fuzzy single-objective function. The answer is related to the problem of ranking the fuzzy numbers.

In fuzzy decision making problems, the concept of maximizing decision was introduced by Bellman and Zadeh [2]. Zimmerman [15] presented a fuzzy approach to multi-objective linear programming problems in his classical paper. Lai and Hwang [6] considered the situations where all parameters are in fuzzy. Lai and Huang [5] assume that the parameters have a triangular possibility distribution. Gani et al. [3] introduced fuzzy linear programming problem based on L-R fuzzy number. Jimenez et al. [4] proposed a method for solving linear programming problems where all coefficients are, in general, fuzzy numbers and using linear ranking technique. Bazaar et al. [1] and Nasseri et al. [9] defined linear programming problems with fuzzy numbers and simplex method is used for finding the optimal

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solution of the fuzzy problem. Rangarajan and Solairaju [11] computed improved fuzzy optimal Hungarian assignment problems with fuzzy numbers by applying Robust ranking techniques to obtain the optimal solution of the fuzzy assignment problem. Pattnaik [10] presented a fuzzy approach to several linear and nonlinear inventory models. Swarup et al. [13] explained the method to obtain post optimal analysis of the different parameters in the linear programming problems.

Looking at the property of representing the preference relationship in fuzzy terms, ranking methods can be classified into two approaches. One of them associates, by means of different functions, each fuzzy number is converted to a single of the real line and then a total crisp order relationship between fuzzy numbers is established. The other approach ranks fuzzy numbers by means of a fuzzy relationship. It allows decision maker to present his preference in a gradual way, a linear programming problem allows it to be handled with different degrees of satisfaction of the constraints. This paper considers fuzzy single-objective linear programming problem whose parameters are fuzzy numbers. The aim of this paper is to introduce Robust ranking technique for defuzzifying the fuzzy parameters and then post optimal analyses are obtained for requirement parameter that permits the interactive participation of the decision maker in all steps of decision process for expressing his opinions in linguistic terms. The major techniques used in the above research articles are summarized in Table 1.

The remainder of this paper is organized as follows. In Section 2, fuzzy numbers and some of the results of applying arithmetic on them are introduced. Assumptions, notations and definitions are provided for the development of the model. In Section 3, Robust ranking technique is introduced for solving fuzzy linear programming problem. In Section 4, a linear programming problem with fuzzy parameters is proposed and a method for solving this problem is explained. A simplex algorithm is developed for this fuzzy model in Section 5. The numerical example is presented to illustrate the development of the model in Section 6. In Section 7 the post optimal analyses are obtained from fuzzy optimal solution. Finally Section 8 deals with the summary and the concluding remarks.

Table 1**:** Major characteristics of fuzzy linear programming models on selected researches

Author(s) and Published Year	Structure of the Model	Objective Model	Model Type	Ranking Function	Post Optimal Study
Zimmermann (1978) [15]	Fuzzy	Single	Cost	Linear	No
Maleki et al. (2000) [8]	Fuzzy	Multi	Profit	Linear	No
Jimenez et al. (2005) [4]	Fuzzy	Multi	Cost	Linear	No
Nasseri et al. (2005) [9]	Fuzzy	Multi	Profit	Linear	No
Present Paper (2015)	Fuzzy	Multi	Profit	Robust	Yes

2 Preliminary

The fundamental notation of fuzzy set theory is reviewed and initiated by Bellman and Zadeh [2]. Below we give definitions taken from [16].

Definition 2.1 *Fuzzy sets:* If X is a collection of objects denoted generally by *x*, then a fuzzy set $\tilde{A}$ in X is defined as a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))/x\epsilon X\}$, where $\mu_{\tilde{A}}(x)$ is called the membership function for the fuzzy set $\tilde{A}$. The membership function maps each element of X to a membership value between 0 and 1.

Definition 2.2 *Support of a fuzzy set;* The support of a fuzzy set $\tilde{A}$ is the set of all points *x* in X such that $\mu_{\tilde{A}}(x) > 0$. That is $support\ (\tilde{A}) = \{x/\mu_{\tilde{A}}(x) > 0\}$.

