Expected Value-Based Method to Determine the Importance of Engineering Characteristics in QFD with Uncertainty Theory

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Abstract

Quality function deployment (QFD) is a systematic method used for developing a product or service, that helps transforming customer requirements (CRs) into appropriate engineering characteristics (ECs). How to obtain the importance of ECs in QFD is a fundamental and crucial issue for efficiently planning resource allocation. Owing to the typical imprecision or vagueness in the QFD process, the uncertain environments and the uncertainty theory are suggested for capturing those linguistic assessments and subjective judgements. Thus, in this paper, a new method for determining the importance of ECs using an uncertain expected value operator is presented, in which the two sets of input data are expressed as uncertain variables, namely, the relative importance of CRs and the relationship matrix between CRs and ECs. An example of a flexible manufacturing system design illustrates the performance and the potential applications of the presented approach.

Keywords: quality function deployment, house of quality, importance of engineering characteristics, uncertainty theory

1 Introduction

Manufacturing enterprises, which are confronted with a more complicated competition on the global scale, realize that the efficient products design and manufacture based on customer needs and expectations are crucial for both of their survival and long-term development. A wide acceptance in the industry to ensure and promote quality during the product development is the use of quality function deployment (QFD), which is a well known customer-driven product development approach originated in Japan in the late 1960s. It is a systematic method that devotes to transforming customer requirements (CRs) to some specific engineering characteristics (ECs) of the product in order to achieve high customer satisfaction. Nowadays, QFD has gained extensive support for helping the design team to maintain a correct focus on true customer requirements and determine how an existing product or service can be improved continuously.

The most typical and significant tool of QFD, the House of Quality (HoQ), is a kind of conceptual map that provides means for interfunctional planning and communications. It mainly consists of the following matrices: customer requirements (CRs, namely what to do) and their importance, engineering characteristics (ECs, namely how to do) and their importance, the relationships between CRs and ECs, and the correlations among all ECs. The process of establishing a HoQ is a quantitative and qualitative analysis procedure, after which we can achieve the conversion from customer feedback to engineering information.

It is generally considered that in the design process, planners should focus on engineering characteristics of a new or existing product or service from the viewpoints of customers’ desires and needs. Therefore, determining the importance of ECs is such a core issue towards successful QFD realization that enterprise resources can then be properly assigned to ECs. Meanwhile, the rankings of the importance of ECs are also key results of QFD as they guide the design team in decision-making, resource allocation, and subsequent QFD analysis. Typically, in order to derive the importance of ECs, two essential prerequisites receive much concern in this field, i.e., obtaining the importance of CRs, as well as the determination of the relationships between CRs and ECs.

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In one aspect, obtaining the importance of CRs is to evaluate the weights of CRs, which is the first step to transform the customer needs in QFD. A classical method to prioritize CRs is the application of analytic hierarchy process (AHP) \[11, 2\]. Afterwards, Karsak et al. \[16\] suggested the use of analytic network process (ANP) to prioritize CRs so that the interdependence among CRs could be considered. However, these methods have strict specifications on the input variables of the QFD process. With reference to the effect that the imprecise information has on CRs, the fuzzy AHP \[18, 19\] and the fuzzy ANP \[8, 17\] were widely employed. Meanwhile, some scholars advocated to revise the expression of the importance of CRs by utilizing an entropy method \[3, 9, 13\]. In the other aspect, the determination of the relationships between CRs and ECs should be concerned, because they are used to convert CRs into ECs, i.e., to transform the importance of CRs to the importance of ECs. On the whole, in the previous literature, individual assessments and judgements to the relationships between CRs and ECs have been expressed as crisp, random or fuzzy variables, and the fuzzy linear and non-linear regressions as effective methods to investigate the functional relationships are frequently applied in the QFD process \[7, 1, 5, 11, 21, 28\].

It can be seen that, all the above studies and researches on deriving the importance of CRs and the relationships between CRs and ECs have laid a good foundation for the determination of the importance of ECs and their rankings. On this basis, numerous studies have been conducted on how to obtain the importance of ECs. Khoo and Ho \[13\] developed a framework of a fuzzy QFD system to address the ambiguity involved in the QFD process. Wang \[31\] used the outranking approach based on possibility and necessity measures to prioritize ECs. Chen et al. \[7\] calculated the technical importance of ECs using a fuzzy weighted average method, and the fuzzy expected value operator proposed by Liu and Liu \[26\] was utilized to prioritize ECs. Geng et al. \[12\] applied the fuzzy ANP and the modified fuzzy logarithmic least squares method to determine the technical importance ratings of ECs. Besides, to expand the application scope, several studies have considered the group decision making in QFD. Kwong et al. \[27\] proposed a fuzzy group decision-making method which integrated the fuzzy weighted average method with a consensus ordinal ranking technique. In their approach, a robustness index was introduced to evaluate the rankings of the importance of ECs. Wang \[32\] presented a group decision-making approach to prioritize ECs under vagueness, which took group decision behaviours of both customers and QFD team members into account. Furthermore, Song et al. \[24\] recently applied an approach to get the importance of ECs by integrating the grey relationship analysis method and the rough set theory.

