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Uncertain Random Approach to Multiobjective Programming Problem Based on Chance Theory

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Abstract

The traditional solution method for the uncertain random multiobjecitve programming (URMOP) problem usually considers the transformation of converting the URMOP problem into a deterministic multiobjective programming (MOP) problem directly, and then solves the deterministic MOP problem, which neglects the nature of the uncertainty and randomness. To avoid this defect, this paper mainly focuses on a novel solution method for the URMOP problem based on chance theory. To present the solution method, we first define relationship between uncertain random variables under some principles, and give the concepts of Pareto efficient solutions of the URMOP problem. Then, on the basis of $\mathcal{P}_{\mathbb{E}}$ principle, the novel solution method by transforming the URMOP problem into uncertain random single objective programming (URSOP) problem is put forward to obtain the Pareto efficient solutions, in which the uncertainty and randomness are taken into account. Finally, the linear weighted method is employed to obtain the URSOP problem, and some theoretical results are also acquired, which can provide the theoretical foundation for solving the optimization problem.

Keywords: chance theory, uncertain random multiobjective programming, Pareto efficient solution, linear weighted method

1 Introduction

The multiobjective programming (MOP) problems have been widely studied by many researchers such as [5, 16, 18]. Since the absolute optimal solutions of the MOP problem which optimize each objective functions simultaneously usually don't exist, the non-inferior solutions are considered, i.e., Pareto optimal solutions. Furthermore, the study on MOP problem becomes more complex because of the indeterministic factors in a practical decision-making process. It is well known that randomness is one kind of indeterministic phenomena. Hence, based on the probability theory, stochastic MOP problem has been presented such as [1, 2, 17, 19]. Unfortunately, when the sample size is too small for us to estimate a probability distribution, the frequently used probability distribution is not always appropriate, especially when the information is vague; we have to invite some domain experts to evaluate their belief degree that each event will occur in this case. Such types of indeterminacy are called uncertainty. A lot of surveys show that human beings usually overweight unlikely events, and the personal belief degree may have much larger variance than the real frequency [12]. Liu [12] declared that it is inappropriate to apply either probability theory or fuzzy set theory to uncertainty, because both theories may lead to counterintuitive results in this case. In order to deal with such kind of uncertain problem, Liu [10] founded the uncertainty theory, which is a branch of mathematics based on normality, duality, subadditivity and product axioms, as a tool to handle uncertainty that is due to imprecision rather than randomness. Based on uncertainty theory, the research works on uncertain MOP problem can be found in [3, 4, 20, 21].

