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Hierarchical Facility Location for the Reverse Logistics Network Design under Uncertainty

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Abstract

One of the most important concerns of reverse logistics network design is to locate interacting facilities in an efficient and cost effective manner, which forms a typical hierarchical system with multiple layer configuration. Considering the hierarchical relationship and flow of waste among the different facility types, both single-flow pattern and multi-flow pattern are discussed in this paper. In order to model the hierarchical facility location problem in an uncertain environment, two types of uncertain programming models, uncertain expected cost minimization model and uncertain α-cost minimization model, are proposed according to different decision criteria. It is shown that these models can be transformed into their deterministic counterparts and then be solved efficiently. Numerical examples are presented for illustration. Moreover, the optimal locations for the reverse logistics network with different flow patterns are compared as well.

Keywords: hierarchical facility, reverse logistics, location, network design

1 Introduction

Reverse logistics is the process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal [21]. It is receiving increased attention as rampant solid waste pollution, frequent energy shortages, and serious materials scarcity are recognized as realities of our modern age [9].

In order to efficiently and cost effectively transport used or defective products from the end user back to the producer or remove environmentally hazardous products from the hands of customers, the reverse logistics network should be well designed, which usually is a typical hierarchical system with multiple layer configuration consisting of interacting facilities [8, 9] . For example, facilities in a typical solid waste disposal system [2], one kind of reverse logistics network, may include collection sites, transfer stations and disposal centers. Due to the size effect in transportation, the transportation costs between higher-level facilities are usually less than those between lower-level facilities. Therefore, locations of the interacting facilities make important effects on the logistics cost and become one of the most important concerns of the reverse logistics network design. For instance, in order to tackle the recycling of construction waste and in particular of sand, Barros et al. [1] proposed a two-level location model and considered its optimization using linear relaxation and heuristics. For comprehensive reviews of the reverse logistics models and the general hierarchical facility location models, the readers may refer to [4] and [20], respectively.

As being reviewed in [20], most of the works on hierarchical facility location problems deal with deterministic cases. However, the real-world situations are much more complicated, and the problem should be considered in an uncertain environment rather than deterministic in a vast range of situations. Particularly for the reverse logistics network design, due to the changing environment and that the network does not actually exist yet when designing it, we often lack observed data, and some parameters of the problem may not be precisely predicted or estimated. In this case, some researchers believed that these nondeterministic phenomena conform to randomness or fuzziness, and hence the probability theory or the fuzzy set theory was

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introduced into the location problem. For example, Zhou and Liu [26] presented a new stochastic models for capacitated location-allocation problem. Wang et al. [23] proposed two decision-making approaches to the multi-objective location problem in a fuzzy environment. Fu [5] designed a reverse logistics network for rubbish recycling in a fuzzy environment.

However, it has been shown that it is inappropriate to describe the nondeterministic phenomena as randomness or fuzziness in many scenarios, particularly those involving the linguistic ambiguity and subjective estimation, since both the probability theory and the fuzzy set theory may lead to counterintuitive results [11]. In the reverse logistics network design problem, before the network being built, no samples are available to estimate the probability distributions of some parameters with respect to the performance of the network. Consequently, we have to invite some domain experts to evaluate the belief degree about the unknown state of nature. The belief degrees evaluated by some domain experts may have much bigger variance than the real frequency. In this case, the probability theory or the fuzzy set theory is no longer suitable, whereas uncertainty theory proposed by Liu [11] provides an alternative appropriate framework to deal with it. Based on uncertainty theory, Zhou et al. [27] proposed a multi-objective goal programming model to formulate the fire station location problem under uncertainty. Gao [6] proposed uncertain models for single facility location problems on networks.

In this paper, we consider the hierarchical facility location problem for reverse logistics network design in an uncertainty environment. Two types of uncertain programming models, uncertain expected cost minimization model and uncertain α-cost minimization model, are proposed according to different decision criteria. Moreover, considering the hierarchical relationship and flow of waste among the different facility types, both single-flow pattern and multi-flow pattern are discussed. Their corresponding optimal locations for the reverse logistics network are compared as well.

The rest of this paper is organized as follows. Section 2 introduces some basic concepts in the uncertainty theory. The description and formulation of the general hierarchical facility location problem for reverse logistics network design are given in Section 3, and then uncertain hierarchical location models are proposed in Sections 4. In Section 5, we transfer the uncertain models into their crisp equivalent models. Numerical examples and the comparisons of the optimal locations with different flow patterns are presented in Section 6.

2 Preliminary

Uncertainty theory, founded by Liu [11, 16], is an efficient tool to deal with nondeterministic information, especially expert data and subjective estimations. By now, it has been applied to many areas, and brought many branches such as uncertain programming [13, 19, 28], uncertain statistics [3, 22], uncertain logic [10, 18], uncertain inference [7, 17], and uncertain process [12, 15].

In this section, we introduce some fundamental concepts and properties of the uncertainty theory, which will be used throughout this paper.

Definition 1 ([11]) *Let $\mathcal{L}$ be a σ-algebra on a nonempty set Γ. A set function $\mathcal{M} : \mathcal{L} \to [0,1]$ is called an uncertain measure if it satisfies the following axioms:*
Axiom 1. *(Normality Axiom) $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ;*
Axiom 2. *(Duality Axiom) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ;*
Axiom 3. *(Subadditivity Axiom) For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots$, we have*

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \le \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}. \tag{1}$$

Besides, the triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space. Moreover, let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \ldots$. Denote

$$\Gamma = \Gamma_1 \times \Gamma_2 \times \cdots, \quad \mathcal{L} = \mathcal{L}_1 \times \mathcal{L}_2 \times \cdots. \tag{2}$$

Then the product uncertain measure $\mathcal{M}$ on the product σ-algebra $\mathcal{L}$ is defined by the following axiom [14].

Axiom 4. (Product Axiom) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \ldots$. The product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\} \tag{3}$$

where Λ_k are arbitrarily chosen events from $\mathcal{L}_k$ for $k = 1, 2, \ldots$, respectively.

Definition 2 ([11]) *An uncertain variable is a measurable function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers, i.e., for any Borel set B of real numbers, the set*

$$\{\xi \in B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \in B\} \tag{4}$$

is an event.

Definition 3 ([11] *Let ξ be an uncertain variable. Its uncertainty distribution is defined by*

$$\Phi(x) = \mathcal{M}\{\xi \leq x\} \tag{5}$$

for any real number x.

