Offshore Decision-making Problems under Fuzzy Environment

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Abstract

Offshore decision-making problems often involve uncertain exchange rates, and the uncertainty may present randomness and/or fuzziness. This paper describes uncertain exchange rates by possibility distributions. A new fuzzy optimization method is developed for offshore decision-making problems, in which the firm’s profit is measured by Lebesgue-Stieltjes (L–S) integral with respect to the credibility distribution of fuzzy exchange rate. When demands are deterministic, we divide the original feasible region into five subregions, and discuss the local optimal solution in each subregion. According to the obtained theoretical results, we design a region decomposition method for the proposed fuzzy optimization model. Finally, we consider an application example about the offshore decision-making problem between China and Brazil to demonstrate our new modeling idea and the efficiency of the solution method.

Keywords: offshore problem, uncertain exchange rate, fuzzy optimization, feasible region decomposition

1 Introduction

Offshoring is the practice of moving a company’s work to a foreign country where labour costs are cheaper than the domestic labour market. With the development of economic globalization, modern companies are facing more and more internal and external pressures, and offshoring becomes one of the most frequent business activities. By employing offshoring, the companies can reduce production cost and improve the core competitiveness. Under this consideration, many companies have contracted with suppliers located in lower-cost countries to help them to manufacture products. However, in the process of offshoring, risks involved in offshoring are significant. The supply chain company may face financial risk, human resource risk, cultural risk, contract risk, exchange rate risk and policy risk, which would lead to the failure of offshoring. Therefore, offshore decision-making problem under uncertainty is a very important issue for research.

In the literature, some researchers have modeled the offshore decision-making problem as a global supply chain network. Huchzermeier and Cohen [4] modeled the exchange rate as a stochastic diffusion process and proposed a multinomial approximation of correlated exchange rate processes. Kazaz et al. [5] examined the impact of foreign exchange risk on the choice of production policies when the allocation decision can be postponed. Kogut and Kulatilaka [6] presented a stochastic dynamic programming model to study how multi-national corporations can benefit from the uncertainty through the coordination of subsidiaries which are geographically dispersed. Li et al. [7] considered a supply contracting problem, in which the buyer firm faces nonstationary stochastic price and demand. Liu and Nagurney [13] studied the impacts of foreign exchange risk and competition intensity on supply chain companies who are involved in offshoring activities. Other research works in this direction can be found in Ding et al. [3], Nagurney and Nagurney [14] and Osei-Bryson and Ngwenyama [15]. In fuzzy community, Chen et al. [2] presented a fuzzy decision-making approach to dealing with the supplier selection problem in supply chain system. Petrovic et al. [16] described a fuzzy supply chain model, in which uncertainties are interpreted and represented by fuzzy sets. Yang and Liu [18] developed a new mean-risk fuzzy optimization method for supply chain network design problem, in which the standard semivariance was suggested to gauge the risk resulted from fuzzy uncertainty.

Based on two-stage fuzzy optimization methods [9][10][12][11], Sun [17] studied global production planning with fuzzy exchange rates. In this paper, we consider a firm’s plans to outsource production to two suppliers. Choosing the offshore supplier may encounter exchange rate risk, but the production cost is significantly lower

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than that in the domestic supplier. Therefore, the firm has to evaluate the trade-off between the two suppliers. We characterize the uncertain exchange rate as a fuzzy variable with known possibility distribution. We use L–S integral \([1]\) with respect to credibility distribution to measure the firm’s profit. Since L–S integral has linear property, its calculation is usually easier than fuzzy Choquet integral. When demand is deterministic, we construct a two-stage fuzzy optimization model for offshoring decision-making problem. We also give a feasible region decomposition method to solve the proposed fuzzy optimization model.

The paper is organized as follows. In Section 2, we present a new two-stage fuzzy optimization model for offshore decision-making problem. In Section 3, we analyze the properties of the proposed optimization model, and design a feasible region decomposition method to solve our two-stage model. Section 4 provides an application example about the offshore decision-making problem between China and Brazil. Finally, Section 5 gives the conclusions of the paper.