Definition 2.3 $\alpha -$ *level of fuzzy set:* The $\alpha -$ cut (or) $\alpha -$ level set of a fuzzy set $\tilde{A}$ is a set consisting of those elements of the universe X whose membership values exceed the threshold level α. It is a subset of its domain that allows representing a fuzzy set in a confidence interval form. That is $\tilde{A}_\alpha = \{x/\mu_{\tilde{A}}(x) \geq \alpha\}$.

Definition 2.4 *Convex fuzzy set:* A fuzzy set $\tilde{A}$ is convex if $\mu_{\tilde{A}}\big(\lambda x_1 + (1 - \lambda x_2) \geq min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\big), x_1, x_2 \epsilon X$ and $\lambda\epsilon[0,1]$. Alternatively, a fuzzy set is convex, if all $\alpha -$level sets are convex.

Definition 2.5 *Convex normalized fuzzy set:* A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set on the real line R such that it exists at least one $x_0\epsilon R$ with $\mu_{\tilde{A}}(x_0) = 1$ and $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 2.6 *Trapezoidal fuzzy numbers:* A fuzzy number is a trapezoidal fuzzy number if the membership function of its be in the following function of it being in the following form: Any trapezoidal fuzzy number by $\tilde{a} = (a^L, a^U, \alpha, \beta)$, where the support of $\tilde{a}$ is $(a^L - \alpha, a^U + \beta)$ and the modal set of $\tilde{a}$ is $[a^L, a^U]$. Let *F(R)* is the set of trapezoidal fuzzy numbers.

Among the various fuzzy numbers, triangular and trapezoidal fuzzy numbers are of the most important. Note that, in this study we only consider trapezoidal fuzzy numbers.

Definition 2.7 *Arithmetic on fuzzy numbers:* Let $\widetilde{a} = (a^L, a^U, \alpha, \beta)$ and $\tilde{b} = (b^L, b^U, \gamma, \theta)$ be two trapezoidal fuzzy numbers and $x \epsilon R$. Then, the results of applying fuzzy arithmetic on the trapezoidal fuzzy numbers as shown in the following:
Image of $\widetilde{a}$: $-\widetilde{a} = (-a^U, -a^L, \beta, \alpha)$;
Addition: $\tilde{a} + \tilde{b} = (a^L + b^L, a^U + b^U, \alpha + \gamma, \beta + \theta)$;
Scalar Multiplication: x> 0, $x\widetilde{a} = (xa^L, xa^U, x\alpha, x\beta)$ and $x < 0$, $x\widetilde{a} = (xa^U, xa^L, -x\alpha, -x\beta)$.

3 Ranking Function

Since there are many ranking function for comparing fuzzy numbers but Robust ranking function is the convenient method for the given fuzzy LPP. Robust ranking technique satisfies compensation, linearity and additive properties and provides results which are consistent with human intuition. For a given convex fuzzy number $\tilde{a}$, the Robust ranking index is defined by $\Re(\tilde{a}) = \int_0^1 0.5(a_\alpha^L, a_\alpha^U)d\alpha$, where (a^L, a^U) is the α- level cut of the fuzzy number $\tilde{a}$. The Robust ranking index $\Re(\tilde{a})$ gives the representative value of the fuzzy number $\tilde{a}$. It satisfies the linearity and additive property.

For solving the problem the fuzzy cost coefficients defuzzify into crisp ones by a fuzzy number ranking method. A convenient method for comparing the fuzzy numbers is the application of ranking functions. A ranking function is a map from *F(R)* into the real line. The orders on *F(R)* are:
$\tilde{a} \geq \tilde{b}$ if and only if $\Re(\tilde{a}) \geq \Re(\tilde{b})$;
$\tilde{a} > \tilde{b}$ if and only if $\Re(\tilde{a}) > \Re(\tilde{b})$;
$\tilde{a} = \tilde{b}$ if and only if $\Re(\tilde{a}) = \Re(\tilde{b})$, where $\tilde{a}$ and $\tilde{b}$ are in *F (R).* It is obvious that $\tilde{a} \leq \tilde{b}$ if and only if $\tilde{b} \geq \tilde{a}$.