Up to now, most of the variables or parameters applied in the QFD process were treated as either crisp data or fuzzy variables. However, it is usually not appropriate enough because both the probability theory and the fuzzy set theory may sometimes lead to counterintuitive results. In order to rationally address human uncertain phenomena which are ubiquitous in the QFD product development process, this paper proposes a new uncertain expected value operator approach for determining the importance of ECs and their rankings in QFD by utilizing the idea of the uncertainty theory, which was founded by Liu \[22\] and refined in Liu \[23\]. In our approach, uncertain variables are used to describe the phenomena where the uncertainty appears in QFD process, i.e., the human evaluation involved in the importance of CRs and the relationships between CRs and ECs.

The rest of the article is organized as follows. In the next section, some important concepts of uncertainty theory are introduced that are used in QFD with uncertainty. In section 3, an uncertain expected value operator method for determining the importance of ECs in the uncertain environment is presented. Finally, section 4 illustrates a numerical example about the design of a flexible manufacturing system, which is presented to demonstrate the performance of the proposed approach.

2 Preliminaries

Uncertainty theory is an efficient tool to deal with the indeterministic information, especially expert data and subjective estimations. In this section, we will introduce some basic concepts of uncertainty theory, which will be applied for calculating the expected value in the sense of uncertain measure. More details are referred to Liu \[25\].

2.1 Uncertainty Theory

**Definition 1** \[22\] Let \(\Gamma\) be a nonempty set, and \(\mathcal{L}\) a \(\sigma\)-algebra over \(\Gamma\). The set function \(\mathcal{M} : \mathcal{L} \to [0, 1]\) is called an uncertain measure if it satisfies:
(i) $\mathcal{M}\{\Gamma\} = 1$ for the universal set $\Gamma$;
(ii) $\mathcal{M}\{A\} + \mathcal{M}\{A^c\} = 1$ for any event $A$;
(iii) For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots$, we have
\[
\mathcal{M}\left\{ \bigcup_{i=1}^{\infty} \Lambda_i \right\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}.
\]

Besides, in order to provide the operational law, Liu (2009) defined the product uncertain measure as follows:
(iv) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \ldots$. Then the product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying
\[
\mathcal{M}\left\{ \prod_{k=1}^{\infty} \Lambda_k \right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\}
\]
where $\Lambda_k$ are arbitrarily chosen events from $\mathcal{L}_k$ for $k = 1, 2, \ldots$, respectively.

Based on the concept of uncertain measure, a formal definition of an uncertain variable is given as follows.

**Definition 2** (\cite{22}) An uncertain variable is a measurable function $\xi$ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers, i.e., for any Borel set $B$ of real numbers, the set
\[
\{\xi \in B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \in B\}
\]
is an event.

In order to describe uncertain variables, the concept of the uncertainty distribution is adopted, and the regular uncertainty distribution is defined.

**Definition 3** (\cite{22}) The uncertainty distribution $\Phi$ of an uncertain variable $\xi$ is defined by
\[
\Phi(x) = \mathcal{M}\{\xi \leq x\}
\]
for any real number $x$.

**Definition 4** (\cite{24}) An uncertainty distribution $\Phi(x)$ is said to be regular if it is a continuous and strictly increasing function with respect to $x$ at which $0 < \Phi(x) < 1$, and
\[
\lim_{x \to -\infty} \Phi(x) = 0, \quad \lim_{x \to +\infty} \Phi(x) = 1.
\]

The inverse function $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of an uncertain variable $\xi$ if it exists and is unique for each $\alpha \in (0, 1)$. The inverse uncertainty distribution plays a crucial role in operations of independent uncertain variables with regular uncertainty distribution.

**Definition 5** (\cite{23}) The uncertain variables $\xi_1, \xi_2, \ldots, \xi_n$ are said to be independent if
\[
\mathcal{M}\left\{ \bigcap_{i=1}^{n} \{\xi_i \in B_i\} \right\} = \bigwedge_{i=1}^{n} \mathcal{M}\{\xi_i \in B_i\}
\]
for any Borel sets $B_1, B_2, \ldots, B_n$ of real numbers.

**Theorem 1** (\cite{24}) Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively, and $f : \mathbb{R}^n \to \mathbb{R}$ a continuous and strictly increasing function. Then the uncertain variable $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ has an inverse uncertainty distribution
\[
\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha)).
\]

The expected value of an uncertain variable is the average value in the sense of uncertain measure, and can be represented by the inverse uncertainty distribution as follows.
Definition 6 ([22]) Let $\xi$ be an uncertain variable. Then the expected value of $\xi$ is defined by

$$E[\xi] = \int_{0}^{+\infty} \mathbb{M}\{\xi \geq x\}dx - \int_{-\infty}^{0} \mathbb{M}\{\xi \leq x\}dx$$

(5)

provided that at least one of the two integrals is finite.