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In a practical decision-making process, we often face a hybrid indeterministic environment where linguistic and frequent nature coexist. For the example of the twofold indeterministic phenomena, we can refer to Zhou [24], Liu [6], Liu [7], Zheng [23], Yazenin [22]. However, under the uncertain random phenomenon, there are few research works except for the literature [13]. In [24], Zhou transformed the uncertain random MOP (URMOP) problem into a deterministic MOP problem first by taking expected value, and then solved the deterministic MOP problem, which can be referred to using the term *multiobjective approach*. Once the original URMOP problem is transformed into deterministic problem, the possible existence of uncertain random dependencies between objectives and the nature of uncertainty and randomness are not taken into account. To avoid this disadvantage, this paper presents a novel solution method for the URMOP problem based on chance theory to generate Pareto efficient solutions of uncertain MOP problem by converting the URMOP problem into uncertain random single objective (URSOP) problem with only one uncertain random objective function. In the real-life world, based on some principle such as $\mathcal{P}_{\mathtt{E}}$ proposed in this paper, URSOP problem can be solved. Compared with the *multiobjective approach*, this solution method can be referred to using the term *uncertain random approach*. Personally speaking, the Pareto efficient solution in URMOP problem should be defined on the uncertain random objectives directly instead of the converted deterministic objectives, which will assure the nature of the uncertainty and randomness of URMOP problem. In order to do this, this paper defines the relationship between uncertain random objectives, in which the symbol $\preceq$ (or $\prec$) is used to denote the relationship. For example, $f(\bar{x},\xi) \preceq f(x^*,\xi)$ means that the valuation of uncertain random objective $f(\bar{x},\xi)$ is lower than or equal to that of uncertain random objective $f(x^*,\xi)$, and $f(\bar{x},\xi) \prec f(x^*,\xi)$ means that the valuation of uncertain random objective $f(\bar{x},\xi)$ is strictly lower than that of uncertain random objective $f(x^*,\xi)$, where the valuation is a function defined under certain principles that determine the value of uncertain random objectives. To deal with the different real-life problems, they need different principles. Hence, according to real-life decision-making process, several principles are proposed in this paper, such as expected-value principle, expected-value minimum-variance principle, α-optimistic value principle and α-pessimistic value principle, which are denoted as $\mathcal{P}_{\mathtt{E}}$, $\mathcal{P}_{\mathtt{EV}}$, $\mathcal{P}_{\alpha_{\mathtt{sup}}}$ and $\mathcal{P}_{\alpha_{\mathtt{inf}}}$, respectively. Due to the mean costs (or profits) are widely used in the real world, the $\mathcal{P}_{\mathtt{E}}$ principle is important. Based on $\mathcal{P}_{\mathtt{E}}$, this paper obtains the URSOP problem by linear weighted method, then proves that the optimal solutions are $\mathcal{P}_{\mathtt{E}}$ Pareto efficient solutions to the original problem. This theoretical results acquired in this paper can provide the theoretical foundation for solving the optimization problem.

This paper is organized as follows. The next section provides a brief review on the related concepts and results in the uncertainty theory and chance theory. Section 3 first defines the relationship between uncertain random variables, then presents the novel method, i.e., *uncertain random approach*. Based on $\mathcal{P}_{\mathtt{E}}$ principle, the URMOP problem is presented and the transformation of converting the URMOP problem into the URSOP problem is also introduced in Section 4. Section 5 discusses the URSOP problem by using linear weighted method, and many theoretical results are acquired. Finally, a brief conclusion is given in Section 6.

2 Preliminary

Let Γ be a nonempty set, and $\mathcal{L}$ a σ-algebra over Γ. Each element Λ in $\mathcal{L}$ is called an event. A set function $\mathcal{M}$ from $\mathcal{L}$ to $[0,1]$ is called an uncertain measure if it satisfies the following axioms [8, 9]:
Axiom 1. (*Normality Axiom*) $\mathcal{M}\{\Gamma\}=1$ for the universal set Γ.
Axiom 2. (*Duality Axiom*) $\mathcal{M}\{\Lambda\}+\mathcal{M}\{\Lambda^c\}=1$ for any event Λ.
Axiom 3. (*Subadditivity Axiom*) For every countable sequence of events $\Lambda_1,\Lambda_2,\ldots$, we have

$$\mathcal{M}\{\bigcup_{i=1}^{\infty}\Lambda_i\} \le \sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}.$$

Axiom 4. (*Product Axiom*) Let $(\Gamma_k,\mathcal{L}_k,\mathcal{M}_k)$ be uncertainty spaces for $k=1,2,\ldots$. The product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\{\prod_{k=1}^{\infty}\Lambda_k\} = \bigwedge_{k=1}^{\infty}\mathcal{M}_k\{\Lambda_k\}$$

where Λ_k are arbitrarily chosen events from $\mathcal{L}_k$ for $k=1,2,...$, respectively. The triplet $(\Gamma,\mathcal{L},\mathcal{M})$ is referred to as a uncertainty space, in which an uncertain variable is defined as follows:

Definition 2.1 ([8]) *An uncertain variable is a measurable function* ξ *from an uncertainty space* $(\Gamma, \mathcal{L}, \mathcal{M})$ *to the set of real numbers, i.e., for any Borel set* B *of real numbers, the set*

$$\{\xi \in B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \in B\}$$

is an event.