4 Fuzzy Linear Programming Problems

However, when formulating a mathematical programming problem which is closely described and represented a real-world decision problem, various factors of the real world system should be reflected in the description of objective functions and constraints which are involving with many parameters, whose possible values may be assigned by the experts. In the conventional approaches, such parameters are required to be fixed at some values in an experimental and subjective manner through the experts' understanding of the nature of the parameters in the problem-formulation process.

It must be observed that, in most of the real-world situations, the possible values of these parameters are often imprecisely known to the experts. With this observation in mind, it would be certainly more appropriate to interpret the experts' understanding of the parameters as fuzzy numerical data which can be represented by means of fuzzy sets of the real line known as fuzzy numbers.

Definition 4.1 *Linear programming:* A linear programming (LP) problem is defined as:

$$Max\ z = cx$$
$$s.t.\ Ax = b$$
$$x \geq 0$$

where $c = (c_1, c_2, \dots, c_n)$, $b = (b_1, b_2, \dots, b_m)^T$ and $A = [a_{ij}]_{m \times n}$.

In the above problem, all of the parameters are crisp. Now, if some of the parameters be fuzzy numbers then fuzzy linear programming is obtained which is defined in the next section.

Definition 4.2 *Fuzzy linear programming:* Suppose in linear programming problem, some parameters be fuzzy numbers, then it is possible to obtain the fuzzy optimal solution for the given problem. The coefficients of the problem in the objective function are fuzzy number, that is the technical coefficients of decision making variables be fuzzy number in [7, 8, 12, 14].

Definition 4.3 *Fuzzy number linear programming:* A fuzzy number linear programming (FNLP) problem is defined as follows:

$$Max\ \tilde{z} = \tilde{c}x$$
$$s.t.\ Ax = b$$
$$x \geq 0$$

where $b \epsilon R^m$, $x \epsilon R^n$, $A \epsilon R^{m \times n}$, $\tilde{c}^T \epsilon((F(R))^n$, and $\mathfrak{R}$ is a Robust ranking function.

Definition 4.4 *Fuzzy feasible solution:* The vector $x \epsilon R^n$ is a feasible solution to FNLP if and only if x satisfies the constraints of the problem.

Definition 4.5 *Fuzzy optimal solution:* A feasible solution x^* is an optimal solution for FNLP, if for all feasible solution x for FNLP, then $\tilde{c}x^* \geq \tilde{c}x$.

Definition 4.6 *Fuzzy basic feasible solution:* The basic feasible solution for FNLP problems is defined as: consider the system $Ax = b$ and $x \geq 0$, where A is an $m \times n$ matrix and b is an m vector. Now, suppose that $rank(A, b) = rank(A) = m$. Partition after possibly rearranging the columns of A as $[B, N]$ where $B,\ m \times m$, is nonsingular. It is obvious that $rank(B) = m$. The point $x = (x_B^T, x_N^T)^T$ where $x_B = B^{-1}b$, $x_N = 0$ is called a basic solution of the system. If $x_B \geq 0$, then x is called a basic feasible solution (BFS) of the system. Here B is called the basic matrix and N is called the non basic matrix.

5 A Fuzzy Number Simplex Algorithm

For the solution of any FNLP by Simplex algorithm, the existence of an initial basic feasible fuzzy solution is always assumed. The steps for the computation of an optimum fuzzy solution are as follows:

Step-1 Check whether the objective function of the given FNLP is to be maximized or minimized. If it is to be minimized then converting it into a problem of maximizing it by using the result $Minimum\ \tilde{z} = -Maximum(-\widetilde{z})$.

Step-2 Check whether all $\widetilde{b_\imath}(i = 1,2, \ldots, m)$ are non-negative. If any one $\tilde{b}_i$ is negative then multiply the corresponding inequation of the constant by -1, so as to get all $\tilde{b}_i$ $(i = 1,2, \ldots, m)$ non-negative.

Step-3 Convert all the inequations of the constraints into equations by introducing slack and/or surplus variables in the constraints. Put the cost of these variables equal to zero.

Step-4 Obtain an initial basic feasible solution to the problem in the problem in the form of $\tilde{x}_B = \tilde{B}^{-1}\tilde{b}$ and put in the first column of the simplex table.