2 Formulation of Offshore Decision-making Problem

In this section, we will construct a two-stage fuzzy optimization model for offshore decision-making problem. The following notations are used in our model.

**Notations**
- \(c\) the unit capacity reservation cost in domestic currency;
- \(x_i\) the reserved capacity units with supplier \(i\), \(i = 1, 2\);
- \(y_i\) the production quantity at the supplier \(i\), \(i = 1, 2\);
- \(c_i\) the unit production cost at supplier \(i\) in stage one (in terms of the currency of supplier \(i\));
- \(p\) the unit selling price of the product in domestic currency;
- \(d\) a known amount of demand for the product;
- \(\tilde{e}\) uncertain exchange rate that converts foreign currency to domestic currency;
- \(Q\) profit in the second stage;
- \(Z\) net profit over two stages.

In the first stage, in order to meet market demand \(d\) for the product, the firm reserves \(x_1\) units of capacity from the domestic supplier and \(x_2\) units of capacity from the offshore supplier. The reservation cost is \(c\) for two suppliers. Thus, the total cost in the first stage is \(c(x_1 + x_2)\).

In the second stage, when the value of the exchange rate \(\tilde{e}\) is known, the firm makes production decisions \(y_1\) and \(y_2\) in two suppliers. The production quantity cannot exceed the reserved capacity at each supplier, which is represented as

\[
y_1 \leq x_1, y_2 \leq x_2.
\]

In addition, the total production quantity cannot exceed the demand,

\[
y_1 + y_2 \leq d.
\]

As a consequence, the profit \(Q\) in the second stage can be expressed as follows:

\[
\begin{align*}
\max Q(x_1, x_2; e) &= (p - c_1)y_1 + (p - c_2\tilde{e})y_2 \\
\text{s.t.:} & \quad 0 \leq y_1 \leq x_1, \\
& \quad 0 \leq y_2 \leq x_2, \\
& \quad y_1 + y_2 \leq d.
\end{align*}
\]

Without loss of generality, we assume \(p - c_1 > c\), so that the domestic supplier will not be eliminated as an option.

The net profit over two stages equals the profit in the second stage minus the expense in the first stage. We construct a two-stage fuzzy optimization model by employing L–S integral with respect to credibility distribution of fuzzy exchange rate. The objective is to obtain the maximum net profit over two stages.

Using the notations above, a two-stage optimization model for the offshore decision-making problem can be built as follows:

\[
\begin{align*}
\max Z(x_1, x_2) &= -c(x_1 + x_2) + \int_{(-\infty, +\infty)} Q(x_1, x_2; e) d\alpha \\
\text{s.t.:} & \quad x_1 \geq 0, \\
& \quad x_2 \geq 0,
\end{align*}
\]
where \( \alpha(e) = \text{Cr}\{ \hat{e} \leq e \} \) with \( \text{Cr} \) being the credibility measure defined in [8], and \( Q(x_1, x_2; e) \) the optimal value function of the following programming problem

\[
\max_{x_1, x_2} Q(x_1, x_2; e) = (p - c_1)y_1 + (p - c_2)e)y_2 \\
\text{s.t.:} \\
0 \leq y_1 \leq x_1, \\
0 \leq y_2 \leq x_2, \\
y_1 + y_2 \leq d.
\]

(5)

3 Feasible Region Decomposition and Solution Method

3.1 Feasible Region Decomposition

When demand \( d \) is deterministic, the feasible region of model (4) can be divided into five subregions.