Theorem 2 ([21]) Let $\xi$ be an uncertain variable with uncertainty distribution $\Phi$. Then

$$E[\xi] = \int_{0}^{+\infty} (1 - \Phi(x))dx - \int_{-\infty}^{0} \Phi(x)dx$$

(6)

Theorem 3 ([21]) Let $\xi$ and $\eta$ be independent uncertain variables with finite expected values. Then for any real numbers $a$ and $b$, we have

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta].$$

(7)

Theorem 4 ([21]) Let $\xi$ be an uncertain variable with a regular uncertainty distribution. Then

$$E[\xi] = \int_{0}^{1} \Phi^{-1}(\alpha)d\alpha$$

(8)

where $\Phi$ and $\Phi^{-1}$ are the uncertainty distribution and the inverse uncertainty distribution of $\xi$, respectively.

To the monotone function of uncertain variables, the expected value can be calculated as follows.

Theorem 5 ([21]) Assume that $\xi_{1}, \xi_{2}, \ldots, \xi_{n}$ are independent uncertain variables with regular uncertainty distributions $\Phi_{1}, \Phi_{2}, \ldots, \Phi_{n}$, respectively. If $f(x_{1}, x_{2}, \ldots, x_{n})$ is strictly increasing with respect to $x_{1}, x_{2}, \ldots, x_{m}$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \ldots, x_{n}$, then the uncertain variable $\xi = f(\xi_{1}, \xi_{2}, \ldots, \xi_{n})$ has an expected value

$$E[\xi] = \int_{0}^{1} f(\Phi_{1}^{-1}(\alpha), \ldots, \Phi_{m}^{-1}(\alpha), \Phi_{m+1}^{-1}(1 - \alpha), \ldots, \Phi_{n}^{-1}(1 - \alpha))d\alpha.$$ 

(9)

2.2 Linear Uncertainty Distribution

Uncertainty distribution is a carrier of incomplete information of uncertain variable. Linear uncertainty distribution as one of the most common uncertainty distributions is introduced in this subsection, and will be used in the numerical example of section 4.

Definition 7 ([23]) An uncertain variable $\xi$ is called linear if it has a linear uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leq a \\ (x - a)/(b - a), & \text{if } a \leq x \leq b \\ 1, & \text{if } x \geq b \end{cases}$$

(10)

denoted by $\mathcal{L}(a, b)$, where $a$ and $b$ are real numbers with $a < b$.

Definition 8 ([22]) Let $\xi$ be an uncertain variable with uncertainty distribution $\Phi$. Then the inverse function $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of $\xi$.

The inverse uncertainty distribution of linear uncertain variable $\mathcal{L}(a, b)$ is

$$\Phi^{-1}(\alpha) = (1 - \alpha)a + \alpha b.$$ 

(11)
3 Determination of the Importance of ECs with Uncertainty

Quality function deployment (QFD) is a planning tool for process and product development, which aims at achieving maximum customer satisfaction by listening to the voice of customers. It employs the matrix, called House of Quality (HoQ), to establish the relationships between CRs and ECs, as shown in Figure 1.

![Figure 1: QFD relationship matrix](image)

As a key strategic tool, HoQ is used for translate CRs into ECs in product improvement and quality management. In this conversion process, both the relative importance of CRs $W_i$ and the relationship measures between CRs and ECs $U_{ij}$ are quite significant coefficients, since the importance of ECs $Y_j$ can be derived from $W_i$ via $U_{ij}$.

In the complex decision process, obtaining the importance of CRs and the relationship measures between CRs and ECs in HoQ are crucial steps, since they are finally transformed into the importance of ECs. Based on the importance and rankings of ECs, a company can purposefully make the product more attractive to customers and thus gains more competitive advantages. However, in the translating process, a large number of subjective judgments have to be made by both customers and QFD team members. Due to the inherently ambiguity in subjective judgments, product planning is becoming more complicated, particularly in an indeterministic environment.

Based on the previous literature, in order to deal with ambiguous information, the fuzzy set theory has been widely suggested and applied for design imprecision in current QFD. Actually, we think that it seems more reasonable to use the uncertainty theory for better capturing those linguistic inputs in the QFD process. On these grounds, a new approach for determining the importance of ECs in QFD by the uncertainty theory is presented in this section.

3.1 Problem Notations

Assume that a product is designed with $m$ CRs, $n$ ECs, $k$ customers and $b$ experts. Before formulating this problem in the uncertain environment, some notations that are used in the paper throughout are shown below:

- $i = 1, 2, \ldots, m$ : the index of customer requirements;
- $j = 1, 2, \ldots, n$ : the index of engineering characteristics;
- $l = 1, 2, \ldots, k$ : the index of customers surveyed in a target market;
- \( s = 1, 2, \ldots, b \): the index of experts involved in the design of a particular product;
- \( CR_i \): the \( i \)th customer requirement, where \( i = 1, 2, \ldots, m \);
- \( EC_j \): the \( j \)th engineering characteristic, where \( j = 1, 2, \ldots, n \);
- \( C_l \): the \( l \)th customer surveyed in a target market, where \( l = 1, 2, \ldots, k \);
- \( D_s \): the \( s \)th expert involved in the product planning, where \( s = 1, 2, \ldots, b \);
- \( W^l_i \): the \( l \)th customer’s individual preference on \( CR_i \), which is an uncertain variable, \( i = 1, 2, \ldots, m, l = 1, 2, \ldots, k \);
- \( \{W^1_t, \ldots, W^s_t, \ldots, W^w_t\} \): A pre-defined uncertain weight set that is used to express the individual weight \( W^l_i \), \( v = 1, 2, \ldots, p \), where \( p \) is the number of linguistic terms in the uncertain set of weights;
- \( W = (W_1, \ldots, W_i, \ldots, W_m)^T \): the uncertain relative importance vector of CRs. Each \( W_i \), \( i = 1, 2, \ldots, m \), is the uncertain relative importance of \( CR_i \) by synthesizing the individual preferences of \( k \) customers on \( CR_i \), which is denoted by \( W^l_i \);
- \( U^s_{ij} \): the relationship measure between \( CR_i \) and \( EC_j \) with respect to \( D_s \), which is also an uncertain variable, \( i = 1, 2, \ldots, m, j = 1, 2, \ldots, n, s = 1, 2, \ldots, b \);
- \( \{U^1_{ij}, \ldots, U^s_{ij}, \ldots, U^w_{ij}\} \): A pre-defined uncertain strength set that is used to evaluate the relationship measures between CRs and ECs, \( t = 1, 2, \ldots, q \), where \( q \) is the number of linguistic terms in the uncertain set of the relationship strength;
- \( U = (U_{ij})_{m \times n} \): the uncertain relationship matrix between CRs and ECs, in which \( U_{ij} \) denotes the uncertain relationship measure between \( CR_i \) and \( EC_j \) by aggregating the individual assessments of \( b \) experts on the relationship between \( CR_i \) and \( EC_j \), namely \( U^s_{ij} \), \( i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \);
- \( Y = (Y_1, \ldots, Y_j, \ldots, Y_n)^T \): the uncertain importance vector of ECs, in which \( Y_j \) is the uncertain importance of \( EC_j \), \( j = 1, 2, \ldots, n \);
- \( Y' = (Y'_1, \ldots, Y'_j, \ldots, Y'_n)^T \): the uncertain relative importance vector of ECs, in which \( Y'_j \), \( j = 1, 2, \ldots, n \) is the normalized uncertain importance of \( EC_j \), i.e., \( Y_j \). It is within the range of 0 to 1.
- \( \Phi_i^l \): the uncertainty distribution of the relative importance of \( CR_i \) evaluated by \( C_l \), i.e., \( W^l_i \), \( i = 1, 2, \ldots, m, l = 1, 2, \ldots, k \);
- \( \Psi^s_{ij} \): the uncertainty distribution of the relationship measure between \( CR_i \) and \( EC_j \) judged by \( D_s \), i.e., \( U^s_{ij} \), \( i = 1, 2, \ldots, m, j = 1, 2, \ldots, n, s = 1, 2, \ldots, b \);
- \( \Phi_i \): the uncertainty distribution of the integrated relative importance of \( CR_i \), i.e., \( W_i \), \( i = 1, 2, \ldots, m \);
- \( \Psi_{ij} \): the uncertainty distribution of the integrated relationship measure between \( CR_i \) and \( EC_j \), i.e., \( U_{ij} \), \( i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \).

3.2 Formulation of the Importance of ECs

Practically speaking, the importance of ECs is closely linked with customer satisfaction. Obtaining the importance of ECs can help product planners to gain a better understanding of what the customers really want, and thereby make the right manufacturing decisions. In this paper, the relative importance of CRs and the relationships between CRs and ECs are aggregated to define the uncertain importance of ECs, which can be expressed as follows,

\[
Y = (W^T U)^T. \tag{12}
\]

The relative weight of each CR is one of key inputs to QFD. Generally, the more important a CR is, the higher weight it should get. CRs are gathered by analyzing questionnaires and surveys with regard to the product. The preferences of different customers on a specific product differ according to personal tastes and
individual needs. By synthesizing the uncertain weights of \( k \) customers, the relative weight of the CRs, can be obtained as

\[
W_i = \frac{1}{k} \sum_{l=1}^{k} W_{i}^l, \quad i = 1, 2, \ldots, m, \tag{13}
\]

which is a weighted average of \( W_{i}^l, l = 1, 2, \ldots, k \), representing a trade-off among the customers surveyed. In QFD, it is natural and reasonable to suppose that \( W_{i}^l, i = 1, 2, \ldots, m, l = 1, 2, \ldots, k \), are independent and nonnegative uncertain variables with regular distributions \( \Phi_{i}^l, i = 1, 2, \ldots, m, l = 1, 2, \ldots, k \), respectively. Therefore, the importance of CRs \( W_i, i = 1, 2, \ldots, m \), are also independent and nonnegative uncertain variables, following regular uncertainty distributions \( \Phi_i, i = 1, 2, \ldots, m \), respectively.

