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Uncertain Spectrum of Temperatures in a Non Homogeneous Fin under Imprecisely Defined Conduction-Convection System

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Abstract

In this article we have presented a modified form of traditional fuzzy finite element method. Here the involved fuzzy numbers are changed into intervals through α -cut. Then the intervals are transformed into crisp form by using some transformation. Crisp representations of intervals are defined by symbolic parameterization. The traditional interval arithmetic is then modified using the crisp representation of intervals. Then the proposed interval arithmetic is extended for fuzzy numbers and this fuzzy arithmetic is used as a tool for uncertain finite element method. In general, the fuzzy finite element method converts the governing differential equation into fuzzy algebraic equations. Fuzzy algebraic equations either give a fuzzy eigenvalue problem or a fuzzy system of linear equations. The proposed method(s) have been used to solve distribution of temperatures in a conduction-convection system for a test problem. Here we have considered both Triangular Fuzzy Number (TFN) and Trapezoidal Fuzzy Number (TRFN) for uncertain parameters involved in the system. Finally the obtained results are compared and it has been seen that the proposed methods are reliable and may be applicable to other heat and mass transfer problems.

Keywords: Triangular Fuzzy Number, Trapezoidal Fuzzy Number, interval arithmetic, heat transfer coefficient, thermal conductivity, ambient temperature

1 Introduction

Heat transfer is a common phenomenon which may be found in various fields of engineering and science. This is actually a multi-dimensional conjugate problem, in which heat conduction takes place not only in the direction orthogonal to the walls (transverse conduction), but also parallel to them (longitudinal conduction). Here conjugate heat transfer refers to a heat transfer process involving an interaction of conduction within a solid body and the convection from the solid surface to fluid moving over the surface. Therefore, a realistic analysis of conjugate heat transfer problems necessitates the coupling of the conduction in the solid and the convection in the fluid. But in actual practice when we deal with conduction-convection system we come across the uncertainty caused due to the involved parameters in the system. These uncertainties occur due to the imprecise data, experimental error and vagueness of involved parameters. This may complicate the problem and the solution procedure become rigid. So, to solve these uncertain systems various authors have used probabilistic and stochastic methods. In this Context Monte Carlo method is an alternate method which is based on the statistical simulation of the random numbers generated on the basis of a specific sampling distribution. Monte Carlo methods have been used to analyse thermal food processes with variable parameters [3, 5, 11, 16, 29, 31]. Deng and Liu [6] implemented Monte Carlo method to solve the direct bio heat transfer problems. They have demonstrated the bio heat transfer problem with transient or space-dependent boundary conditions, blood perfusion, metabolic rate, and volumetric heat source for tissue. On the other hand various numerical techniques are proposed viz. finite difference method (FDM), finite volume method (FVM) and finite element method (FEM) [13, 20] to handle the said problems. Magnus et al. [13] used finite difference method in his paper to model and solve the governing ground water flow rates, flow direction and hydraulic heads through an aquifer.

Muhieddine et al. [20] described one dimensional phase change problem. They have used vertex centred finite volume method to solve the problem. Edward and Robert [32] used Finite Element Method (FEM) to solve heat

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conduction problem. A non-iterative, finite element-based inverse method for estimating surface heat flux histories on thermally conducting bodies is developed by Ling et al. [18]. They considered both linear and non-linear problems, and sequentially minimized the least square error norm between corresponding sets of measured and computed temperatures.

Further Onate et al. [10] used Galarkin finite element method for convective–diffusive problems with sharp gradients using finite calculus. In view of the above literatures, it reveals that the traditional finite element method may easily be used where the parameters or the values are exact that is in crisp form. But in actual practice the values may be in a region of possibility or we can say the values are uncertain. The above discussed uncertain parameters give uncertain model predictions. Although the uncertainty may be reduced by appropriate experiments but still it may also give the variability in the parameters. Nicolaï and De Baerdemaeker [23] and Nicolaï et al. [24] used finite element method for heat conduction problem considering uncertain physical parameters. Further Nicolaï et al. [25,26] found the temperature in heat conduction problem for randomly varying parameters with respect to time. They have used a variance propagation technique to compute the mean and covariance of the temperatures.

In particular, it is very difficult to get a large number of experimental data so we need an alternative method in which we may handle the uncertainty considering few experimental data. In this context Zadeh [34] proposed an alternate idea that is fuzzy number to handle uncertain values. Hence we need the help of interval/fuzzy analysis for handling this type of data. The direct implementation of interval or fuzzy becomes more complex and the computation is also a difficult task. So to avoid such difficulty various authors tried different techniques to handle such difficulty. Dong and Shah [8] proposed vertex method for computing functions of fuzzy variables. Dong and Wong [9] used Fuzzy Weighted Average Method (FWAM). Yang et al. [33] discussed the calculation of functions with fuzzy numbers. They developed methods which require less computation than the FWAM. Klir [14] revised fuzzy arithmetic by considering the relevant requisite constraints. Hanss [12] gave a transformation method based on the concept of -cut where the fuzzy arithmetic is reduced to interval computation.