Definition 2.2 ([9]) *The uncertain variables* $\xi_1, \xi_2, \ldots, \xi_n$ *are said to be independent if*

$$\mathcal{M}\{\bigcap_{i=1}^{n}(\xi_i \in B_i)\} = \bigwedge_{i=1}^{n} \mathcal{M}\{\xi_i \in B_i\}$$

for any Borel sets $B_1, B_2, \ldots, B_n$ *of real numbers.*

Definition 2.3 ([8]) *The uncertainty distribution* Φ *of an uncertain variable* ξ *is defined by*

$$\Phi(x) = \mathcal{M}\{\xi \le x\}$$

for any real number x.

Definition 2.4 ([11]) *Let* ξ *be an uncertain variable with regular uncertainty distribution* Φ. *Then the inverse function* Φ^{-1} *is called the inverse uncertainty distribution of* ξ.

Definition 2.5 ([8]) *Let* ξ *be an uncertain variable. Then the expected value of* ξ *is defined by*

$$E[\xi] = \int_0^{\infty} \mathcal{M}\{\xi \ge x\}dr - \int_{-\infty}^{0} \mathcal{M}\{\xi \le x\}dr$$

provided that at least one of the two integrals is finite.

Definition 2.6 ([8]) *Let* ξ *be an uncertain variable with finite expected value* e. *Then the variance of* ξ *is* $V[\xi] = E[(\xi - e)^2]$.

Theorem 2.1 ([10]) *Let* $\xi_1, \xi_2, \ldots, \xi_n$ *be uncertain variables, and* f *a real-valued measurable function. Then* $f(\xi_1, \xi_2, \cdots, \xi_n)$ *is an uncertain variable.*

Theorem 2.2 ([11]) *Let* ξ *be an uncertain variable with regular uncertainty distribution* Φ. *If the expected value exists, then*

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha)d\alpha.$$

Theorem 2.3 ([11]) *Let* $\xi_1, \xi_2, \ldots, \xi_n$ *be independent uncertain variables with regular uncertainty distributions* $\Phi_1, \Phi_2, \ldots, \Phi_n$, *respectively. If the function* $f(x_1, x_2, \cdots, x_n)$ *is strictly increasing with respect to* $x_1, x_2, \ldots, x_m$ *and strictly decreasing with respect to* $x_{m+1}, x_{m+2}, \ldots, x_n$, *then* $\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$ *is an uncertain variable with inverse uncertainty distribution*

$$\Psi^{-1}(\alpha) = f\left(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \cdots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \Phi_{m+2}^{-1}(1-\alpha), \cdots, \Phi_n^{-1}(1-\alpha)\right).$$

Definition 2.7 ([14]) *An uncertain random variable is a measurable function* ξ *from a chance space* $(\Gamma, \mathcal{L}, \mathcal{M}) \times (\Omega, \mathcal{A}, \Pr)$ *to the set of real numbers such that* $\{\xi \in B\}$ *is an event in* $\mathcal{L} \times \mathcal{A}$ *for any Borel set* B.

Definition 2.8 ([14]) *Let* $(\Gamma, \mathcal{L}, \mathcal{M}) \times (\Omega, \mathcal{A}, \Pr)$ *be a chance space, and let* $\Theta \in \mathcal{L} \times \mathcal{A}$ *be an event. Then the chance measure of* Θ *is defined as*

$$\mathrm{Ch}\{\Theta\} = \int_0^1 \Pr\{\omega \in \Omega | \mathcal{M}\{\gamma \in \Gamma | (\gamma, \omega) \in \Theta\} \ge x\}dx.$$

Definition 2.9 ([14]) *Let* ξ *be an uncertain random variable. Then its chance distribution is defined by*

$$\Phi(x) = \mathrm{Ch}\{\xi \le x\}$$

for any real number x.

Definition 2.10 ([14]) *Let ξ be an uncertain random variable. Then its expected value is defined by*

$$E[\xi] = \int_0^{\infty} \mathrm{Ch}\{\xi \geq x\}dr - \int_{-\infty}^{0} \mathrm{Ch}\{\xi \leq x\}dr$$

provided that at least one of the two integrals is finite.