Step-5 Compute the net evaluations $\tilde{\Delta}_j = \tilde{z}_j - \tilde{c}_j$ $(j = 1,2, \ldots, n)$ by using the relation $\tilde{\Delta}_j = \tilde{c}_B\tilde{y}_j - c_j$. Examine the sign $\tilde{\Delta}_j$.

i) If all $\tilde{\Delta}_j \geq \tilde{0}$ then the initial basic feasible fuzzy solution $\tilde{x}_B$ is an optimum basic feasible fuzzy solution.

ii) If at least one $\tilde{\Delta}_j < \tilde{0}$, proceed on to the next step.

Step-6 If there are more than one negative $\tilde{\Delta}_j$, then choose the most negative of them. Let it be $\tilde{\Delta}_r$ for some *j=r*.

i) If all $\tilde{y}_{ir} \leq 0, (i = 1,2, \ldots, m)$, then there is an unbounded solution to the given problem.

ii) If at least one $\tilde{y}_{ir} > 0, (i = 1,2, \ldots, m)$, then the corresponding vector $\tilde{y}_r$ enter the basis $\tilde{y}_B$.

Step-7 Compute the $\tilde{x}_{Bi}/\tilde{y}_{ir}, i = 1,2, \ldots, m$ and choose minimum of them. Let minimum of these ratios be $\tilde{x}_{Br}/\tilde{y}_{kr}$. Then the vector $\tilde{y}_k$ will level the basis $\tilde{y}_B$. The common element $\tilde{y}_{kr}$, which in the k^th row and r^th column is known as leading number of the table.

Step-8 Convert the leading number to unit number by dividing its row by the leading number itself and all other number itself and all other elements in its column to zero.

$$\tilde{\boldsymbol{y}} \approx \tilde{y}_{ij} - \left(\frac{\tilde{y}_{kj}}{\tilde{y}_{kr}}\right)\tilde{y}_{ir}, i = 1,2, \ldots, m+1; i \neq k \text{ and } \tilde{\boldsymbol{y}}_{\boldsymbol{kj}} \approx \left(\frac{\tilde{y}_{kj}}{\tilde{y}_{kr}}\right), j = 0,1,2, \ldots, n.$$

Step-9 Go to step 5 and repeat the computational procedure until either an optimum solution is obtained or this is an indication of an unbounded solution.

6 Numerical Examples

$$\text{Max } \tilde{z} = (3,5,6,7)x_1 + (5,8,11,12)x_2$$

such that

$$-2x_1 + 3x_2 \leq 6$$
$$5x_1 + 3x_2 \leq 15$$
$$\forall\, x_1, x_2 \geq 0.$$

The optimal solution of the given fuzzy linear programming problem is evaluated in the Table 2.