\( A = \{ x_1 \geq 0, x_2 \geq 0 \mid 0 \leq x_1 + x_2 < d \} \),

\( B = \{ x_1 \geq 0, x_2 \geq 0 \mid x_1 \leq d, x_2 \leq d, d \leq x_1 + x_2 \leq 2d \} \),

\( C = \{ x_1 \geq 0, x_2 \geq 0 \mid x_1 < d, x_2 > d \} \),

\( D = \{ x_1 \geq 0, x_2 \geq 0 \mid x_1 > d, x_2 < d \} \),

\( E = \{ x_1 \geq 0, x_2 \geq 0 \mid x_1 > d, x_2 > d \} \).

In these five subregions, the analytical expressions of the second value function \( Q(x_1, x_2; e) \) are available. Therefore, we can solve five subproblems instead of solving original model (4) directly.

Note that the demand is deterministic and the firm cannot reserve more than \( d \) from any supplier. Therefore, the optimal solutions are not in the subregions \( C, D \) and \( E \). In the following, we focus our attention on domains \( A \) and \( B \), and assume fuzzy exchange rate \( \hat{e} \) has a general possibility distribution.

We use the following notations in our analysis,

\( P_{A1} = \int_{[0, +\infty)} (p - c_1) d\alpha, \quad P_{A2} = \int_{[0, p/c_2]} (p - c_2 e) d\alpha \),

\( P_{B1} = \int_{c_1/c_2, p/c_2} (c_2 e - c_1) d\alpha + \int_{[p/c_2, +\infty)} (p - c_1) d\alpha \),

\( P_{B2} = \int_{[0, c_1/c_2]} (c_1 - c_2 e) d\alpha \).

Theorem 1. In domain \( A = \{ x_1 \geq 0, x_2 \geq 0 \mid 0 \leq x_1 + x_2 < d \} \), we have the following assertions about local optimal solution.

(i) \( x_1 = 0, x_2 = 0 \) if and only if \( P_{A1} \leq c, P_{A2} \leq c \);  
(ii) \( x_1 = d, x_2 = 0 \) if and only if \( P_{A1} > c, P_{A1} \geq P_{A2} \);  
(iii) \( x_1 = 0, x_2 = d \) if and only if \( P_{A2} > c, P_{A1} \leq P_{A2} \).

Proof. In region \( A = \{ x_1 \geq 0, x_2 \geq 0 \mid 0 \leq x_1 + x_2 < d \} \), the analytical expression of the second value function \( Q(x_1, x_2; e) \) is

\[
Q(x_1, x_2; e) = \begin{cases} 
(p - c_1)x_1 + (p - c_2 e)x_2, & 0 \leq e < p/c_2, \\
(p - c_1)x_1, & e \geq p/c_2.
\end{cases}
\]

Therefore, the objective function \( Z(x_1, x_2) \) has the following analytical expression:

\[
Z(x_1, x_2) = -c(x_1 + x_2) + \int_{[0, p/c_2]} \{ (p - c_1)x_1 + (p - c_2 e)x_2 \} d\alpha + \int_{[p/c_2, +\infty)} (p - c_1)x_1 d\alpha \\
= [-c + \int_{[0, +\infty)} (p - c_1) d\alpha] x_1 + [-c + \int_{[0, p/c_2]} (p - c_2 e) d\alpha] x_2 \\
= [-c + P_{A1}] x_1 + [-c + P_{A2}] x_2.
\]

Note that the second-order partial derivatives with respect to \( x_1 \) and \( x_2 \) are zero. Thus, the following first-order partial derivatives are sufficient to determine the optimal behavior of \( x_1 \) and \( x_2 \):

\[
\frac{\partial Z(x_1, x_2)}{\partial x_1} \bigg|_{(x_1, x_2) \in A} = -c + P_{A1}, \\
\frac{\partial Z(x_1, x_2)}{\partial x_2} \bigg|_{(x_1, x_2) \in A} = -c + P_{A2}.
\]
To obtain maximum $Z$, we compare the first-order partial derivatives of $x_1$ and $x_2$. When $-c + P_{A1} ≥ -c + P_{A2}$ and $-c + P_{A1} > 0$, we have the local optimal solution $x_1 = d, x_2 = 0$. When $-c + P_{A1} ≤ -c + P_{A2}$ and $-c + P_{A2} > 0$, we have $x_1 = 0, x_2 = d$. In other cases, we have $x_1 = 0, x_2 = 0$. The proof of theorem is complete.