For example, an uncertain variable ξ is called *linear* if it has a linear uncertainty distribution (see Figure. 1)

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leq a \\ (x-a)/(b-a), & \text{if } a \leq x \leq b \\ 1, & \text{if } x \geq b, \end{cases} \tag{6}$$

denoted by $\mathcal{L}(a, b)$, where a and b are real numbers with $a < b$.

An uncertain variable ξ is called *zigzag* if it has a zigzag uncertainty distribution (see Figure 2)

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leq a \\ (x-a)/2(b-a), & \text{if } a \leq x \leq b \\ (x+c-2b)/2(c-b), & \text{if } b \leq x \leq c \\ 1, & \text{if } x \geq c, \end{cases} \tag{7}$$

denoted by $\mathcal{Z}(a, b, c)$, where a, b and c are real numbers with $a < b < c$.

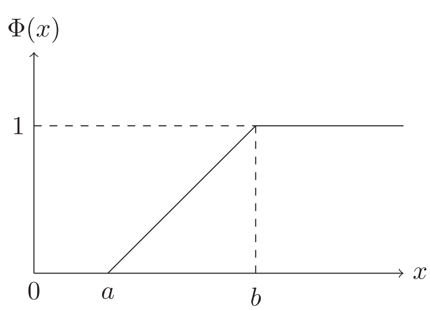

Figure 1: Linear uncertainty distribution

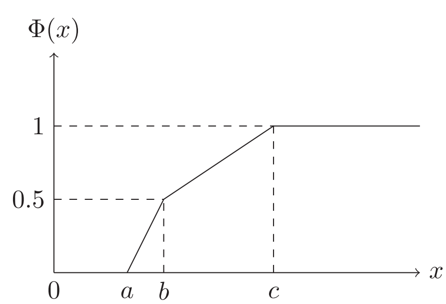

Figure 2: Zigzag uncertainty distribution

An uncertainty distribution Φ is said to be *regular* if its inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0, 1)$. It is clear that the linear and zigzag uncertainty distributions are both regular. The inverse uncertainty distribution of a linear uncertain variable $\xi \sim \mathcal{L}(a, b)$ is

$$\Phi^{-1}(\alpha) = (1-\alpha)a + \alpha b, \tag{8}$$

while the inverse uncertainty distribution of a zigzag uncertain variable $\xi \sim \mathcal{Z}(a,b,c)$ is

$$\Phi^{-1}(\alpha) = \begin{cases} a + 2(b-a)\alpha, & \text{if } \alpha \le 0.5 \\ 2b - c + 2(c-b)\alpha, & \text{if } \alpha \ge 0.5. \end{cases} \tag{9}$$

Definition 4 ([14]) *The uncertain variables ξ_1, ξ_2, $\ldots$, ξ_n are said to be independent if*

$$\mathcal{M}\left\{\bigcap_{i=1}^{n}\{\xi_i \in B_i\}\right\} = \bigwedge_{i=1}^{n} \mathcal{M}\{\xi_i \in B_i\} \tag{10}$$

for any Borel sets $B_1, B_2, \ldots, B_n$ of real numbers.

Theorem 1 ([16]) *Let ξ_1, ξ_2, $\ldots$, ξ_n be independent uncertain variables with regular uncertainty distributions Φ_1, Φ_2, $\ldots$, Φ_n, respectively, and $f : \Re^n \to \Re$ a continuous and strictly increasing function. Then the uncertain variable $\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$ has an inverse uncertainty distribution*

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \cdots, \Phi_n^{-1}(\alpha)). \tag{11}$$

Definition 5 ([11]) *Let ξ be an uncertain variable. Then the expected value of ξ is defined by*

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \ge r\}\mathrm{d}r - \int_{-\infty}^{0} \mathcal{M}\{\xi \le r\}\mathrm{d}r \tag{12}$$

provided that at least one of the two integrals is finite.

The expected value of a linear uncertain variable $\xi \sim \mathcal{L}(a,b)$ is

$$E[\xi] = \frac{a+b}{2}, \tag{13}$$

while the zigzag uncertain variable $\xi \sim \mathcal{Z}(a,b,c)$ has an expected value

$$E[\xi] = \frac{a+2b+c}{4}. \tag{14}$$

Theorem 2 ([16]) *Let ξ and η be independent uncertain variables with finite expected values. Then for any real numbers a and b, we have*

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta]. \tag{15}$$

3 Problem Description and Formulation

The problem is to design a reverse logistics network for a city to recycle waste generated by residents in the urban area. Now, we consider a reverse logistics network for waste recycling composed of 3 levels of hierarchical facilities, including collection sites, transfer stations and disposal centers. The lowest level of facilities (i.e., collection sites) is denoted as level 1 whereas the highest level (i.e., disposal centers) is denoted as level 3. The residential points, i.e., waste generation points, are assigned to level 0. The hierarchical structure of this reverse logistics network is shown in Figure 3.

Waste is collected from the residential points and carried to collection sites or transfer stations. Then, the waste in collection sites is shipped to transfer stations by trucks. Finally, it is transported to disposal centers by larger trucks. Due to the size effect in transportation, the transportation costs between the higher-level facilities are usually less than those between lower-level facilities. Therefore, the location of different types of facilities makes important effects on the total logistics cost.

In this problem, the locations of residential points and waste disposal centers are fixed. The objective is to locate the collection sites and transfer stations in the most effective way that makes the total logistics cost minimized.

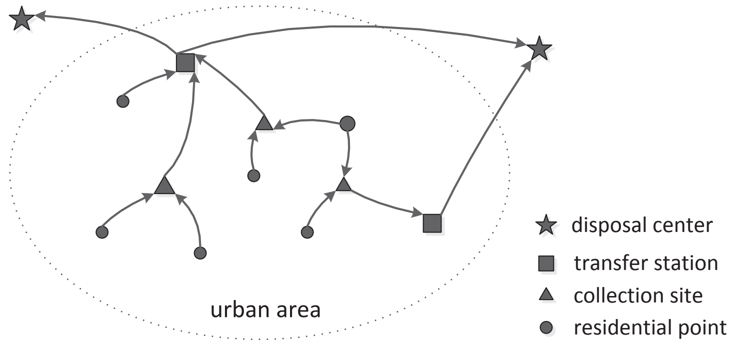

Figure 3: The hierarchical structure of a typical reverse logistics network

In order to formulate mathematical models for this problem, we introduce some indices, parameters, and decision variables as follows:

– i: the index of residential points, and denote by I the set of all points;
– j: the index of candidate collection sites, and denote by J the set of all sites;
– k: the index of candidate transfer stations, and denote by K the set of all stations;
– l: the index of disposal centers, and denote by L the set of all centers;
– D_i: the amount of waste generated by residential point i;
– M_j^1: the capacity of candidate collection site j;
– M_k^2: the capacity of candidate transfer station k;
– M_l^3: the capacity of disposal center l;
– F_j^1: the fixed cost of opening collection site j;
– F_k^2: the fixed cost of opening transfer station k;
– d_{ij}^1: the distance between residential point i and collection site j;
– d_{jk}^2: the distance between collection site j and transfer station k;
– d_{kl}^3: the distance between transfer station k and disposal center l;
– d_{ik}^4: the distance between residential point i and transfer station k;
– c_{ij}^1: the cost of unit flow from residential point i to collection site j;
– c_{jk}^2: the cost of unit flow from collection site j to transfer station$k$;
– c_{kl}^3: the cost of unit flow from transfer station k to disposal center l;
– c_{ik}^4: the cost of unit flow from residential point i to transfer station k;

Decision variables:

– x_{ij}^1: the flow amount from residential point i to collection site j;
– x_{jk}^2: the flow amount from collection site j to transfer station$k$;
– x_{kl}^3: the flow amount from transfer station k to disposal center l;
– x_{ik}^4: the flow amount from residential point i to transfer station k;
– Y_j^1: binary variable, and $Y_j^1 = 1$ if collection site j is open, otherwise $Y_j^1 = 0$;
– Y_k^2: binary variable, and $Y_k^2 = 1$ if transfer station k is open, otherwise $Y_k^2 = 0$.

Considering the hierarchical relationship and flow of waste among the different facility types, we discuss the single-flow pattern and the multi-flow pattern respectively in the following sections.

3.1 Single-Flow Pattern

Flow pattern describes the feature of how the waste flows through different levels of facilities in the hierarchical network [20]. In the single-flow pattern, the flow starts from level 0, passes through all levels, and ends at the highest level. In other words, the waste is collected from the residential points and carried to the collection sites, and then it is transported to the transfer stations and the disposal centers, sequentially. The flow between facilities out of this sequence is not allowed. Figure 4 illustrates the single-flow pattern of the reverse logistics network, and some indices, parameters, and decision variables related to this pattern are labeled in the figure as well.

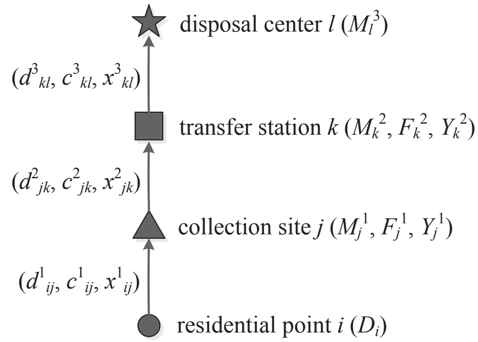

Figure 4: Single-flow pattern

The total logistics cost of this pattern consists of the transportation cost from residential points to collection sites, from collection sites to transfer stations, and from transfer stations to disposal centers, and the fixed cost of opening the collection sites and transfer stations. Denote by C_{SF} the total logistics cost of the single-flow pattern. It can be formulated as follows:

$$C_{\mathrm{SF}} = \sum_{i\in I}\sum_{j\in J} c^1_{ij} d^1_{ij} x^1_{ij} + \sum_{j\in J}\sum_{k\in K} c^2_{jk} d^2_{jk} x^2_{jk} + \sum_{k\in K}\sum_{l\in L} c^3_{kl} d^3_{kl} x^3_{kl} + \sum_{j\in J} F^1_j Y^1_j + \sum_{k\in K} F^2_k Y^2_k. \tag{16}$$

Then, in order to decide the location of hierarchical facilities with minimal total logistics cost, we have the following single-flow model for this problem,

$$\left\{\begin{array}{lll}
\min\ C_{\mathrm{SF}} & & \\
\text{subject to:} & & \\
\quad \sum_{i\in I} x^1_{ij} = \sum_{k\in K} x^2_{jk}, & \forall j \in J & (17.1)\\
\quad \sum_{j\in J} x^2_{jk} = \sum_{l\in L} x^3_{kl}, & \forall k \in K & (17.2)\\
\quad \sum_{j\in J} x^1_{ij} \geq D_i, & \forall i \in I & (17.3)\\
\quad \sum_{k\in K} x^2_{jk} \leq M^1_j Y^1_j, & \forall j \in J & (17.4)\\
\quad \sum_{l\in L} x^3_{kl} \leq M^2_k Y^2_k, & \forall k \in K & (17.5)\\
\quad \sum_{k\in K} x^3_{kl} \leq M^3_l, & \forall l \in L & (17.6)\\
\quad x^1_{ij}, x^2_{jk}, x^3_{kl} \geq 0, & \forall i \in I, \forall j \in J, \forall k \in K, \forall l \in L & (17.7)\\
\quad Y^1_j, Y^2_k \in \{0,1\}, & \forall j \in J, \forall k \in K. & (17.8)
\end{array}\right. \tag{17}$$

Constraints (17.1) and (17.2) ensure that the total amount of waste flowed into a facility is equivalent to that flowed out, for each collection site and transfer station, respectively. Constraint (17.3) ensures that the waste generated by each residential point can be collected by collection sites. Constraints (17.4), (17.5) and (17.6) are the facility capacity constraints for each collection site, transfer station and disposal center, respectively. Constraint (17.7) is the non-negativity for each flow variable, and (17.8) is the constraint for binary variables.

3.2 Multi-Flow Pattern

In the multi-flow pattern of this reverse logistics network, waste generated by the residential points is carried to collection sites or transfer stations. In other words, it is allowed to ship the waste to transfer stations directly and not passing through the collection sites. Figure 5 illustrates the multi-flow pattern, and the indices, parameters, and decision variables related to this pattern are also labeled in the figure. It is clear that the single-flow pattern is more restricted and simple than the multi-flow pattern, and it can be treated as a special case of the multi-flow pattern.