Similar to the relative weights of CRs, by aggregating the evaluation of \( b \) experts, the final relationship measure between the \( i \)th CR and the \( j \)th EC can be derived from the following equation,

\[
U_{ij} = \frac{1}{b} \sum_{s=1}^{b} U_{ij}^s, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n, \tag{14}
\]

which is also a weighted average of \( U_{ij}^s, s = 1, 2, \ldots, b \), representing a balance of the relationship measures between CRs and ECs judged by all the consulted experts. In QFD, we are also allowed to assume that \( U_{ij}^s, i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n, \quad s = 1, 2, \ldots, b \), are independent and nonnegative uncertain variables with regular uncertainty distributions \( \Psi_{ij}^s, i = 1, 2, \ldots, m, \quad l = 1, 2, \ldots, k, \quad s = 1, 2, \ldots, b \), respectively. Therefore, the relationship measures between CRs and ECs, i.e., \( U_{ij}, i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \), are independent and nonnegative uncertain variables too, following regular uncertainty distributions \( \Psi_{ij}, i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \), respectively.

Now, let us consider the importance of ECs. Through survey data collection of both customers and experts as well as the above Eqs. (13) and (14), we can get the importance of CRs and the relationship measures between CRs and ECs. Then, according to Eq. (12), the uncertain importance of ECs, denoted by \( Y_j \), can be expressed as follows,

\[
Y_j = \sum_{i=1}^{m} W_i U_{ij}, \quad j = 1, 2, \ldots, n. \tag{15}
\]

As such, CRs are successfully transformed into ECs. Since \( W_i \) and \( U_{ij} \) are uncertain variables, \( Y_j \) is also an uncertain variable. Then in the next subsection, our discussion will refer to how to measure and calculate the importance of ECs, i.e., \( Y_j, j = 1, 2, \ldots, n \), in details.

### 3.3 An Uncertain Expected Value Operator Method

Notably, although the importance of ECs expressed by uncertain variables seems quite appropriate in the practical environment, comparing uncertain variables is not as straightforward as comparing crisp variables. For this reason, an uncertain expected value operator method is proposed to measure the importance of ECs in order to make it meaningful.

As shown in Eq. (13), the expected value of the uncertain importance of ECs, i.e., \( Y_j \), can be given by

\[
E[Y_j] = E \left[ \sum_{i=1}^{m} W_i U_{ij} \right], \quad j = 1, 2, \ldots, n. \tag{16}
\]

After that, according to Definition 8, the expected value of \( Y_j \) is defined by

\[
E[Y_j] = \int_{0}^{+\infty} \mathbb{M} \left\{ \sum_{i=1}^{m} W_i U_{ij} \geq x \right\} dx - \int_{-\infty}^{0} \mathbb{M} \left\{ \sum_{i=1}^{m} W_i U_{ij} \leq x \right\} dx, \tag{17}
\]

which is described by the uncertain measure.

In addition, as mentioned above, in the QFD process, \( W_1, W_2, \ldots, W_m, U_{1j}, U_{2j}, \ldots, U_{mj} \) are independent uncertain variables with regular uncertainty distributions \( \Phi_1, \Phi_2, \ldots, \Phi_m, \Psi_{1j}, \Psi_{2j}, \ldots, \Psi_{mj} \), respectively. Moreover,

\[
Y_j = \sum_{i=1}^{m} W_i U_{ij}
\]
is strictly increasing with respect to $W_1, W_2, \ldots, W_m, U_{1j}, U_{2j}, \ldots, U_{mj}$. Then based on Theorem 2, the expected value of the uncertain importance of $EC_j$ Eq. (17) can be rewritten as

$$E[Y_j] = \int_0^1 \left( \sum_{i=1}^{m} \Phi_i^{-1}(r)\Psi_{ij}^{-1}(r) \right) dr,$$

(18)

which is represented by the inverse uncertainty distribution of $Y_j$.

Since we know that $W_1U_{1j}, W_2U_{2j}, \ldots, W_mU_{mj}$ are independent of each other according to the interpretation above, on the basis of Theorem 3, $E[Y_j]$ can be calculated by

$$E[Y_j] = \sum_{i=1}^{m} E[W_iU_{ij}] = \sum_{i=1}^{m} \int_0^1 \Phi_i^{-1}(r)\Psi_{ij}^{-1}(r) dr.$$

(19)

Nevertheless in fact, in the construction process of a product, designers often have to take large amounts to of ECs into account. Hence, rating the importance of ECs is essential, because it has a great help on preferring some valuable ECs to make emphatic improvements and filtering, deleting, and redefining ECs. Given this, for convenience of ranking the importance of ECs, it is necessary to normalize the relative uncertain importance of ECs into account. Hence, rating the importance of ECs is essential, because it has a great help on

$$Y'_j = \frac{E[Y_j]}{\sum_{j=1}^{n} E[Y_j]}, \quad j = 1, 2, \ldots, n,$$

(20)

where $0 < Y'_j < 1$. Through this step, all of the importance of ECs can be scaled from 0 to 1.

Up to now, based on the concepts of the uncertainty theory, the importance of ECs has been determined by utilizing the presented uncertain expected value operator method.