Theorem 2.4 ([14]) *Let ξ be an uncertain random variable with regular chance distribution Φ. Then*

$$E[\xi] = \int_0^1 \Phi^{-1}\alpha d\alpha.$$

Theorem 2.5 ([14]) *Let $\xi_1, \xi_2, \ldots, \xi_n$ be uncertain random variables on the chance space $(\Gamma, \mathcal{L}, \mathcal{M}) \times (\Omega, \mathcal{A}, \Pr)$, and let $f : \Re^n \to \Re$ be a measurable function. Then $\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$ is an uncertain random variable determined by*

$$\xi(\gamma, \omega) = f(\xi_1(\gamma, \omega), \xi_2(\gamma, \omega), \cdots, \xi_n(\gamma, \omega))$$

for all $(\gamma, \omega) \in \Gamma \times \Omega$.

Theorem 2.6 ([15]) *Let $\eta_1, \eta_2, \ldots, \eta_m$ be independent random variables with probability distributions Ψ_1, $\Psi_2, \ldots, \Psi_m$, respectively, and let $\tau_1, \tau_2, \ldots, \tau_n$ be uncertain variables (not necessarily independent), then the uncertain random variable*

$$\xi = f(\eta_1, \eta_2, \cdots, \eta_m, \tau_1, \tau_2, \cdots, \tau_n)$$

has an expected value

$$E[\xi] = \int_{\Re^m} E[f(y_1, y_2, \cdots, y_m, \tau_1, \tau_2, \cdots, \tau_n)]d\Psi_1(y_1) \cdots \Psi_m(y_m)$$

where $E[f(y_1, y_2, \cdots, y_m, \tau_1, \tau_2, \cdots, \tau_n)]$ is the expected value of the uncertain variable $f(y_1, y_2, \cdots, y_m, \tau_1,$ $\tau_2, \cdots, \tau_n)$ for any real numbers $y_1, y_2, \ldots, y_m$.

Theorem 2.7 ([15]) *Assume η_1 and η_2 are random variables, τ_1 and τ_2 are independent uncertain variables, and f_1 and f_2 are measurable functions. Then*

$$E[f_1(\eta_1, \tau_1) + f_2(\eta_2, \tau_2)] = E[f_1(\eta_1, \tau_1)] + E[f_2(\eta_2, \tau_2)].$$

3 The Relationship Between Uncertain Random Variables

Definition 3.1 *Let ξ and η be two uncertain random variables. We say $\xi \preceq$ (or $\prec$) η if and only if $\mathcal{P}[\xi] \leq$ (or $<$)$\mathcal{P}[\eta]$.*

The symbol $\xi \preceq \eta$ (or $\xi \prec \eta$) means that the valuation of ξ is lower than or equal to (or strictly lower than) that of η, and $\mathcal{P}$ denotes the principle employed to define the valuation of uncertain random variables. In addition, different real-life problems call for different meanings of valuation to satisfy its need in practical application. Thus, corresponding principle $\mathcal{P}$ should be proposed to define this valuation according to the real context of problem.

Definition 3.2 (Expected-value principle $\mathcal{P}_{\mathtt{E}}$) *Let ξ and η be two uncertain random variables. We say $\xi \preceq$ (or $\prec$) η if and only if $E[\xi] \leq$ (or $<$) $E[\eta]$, where $E[\cdot]$ denotes the expected value of uncertain random variable.*

Definition 3.3 (Expected-value minimum-variance principle $\mathcal{P}_{\mathtt{EV}}$) *Let ξ and η be two uncertain random variables. We say $\xi \preceq$ (or $\prec$) η if and only if $E[\xi] \leq$ (or $<$) $E[\eta]$ and $V[\xi] \leq$ (or $<$) $V[\eta]$.*