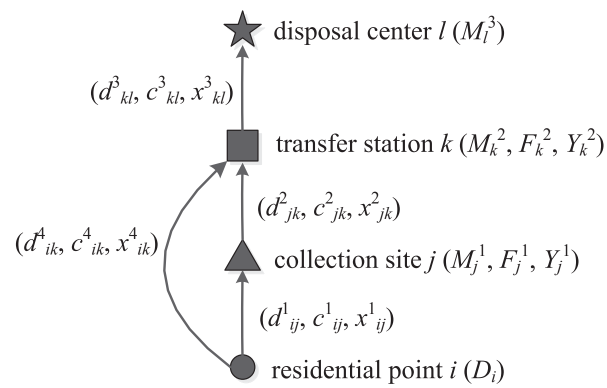

Figure 5: Multi-flow pattern

Denote by $C_{\rm MF}$ the total logistics cost of this multi-flow pattern. It can be formulated as follows:

$$\begin{aligned} C_{\rm MF} = & \sum_{i\in I}\sum_{j\in J} c_{ij}^1 d_{ij}^1 x_{ij}^1 + \sum_{j\in J}\sum_{k\in K} c_{jk}^2 d_{jk}^2 x_{jk}^2 + \sum_{k\in K}\sum_{l\in L} c_{kl}^3 d_{kl}^3 x_{kl}^3 + \sum_{i\in I}\sum_{k\in K} c_{ik}^4 d_{ik}^4 x_{ik}^4 \\ & + \sum_{j\in J} F_j^1 Y_j^1 + \sum_{k\in K} F_k^2 Y_k^2 \end{aligned} \tag{18}$$

Then we have the following model for the hierarchical facility location problem with multi-flow pattern,

$$\left\{ \begin{array}{lll} \min\ C_{\rm MF} & & \\ \text{subject to:} & & \\ \qquad (17.1), (17.4) - (17.8) & & \\ \qquad \sum\limits_{j\in J} x_{jk}^2 + \sum\limits_{i\in I} x_{ik}^4 = \sum\limits_{l\in L} x_{kl}^3, & \forall k \in K & (19.1) \\ \qquad \sum\limits_{j\in J} x_{ij}^1 + \sum\limits_{k\in K} x_{ik}^4 \geq D_i, & \forall i \in I & (19.2) \\ \qquad x_{ik}^4 \geq 0, & \forall i \in I, \forall k \in K. & (19.3) \end{array} \right. \tag{19}$$

Constraint (19.1) ensures that the total amount of waste flowed into each transfer station from the collection sites and the residential points is equivalent to that flowed out to disposal centers. Since multi-flow is considered, constraint (19.2) ensures that the waste generated by each residential point can be serviced by collection sites or transfer stations. Constraint (19.3) is the non-negativity for the flows from residential points to transfer stations.

4 Uncertain Hierarchical Location Models

Models (17) and (19) formulate the general case of the hierarchical facility location problem for reverse logistics network design with single-flow pattern and multi-flow pattern, respectively. Now, we consider the problem in an uncertain environment.

Due to the changing environment and that the network does not actually exist yet, we often lack observed data, and some parameters of the problem may not be precisely predicted or estimated when designing the network. In this case, as mentioned in the section of introduction, we have to invite some domain experts to evaluate the belief degree about the unknown state, which makes the probability theory or the fuzzy set theory no longer appropriate to model the problem, whereas the uncertainty theory provides an alternative appropriate framework to deal with it.

Therefore, we assume all the cost-related parameters (i.e., c^1_{ij}, c^2_{jk}, c^3_{kl}, , c^4_{ik}, F^1_j and F^2_k) and the amount of waste generated by residential point (i.e., D_i) to be uncertain variables in this problem. Now, it is clear that models (17) and (19) are not well-defined since the objectives and constraints involving uncertain variables are not clear. In the following, we will present a framework of uncertain programming, which was first proposed by Liu [13], for the hierarchical facility location problem by providing two types of uncertain programming models according to different criteria.

4.1 Uncertain Expected Cost Minimization Model

Since the cost-related parameters and D_i are uncertain variables, the total logistics costs C_{SF} and C_{MF} also become uncertain variables. Then, a natural decision criterion for this problem is to minimize the expected total logistics cost, i.e., $E[C_{\mathrm{SF}}]$ and $E[C_{\mathrm{MF}}]$.

As for the constraints involving uncertain variables, since they do not define crisp constraints, we hope these constraints hold at least at some confidence levels following from the idea of chance constraint. With a given confidence level α, we have the following chance constraints,

$$\mathcal{M}\left\{\sum_{j\in J} x^1_{ij} \geq D_i\right\} \geq \alpha \tag{20}$$

and

$$\mathcal{M}\left\{\sum_{j\in J} x^1_{ij} + \sum_{k\in K} x^4_{ik} \geq D_i\right\} \geq \alpha \tag{21}$$

with respect to constraints (17.3) and (19.2), respectively.

Consequently, for the hierarchical facility location problem with single-flow pattern, in order to obtain a decision minimizing the expected total logistics cost subject to a set of chance constraints with a confidence level α, we have the following uncertain expected cost minimization model,

$$\begin{cases} \min\ E[C_{\mathrm{SF}}] \\ \text{subject to:} \\ \qquad (17.1), (17.2), (17.4) - (17.8) \\ \qquad \mathcal{M}\left\{\sum_{j\in J} x^1_{ij} \geq D_i\right\} \geq \alpha, \qquad \forall i \in I. \end{cases} \tag{22}$$

Similarly, for the problem with multi-flow pattern, we obtain the uncertain expected cost minimization model as follows,

$$\begin{cases} \min\ E\left[C_{\mathrm{MF}}\right] \\ \text{subject to:} \\ \qquad (17.1), (17.4) - (17.8), (19.1), (19.3) \\ \qquad \mathcal{M}\left\{\sum_{j\in J} x^1_{ij} + \sum_{k\in K} x^4_{ik} \geq D_i\right\} \geq \alpha, \qquad \forall i \in I. \end{cases} \tag{23}$$

4.2 Uncertain α-cost Minimization Model

The second decision criterion for this problem in an uncertain environment is to minimize the so-called α-cost, which is also extensively applied to solve the practical optimization problems with uncertain variables (e.g., [24, 25]).

Definition 6 *The α-costs of an uncertain hierarchical facility location problem with single-flow pattern and multi-flow pattern are defined as*

$$C_{\mathrm{SF}}(\alpha) = \min\left\{C^* \mid \mathcal{M}\left\{C_{\mathrm{SF}} \le C^*\right\} \ge \alpha\right\} \tag{24}$$

and

$$C_{\mathrm{MF}}(\alpha) = \min\left\{C^* \mid \mathcal{M}\left\{C_{\mathrm{MF}} \le C^*\right\} \ge \alpha\right\} \tag{25}$$

respectively, where α is the predetermined confidence level.