In many practical planning processes, it is unrealistic to expect customers to provide much elaborate information timely. Therefore, this requires the design team to extract useful information from a large amount of subjective evaluations offered by target customers. Our method provides a way to determine the importance of ECs according to the relative importance of CRs and the relationships between CRs and ECs. It can help the designers to meet customer demands with lower cost and shorter development time, and finally achieve higher returns.

4 Numerical Example

In this section, the design of a flexible manufacturing system (FMS) [12, 13, 14] is applied in this paper to demonstrate the application of the proposed approach for determining the importance of ECs. Questionnaire surveys are conveyed on design experts and professional customers to elicit the major customer requirements.


In light of the design team’s experience and expert knowledge, ten major ECs are identified corresponding to the eight major CRs, namely, “automatic gauging” (EC1), “the tool change system” (EC2), “tool monitoring system” (EC3), “coordinate measuring machine” (EC4), “automated guided vehicle” (EC5), “conveyor” (EC6), “programmable logic controller” (EC7), “storage and retrieval system” (EC8), “modular fixtureing” (EC9), and “robots” (EC10).

The relative uncertain importance of CRs is classified into seven levels to describe the difference of importance, that is, very unimportant, quite unimportant, unimportant, some important, moderately important, important, and very important. A pre-defined uncertain weight set \{\(W_1^*, W_2^*, W_3^*, W_4^*, W_5^*, W_6^*, W_7^*\)\} is used to quantize these seven linguistic terms. Similarly, the relationships between CRs and ECs are linguistically judged as none, weak, moderate, strong, or very strong, which can be expressed by a pre-defined uncertain strength set \{\(U_1^*, U_2^*, U_3^*, U_4^*, U_5^*\)\}.

It is supposed that the uncertain importance of CRs $W^*_v$, $v = 1, 2, \ldots, 7$, and the uncertain relationships between CRs and ECs $U_t^*$, $t = 1, 2, \ldots, 5$, are linear uncertain variables, given in Table I.
Suppose that ten customers are surveyed in the target market, represented by $C_l, l = 1, 2, \ldots, 10$, utilizing uncertain variables $W_i^v, v = 1, 2, \ldots, 7$, to express their personal evaluations on each CR as summarized in Table 2. After synthesizing individual weights of the ten customers on the eight CRs by Eq. (21), the final relative importance of eight CRs $W$ can be obtained as $W_1 \sim \mathcal{L}(0.81, 0.94), W_2 \sim \mathcal{L}(0.5, 0.65), W_3 \sim \mathcal{L}(0.71, 0.86), W_4 \sim \mathcal{L}(0.67, 0.81), W_5 \sim \mathcal{L}(0.52, 0.66), W_6 \sim \mathcal{L}(0.66, 0.82), W_7 \sim \mathcal{L}(0.62, 0.77)$, and $W_8 \sim \mathcal{L}(0.73, 0.87)$, which are shown on the left wall of HoQ in Table 4.

Table 2: The relative uncertain weights of eight CRs assessed by ten customers

<table>
<thead>
<tr>
<th>CR1</th>
<th>CR2</th>
<th>CR3</th>
<th>CR4</th>
<th>CR5</th>
<th>CR6</th>
<th>CR7</th>
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<td>$W_5^*$</td>
<td>$W_4^*$</td>
<td>$W_7^*$</td>
<td>$W_3^*$</td>
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<tr>
<td>$W_1^*$</td>
<td>$W_5^*$</td>
<td>$W_6^*$</td>
<td>$W_4^*$</td>
<td>$W_5^*$</td>
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<td>$W_4^*$</td>
<td>$W_6^*$</td>
<td>$W_4^*$</td>
<td>$W_6^*$</td>
</tr>
</tbody>
</table>

Meanwhile, it is assumed that seven experts, denoted by $D_s, s = 1, 2, \ldots, 7$, are involved in evaluating the relationships between CRs and ECs by using uncertain variables $U_i^*, i = 1, 2, \ldots, 5$. After aggregating all the assessments of each expert using Eq. (22), the relationship matrix of eight CRs and ten ECs $U$ can be obtained, as shown in the room of HoQ in Table 4.

Therefore, the uncertain importance of EC1, denoted by $Y_1$, can be expressed as follows,

$$Y_1 = \sum_{i=1}^{8} W_i U_{i1} = W_1 U_{11} + W_2 U_{21} + W_3 U_{31} + W_4 U_{41} + W_5 U_{51} + W_6 U_{61} + W_7 U_{71} + W_8 U_{81}.$$  (21)
### Table 3: The Relationship Matrix between CRs and ECs, and Relative Importance of CRs