Definition 3.4 (α-optimistic value principle $\mathcal{P}_{\alpha_{\sup}}$) *Let ξ and η be two uncertain random variables. We say $\xi \preceq$ (or $\prec$) η if and only if $\xi_{\sup}(\alpha) \leq$ (or $<$) $\eta_{\sup}(\alpha)$ for a given confidence level $\alpha \in (0, 1]$, where $\xi_{\sup}(\alpha)$ and $\eta_{\sup}(\alpha)$ denote the α-optimistic value of uncertain random variables ξ and η, respectively.*

Definition 3.5 (**α-pessimistic value principle** $\mathcal{P}_{\alpha_{\inf}}$) *Let ξ and η be two uncertain random variables. We say $\xi \preceq$ (or $\prec$) η if and only if $\xi_{\inf}(\alpha) \le$ (or $<$) $\eta_{\inf}(\alpha)$ for a given confidence level $\alpha \in (0,1]$, where $\xi_{\inf}(\alpha)$ and $\eta_{\inf}(\alpha)$ denote the α-pessimistic value of uncertain random variables ξ and η, respectively.*

Due to that the expected value is widely used in the real world, the following contents will especially discuss the solution to the multiobjective problem based on $\mathcal{P}_{\mathtt{E}}$ principle, and the other $\mathcal{P}$ principles can be also studied according to different real-life problems.

Theorem 3.1 *Let ξ and η be two uncertain random variables with regular chance distributions Φ and Ψ, respectively. If $\xi \preceq$ (or $\prec$) η, then for any real number $\lambda > 0$, we have*

$$\lambda\xi \preceq (or \prec)\ \lambda\eta.$$

Proof: Since $\xi \preceq$ (*or* $\prec$) η, it follows from the definition of $\mathcal{P}_{\mathtt{E}}$ principle that

$$E[\xi] \le (or <)\ E[\eta].$$

By Theorem 2.4, we have

$$\int_0^1 \Phi^{-1}(\alpha)d\alpha \le (or <) \int_0^1 \Psi^{-1}(\alpha)d\alpha$$

For any real number $\lambda > 0$, we can obtain that

$$\lambda\int_0^1 \Phi^{-1}(\alpha)d\alpha \le (or <)\lambda \int_0^1 \Psi^{-1}(\alpha)d\alpha,$$

namely,

$$\int_0^1 \lambda\Phi^{-1}(\alpha)d\alpha \le (or <) \int_0^1 \lambda\Psi^{-1}(\alpha)d\alpha.$$

It follows from the definition of expected value of uncertain random variable that

$$E[\lambda\xi] = \int_0^1 \lambda\Phi^{-1}(\alpha)d\alpha,$$

and

$$E[\lambda\eta] = \int_0^1 \lambda\Psi^{-1}(\alpha)d\alpha.$$

Evidently,

$$E[\lambda\xi] \le (or <)\ E[\lambda\eta],$$

which implies that $\lambda\xi \preceq$ (*or* $\prec$) $\lambda\eta$. The theorem is proved.

Theorem 3.2 *Assume that ζ_i, ς_i, $i = 1,2$, are random variables, τ_i, ι_i, $i = 1,2$, are independent uncertain variables, and f_1, f_2, g_1 and g_2 are measurable functions. If the uncertain random variable $\xi_i = f_i(\zeta_i, \tau_i)$ and $\eta_i = g_i(\varsigma_i, \iota_i)$ satisfy the following condition*

$$\xi_1 \preceq (or \prec)\ \eta_1,\ \xi_2 \preceq (or \prec)\ \eta_2,$$

then we have

$$\xi_1 + \xi_2 \preceq (or \prec)\ \eta_1 + \eta_2.$$

Proof: Since $\xi_1 \prec \eta_1,\ \xi_2 \preceq \eta_2$, according to $\mathcal{P}_{\mathtt{E}}$ principle, we have

$$E[\xi_1] \le (or <)\ E[\eta_1],\ E[\xi_2] \le (or <)\ E[\eta_2].$$

Evidently,

$$E[\xi_1] + E[\xi_2] \le (or <)\ E[\eta_1] + E[\eta_2].$$

It follows from Theorem 2.7 that

$$E[\xi_1 + \xi_2] = E[\xi_1] + E[\xi_2], E[\eta_1 + \eta_2] = E[\eta_1] + E[\eta_2],$$

which implies that

$$E[\xi_1 + \xi_2] \le E[\eta_1 + \eta_2].$$

By the definition of $\mathcal{P}_{\mathtt{E}}$ principle, we can obtain

$$\xi_1 + \xi_2 \preceq (or \prec)\ \eta_1 + \eta_2.$$

The theorem is proved.