Following from the concept of α-cost, in order to obtain a decision minimizing the α-cost subject to a set of chance constraints with confidence level α, we have the following uncertain α-costs minimization models,

$$\begin{cases} \min\ C_{\mathrm{SF}}(\alpha) \\ \text{subject to:} \\ \qquad (17.1), (17.2), (17.4) - (17.8) \\ \qquad \mathcal{M}\left\{\displaystyle\sum_{j\in J} x^1_{ij} \ge D_i\right\} \ge \alpha, \qquad \forall i \in I, \end{cases} \tag{26}$$

and

$$\begin{cases} \min\ C_{\mathrm{MF}}(\alpha) \\ \text{subject to:} \\ \qquad (17.1), (17.4) - (17.8), (19.1), (19.3) \\ \qquad \mathcal{M}\left\{\displaystyle\sum_{j\in J} x^1_{ij} + \sum_{k\in K} x^4_{ik} \ge D_i\right\} \ge \alpha, \qquad \forall i \in I, \end{cases} \tag{27}$$

respectively for the hierarchical facility location problem with single-flow pattern and multi-flow pattern.

5 Crisp Equivalent Models

In this section, the uncertain hierarchical location models proposed above are transformed to crisp equivalent models based on the properties of the uncertainty theory. It is shown that the hierarchical facility location problem with uncertain variables can be handled eventually within the framework of deterministic linear programming and requires no particular solving methods.

5.1 Expected Cost Minimization Model

For the expected total logistics cost, it is easy to get the following conclusion,

Theorem 3 *Suppose that all the cost-related parameters c^1_{ij}, c^2_{jk}, c^3_{kl}, c^4_{ik}, F^1_j and F^2_k are independent uncertain variables. Then the expected total logistics cost of the hierarchical facility location problem with single-flow pattern and multi-flow pattern are*

$$\begin{aligned} E[C_{\mathrm{SF}}] = &\sum_{i\in I}\sum_{j\in J} d^1_{ij}x^1_{ij}E\left(c^1_{ij}\right) + \sum_{j\in J}\sum_{k\in K} d^2_{jk}x^2_{jk}E\left(c^2_{jk}\right) + \sum_{k\in K}\sum_{l\in L} d^3_{kl}x^3_{kl}E\left(c^3_{kl}\right) \\ &+ \sum_{j\in J} Y^1_j E\left(F^1_j\right) + \sum_{k\in K} Y^2_k E\left(F^2_k\right) \end{aligned} \tag{28}$$

and

$$\begin{aligned} E[C_{\mathrm{MF}}] = &\sum_{i\in I}\sum_{j\in J} d^1_{ij}x^1_{ij}E\left(c^1_{ij}\right) + \sum_{j\in J}\sum_{k\in K} d^2_{jk}x^2_{jk}E\left(c^2_{jk}\right) + \sum_{k\in K}\sum_{l\in L} d^3_{kl}x^3_{kl}E\left(c^3_{kl}\right) \\ &+ \sum_{i\in I}\sum_{k\in K} d^4_{ik}x^4_{ik}E\left(c^4_{ik}\right) + \sum_{j\in J} Y^1_j E\left(F^1_j\right) + \sum_{k\in K} Y^2_k E\left(F^2_k\right), \end{aligned} \tag{29}$$

respectively.

Proof: It follows from Theorem 2 immediately.

□

For the chance constraints in the above uncertain hierarchical location models, we also have the following conclusion.

Theorem 4 *Suppose that the amount of waste generated by residential point, D_i $(i \in I)$, is an uncertain variable with regular distribution Φ_i. Then the chance constraint*

$$\mathcal{M}\left\{\sum_{j\in J} x_{ij}^1 \geq D_i\right\} \geq \alpha, \tag{30}$$

holds if and only if

$$\Phi_i^{-1}(\alpha) \leq \sum_{j\in J} x_{ij}^1, \tag{31}$$

where Φ_i^{-1} is the inverse uncertainty distribution of D_i.

Proof: It follows from definition of regular uncertainty distribution and inverse uncertainty distribution immediately.

□

Then, following from Theorems 3 and 4, we get that the uncertain expected cost minimization model (22) for the single-flow pattern is equivalent to the following deterministic model,

$$\begin{cases} \min \sum\limits_{i\in I}\sum\limits_{j\in J} d_{ij}^1 x_{ij}^1 E\left(c_{ij}^1\right) + \sum\limits_{j\in J}\sum\limits_{k\in K} d_{jk}^2 x_{jk}^2 E\left(c_{jk}^2\right) + \sum\limits_{k\in K}\sum\limits_{l\in L} d_{kl}^3 x_{kl}^3 E\left(c_{kl}^3\right) \\ \qquad + \sum\limits_{j\in J} Y_j^1 E\left(F_j^1\right) + \sum\limits_{k\in K} Y_k^2 E\left(F_k^2\right) \\ \text{subject to:} \\ \qquad (17.1), (17.2), (17.4) - (17.8) \\ \qquad \Phi_i^{-1}(\alpha) \leq \sum\limits_{j\in J} x_{ij}^1, \qquad \forall i \in I. \end{cases} \tag{32}$$

Similarly, we can get the equivalent deterministic model of (23) for the multi-flow pattern as follows,

$$\begin{cases} \min \sum\limits_{i\in I}\sum\limits_{j\in J} d_{ij}^1 x_{ij}^1 E\left(c_{ij}^1\right) + \sum\limits_{j\in J}\sum\limits_{k\in K} d_{jk}^2 x_{jk}^2 E\left(c_{jk}^2\right) + \sum\limits_{k\in K}\sum\limits_{l\in L} d_{kl}^3 x_{kl}^3 E\left(c_{kl}^3\right) \\ \qquad + \sum\limits_{i\in I}\sum\limits_{k\in K} d_{ik}^4 x_{ik}^4 E\left(c_{ik}^4\right) + \sum\limits_{j\in J} Y_j^1 E\left(F_j^1\right) + \sum\limits_{k\in K} Y_k^2 E\left(F_k^2\right) \\ \text{subject to:} \\ \qquad (17.1), (17.4) - (17.8), (19.1), (19.3) \\ \qquad \Phi_i^{-1}(\alpha) \leq \sum\limits_{j\in J} x_{ij}^1 + \sum\limits_{k\in K} x_{ik}^4, \qquad \forall i \in I. \end{cases} \tag{33}$$

It is easy to see that models (32) and (33) are deterministic linear programming models. Therefore, they can be solved effectively by using some well developed software packages, for example, LINGO and MATLAB.

5.2 α-cost Minimization Model

For the α-cost of an uncertain hierarchical facility location problem, we can get the following theorem based on the operational law of independent uncertain variables (see Theorem 1).

