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Hesitation Degree of Intuitionistic Fuzzy Sets in a New Cosine Similarity Measure

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Abstract

Cosine similarity measure between fuzzy sets was made a breakthrough based on the idea of Bhattacharya on a measure of divergence between two multinomial populations. The measure was extended by Ye in 2011 specifically for measuring similarity between intuitionistic fuzzy sets (IFSs). The IFS is characterized by the notions of membership degree, non membership degree and the degree of hesitation as vector representations in vector space. Ye proposed a cosine similarity measure and weighted cosine similarity measure for IFSs. However, the hesitation degree of IFS is excluded in Ye similarity measure. This paper proposes a new cosine similarity measure and weighted cosine similarity measure for IFSs by considering membership degree, non membership degree and hesitation degree concurrently. The hesitation degree is added to the new cosine similarity measures without compromising the notions of membership degree and non membership degree. Four numerical examples in pattern recognition are provided to illustrate the feasibility of the proposed methods.

Keywords: intuitionistic fuzzy sets, cosine similarity measure, pattern recognition, degree of hesitation

1 Introduction

The theory of similarity measure. A similarity measure is an important tool for determining the degree of similarity between two objects. Since Atanassov [1] extended fuzzy sets to IFS, many different similarity measures between IFSs have been proposed in the literature. Li and Cheng [6] discussed some similarity measures on IFSs and proposed a similarity measure between IFSs which is the first one to be applied to pattern recognition problems. About a year later, Liang and Shi [7] proposed several similarity measures to differentiate different IFSs and discussed the relationships between these measures. Mitchell [8] interpreted IFSs as ensembles of ordered fuzzy sets from a statistical viewpoint to modify Li and Cheng's measures [6]. Based on the extension of the Hamming distance to fuzzy sets, Szmidt and Kacprzyk [11] introduced the Hamming distance between IFSs and proposed a similarity measure between IFSs based on the distance. In other research, Hung and Yang [4] proposed another method to calculate the distance between IFSs based on the Hausdorff distance and then used this distance to generate several similarity measures between IFSs that are suited to be used in linguistic variables. Hung and Yang [5] also proposed a method to calculate the degree of similarity between IFSs, in which the proposed similarity measures are induced by Lp metric. The latest development in these measures is cosine similarity measure.

Formulation of the problem. Ye [15] introduced cosine similarity measure and weighted cosine similarity measure for IFSs after considering the advantages of membership degree and non membership degree as vector representations. Indeed, it is an extension of cosine similarity measure of fuzzy sets. Despite the success of his similarity measures, the role of hesitation degree in IFSs is neglected.

The importance of hesitation degree. The new cosine similarity measure proposed in this paper does consider the hesitation degree as a vector representation in the formula. The inclusion of hesitation degree might produce more comprehensive judgment due to the representations of incomplete knowledge in defining the membership function. The importance of hesitation degree in IFS and the recent development in cosine similarity measures motivate a new idea in the possibility of integrating of these two notions. Szmidt and Kacpryzk [10] also stressed the necessity of

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taking into consideration a third parameter (hesitation degree), which arises due to the lack of knowledge or 'personal error' [3].

What we do in this paper. In this paper, we propose a new cosine similarity measure and a new weighted cosine similarity measure for IFSs by considering hesitation degree. This novel cosine similarity measure for IFSs is proposed based on the precedence research on cosine similarity measure (angular coefficient) between fuzzy sets and IFSs. The rest of the paper is organized as follows. In the next section, basic notions and definitions on fuzzy sets, IFSs and cosine measure in fuzzy sets and cosine similarity measures in IFSs are elucidated. A new similarity measure is proposed in Section 3. The applications of the proposed methods in pattern recognitions are furnished in Section 4. Finally, conclusion is appeared in the last section.

2 Preliminaries

Theory of fuzzy sets and intuitionistic fuzzy sets (IFS). Fuzzy set theory, a well-known theory was proposed by Zadeh [16] and defines set membership as a possibility distribution. The general rule for this can expressed as:

$$f : [0,1]^n \to [0,1] \tag{1}$$

where n is some number of possibilities. This basically states that we can take n possible events and use f to generate as single possible outcome. In fuzzy set theory, the degree of belonging of element to the set is represented by a membership value in the real interval [0, 1] and there exists degree of non-membership which is complementary in nature. One of the extensions of fuzzy sets is IFSs. IFS have been found to be highly useful to deal with vagueness and IFSs A in X is defined by Atanassov [1] as:

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}, \tag{2}$$

where $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ with the condition $0 \le \mu_A(x) + \nu_A(x) \le 1$. The numbers $\mu_A(x)$ and $\nu_A(x)$ represent respectively the membership degree and non-membership degree of the element x to the set A. For each IFSs in X, if $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x), x \in X$, then $\pi_A(x)$ is called the intuitionistic index of the element x in the set A. It is a hesitancy degree of x to A. It is obvious that $0 \le \pi_A(x) \le 1, x \in X.$ For two IFS

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$$

and

$$B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\},$$

two relations are defined as follows [1]:

1. $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \le \nu_B(x)$ for any $x \in X$;
2. $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ for any $x \in X$.