4 The URMOP Problem under $\mathcal{P}_E$ Principle

4.1 Pareto Efficient Solution

We present the URMOP problem as follows,

$$\begin{cases} \min_{x\in R^n} \mathbf{F}(x,\xi) = (f_1(x,\xi), f_2(x,\xi), \cdots, f_p(x,\xi)) \\ \text{subject to :} \\ \qquad g_i(x,\eta) \le 0, i = 1,2,\ldots,m \end{cases} \tag{1}$$

where $x \in R^n$ is a vector of decision variables of the problem; $\xi = h(\zeta,\tau)$ and $\eta = l(\varsigma,\iota)$ are uncertain random variables, h and l are Borel measurable functions, the components ζ, ς are random variables, and τ, ι, are independent uncertain variables, respectively.

In real decision-making process, the objectives in URMOP problem are usually in conflict, and we can't simultaneously minimize all the objective functions. In order to deal with this difficulty, the concepts of $\mathcal{P}_E$ Pareto efficient solution (or weakly efficient solution) are introduced.

Definition 4.1 *A feasible solution x^* is said to be $\mathcal{P}_E$ Pareto efficient to the URMOP problem if there is no feasible solution x such that*

$$\boldsymbol{F}(x,\xi) \preceq \boldsymbol{F}(x^*,\xi),$$

that is to say,

$$f_k(x,\xi) \preceq f_k(x^*,\xi), k = 1,2,\ldots,p$$

and $f_{k_0}(x,\xi) \prec f_{k_0}(x^*,\xi)$ for at least one index k_0.

Definition 4.2 *A feasible solution x^* is said to be $\mathcal{P}_E$ Pareto weakly efficient solution to the uncertain URMOP problem if there is no feasible solution x such that*

$$\boldsymbol{F}(x,\xi) \prec \boldsymbol{F}(x^*,\xi),$$

that is to say,

$$f_k(x,\xi) \prec f_k(x^*,\xi), k = 1,2,\ldots,p.$$

4.2 Uncertain Random Method

In order to solve the URMOP problem, the method of transforming the original URMOP problem into URSOP problem is employed by using a real-valued measurable function U defined on *p-ary* space. In some cases, we can prove that the optimal solution to the URSOP problem is Pareto efficient solution (or weakly efficient solution) to the URMOP problem. The specific transformation is as follows,

$$\begin{cases} \min_{x\in R^n} s(x,\xi) = U(\mathbf{F}(x,\xi)) = U(f_1(x,\xi), f_2(x,\xi), \cdots, f_p(x,\xi)) \\ \text{subject to :} \\ \qquad \text{Ch}\{g_i(x,\eta) \le 0\} \ge \alpha_i, i = 1,2,\ldots,m. \end{cases} \tag{2}$$

Note that since the uncertain random constraints $g_i(x,\eta) \le 0, i = 1,2,\ldots,m$ do not define a crisp feasible set, they have been converted into chance constraints with confidence levels $\alpha_1, \alpha_2, \ldots, \alpha_m$ in problem (2), which is a crisp feasible set.

Theorem 4.1 *Let U be a real-valued measurable function, then $s(x,\xi)$ is an uncertain random variable in problem 4.2.*

Proof: By Theorem 2.5, the theorem is easy to proved.

This theorem guarantees the uncertain random nature of URMOP problem. In order to solve the URSOP problem, its optimal solution should be defined firstly.