Theorem 5 *Suppose that all the cost-related parameters c_{ij}^1, c_{jk}^2, c_{kl}^3, c_{ik}^4, F_j^1 and F_j^2 are independent uncertain variables with regular distribution Ψ_{ij}^1, Ψ_{jk}^2, Ψ_{kl}^3, Ψ_{ik}^4, Υ_j^1 and Υ_k^2, respectively. Then for a given confidence level α, the α-cost of the hierarchical facility location problem with single-flow pattern and multi-flow pattern are*

$$\begin{aligned} C_{\mathrm{SF}}(\alpha) = &\sum_{i\in I}\sum_{j\in J} d_{ij}^1 x_{ij}^1 (\Psi_{ij}^1)^{-1}(\alpha) + \sum_{j\in J}\sum_{k\in K} d_{jk}^2 x_{jk}^2 (\Psi_{jk}^2)^{-1}(\alpha) + \sum_{k\in K}\sum_{l\in L} d_{kl}^3 x_{kl}^3 (\Psi_{kl}^3)^{-1}(\alpha) \\ &+ \sum_{j\in J} Y_j^1 (\Upsilon_j^1)^{-1}(\alpha) + \sum_{k\in K} Y_k^2 (\Upsilon_k^2)^{-1}(\alpha) \end{aligned} \tag{34}$$

and

$$\begin{aligned} C_{\mathrm{MF}}(\alpha) = &\sum_{i\in I}\sum_{j\in J} d_{ij}^1 x_{ij}^1 (\Psi_{ij}^1)^{-1}(\alpha) + \sum_{j\in J}\sum_{k\in K} d_{jk}^2 x_{jk}^2 (\Psi_{jk}^2)^{-1}(\alpha) + \sum_{k\in K}\sum_{l\in L} d_{kl}^3 x_{kl}^3 (\Psi_{kl}^3)^{-1}(\alpha) \\ &+ \sum_{i\in I}\sum_{k\in K} d_{ik}^4 x_{ik}^4 (\Psi_{ik}^4)^{-1}(\alpha) + \sum_{j\in J} Y_j^1 (\Upsilon_j^1)^{-1}(\alpha) + \sum_{k\in K} Y_k^2 (\Upsilon_k^2)^{-1}(\alpha), \end{aligned} \tag{35}$$

where $(\Psi_{ij}^1)^{-1}$, $(\Psi_{jk}^2)^{-1}$, $(\Psi_{kl}^3)^{-1}$, $(\Psi_{ik}^4)^{-1}$, $(\Upsilon_j^1)^{-1}$ and $(\Upsilon_k^2)^{-1}$ are the inverse uncertainty distributions of c_{ij}^1, c_{jk}^2, c_{kl}^3, c_{ik}^4, F_j^1 and F_j^2, respectively.

Proof: According to equation (16), C_{SF} is a strictly increasing function with respect to c_{ij}^1, c_{jk}^2, c_{kl}^3, F_j^1 and F_j^2. Following from Theorem 1, we have that C_{SF} is an uncertain variable with inverse uncertainty distribution

$$\begin{aligned} \Gamma_{\mathrm{SF}}^{-1}(\alpha) = &\sum_{i\in I}\sum_{j\in J} d_{ij}^1 x_{ij}^1 (\Psi_{ij}^1)^{-1}(\alpha) + \sum_{j\in J}\sum_{k\in K} d_{jk}^2 x_{jk}^2 (\Psi_{jk}^2)^{-1}(\alpha) + \sum_{k\in K}\sum_{l\in L} d_{kl}^3 x_{kl}^3 (\Psi_{kl}^3)^{-1}(\alpha) \\ &+ \sum_{j\in J} Y_j^1 (\Upsilon_j^1)^{-1}(\alpha) + \sum_{k\in K} Y_k^2 (\Upsilon_k^2)^{-1}(\alpha) \end{aligned} \tag{36}$$

where Γ_{SF} denotes the uncertainty distribution of C_{SF}.

From Definition 6, we get $C_{\mathrm{SF}}(\alpha) = \Gamma_{\mathrm{SF}}^{-1}(\alpha)$, so equation (34) is verified.

Similarly, equation (35) can be verified.

□

Following from Theorems 5 and 4, we get that the uncertain α-cost minimization model (26) for the single-flow pattern is equivalent to the following deterministic model,

$$\begin{cases} \min \sum_{i\in I}\sum_{j\in J} d_{ij}^1 x_{ij}^1 (\Psi_{ij}^1)^{-1}(\alpha) + \sum_{j\in J}\sum_{k\in K} d_{jk}^2 x_{jk}^2 (\Psi_{jk}^2)^{-1}(\alpha) + \sum_{k\in K}\sum_{l\in L} d_{kl}^3 x_{kl}^3 (\Psi_{kl}^3)^{-1}(\alpha) \\ \qquad + \sum_{j\in J} Y_j^1 (\Upsilon_j^1)^{-1}(\alpha) + \sum_{k\in K} Y_k^2 (\Upsilon_k^2)^{-1}(\alpha) \\ \text{subject to:} \\ \qquad (17.1), (17.2), (17.4) - (17.8) \\ \qquad \Phi_i^{-1}(\alpha) \le \sum_{j\in J} x_{ij}^1, \qquad \forall i \in I. \end{cases} \tag{37}$$

Similarly, we can get the equivalent deterministic model of (27) for the multi-flow pattern as follows,

$$\begin{cases} \min \sum_{i\in I}\sum_{j\in J} d_{ij}^1 x_{ij}^1 (\Psi_{ij}^1)^{-1}(\alpha) + \sum_{j\in J}\sum_{k\in K} d_{jk}^2 x_{jk}^2 (\Psi_{jk}^2)^{-1}(\alpha) + \sum_{k\in K}\sum_{l\in L} d_{kl}^3 x_{kl}^3 (\Psi_{kl}^3)^{-1}(\alpha) \\ \qquad + \sum_{i\in I}\sum_{k\in K} d_{ik}^4 x_{ik}^4 (\Psi_{ik}^4)^{-1}(\alpha) + \sum_{j\in J} Y_j^1 (\Upsilon_j^1)^{-1}(\alpha) + \sum_{k\in K} Y_k^2 (\Upsilon_k^2)^{-1}(\alpha) \\ \text{subject to:} \\ \qquad (17.1), (17.4) - (17.8), (19.1), (19.3) \\ \qquad \Phi_i^{-1}(\alpha) \le \sum_{j\in J} x_{ij}^1 + \sum_{k\in K} x_{ik}^4, \qquad \forall i \in I. \end{cases} \tag{38}$$

Models (37) and (38) are also deterministic linear programming models, and can be solved effectively by using some well developed software packages.