**Theorem 2.** In domain $B = \{x_1 ≥ 0, x_2 ≥ 0 \mid x_1 ≤ d, x_2 ≤ d, d ≤ x_1 + x_2 ≤ 2d\}$, we have the following assertions about local optimal solution:

(i) $x_1 = 0, x_2 = d$ if and only if $P_{B1} ≤ c, P_{B1} ≤ P_{B2}$;
(ii) $x_1 = d, x_2 = 0$ if and only if $P_{B2} ≤ c, P_{B1} ≥ P_{B2}$;
(iii) $x_1 = d, x_2 = d$ if and only if $P_{B1} > c, P_{B2} > c$.

**Proof.** In domain $B = \{x_1 ≥ 0, x_2 ≥ 0 \mid d ≤ x_1 + x_2 ≤ 2d\}$, the analytical expression of the second value function $Q(x_1, x_2; e)$ is

$$Q(x_1, x_2; e) = \begin{cases} (p - c_1)(d - x_2) + (p - c_2)e x_2, & 0 ≤ e < c_1/c_2, \\ (p - c_1)x_1 + (p - c_2e)(d - x_1), & c_1/c_2 ≤ e < p/c_2, \\ (p - c_1)x_1, & e ≥ p/c_2. \end{cases}$$

Therefore, the objective function $Z(x_1, x_2)$ has the following analytical expression:

$$Z(x_1, x_2) = -c(x_1 + x_2) + \int_{[0,c_1/c_2]} ((p - c_1)(d - x_2) + (p - c_2e)x_2) d\alpha$$
$$+ \int_{[c_1/c_2,p/c_2]} ((p - c_1)x_1 + (p - c_2e)(d - x_1)) d\alpha + \int_{[p/c_2,\infty]} (p - c_1)x_1 d\alpha$$
$$= [-c + \int_{[0,c_1/c_2]} (c_2e - c_1) d\alpha + \int_{[p/c_2,\infty]} (p - c_1) d\alpha] x_1 + [-c + \int_{[0,c_1/c_2]} (c_1 - c_2e) d\alpha] x_2$$
$$+ \int_{[0,c_1/c_2]} d(p - c_1) d\alpha + \int_{[c_1/c_2,p/c_2]} d(p - c_2e) d\alpha$$
$$= [-c + P_{B1}] x_1 + [-c + P_{B2}] x_2 + \int_{[0,c_1/c_2]} d(p - c_1) d\alpha + \int_{[c_1/c_2,p/c_2]} d(p - c_2e) d\alpha.$$

The first-order partial derivatives of $Z(x_1, x_2)$ are

$$\frac{\partial Z(x_1, x_2)}{\partial x_1} \bigg|_{(x_1, x_2) \in B} = -c + P_{B1}, \quad \frac{\partial Z(x_1, x_2)}{\partial x_2} \bigg|_{(x_1, x_2) \in B} = -c + P_{B2}.$$

When $-c + P_{B1} ≤ -c + P_{B2}$ and $-c + P_{B1} < 0$, we have the local optimal solution $x_1 = 0, x_2 = d$. When $-c + P_{B1} ≥ -c + P_{B2}$ and $-c + P_{B2} < 0$, we have $x_1 = d, x_2 = 0$. In other cases, we have $x_1 = d, x_2 = d$. The proof of theorem is complete.