Uncertain parameters for the present purpose have been addressed in the form of a fuzzy number and using alpha-cut definition of fuzzy set various intervals are constructed for the specified fuzzy number. The interval values are transformed into crisp form using a proposed transformation. Then we present the traditional finite element procedure [15, 21, 22] for solving the problem by taking these parameters in interval. Next interval/fuzzy finite element technique is described for the said problem. Here we operate interval/fuzzy parameters through finite element method using proposed arithmetic. Fuzzy finite element method results into a set of algebraic equations. These set of algebraic equations will be a fuzzy system of linear simultaneous equations in this case. Matinfar et al. [19] used householder decomposition method to solve fuzzy linear equations and they considered only the right hand side column vector as fuzzy and solved some example problems. For the fuzzy coefficient matrix , Panahi et al. [27] obtained lower triangular and upper triangular matrix separately. Senthilkumar and Rajendran [28] considered symmetric coefficient matrix to solve Fuzzy Linear System (FFLS) of equations. They decomposed the coefficient matrix by using Cholesky method. However Vijayalakshmi and Sattanathan [30] introduced Symmetric times Triangular (ST) decomposition procedure to solve fully fuzzy system of linear equations. Behera and Chakraverty [7] proposed a method to solve fuzzy real system of linear equations by solving two $n \times n$ crisp systems of linear equations. Here the coefficient matrix is considered as real crisp, whereas an unknown variable vector and right hand side vector are considered as fuzzy. The general system is solved by adding and subtracting the left and right bounds of the vectors respectively. Further, Behera and Chakraverty [1, 2] solved complex fuzzy system of linear equations also.

In the present paper a simple arithmetic is presented to handle fuzzy system of linear equations. We have used a modified form of fuzzy finite element method [4] to solve the uncertain heat conduction-convection problem. Here we have considered both Triangular Fuzzy Number (TFN) and Trapezoidal Fuzzy Number (TRFN) for imprecise parameters involved in the problem. The utility and application of the proposed method is discussed by considering a nonhomogeneous fin in conduction-convection system. The obtained result lies in an uncertain region. These uncertain solutions are discussed broadly along with its special cases. It is found that the proposed fuzzy finite element method is efficient and simple to handle the said problem and the authors believe the proposed procedure may very well be used for other heat and mass transfer problems also.

2 Finite Element Formulation for Tapered Fin

Let us consider a tapered fin with plane surfaces on the top and bottom. The fin also loses heat to the ambient via the tip. The thickness of the fin varies linearly from d_1 at the base to d_2 at the tip. The width b remains constant throughout the fin and L is the length.

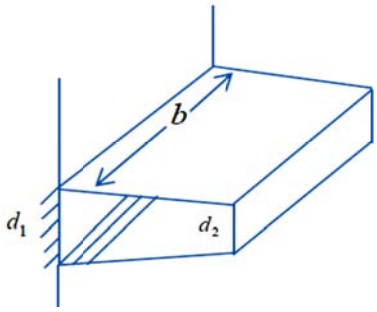

Figure 1: Model diagram of tapered fin

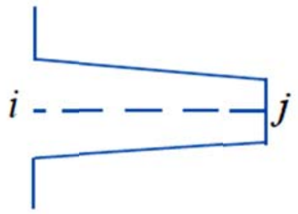

Figure 2: Tapered fin having two nodes

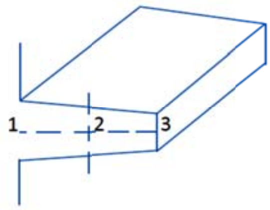

Figure 3: Two element discretization of tapered fin

Here, a typical element e_1 is having nodes i and j respectively as shown in Figure 1. Then the corresponding area (A_i and A_j) and perimeter (P_i and P_j) for nodes i and j respectively are

$$A_i = bd_i ;\ A_j = bd_j ;\ P_i = 2(b+d_i) ;\ P_j = 2(b+d_j) . \tag{1}$$

The shape functions for each element are $(1-x/L)$ and x/L and the area of the fin varies linearly. So the area A of tapered fin may be expressed as the following:

$$A = A_i\left(1-\frac{x}{L}\right) + A_j\left(\frac{x}{L}\right) = A_i - \frac{A_i - A_j}{L}x. \tag{2}$$

Similarly perimeter P of tapered fin may be written as

$$P = P_i\left(1-\frac{x}{L}\right) + P_j\left(\frac{x}{L}\right) = P_i - \frac{P_i - P_j}{L}x. \tag{3}$$

The stiffness matrix for the corresponding tapered fin is [17]

$$[K] = \frac{k}{l}\left(\frac{A_i + A_j}{2}\right)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hl}{12}\begin{bmatrix} 3P_i + P_j & P_i + P_j \\ P_i + P_j & P_i + 3P_j \end{bmatrix}, \tag{4}$$

and the force vector is [17]

$$\{f\} = \frac{Gl}{6}\begin{Bmatrix} 2A_i + A_j \\ A_i + 2A_j \end{Bmatrix} - \frac{ql}{6}\begin{Bmatrix} 2P_i + P_j \\ P_i + 2P_j \end{Bmatrix} + \frac{hT_a l}{6}\begin{Bmatrix} 2P_i + P_j \\ P_i + 2P_j \end{Bmatrix} + hT_a A\begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \tag{5}$$

where G is heat source per unit volume, q is heat flux, h is heat transfer coefficient, k is thermal conductivity, T_a is ambient temperature and l is length of each element. Here in Eq.(5) the last term is valid only for the element at the end face with area A. For all other element the last term of the Eq.(5) is zero.