6 Numerical Examples

In this section, we will present some numerical examples to illustrate the proposed models. The results of locations for the reverse logistics network with different flow patterns are compared as well.

6.1 Example Description

We consider the hierarchical facility location problem with 15 residential points, 8 candidate collection sites, 4 candidate transfer stations and 2 disposal centers, whose locations are shown in Figure 6.

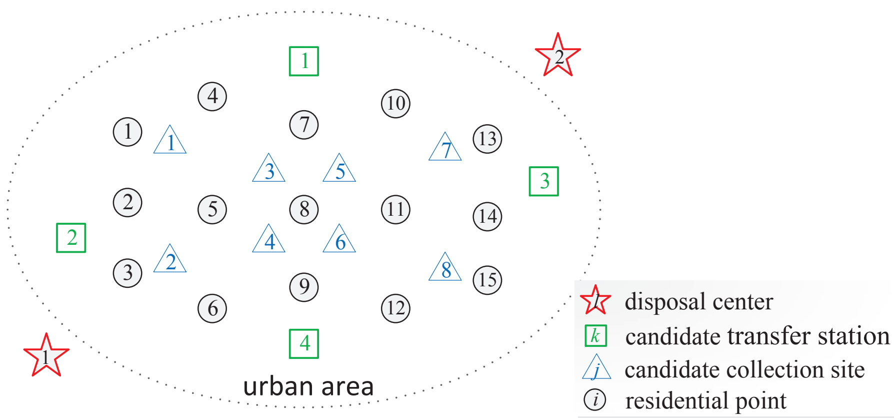

Figure 6: Locations of the candidate facilities for the numerical examples

The distances of all pairs of residential points and collection sites, residential points and transfer stations, collection sites and transfer stations, transfer stations and disposal centers are listed in Tables 1, 2, 3 and 4, respectively.

The transportation costs are all assumed to be linear uncertain variables, and the fixed costs of opening these facilities are zigzag uncertain variables. For simplicity, the cost-related parameters are set as follows: $\Psi^1_{ij} \sim \mathcal{L}(5,8)$, $\Psi^2_{jk} \sim \mathcal{L}(2,4)$, $\Psi^3_{kl} \sim \mathcal{L}(1,3)$, $\Psi^4_{ik} \sim \mathcal{L}(5,8)$, $\Upsilon^1_j \sim \mathcal{Z}(220,235,260)$, $\Upsilon^2_k \sim \mathcal{Z}(950,970,1020)$, for all $1 \le i \le 15$, $1 \le j \le 8$, $1 \le k \le 4$, $1 \le l \le 2$.

The amount of waste generated by residential point, D_i, is a linear uncertain variable with uncertainty distribution $\Phi_i \sim \mathcal{L}(10,20)$, for all $1 \le i \le 15$. The capacity of each candidate collection site is 50. The capacity of each candidate transfer station is 120. The capacity of these two disposal centers are $M^3_1 = 150$ and $M^3_2 = 200$, respectively.

Table 1: The distances from residential point i to collection site j

$j\backslash i$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	5	10	22	10	11	30	25	27	35	38	39	42	42	45	50
2	22	10	5	30	11	10	35	27	25	42	39	38	50	45	42
3	15	15	25	12	8	17	6	5	15	16	14	20	25	25	30
4	25	15	15	17	8	12	15	5	6	20	14	16	30	25	25
5	25	25	30	12	14	20	6	5	15	12	8	17	15	15	25
6	30	25	25	20	14	12	15	5	6	17	8	12	25	15	15
7	42	45	50	38	39	42	25	27	35	10	11	30	5	10	22
8	50	45	42	42	39	38	35	27	25	30	11	10	22	10	5

6.2 Expected Cost Minimization

We first consider the decision criterion that minimize the expected total logistics cost subject to a set of chance constraints. The confidence level of the chance constraints is set to be 0.8.

Table 2: The distances from residential point i to transfer station k

$k\backslash i$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	30	40	50	15	35	50	7	22	35	15	35	50	30	40	50
2	20	8	8	30	22	24	45	35	35	50	45	45	60	55	55
3	55	55	60	45	45	50	35	35	45	24	22	30	8	8	20
4	50	40	30	50	35	15	35	22	7	50	35	15	50	40	30

Table 3: The distances from collection site j to transfer station k

$k\backslash j$	1	2	3	4	5	6	7	8
1	30	48	20	35	20	35	30	48
2	25	15	35	30	40	38	52	50
3	50	52	38	40	30	35	15	25
4	48	30	35	20	35	20	48	30

Table 4: The distances from transfer station k to disposal center l

$l\backslash k$	1	2	3	4
1	160	50	190	80
2	70	80	50	150

According to (13) and (14), we have $E(c^1_{ij}) = 6.5$, $E(c^2_{jk}) = 3$, $E(c^3_{kl}) = 2$, $E(c^4_{ik}) = 6.5$, $E(F^1_j) = 237.5$ and $E(F^2_j) = 977.5$. According to (8), we get $\Phi_i^{-1}(0.8) = 18$.

Taking these values into model (32), we can get a deterministic programming model for the hierarchical facility location problem as shown in Figure 6 with single-flow pattern as follows,

$$\left\{\begin{array}{ll}
\multicolumn{2}{l}{\min\ 6.5\sum_{i=1}^{15}\sum_{j=1}^{8} d^1_{ij}x^1_{ij} + 3\sum_{j=1}^{8}\sum_{k=1}^{4} d^2_{jk}x^2_{jk} + 2\sum_{k=1}^{4}\sum_{l=1}^{2} d^3_{kl}x^3_{kl} + 237.5\sum_{j=1}^{8} Y^1_j + 977.5\sum_{k=1}^{4} Y^2_k} \\
\multicolumn{2}{l}{\text{subject to:}} \\
\qquad \sum_{i=1}^{15} x^1_{ij} = \sum_{k=1}^{4} x^2_{jk}, & \forall 1 \le j \le 8 \\
\qquad \sum_{j=1}^{8} x^2_{jk} = \sum_{l=1}^{2} x^3_{kl}, & \forall 1 \le k \le 4 \\
\qquad \sum_{k=1}^{4} x^2_{jk} \le 50Y^1_j, & \forall 1 \le j \le 8 \\
\qquad \sum_{l=1}^{2} x^3_{kl} \le 120Y^2_k, & \forall 1 \le k \le 4 \\
\qquad \sum_{k=1}^{4} x^3_{kl} \le M^3_l, & \forall 1 \le l \le 2 \\
\qquad \sum_{j=1}^{8} x^1_{ij} \ge 18, & \forall 1 \le i \le 15 \\
\qquad x^1_{ij}, x^2_{jk}, x^3_{kl} \ge 0, & \forall 1 \le i \le 15, 1 \le j \le 8, \le k \le 4, 1 \le l \le 2 \\
\qquad Y^1_j, Y^2_k \in \{0, 1\}, & \forall 1 \le j \le 8, \le k \le 4.
\end{array}\right. \tag{39}$$

Solving this model by LINGO, we obtain that the optimal locations of collection sites and transfer stations are 1, 2, 4, 5, 7, 8, and 1, 2, 3, respectively. The corresponding expected total logistics cost is 63676.5.