<table>
<thead>
<tr>
<th>CRs</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
<th>C12</th>
<th>C13</th>
<th>C14</th>
<th>C15</th>
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</thead>
<tbody>
<tr>
<td>CR1</td>
<td>0.73087</td>
<td>0.76277</td>
<td>0.66038</td>
<td>0.67255</td>
<td>0.72250</td>
<td>0.62004</td>
<td>0.65495</td>
<td>0.59537</td>
<td>0.62471</td>
<td>0.64550</td>
<td>0.70257</td>
<td>0.65890</td>
<td>0.61548</td>
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</tr>
<tr>
<td>CR2</td>
<td>0.76277</td>
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<td>0.63950</td>
<td>0.63500</td>
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<td>0.62544</td>
<td>0.61504</td>
<td>0.69537</td>
<td>0.70257</td>
<td>0.72500</td>
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<tr>
<td>CR3</td>
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<td>0.63950</td>
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<td>0.67528</td>
<td>0.62544</td>
<td>0.61504</td>
<td>0.69537</td>
<td>0.70257</td>
<td>0.72500</td>
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<tr>
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<td>0.63500</td>
<td>0.69000</td>
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<td>0.67528</td>
<td>0.62544</td>
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<td>0.69537</td>
<td>0.70257</td>
<td>0.72500</td>
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<tr>
<td>CR5</td>
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<td>0.67528</td>
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<td>0.70519</td>
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<td>0.69000</td>
<td>0.67528</td>
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<td>0.70519</td>
<td>0.69000</td>
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<td>0.67528</td>
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<tr>
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<td>0.61504</td>
<td>0.61504</td>
<td>0.61504</td>
<td>0.69000</td>
<td>0.70519</td>
<td>0.70519</td>
<td>0.69000</td>
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<tr>
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<td>0.69537</td>
<td>0.69537</td>
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<tr>
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<tr>
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<td>0.70257</td>
<td>0.70257</td>
<td>0.70257</td>
<td>0.67528</td>
<td>0.67528</td>
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</tr>
<tr>
<td>CR12</td>
<td>0.65890</td>
<td>0.70257</td>
<td>0.70257</td>
<td>0.70257</td>
<td>0.67528</td>
<td>0.67528</td>
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<tr>
<td>CR13</td>
<td>0.61548</td>
<td>0.70257</td>
<td>0.70257</td>
<td>0.70257</td>
<td>0.67528</td>
<td>0.67528</td>
<td>0.67528</td>
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<td>CR14</td>
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</tbody>
</table>

**Customer Requirements** & Engineering Characteristics

The relationship matrix between Customer Requirements (CRs) and Engineering Characteristics (ECs), and relative importance of CRs.
Since the relative weights of CRs and the relationship measures between CRs and ECs are uncertain variables, the proposed uncertain expected value operator method is employed to calculate the importance of ECs. Then according to Eq. (17), the expected value of $Y_1$ is defined by

$$E[Y_1] = E\left[\sum_{i=1}^{8} W_i U_{1i}\right]$$

$$= \int_0^{+\infty} \mathcal{M}\left\{\sum_{i=1}^{8} W_i U_{1i} \geq x\right\} dx - \int_{-\infty}^{0} \mathcal{M}\left\{\sum_{i=1}^{8} W_i U_{1i} \leq x\right\} dx. \quad (22)$$

As $W_1, W_2, \ldots, W_8, U_{11}, U_{21}, \ldots, U_{81}$ are independent and nonnegative linear uncertain variables with linear uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_8, \Psi_{11}, \Psi_{21}, \ldots, \Psi_{81}$, respectively, and

$$Y_1 = \sum_{i=1}^{8} W_i U_{1i}$$

is a strictly increasing function with respect to $W_1, W_2, \ldots, W_8, U_{11}, U_{21}, \ldots, U_{81}$, in accordance with Eq. (18), the expected value of the uncertain importance of $EC_1$ can be rewritten as

$$E[Y_1] = \int_0^{1} \left[ \sum_{i=1}^{8} \Phi_i^{-1}(r) \Psi_{1i}^{-1}(r) \right] dr$$

$$= \int_0^{1} \left[ \Phi_1^{-1}(r)\Psi_{11}^{-1}(r) + \Phi_2^{-1}(r)\Psi_{21}^{-1}(r) + \cdots + \Phi_8^{-1}(r)\Psi_{81}^{-1}(r) \right] dr. \quad (23)$$

Besides, $W_1U_{11}, W_2U_{21}, \ldots, W_8U_{81}$ are independent of each other, then according to Eq. (19), the expected value of $Y_1$ can be calculated as follows,

$$E[Y_1] = \sum_{i=1}^{8} E[W_i U_{1i}]$$

$$= \sum_{i=1}^{8} \int_0^{1} \left[ \Phi_i^{-1}(r)\Psi_{1i}^{-1}(r) \right] dr$$

$$= \int_0^{1} \Phi_1^{-1}(r)\Psi_{11}^{-1}(r) dr + \int_0^{1} \Phi_2^{-1}(r)\Psi_{21}^{-1}(r) dr + \cdots + \int_0^{1} \Phi_8^{-1}(r)\Psi_{81}^{-1}(r) dr. \quad (24)$$