Next considering the above finite element formulation of tapered fin, we have taken a numerical example of the same and obtained the corresponding results under uncertain environment.

3 Interval Arithmetic

In view of the above literature we may now consider that parameters involved in various systems may be uncertain and so we need interval arithmetic to operate such values. The interval form is then may be written as

$$[\underline{x},\overline{x}] = \{x : x \in \Re, \underline{x} \le x \le \overline{x}\}$$

where $\underline{x}$ is the left value and $\overline{x}$ is the right value of the interval respectively. Let $m = (\underline{x}+\overline{x})/2$ is the mid value and $w = \overline{x} - \underline{x}$ is the width of the interval $[\underline{x},\overline{x}]$.

Let $[\underline{x},\overline{x}]$ and $[\underline{y},\overline{y}]$ be two intervals. Then

i. $[\underline{x},\overline{x}] + [\underline{y},\overline{y}] = [\underline{x}+\underline{y}, \overline{x}+\overline{y}]$;

ii. $[\underline{x},\overline{x}] - [\underline{y},\overline{y}] = [\underline{x}-\overline{y}, \overline{x}-\underline{y}]$;

iii. $[\underline{x},\overline{x}] \times [\underline{y},\overline{y}] = [\min\{\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\overline{y}\}, \max\{\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\overline{y}\}]$;

iv. $[\underline{x},\overline{x}] \div [\underline{y},\overline{y}] = [\min\{\underline{x}\div\underline{y}, \underline{x}\div\overline{y}, \overline{x}\div\underline{y}, \overline{x}\div\overline{y}\}, \max\{\underline{x}\div\underline{y}, \underline{x}\div\overline{y}, \overline{x}\div\underline{y}, \overline{x}\div\overline{y}\}]$.

Now we define a modified form of the interval arithmetic [1, 2] as follows.

Let us consider $[\underline{a},\overline{b}]$ be an arbitrary interval then $[\underline{a},\overline{b}]$ can be written as

$$[\underline{a},\overline{b}] = a + \frac{w}{n} = l \text{; where } w \text{ is the width of the interval and } n \in [1,\infty).$$

If all the values of the interval are in R^+ or R^-, then the arithmetic rules may be written as (developed by Chakraverty and Nayak [33]):

i. $[\underline{x},\overline{x}] + [\underline{y},\overline{y}] = [\min\{\lim_{n\to\infty} l_1 + \lim_{n\to\infty} l_2, \lim_{n\to 1} l_1 + \lim_{n\to 1} l_2\}, \max\{\lim_{n\to\infty} l_1 + \lim_{n\to\infty} l_2, \lim_{n\to 1} l_1 + \lim_{n\to 1} l_2\}]$;

ii. $[\underline{x},\overline{x}] - [\underline{y},\overline{y}] = [\min\{\lim_{n\to\infty} l_1 - \lim_{n\to 1} l_2, \lim_{n\to 1} l_1 - \lim_{n\to\infty} l_2\}, \max\{\lim_{n\to\infty} l_1 - \lim_{n\to 1} l_2, \lim_{n\to 1} l_1 - \lim_{n\to\infty} l_2\}]$;

iii. $[\underline{x},\overline{x}] \times [\underline{y},\overline{y}] = [\min\{\lim_{n\to\infty} l_1 \times \lim_{n\to\infty} l_2, \lim_{n\to 1} l_1 \times \lim_{n\to 1} l_2\}, \max\{\lim_{n\to\infty} l_1 \times \lim_{n\to\infty} l_2, \lim_{n\to 1} l_1 \times \lim_{n\to 1} l_2\}]$;

iv. $[\underline{x},\overline{x}] \div [\underline{y},\overline{y}] = [\min\{\lim_{n\to\infty} l_1 \div \lim_{n\to 1} l_2, \lim_{n\to 1} l_1 \div \lim_{n\to\infty} l_2\}, \max\{\lim_{n\to\infty} l_1 \div \lim_{n\to 1} l_2, \lim_{n\to 1} l_1 \div \lim_{n\to\infty} l_2\}]$.

The above form of the interval arithmetic has great utility over the traditional interval arithmetic. Using the above transformation the interval values are transformed into crisp form and then mathematical limit is operated. The generalised versions of transformed crisp values may easily be handled. This arithmetic may be extended for fuzzy numbers which is described in the succeeding sections.