Table 2: Optimal values for the proposed fuzzy linear programming model

$\tilde{\boldsymbol{C}}_j$			(3,5,6,7)	(5,8,11,12)	(0,0,0,0)	(0,0,0,0)	
$\tilde{\boldsymbol{C}}_{\boldsymbol{B}}$	$\tilde{\boldsymbol{Y}}_{\boldsymbol{B}}$	$\tilde{\boldsymbol{X}}_{\boldsymbol{B}}$	$\tilde{\boldsymbol{y}}_1$	$\tilde{\boldsymbol{y}}_2$	$\tilde{\boldsymbol{y}}_3$	$\tilde{\boldsymbol{y}}_4$	**Min ratio**
(0,0,0,0)	$\tilde{y}_3$	6	-2	$\boxed{3}$	1	0	2→
(0,0,0,0)	$\tilde{y}_4$	15	5	3	0	1	3
$\tilde{z}_j$		(0,0,0,0)	(-7,-6,-5,-3)	(-12,-11,-8,-5)↑	(0,0,0,0)	(0,0,0,0)	$\tilde{\Delta}_j$
$\tilde{\boldsymbol{C}}_{\boldsymbol{B}}$	$\tilde{\boldsymbol{Y}}_{\boldsymbol{B}}$	$\tilde{\boldsymbol{X}}_{\boldsymbol{B}}$	$\tilde{\boldsymbol{y}}_1$	$\tilde{\boldsymbol{y}}_2$	$\tilde{\boldsymbol{y}}_3$	$\tilde{\boldsymbol{y}}_4$	**Min ratio**
(5,8,11,12)	$\tilde{y}_2$	2	$\frac{-2}{3}$	1	$\frac{1}{3}$	0	-
(0,0,0,0)	$\tilde{y}_4$	9	$\boxed{7}$	0	-1	1	$\frac{9}{7}$→
$\tilde{z}_j$		(10,16,22,24)	(-15,-$\frac{40}{3}$,-$\frac{31}{3}$,-$\frac{19}{3}$)↑	(0,0,0,0)	$(\frac{5}{3},\frac{8}{3},\frac{11}{3},\frac{12}{3})$	(0,0,0,0)	$\tilde{\Delta}_j$
$\tilde{\boldsymbol{C}}_{\boldsymbol{B}}$	$\tilde{\boldsymbol{Y}}_{\boldsymbol{B}}$	$\tilde{\boldsymbol{X}}_{\boldsymbol{B}}$	$\tilde{\boldsymbol{y}}_1$	$\tilde{\boldsymbol{y}}_2$	$\tilde{\boldsymbol{y}}_3$	$\tilde{\boldsymbol{y}}_4$	**Min ratio**
(5,8,11,12)	$\tilde{y}_2$	$\frac{20}{7}$	0	1	$\frac{5}{21}$	$\frac{2}{21}$	-
(3,5,6,7)	$\tilde{y}_1$	$\frac{9}{7}$	1	0	$\frac{-1}{7}$	$\frac{1}{7}$	-
$\tilde{z}_j$		$(\frac{127}{7},\frac{205}{7},\frac{274}{7},\frac{303}{7})$	(0,0,0,0)	(0,0,0,0)	$(\frac{4}{21},\frac{22}{21},\frac{40}{21},\frac{52}{21})$	$(\frac{19}{21},\frac{31}{21},\frac{40}{21},\frac{45}{21})$	$\tilde{\Delta}_j \geq 0$

Therefore, the fuzzy optimal solution is

$$\tilde{x}_1^* = \frac{9}{7} \approx 0.1.2857,\ \tilde{x}_2^* = \frac{20}{7} \approx 2.8571$$

and

$$\tilde{z}^* = (\frac{127}{7},\frac{205}{7},\frac{274}{7},\frac{303}{7}) = \frac{911}{28} \approx 32.5357.$$

7 Post Optimal Analyses

The investigations that deal with changes in the optimum solutions due to discrete variations in the parameter b_i and the fuzzy parameter $\tilde{c}_j$ is called post optimal.

7.1 Discrete Variation in b

The investigations that deal with changes in the optimum solutions due to discrete variations in the parameter b_i is called post optimal analysis.

Consider the fuzzy linear programming problem $Maximize\ \tilde{z} = \tilde{\boldsymbol{C}}^T\tilde{\boldsymbol{X}}$ subject to $\boldsymbol{A}\tilde{\boldsymbol{X}} = \boldsymbol{b}$ and $\tilde{\boldsymbol{X}} \geq \boldsymbol{0}$. Let the component b_k of the vector **b** be changed to $b_k + \Delta b_k$, hence range of Δb_k, so that the optimum solution $\tilde{\boldsymbol{X}}_B^*$ also remains feasible is

$$\text{Max}_{b_{ik}>0}\left\{\frac{-\tilde{x}_{Bi}}{b_{ik}}\right\} \le \Delta b_k \le \text{Min}_{b_{ik}<0}\left\{\frac{-\tilde{x}_{Bi}}{b_{ik}}\right\}.$$

From Table 2 it is observed that

$$\tilde{\boldsymbol{x}}_B = \left[\frac{20}{7}, \frac{9}{7}\right] \text{ and } \boldsymbol{B}^{-1} = [\tilde{y}_2, \tilde{y}_1] = \begin{bmatrix} \frac{5}{21} & \frac{2}{21} \\ \frac{-1}{7} & \frac{1}{7} \end{bmatrix}.$$