Afterwards, in line with Eq. (20), the parameters corresponding to the linear uncertainty distributions $\Phi_i(r)$ and $\Psi_{1i}(r), i = 1, 2, \ldots, 8$, as shown in Table 3, can be brought into the above Eq. (22). Then we can obtain

$$E[Y_1] = \int_0^{1} \Phi_1^{-1}(r)\Psi_{11}^{-1}(r) dr + \int_0^{1} \Phi_2^{-1}(r)\Psi_{21}^{-1}(r) dr + \cdots + \int_0^{1} \Phi_8^{-1}(r)\Psi_{81}^{-1}(r) dr$$

$$= \int_0^{1} [(1 - r) \times 0.81 + r \times 0.94][(1 - r) \times 0.425 + r \times 0.65] dr$$

$$+ \int_0^{1} [(1 - r) \times 0.5 + r \times 0.65][(1 - r) \times 0.17 + r \times 0.375] dr$$

$$+ \cdots$$

$$+ \int_0^{1} [(1 - r) \times 0.73 + r \times 0.87][(1 - r) \times 0.32 + r \times 0.535] dr$$

$$= 3.043. \quad (25)$$

Following the similar calculation of the importance of $EC_1$, the expected values of the uncertain importance of $EC_2, EC_3, \ldots, EC_{10}$ can also be obtained. The results are summarized in the first line of Table 4.
Table 4: The uncertain expected values and the ordinal ranking results of the importance of ten ECs

<table>
<thead>
<tr>
<th>EC</th>
<th>EC1</th>
<th>EC2</th>
<th>EC3</th>
<th>EC4</th>
<th>EC5</th>
<th>EC6</th>
<th>EC7</th>
<th>EC8</th>
<th>EC9</th>
<th>EC10</th>
</tr>
</thead>
<tbody>
<tr>
<td>E[Y_j]</td>
<td>3.043</td>
<td>2.858</td>
<td>3.528</td>
<td>2.912</td>
<td>1.962</td>
<td>2.409</td>
<td>3.452</td>
<td>2.686</td>
<td>2.324</td>
<td>2.010</td>
</tr>
<tr>
<td>Y'_j</td>
<td>0.112</td>
<td>0.105</td>
<td>0.130</td>
<td>0.107</td>
<td>0.072</td>
<td>0.089</td>
<td>0.127</td>
<td>0.099</td>
<td>0.086</td>
<td>0.074</td>
</tr>
<tr>
<td>Ranking</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>4</td>
<td>10</td>
<td>7</td>
<td>2</td>
<td>6</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

In the meantime, for purpose of ranking ECs, we normalize $E[Y_j], j = 1, 2, \ldots, 10$, according to Eq. (26) to get the relative importance of ECs. For example, the normalized expected value of $Y_1$, i.e., $Y'_1$, can be calculated by


Likewise, similar calculations for $Y'_2, Y'_3, \ldots, Y'_{10}$ can be done and the results are listed in the second line of Table 4. Then the relative importance of ECs are sorted in accordance with the values of $Y'_j$. The larger the value of $Y'_j$ is, the higher ranking EC $j$ can get. Therefore, it can be seen that the rankings of the importance of ECs based on the proposed approach and the stated criterion can be obtained as follows:

$$EC_3 \succ EC_7 \succ EC_1 \succ EC_4 \succ EC_2 \succ EC_8 \succ EC_6 \succ EC_9 \succ EC_{10} \succ EC_5,$$

where $\succ$ means “is more preferred than”. The third line of Table 4 shows the above rankings of ten ECs.

For an EC, owning a large value of importance or ranking indicates that the company should put more energy and efforts on it in order to ensure the corresponding product development and improvement successful, as well as the customers satisfied. By contrast, if the importance or ranking of an EC is not so high, the company should prefer to disregard or eliminate it and then try to redefine a new effective EC rather than keep much concern on it. In our case, we can see that “tool monitoring system” (EC3), “programmable logic controller” (EC7) and “automatic gauging” (EC1) have greater priorities than the other ECs, which means that the product designers should pay more attention on achieving and promoting these three engineering qualities. On the other hand, as “automated guided vehicle” (EC5) and “robots” (EC10) are given the lowest rankings, the product designers should consider removing them timely, and it is necessary to discover new ECs to replace them.

In the end, we can draw a conclusion that the proposed uncertain expected value operator method has a good performance in reflecting the imprecise human language information to the uncertain environments. This method can not only achieve reasonable and useful results, but also benefit for supporting the continuous product quality improvement in the QFD process practically. Once the importance of ECs and their rankings are determined, they would make the technical staffs of the enterprise more clear and convenient to satisfy and even track the customer requirements in designing or improving a product.

5 Conclusions

In this paper, a novel approach to ascertain the importance of ECs in uncertain environments was proposed. To sum up, our contributions to the related research area mainly lies in the following two aspects: (i) In the
proposed QFD problem, the uncertainty theory was applied to account for the imprecision or vagueness of human languages. The linguistic inputs including the relative importance of CRs and relationships between CRs and ECs in the QFD process were represented by uncertain variables, as well as the importance of ECs; (ii) We introduced an uncertain expected value operator method for determining the importance of ECs and their rankings, the performance of which was well certified by a practical product development example. This method could provide effective supports for both of the quality assurance in product development and the continuous quality progress improvement in product redesign.

In the future research, the correlations among ECs, the benchmarking information compared to competitors or other types of uncertain variables can be taken into consideration to enrich our study in the area of determination of the importance of ECs and its applications.

Acknowledgments

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References