4 Fuzzy Number

A fuzzy number is a convex, normalized fuzzy set $\tilde{A} \subseteq R$ which is piecewise continuous. A fuzzy number $\tilde{A} = [a^L, a^N, a^R]$ is said to be triangular fuzzy number when the membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \le a^L \\ \dfrac{x-a^L}{a^N - a^L}, & a^L \le x \le a^N \\ \dfrac{a^R - x}{a^R - a^N}, & a^N \le x \le a^R \\ 0, & x \ge a^R. \end{cases}$$

The fuzzy numbers may be represented as an ordered pair form $[\underline{f}(\alpha), \overline{f}(\alpha)]$, $0 \le \alpha \le 1$ where $\underline{f}(\alpha)$ and $\overline{f}(\alpha)$ are left and right monotonic increasing and decreasing functions over [0, 1] respectively. The Triangular Fuzzy Number (TFN) $\tilde{A} = [a^L, a^N, a^R]$ may be transformed into interval form by using $\alpha-$cut as follow.

$$\tilde{A} = [a^L, a^N, a^R] = [a^L + (a^N - a^L)\alpha,\ a^R - (a^R - a^N)\alpha], \quad \alpha \in [0,1].$$

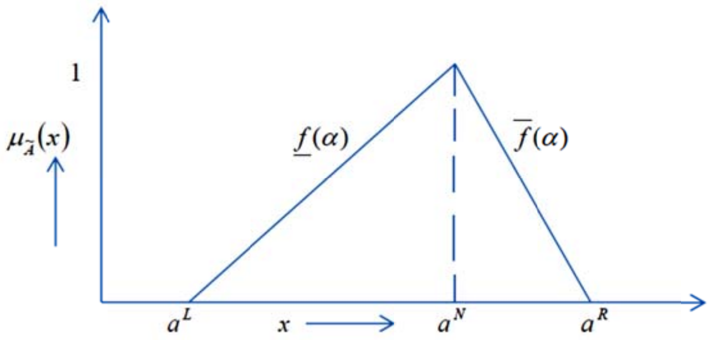

Figure 4: Triangular Fuzzy Number (TFN)

A fuzzy number $\tilde{A}=[a^L, a^{NL}, a^{NR}, a^R]$ is said to be trapezoidal fuzzy number when the membership function is given by

$$\mu_{\tilde{A}}(x)=\begin{cases} 0, & x \le a^L \\ \dfrac{x-a^L}{a^{NL}-a^L}, & a^L \le x \le a^{NL} \\ 1, & a^{NL} \le x \le a^{NR} \\ \dfrac{a^R-x}{a^R-a^{NR}}, & a^{NR} \le x \le a^R \\ 0, & x \ge a^R. \end{cases}$$

The Trapezoidal Fuzzy Number (TRFN) in interval form may also be represented as

$$\tilde{A}=[a^L, a^{NL}, a^{NR}, a^R]=[a^L+(a^{NL}-a^L), a^R-(a^R-a^{NR})], \quad \alpha \in [0,1].$$

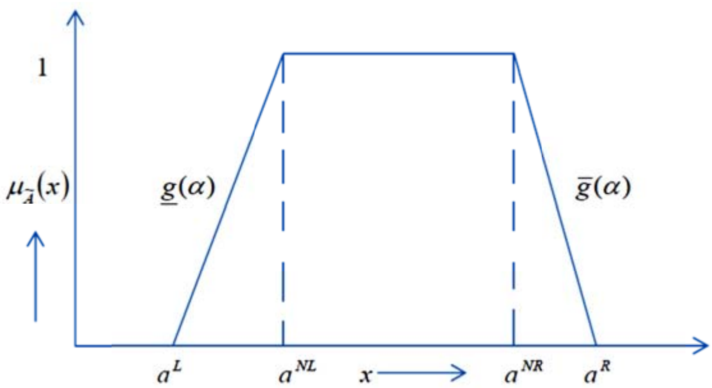

Figure 5: Trapezoidal Fuzzy Number (TRFN)

Let us consider two fuzzy numbers $x=[\underline{x}(\alpha), \overline{x}(\alpha)]$ and $y=[\underline{y}(\alpha), \overline{y}(\alpha)]$ and a scalar k, then

a) $x=y$ if and only if $\underline{x}(\alpha)=\underline{y}(\alpha)$ and $\overline{x}(\alpha)=\overline{y}(\alpha)$;

b) $x+y=[\underline{x}(\alpha)+\underline{y}(\alpha), \overline{x}(\alpha)+\overline{y}(\alpha)]$;

c) $kx=\begin{cases} [k\underline{x}(\alpha), k\overline{x}(\alpha)], & k \ge 0 \\ [k\overline{x}(\alpha), k\underline{x}(\alpha)], & k < 0. \end{cases}$

Combining both the fuzzy number and interval arithmetic we may represent the fuzzy arithmetic as defined below [4].