### 3.2 Solution Method

According to Theorems 1 and 2, we can find the global optimal solution in region A or B. We now design a feasible region decomposition method to find the global optimal solution. First, we divide the feasible region into five subregions. Then, we solve model (4) in regions A and B to find the local optimal solutions, and compute the corresponding objective values. Finally, we compare the obtained objective values in regions A and B, and the local optimal solution with the larger objective value is the global optimal solution. The solution method is summarized as follows:

**Step 1:** Divide the feasible region into five domains: $A = \{x_1 ≥ 0, x_2 ≥ 0 \mid 0 ≤ x_1 + x_2 < d\}$, $B = \{x_1 ≥ 0, x_2 ≥ 0 \mid x_1 ≤ d, x_2 ≤ d, d ≤ x_1 + x_2 ≤ 2d\}$, $C = \{x_1 ≥ 0, x_2 ≥ 0 \mid x_1 < d, x_2 ≥ d\}$, $D = \{x_1 ≥ 0, x_2 ≥ 0 \mid x_1 > d, x_2 < d\}$, and $E = \{x_1 ≥ 0, x_2 ≥ 0 \mid x_1 > d, x_2 > d\}$.
Step 2: Find the local optimal solutions in domains A and B, and compute the corresponding objective values.

In domain A, we calculate $P_{A1}$ and $P_{A2}$ by MATLAB. By Theorem 1, we have:

- If $P_{A1} \leq c, P_{A2} \leq c$, then local optimal solution is $x_1 = x_2 = 0$ with objective value 0;
- If $P_{A1} > c, P_{A1} \geq P_{A2}$, then local optimal solution is $x_1 = d, x_2 = 0$ and objective value is $(P_{A1} - c)d$;
- If $P_{A2} > c, P_{A1} \leq P_{A2}$, then local optimal solution is $x_1 = 0, x_2 = d$ and objective value is $(P_{A2} - c)d$.

In domain B, we calculate $P_{B1}$ and $P_{B2}$ by MATLAB. By Theorem 2, we have:

- If $P_{B1} \leq c, P_{B1} \leq P_{B2}$, then local optimal solution is $x_1 = 0, x_2 = d$ and objective value is
  \[ \left( P_{B2} - c \right) d + \int_{0,c_1/c_2}^1 d(p - c_1) d\alpha + \int_{c_1/c_2,y_1}^1 d(p - c_2 e) d\alpha; \]
- If $P_{B2} \leq c, P_{B1} \geq P_{B2}$, then local optimal solution is $x_1 = d, x_2 = 0$ and objective value is
  \[ \left( P_{B1} - c \right) d + \int_{0,c_1/c_2}^1 d(p - c_1) d\alpha + \int_{c_1/c_2,y_1}^1 d(p - c_2 e) d\alpha; \]
- If $P_{B1} > c, P_{B2} > c$, then local optimal solution is $x_1 = d, x_2 = d$ and objective value is
  \[ \left( P_{B1} - c \right) d + \left( P_{B2} - c \right) d + \int_{0,c_1/c_2}^1 d(p - c_1) d\alpha + \int_{c_1/c_2,y_1}^1 d(p - c_2 e) d\alpha. \]

Step 3: Compare the objective values in domains A and B, and the local optimal solution corresponding to the larger one is the global optimal solution.

In the next section, we will provide an application example to demonstrate our new modeling idea and the efficiency of the solution method.

4 An Application Example

4.1 Problem Description

Brazil is the world’s leading coffee grower and producer, and it has long been known as “Coffee Kingdom”. Suppose that a coffee company in Shanghai outsources their production operations to a domestic supplier and a Brazilian supplier, and sells their products to the domestic market. The basic unit of currency in Brazil is Real. The exchange rate (converts Real to Yuan) changes over time. The change curve of the exchange rate during a period is plotted in Figure 1. We use a lognormal possibility distribution with parameters $m = 1, \sigma = 0.4$ to character the uncertain exchange rate $\hat{e}$, that is

\[ \ln \hat{e} \sim n(1, 0.4). \] (6)
The credibility distribution of the fuzzy exchange rate $\tilde{e}$ is
\[
\alpha(r) = Cr\{\tilde{e} \leq r\} = \begin{cases} 
\frac{2}{\pi} e^{-\frac{(\ln r - \mu)^2}{2\sigma^2}}, & \ln r \leq m, \\
1 - \frac{2}{\pi} e^{-\frac{(\ln r - \mu)^2}{2\sigma^2}}, & \ln r > m.
\end{cases}
\]  
(7)