Similarly, we can solve the problem with multi-flow pattern (which refers to model (33)), and get the results as follows. The optimal locations of collection sites become 1, 2, 4, 7, 8, and these of transfer stations are still 1, 2, 3. The corresponding expected total logistics cost is 58129.

6.3 α-cost Minimization

Now, we consider the decision criterion that minimize the α-cost of the hierarchial facility problem. The confidence level α is also set to be 0.8.

According to (8) and (9), we have $(\Psi^1_{ij})^{-1}(0.8) = 7.4$, $(\Psi^2_{jk})^{-1}(0.8) = 3.6$, $(\Psi^3_{kl})^{-1}(0.8) = 2.6$, $(\Psi^4_{ik})^{-1}(0.8) = 7.4$, $(\Upsilon^1_j)^{-1}(0.8) = 250$, and $(\Upsilon^2_k)^{-1}(0.8) = 1000$.

Similarly with solving the expected cost minimization problem, we can take the values of related parameters into models (37) and (38), and obtain the optimal locations of the facilities for single-flow pattern and multi-flow pattern, respectively.

For the single-flow pattern, the optimal locations of collection sites and transfer stations are 1, 2, 4, 5, 7, 8, and 1, 2, 3, respectively. The corresponding expected total logistics cost is 77684.4.

For the multi-flow pattern, the optimal locations of collection sites and transfer stations are 1, 2, 4, 7, 8, and 1, 2, 3, respectively. The corresponding expected total logistics cost is 70990.4.

6.4 Comparisons of These Two Flow Patterns

The results of the problem with the single-flow pattern and the multi-flow pattern under different decision criteria are shown in Table 5. It shows that since the single-flow pattern is more restricted than the multi-flow pattern, more facilities are located in the single-flow pattern, and the objective value of the network with the single-flow pattern is also always more than that of the multi-flow pattern.

Table 5: The results of the numerical examples

Decision criteria	Flow patterns	Collection sites	Transfer stations	Objective values
Expected cost minimization	Single-flow	1, 2, 4, 5, 7, 8	1, 2, 3	63676.5
	Multi-flow	1, 2, 4, 7, 8	1, 2, 3	58129.0
α-cost minimization	Single-flow	1, 2, 4, 5, 7, 8	1, 2, 3	77684.4
	Multi-flow	1, 2, 4, 7, 8	1, 2, 3	70990.4

The optimal locations for single-flow pattern and multi-flow pattern are shown in Figures 7 and 8, respectively. The flows flowing into higher-level facilities (transfer stations and disposal centers) are also shown in the figures. To make the illustrations more clearly, the flows between residential points and collection sites are omitted.

7 Conclusions

As a typical hierarchical facility location problems, it was concerned in this paper the problem of locating interacting facilities for the reverse logistics network design in an uncertain environment, and then two types of uncertain programming models were proposed to deal with such situations according to different decision criteria. Based on the uncertainty theory founded by Liu [11], they were transformed into crisp equivalent programming models and solved efficiently with the aid of some well developed optimization software packages. The methods presented in this paper illustrated alternative modeling techniques for the uncertain hierarchical facility location problems when there were nondeterministic factors that cannot be precisely determined or statistically estimated.

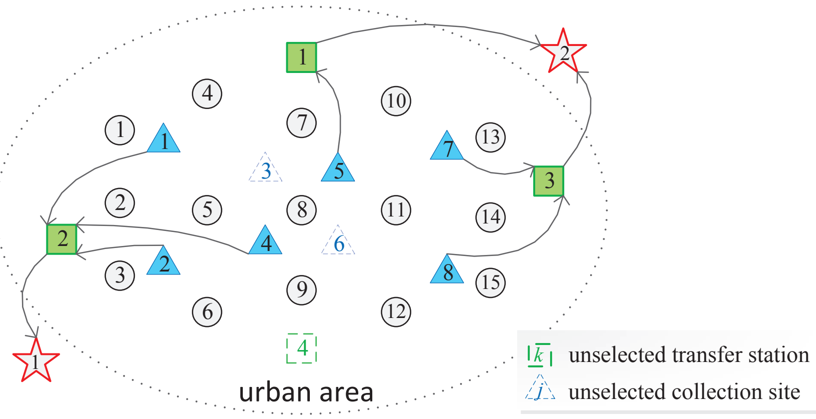

Figure 7: The optimal locations for single-flow pattern

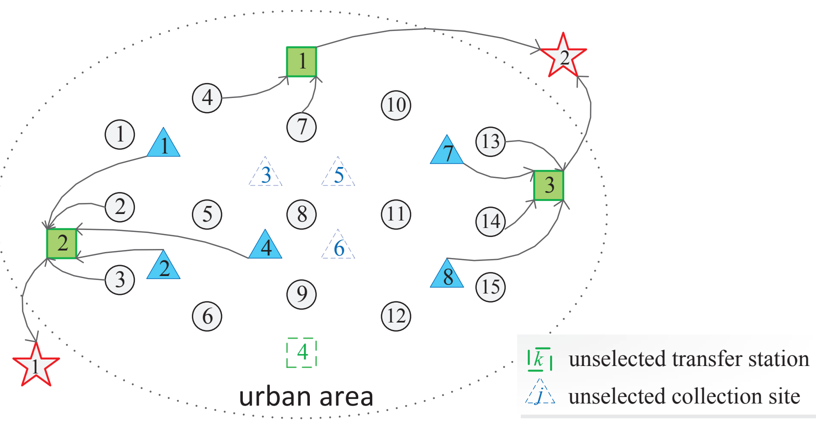

Figure 8: The optimal locations for multi-flow pattern

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