Definition 4.3 *A feasible solution x^* is called an optimal solution to the URSOP problem* (2) *if*

$$s(x^*,\xi) \preceq s(x,\xi)$$

for any feasible solution x.

Obviously, the optimal solution to problem (2) is also defined under the relationship between uncertain random variables.

Based on the $\mathcal{P}_{\mathtt{E}}$ principle, the equivalent URSOP problem can be obtained as follows,

$$\begin{cases} \min\limits_{x\in R^n} s(x,\xi) = U\big(f_1(x,\xi), f_2(x,\xi), \cdots, f_p(x,\xi)\big) \\ \text{subject to :} \\ \qquad \text{Ch}\{g_i(x,\eta_i)\le 0\} \ge \alpha_i, i=1,2,\ldots,m. \end{cases} \tag{3}$$

From the decision-making process, it is evident that *uncertain random approach* is different from that in *multiobjective approach*, and the results obtained using these two approaches are usually different. There are many transformation methods, and the well-known method is the linear weighted method which is a compromise method by weighting the objective functions. Thus the compromise model is set up in the following part.

5 Linear Weighted Method

Based on linear weighted method, the URSOP problem can be rewritten as follows,

$$\begin{cases} \min\limits_{x\in R^n} s(x,\xi) = \sum\limits_{j=1}^{p} \lambda_j f_j(x,\xi) \\ \text{subject to :} \\ \qquad \text{Ch}\{g_i(x,\eta)\le 0\} \ge \alpha_i, i=1,2,\ldots,m \end{cases} \tag{4}$$

where $\lambda \in \Lambda^{++} = \{\lambda = (\lambda_1, \cdots, \lambda_p)^T | \lambda_j > 0, \sum_{j=1}^{p} \lambda_j = 1\}$.

Theorem 5.1 *The optimal solution x^* to URSOP problem* (4) *under $\mathcal{P}_{\mathtt{E}}$ principle is $\mathcal{P}_{\mathtt{E}}$ Pareto efficient solution to the original URMOP problem* (1).

Proof: Assume that x^* is the optimal solution to URSOP problem (4), which isn't the $\mathcal{P}_{\mathtt{E}}$ Pareto efficient solution to the original URMOP problem (1). By Definition 4.1, there exists $\bar{x}$ such that $f_k(\bar{x},\xi) \preceq f_k(x^*,\xi)$, and $f_{k_0}(\bar{x},\xi) \prec f_{k_0}(x^*,\xi)$ for at least one index $k_0, 1 \le k_0 \le k$.

Since $\lambda \in \Lambda^{++} = \{\lambda = (\lambda_1, \cdots, \lambda_p)^T | \lambda_k > 0, \sum_{k=1}^{p} \lambda_k = 1\}$, and $f_{k_0}(\bar{x},\xi) \prec f_{k_0}(x^*,\xi)$, it follows from the definition $\mathcal{P}_{\mathtt{E}}$ principle and Theorem 3.1, Theorem 3.2 that

$$\sum_{k=1}^{p} \lambda_k f_j(\bar{x},\xi_k) \prec \sum_{j=k}^{p} \lambda_k f_k(x^*,\xi_k),$$

that is to say, $s(\bar{x},\xi) \prec s(x^*,\xi)$. It follows from Definition 4.3 that x^* is not the optimal solution to URSOP problem (4), which contradicts with the previous hypothesis that x^* is the optimal solution. Hence, x^* is $\mathcal{P}_{\mathtt{E}}$ Pareto efficient solution to the original URMOP problem (1). The theorem is proved.

6 Conclusion

In this paper, we focused on the *uncertain random approach* based on chance theory. We first presented the order relationship between uncertain random variables, and gave several definitions of the relationship under different principle. Then, based on $\mathcal{P}_{\mathtt{E}}$ principles, the transformation of converting the URMOP problem into URSOP problem was introduced, and the Pareto efficient solution was also defined. Finally, the linear weighted method was employed to obtain the URSOP problem, and some theoretical results were also acquired, which can provide the theoretical foundation for designing the optimization algorithm.

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