If the required data for the example is as follows: $d = 5000$, $c = 10$ Yuan, $p = 100$ Yuan, $c_1 = 80$ Yuan and $c_2 = 20$ Real, then the following two-stage optimization model is built for the coffee company,
\[
\begin{align*}
\max Z(x_1, x_2) &= -10(x_1 + x_2) + \int_{(-\infty, +\infty)} [20y_1 + (100 - 20c)y_2] dCr\{\tilde{e} \leq r\} \\
\text{s.t.:} & \quad 0 \leq y_1 \leq x_1, \\
& \quad 0 \leq y_2 \leq x_2, \\
& \quad y_1 + y_2 \leq 5000.
\end{align*}
\]  
(8)

### 4.2 Computational Results

In this section, we solve model (8) by the feasible region decomposition method. First, we divide the feasible region of model (8) into five domains:
\[
\begin{align*}
A &= \{x_1 \geq 0, x_2 \geq 0 \mid 0 \leq x_1 + x_2 < 5000\}, \\
B &= \{x_1 \geq 0, x_2 \geq 0 \mid x_1 \leq 5000, x_2 \leq 5000, 5000 \leq x_1 + x_2 \leq 10000\}, \\
C &= \{x_1 \geq 0, x_2 \geq 0 \mid x_1 < 5000, x_2 > 5000\}, \\
D &= \{x_1 \geq 0, x_2 \geq 0 \mid x_1 > 5000, x_2 < 5000\}, \\
E &= \{x_1 \geq 0, x_2 \geq 0 \mid x_1 > 5000, x_2 > 5000\}.
\end{align*}
\]

Then we use MATLAB software to calculate the values of $P_{A1}, P_{A2}, P_{B1}$ and $P_{B2}$. On the basis of Theorems 1 and 2, we obtain the local optimal solutions and their objective values in domains A and B, and reported them in Tables 1 and 2, respectively.

#### Table 1: The results in domain A

<table>
<thead>
<tr>
<th>$P_{A1}$</th>
<th>$P_{A2}$</th>
<th>Optimal solution</th>
<th>Objective value</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>40.3015</td>
<td>(0, 5000)</td>
<td>151507.5 Yuan</td>
</tr>
</tbody>
</table>

#### Table 2: The results in domain B

<table>
<thead>
<tr>
<th>$P_{B1}$</th>
<th>$P_{B2}$</th>
<th>Optimal solution</th>
<th>Objective value</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.6101</td>
<td>24.9115</td>
<td>(0, 5000)</td>
<td>151507 Yuan</td>
</tr>
</tbody>
</table>

Table 1 shows that in domain A the local optimal solution is $x_1 = 0, x_2 = 5000$ and the objective value is 151507.5 Yuan. Table 2 shows that in domain B we have the same solution results as in domain A. Therefore, the global optimal solution is $x_1 = 0, x_2 = 5000$, that is, the company should reserve only with the Brazilian supplier.

### 5 Conclusions

In the present paper, we studied offshore problems in fuzzy decision-making system, and obtained the following new results.

Firstly, we described the exchange rate by a fuzzy variable with known possibility distribution, and employed L-S integral with respect to credibility distribution to measure the firm’s profit.

Secondly, we built a two-stage fuzzy optimization model for offshore decision-making problem, in which the objective is to maximize the profit over the two stages. When the demand is deterministic, we obtained some theoretical results about the local optimal solution in each subregion.

Finally, we designed a feasible region decomposition method for the proposed fuzzy optimization model. We also provided an application example about offshore decision-making problem between China and Brazil to demonstrate our new modeling idea and the efficiency of the solution method.

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