1. $[\underline{x}(\alpha), \overline{x}(\alpha)] + [\underline{y}(\alpha), \overline{y}(\alpha)]$

   $= [\min\{\lim_{n\to\infty} m_1 + \lim_{n\to\infty} m_2, \lim_{n\to 1} m_1 + \lim_{n\to 1} m_2\}, \max\{\lim_{n\to\infty} m_1 + \lim_{n\to\infty} m_2, \lim_{n\to 1} m_1 + \lim_{n\to 1} m_2\}];$

2. $[\underline{x}(\alpha), \overline{x}(\alpha)] - [\underline{y}(\alpha), \overline{y}(\alpha)]$

   $= [\min\{\lim_{n\to\infty} m_1 - \lim_{n\to 1} m_2, \lim_{n\to 1} m_1 - \lim_{n\to\infty} m_2\}, \max\{\lim_{n\to\infty} m_1 - \lim_{n\to 1} m_2, \lim_{n\to 1} m_1 - \lim_{n\to\infty} m_2\}];$

3. $[\underline{x}(\alpha), \overline{x}(\alpha)] \times [\underline{y}(\alpha), \overline{y}(\alpha)]$

   $= [\min\{\lim_{n\to\infty} m_1 \times \lim_{n\to\infty} m_2, \lim_{n\to 1} m_1 \times \lim_{n\to 1} m_2\}, \max\{\lim_{n\to\infty} m_1 \times \lim_{n\to\infty} m_2, \lim_{n\to 1} m_1 \times \lim_{n\to 1} m_2\}];$

4. $[\underline{x}(\alpha), \overline{x}(\alpha)] \div [\underline{y}(\alpha), \overline{y}(\alpha)]$

   $= [\min\{\lim_{n\to\infty} m_1 \div \lim_{n\to 1} m_2, \lim_{n\to 1} m_1 \div \lim_{n\to\infty} m_2\}, \max\{\lim_{n\to\infty} m_1 \div \lim_{n\to 1} m_2, \lim_{n\to 1} m_1 \div \lim_{n\to\infty} m_2\}]$

where

$$[\underline{x}(\alpha), \overline{x}(\alpha)] = \left\{ \underline{x}(\alpha) + \frac{\overline{x}(\alpha) - \underline{x}(\alpha)}{n} = m \,\middle|\, \underline{x}(\alpha) \le m \le \overline{x}(\alpha), n \in [1, \infty) \right\}.$$

Here the uncertain parameters are handled by using fuzzy values in place of classical (crisp) values. So we may combine the proposed fuzzy arithmetic and finite element method to develop Fuzzy Finite Element Method (FFEM) and the same can be used to solve various problems of engineering and science. As such in the succeeding sections the developed fuzzy finite element method is discussed by taking a simple test problem viz. the variation of uncertain spectrum of temperatures in a non-homogeneous (tapered) fin under imprecisely defined conduction-convection system.

5 Numerical Investigation

Here we have considered a tapered fin where the thickness, d varies from the base $2mm$ to the tip $1mm$ (Figure 1). The base temperature is maintained at $100^\circ C$. The total length of the fin is $20mm$ and width b is $3mm$. Further corresponding data for this problem is provided in Tables 1 and 2.

Table 1: Fuzzy parameters (Triangular Fuzzy Number)

Parameters	Notations	Crisp value	Fuzzy value
Heat transfer coefficient	h	$120 W/m^2\,^\circ C$	$[115, 120, 125] W/m^2\,^\circ C$
Thermal conductivity	k	$200 W/m-^\circ C$	$[195, 200, 205] W/m-^\circ C$
Ambient temperature	T_a	$25^\circ C$	$[20, 25, 30]^\circ C$

Table 2: Fuzzy parameters (Trapezoidal Fuzzy Number)

Parameters	Notations	Crisp value	Fuzzy value
Heat transfer coefficient	h	$120 W/m^2\,^\circ C$	$[115, 119, 121, 125] W/m^2\,^\circ C$
Thermal conductivity	k	$200 W/m-^\circ C$	$[195, 200, 205] W/m-^\circ C$
Ambient temperature	T_a	$25^\circ C$	$[20, 24, 26, 30]^\circ C$

Initially the nodal temperatures of tapered fin under conduction-convection system is analysed for crisp values which is obvious but investigated for the sake of completeness. The computed nodal temperatures for different number of discretizations of the same domain are given in Table 3.

Table 3: Nodal temperatures of tapered fin (crisp value)

Elements / Temperatures	2 elements in $^{\circ}C$	4 elements in $^{\circ}C$	8 elements in $^{\circ}C$	16 elements in $^{\circ}C$
T_1	100	100	100	100
T_2	**90.2798**	94.59511	97.16331	98.54563
T_3	86.22617	**90.40015**	94.61663	97.16265
T_4		87.59145	92.37163	95.85208
T_5		86.53492	**90.44605**	94.61518
T_6			88.86572	93.45358
T_7			87.66768	92.36925
T_8			86.89835	91.36401
T_9			86.64014	**90.44135**
T_{10}				89.60477
T_{11}				88.85846
T_{12}				88.20742
T_{13}				87.65763
T_{14}				87.21625
T_{15}				86.89188
T_{16}				86.69491
T_{17}				86.63797

Corresponding nodal temperatures for different number of discretization have been presented graphically in Figure 6.