The individual effects of changes in $b_1 and\ b_2$ where $\boldsymbol{b} = [\ b_1\ b_2\ ]$ such that the optimality of the basic feasible solution is not violated, are given by

$$\text{Max}_{b_{ik}>0}\left\{\frac{-\tilde{x}_{Bi}}{b_{ik}}\right\} \le \Delta b_k \le \text{Min}_{b_{ik}<0}\left\{\frac{-\tilde{x}_{Bi}}{b_{ik}}\right\},$$

$$\text{Max}_{b_{11}>0}\left\{\frac{-\frac{20}{23}}{\frac{5}{7}}\right\} \le \Delta b_1 \le \text{Min}_{b_{12}<0}\left\{\frac{\frac{-9}{7}}{\frac{-1}{7}}\right\} = -4 \le \Delta b_1 \le 9$$

and

$$\text{Max}_{b_{22}>0}\left\{\frac{\frac{-20}{7}}{\frac{2}{21}}, \frac{\frac{-9}{7}}{\frac{1}{7}}\right\} \le \Delta b_2 = \text{Max}_{b_{22}>0}\{-30, -9\} \le \Delta b_2.$$

Hence, $-4 \le \Delta b_1 \le 9$ and $-9 \le \Delta b_2$. Now, since $b_1 = 6\ and\ b_2 = 15$, the required range of variation is $2 \le b_1 \le 15$ and $6 \le b_2$.

7.2 Discrete Variation in $\tilde{c}$

The investigations that deal with changes in the optimum solutions due to discrete variations in the fuzzy parameter $\tilde{c}_j$ is called post optimal analysis. Considering the fuzzy linear programming problem $Maximize\ \tilde{z} = \widetilde{\boldsymbol{C}}^T\widetilde{\boldsymbol{X}}$ subject to $\boldsymbol{A}\widetilde{\boldsymbol{X}} = \boldsymbol{b}$ and $\widetilde{\boldsymbol{X}} \ge \boldsymbol{0}$. There are two possibilities: i) $\tilde{c}_1 \notin \tilde{\boldsymbol{c}}_{\boldsymbol{B}}$ and ii) $\tilde{c}_1 \epsilon \tilde{\boldsymbol{c}}_{\boldsymbol{B}}$.

i) $\tilde{c}_1 \notin \tilde{\boldsymbol{c}}_{\boldsymbol{B}}$, then the current solutions remain optimum for the new problem if $\tilde{z}_k - \left(\tilde{c}_k + \widetilde{\Delta c}_k\right) \ge 0$. Further, since $\tilde{z}$ is independent of $\tilde{c}_k$, the value of the objective function and the fuzzy optimum solution will be remain unchanged.

ii) $\tilde{c}_1 \epsilon \tilde{\boldsymbol{c}}_{\boldsymbol{B}}$, the current basic feasible solutions remain optimum for the new problem, $\tilde{\Delta}_j \ge 0$. That is

$$\text{Max}\left\{\frac{-\tilde{\Delta}_j}{\tilde{y}_{kj}>0}\right\} \le \Delta\tilde{c}_k \le \text{Min}\left\{\frac{-\tilde{\Delta}_j}{\tilde{y}_{kj}<0}\right\}.$$

From the Table 2 it is observed that:

Variation in $\tilde{\boldsymbol{c}}_1$: since $\tilde{c}_1 \epsilon \tilde{\boldsymbol{c}}_{\boldsymbol{B}}$, the range of $\widetilde{\Delta c}_1$ is given by

$$\text{Max}_{y_{1j}>0}\left\{\frac{-\tilde{\Delta}_j}{y_{1j}}\right\} \le \widetilde{\Delta c}_1 \le \text{Min}_{y_{1j}<0}\left\{\frac{-\tilde{\Delta}_j}{y_{1j}}\right\} = Max\left[\frac{-135/84}{1/7}\right] \le \widetilde{\Delta c}_1 \le Min\left[\frac{-59/42}{-1/7}\right]$$

$$= \frac{-45}{4} \le \widetilde{\Delta c}_1 \le \frac{59}{6} = -11.25 \le \widetilde{\Delta c}_1 \le 9.833.$$