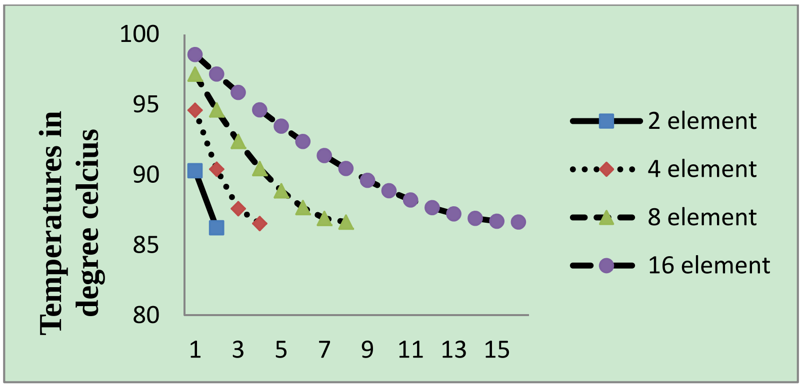

Figure 6: Nodal temperatures of tapered fin (crisp values)

Now considering the imprecise values as fuzzy, mentioned above, we get a series of nodal temperatures depending upon the value of α. When the value of α becomes one (that is crisp) we get the series of nodal temperatures which are presented in Table 3. Similarly when the value of α is zero then the obtained results are mentioned in Table 4. It is worth mentioning that the value of $\alpha = 0$ gives the interval results.

Table 4: Nodal temperatures of tapered fin (Triangular fuzzy values)

Temperatures \ Elements		2 elements in $^{\circ}C$	4 elements in $^{\circ}C$	8 elements in $^{\circ}C$	16 elements in $^{\circ}C$
T_1	Left	100	100	100	100
	Right	100	100	100	100
T_2	Left	**89.7674**	94.31276	97.01493	98.47007
	Right	**90.81915**	94.89252	97.31966	98.62527
T_3	Left	85.47571	**89.89299**	94.33342	97.01486
	Right	87.01505	**90.93384**	94.91488	97.31839
T_4	Left		86.92682	91.96786	95.63545
	Right		88.29033	92.79662	96.08028
T_5	Left		85.79875	**89.937**	94.3332
	Right		87.30858	**90.9816**	94.91216
T_6	Left			88.26804	93.10981
	Right			89.49425	93.81553
T_7	Left			86.99991	91.9674
	Right			88.36966	92.79225
T_8	Left			86.18306	90.90804
	Right			87.65007	91.84387
T_9	Left			85.90237	**89.93525**
	Right			87.41529	**90.97385**
T_{10}	Left				89.05274
	Right				90.1855
T_{11}	Left				88.26489
	Right				89.48277
T_{12}	Left				87.57698
	Right				88.8704
T_{13}	Left				86.99527
	Right				88.35404
T_{14}	Left				86.5273
	Right				87.9405
T_{15}	Left				86.18209
	Right				87.63794
T_{16}	Left				85.97054
	Right				87.4562
T_{17}	Left				85.90588
	Right				87.40727

Corresponding nodal temperatures of Table 4 for different number of discretizations with fuzzy parameters are also shown in Figure 7.

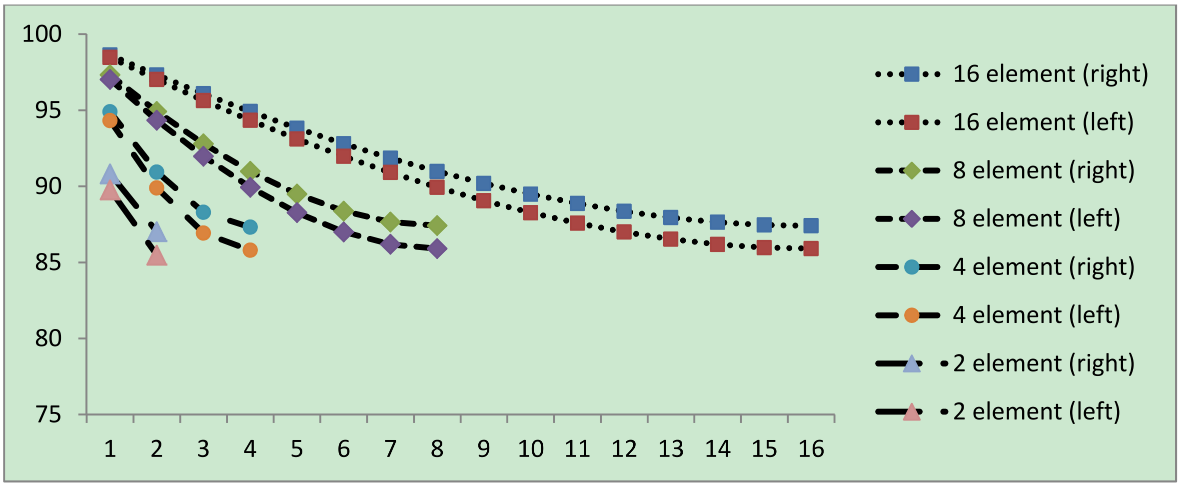

Figure 7: Nodal temperatures of tapered fin (left and right values)

6 Discussion

Here the base of the tapered fin is maintained at a constant temperature of $100^{\circ}C$. Width of the fin is varying linearly so the surface of contact changes with the ambient temperatures. Now considering the above criteria, the uncertain temperatures are obtained when the system is at equilibrium. Firstly the crisp parameters are considered to study the variation of temperature for a tapered fin. The domain is discretized into various numbers of elements and the nodal temperatures are obtained. It is observed that the temperature decreases as we move away from the base of the fin. To observe the above variation of temperature through the domain we have considered 2, 4, 8 and 16 element discretization. The resultant temperatures (crisp) for various number of discretization are presented in Table 3. In view of Table 3 let us fix the node which divides the domain into two equal parts for different number of discretization and we call it as central node. The corresponding temperatures of the central node occur at T_2, T_3, T_5 and T_9 for 2, 4, 8 and 16 elements discretization respectively. It is observed that the central nodal temperature converges as we vary the number of discretization. Figure 6 shows that the distribution graph of temperatures become steeper with more number of discretization of the considered tapered fin. So, one may get the better distribution of temperature along the domain by considering more number of discretized elements for the same domain, as expected. The above is well known although but to investigate the corresponding uncertain results we do study the usual case of the crisp problem.

As mentioned earlier, in actual practice the involved parameters may not be crisp, rather it may be uncertain. To handle such uncertainty here the parameters are taken as fuzzy/interval which are given in Tables 1 (TFN) and 2 (TRFN) respectively. The fuzzy values are converted into α -cut form. These values give a piecewise normalised function. If the normality is considered then the temperature will be equivalent to the temperature for crisp form. So the results varies over the value of α which will give the results in term of fuzzy numbers.

The variation of temperatures using the data of Table 1 (TFN) is given in Table 4. It may be noted that the left and right values of the resultant uncertain temperatures are given in this Table 4. For better visualization of the values these are plotted in Figure 7. The widths of the uncertain results are studied next. We get a narrow width for various temperatures in different discretization and it maintains a regular pattern. Results are compared by considering the central node of the tapered fin. Corresponding triangular fuzzy temperatures at central node are also presented in Figures 8(a) to 8(d).

Further trapezoidal fuzzy values are taken for uncertain parameters involved in the system. Using proposed fuzzy finite element method the nodal temperatures at central node are shown in Figure 9. It may be seen from Figure 9 that the left and right temperatures for different discretization converge which shows the efficacy of the proposed method.

It may be a point to be noted that the reliability of the fuzzy results can be seen in the special cases viz. crisp and interval which are derived from the fuzzy values. As such three cases are reported with respect to the above.

Case-1

Here we have considered only left monotonic increasing functions of the resultant temperatures. The resulting temperatures vary with the value of membership functions. Assigning zero for the value of α we get the left bound of the uncertain fuzzy temperatures. Similarly if the value of α is taken as one then we get right bound of the left monotonic increasing functions which are the centre value of TFN.

Case-2

In this case only right monotonic decreasing functions of the resultant temperatures are considered. Resulting temperatures vary with the value of membership functions. Assigning zero for the value of α we get the right bound of the uncertain fuzzy temperatures. Similarly if the value of α is taken as one then we get left bound of the right monotonic decreasing functions which are the centre value of TFN.

Case-3

Now let us consider the case where the value of α is one for both the left and right monotonic functions. We observe that the resultant temperatures become same for both monotonic functions. If we consider TFN then we find that this is nothing but the centre value. For TRFN we get two different values for both monotonic functions. Here we get an interval of temperatures where the membership functions are normalised.