Variation in $\tilde{\boldsymbol{c}}_2$: since $\tilde{c}_2 \epsilon \tilde{\boldsymbol{c}}_{\boldsymbol{B}}$, the range of $\widetilde{\Delta c}_2$ is given by

$$\text{Max}_{y_{2j}>0}\left\{\frac{-\tilde{\Delta}_j}{y_{2j}}\right\} \le \widetilde{\Delta c}_2 \le \text{Min}_{y_{2j}<0}\left\{\frac{-\tilde{\Delta}_j}{y_{2j}}\right\} = Max\left[\frac{-\frac{59}{21}}{\frac{5}{21}}, \frac{-\frac{135}{84}}{\frac{2}{21}}\right] \le \widetilde{\Delta c}_2 \le \infty\ ]$$

$$= Max\left[\frac{-59}{5}, \frac{-135}{8}\right] \le \widetilde{\Delta c}_2 \le \infty\ ] = -\frac{59}{5} \le \widetilde{\Delta c}_2 \le \infty = -11.8 \le \widetilde{\Delta c}_2 \le \infty.$$

Hence, $-11.25 \le \widetilde{\Delta c}_1 \le 9.833$ and $-11.8 \le \widetilde{\Delta c}_2 \le \infty$. Now, since $\tilde{c}_1 = (3,5,6,7) and\ \tilde{c}_2 = (5,8,11,12)$ the required range of variation is $-6 \le \tilde{c}_1 \le 15.083$ and $-2.55 \le \tilde{c}_2 \le \infty$. Figure 1 represents the mesh plot of the three dimensional figure with fuzzy total profit, $\tilde{x}_1$ and $\tilde{x}_2$ respectively.

8 Conclusions

In this paper Robust ranking technique has been implemented, for a linear programming problem with fuzzy parameters, which allows for taking a decision interactively with the decision maker in fuzzy environment. The decision maker has also some additional information about the violation of the requirement vector and profit vector in the constraints and about the compatibility of the optimal solution of the given FNLP with single objective function. The decision maker can interfere in all the steps of the decision process of fuzzy simplex algorithm which makes the approach very useful to be applied in a lot of physical- real-world problems where the information is uncertain with nonrandom. Illustrative numerical example and post optimal analyses have been provided to demonstrate the feasibility of the proposed method.

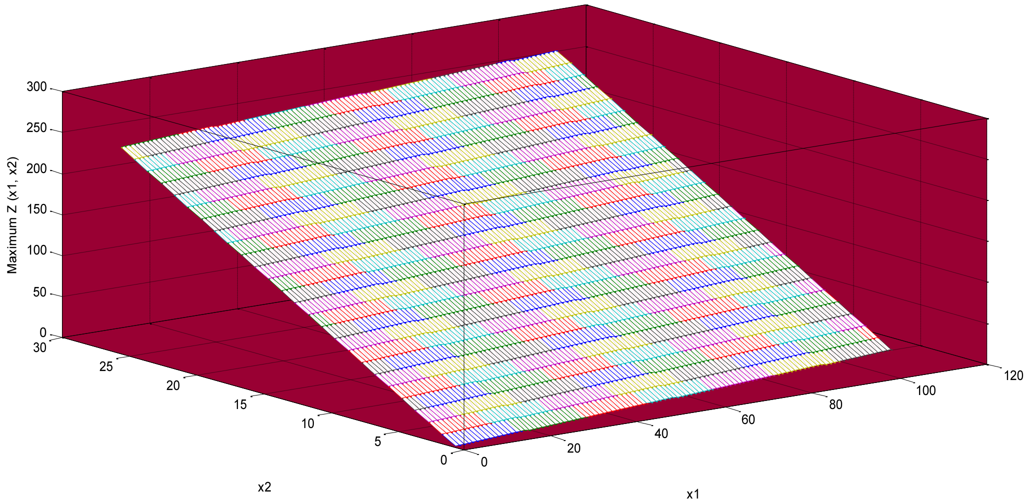

Figure 1: Mesh plot of fuzzy total profit $z(\tilde{x}_{1,}\tilde{x}_2)$, $\tilde{x}_1$ and $\tilde{x}_2$

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