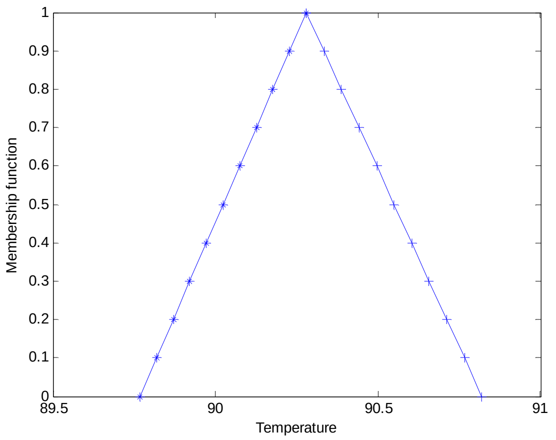

Figure 8(a): Two element discretization

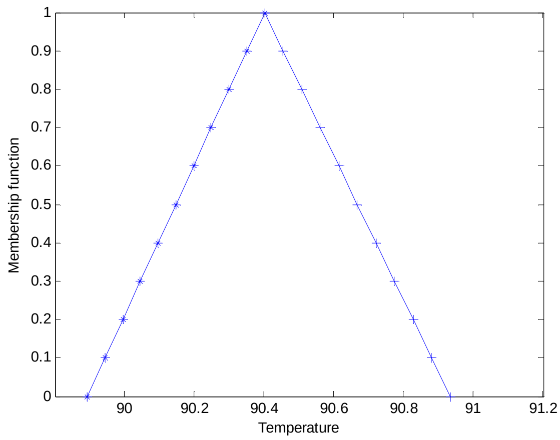

Figure 8(b): Four element discretization

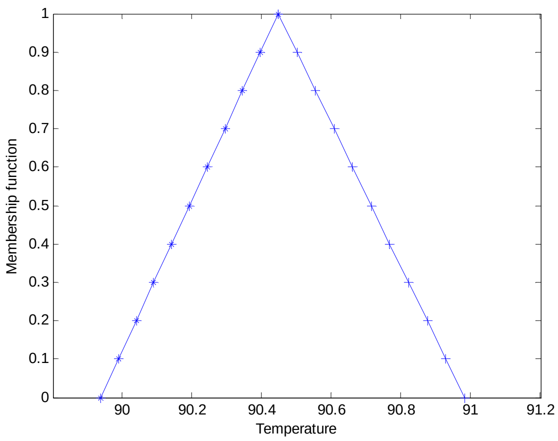

Figure 8(c): Eight element discretization

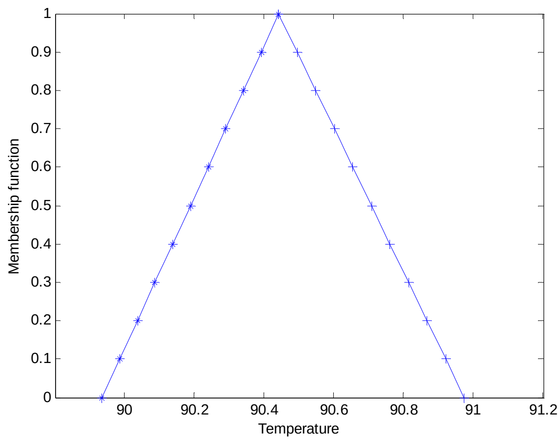

Figure 8(d): Sixteen element discretization

Figure 8: Triangular fuzzy temperature at central node of the tapered fin for different number of discretization

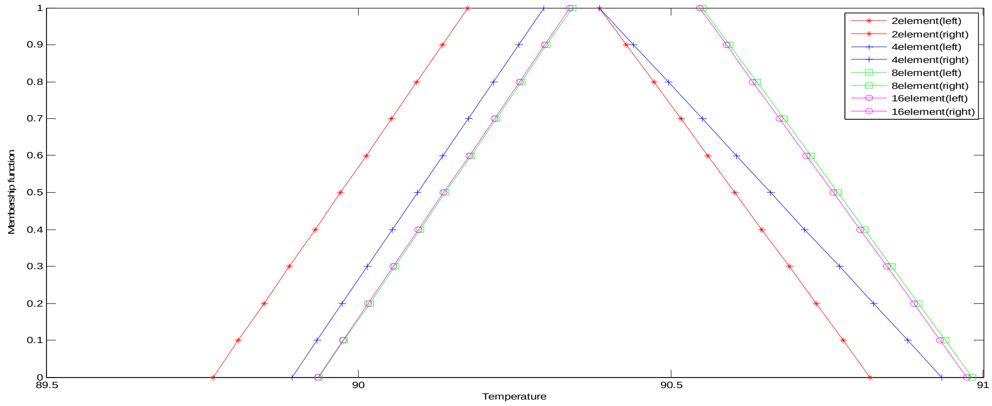

Figure 9: Central nodal temperatures (TRFN) for various discretization of the said domain

7 Conclusion

In general the uncertainties involve in various fields of science and engineering may dictate the final solution thereby it effects the design steps. This paper has undertaken the challenge of uncertainty having in term of fuzzy/interval. Accordingly best suited method is proposed and validated by considering an example problem of convection-conduction system. The given system is solved by using a proposed fuzzy finite element method. Main advantage of this proposed fuzzy finite element method is that if the parameters are taken uncertain viz. fuzzy then we may predict the possibility of temperature distribution at any nodal points of the domain. This concept may be generalised for more number of element discretization through computer program and one may get better distribution for variation of nodal temperatures.

The main purpose of this article is to present an alternative non probabilistic method to manage various engineering and science problems. Here the traditional interval arithmetic is modified for the said problem and a simpler method is proposed to compute interval arithmetic. The idea of modified interval arithmetic is then extended for uncertain fuzzy numbers also. As such uncertain parameters are taken as fuzzy. Then the fuzzy numbers are converted into interval using α -cut techniques. This fuzzy numbers contain left monotonically increasing and right monotonically decreasing functions respectively. Here two types of fuzzy numbers viz. TFN and TRFN have been considered for the investigation. It is found that the fuzzy finite element method with the proposed interval computation is simpler to handle and also efficient. Hence it may be used as a tool for various other heat transfer systems.

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