The existing cosine similarity measure for fuzzy sets and IFSs. Cosine similarity measures are defined as the inner product of two vectors divided by the product of their lengths [2, 9]. Assume that $A = \{\mu_A(x_1), \mu_A(x_2), ..., \mu_A(x_n)\}$ and $B = \{\mu_B(x_1), \mu_B(x_2), ..., \mu_B(x_n)\}$ are two fuzzy sets in the universe of discourse $X = \{x_1, x_2, ..., x_n\}, x_i \in X$. A cosine similarity measure (angular coefficient) based on Battacharya's distance [2, 9] between the fuzzy sets A and B can be defined as

$$C_F(A,B) = \frac{\sum_{i-1}^{n} \mu_A(x_i)\mu_B(x_i)}{\sqrt{\sum_{i=1}^{n} \mu_A^2(x_i)} \sqrt{\sum_{i=1}^{n} \mu_B^2(x_i)}}. \tag{3}$$

The cosine similarity measure takes value in the interval [0, 1]. It is undefined if $\mu_A(x_i) = 0$ and/or $\mu_B(x_i) = 0$ $(i = 1, 2, ..., n)$.

Based on the extension of the cosine measure between fuzzy sets, a cosine similarity measure between two IFSs A and B is defined. Assume that there are two IFSs A and B in a universe discourse $X = \{x_1, x_2, ..., x_n\}$ [15],

$$C_{IFS}(A,B)=\frac{1}{n}\frac{\sum_{i-1}^{n}\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)}{\sqrt{\sum_{i=1}^{n}\mu_A^2(x_i)+\nu_A^2(x_i)}\sqrt{\sum_{i=1}^{n}\mu_B^2(x_i)+\nu_B^2(x_i)}}. \tag{4}$$

The cosine similarity measures of two IFSs *A* and *B* satisfies the following properties:

1. $0 \le C_{IFS}(A,B) \le 1$;
2. $C_{IFS}(A,B) = C_{IFS}(B,A)$;
3. $C_{IFS}(A,B) = 1$ if $A = B$, i.e., $\mu_A(x_i)=\mu_B(x_i)$ and $\nu_A(x_i)=\nu_B(x_i)$ for $i=1,2,...,n$.

If we consider the weights of x_i, a weighted cosine similarity measure between IFSs *A* and *B* is defined as follows [15]:

$$C_{IFS}(A,B)=\frac{1}{n}\sum_{i=1}^{n}w_i\frac{\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A^2(x_i)+\nu_A^2(x_i)}\sqrt{\mu_B^2(x_i)+\nu_B^2(x_i)}} \tag{5}$$

where $w_i \in [0,1], i=1,2,...,n$ and $\sum_{i=1}^{n} w_i = 1$. If $w_i = 1/n, i=1,2,...,n,$ then there is $C_{WIFS}(A,B)=C_{IFS}(A,B)$. The weighted cosine similarity measure of two IFSs *A* and *B* also satisfies the following properties:

1. $0 \le C_{WIFS}(A,B) \le 1$;
2. $C_{WIFS}(A,B) = C_{WIFS}(B,A)$;
3. $C_{WIFS}(A,B) = 1$ if $A = B$, i.e., $\mu_A(x_i)=\mu_B(x_i)$ and $\nu_A(x_i)=\nu_B(x_i)$ for $i=1,2,...,n$.

Clearly the equation (4) and equation (5) do consider membership degree and non membership degree in IFSs. The two equations seem incomplete when the importance of hesitation degree is neglected.

3 A New Cosine Similarity Measure for IFSs

The new cosine similarity measure. Assume that there are two IFSs *A* and *B* in the universe of discourse $X=\{x_1,x_2,...,x_n\}$. The IFS *A* is characterized by the degree of membership, $\mu_A(x_i)$, degree of non-membership, $\nu_A(x_i)$ and degree of hesitation, $\pi_A(x_i)$ for $i=1,2,3,...,n$, which can be considered as vector representations with *n* elements: $\mu_A=(\mu_A(x_1),\mu_A(x_2),..,\mu_A(x_n)),\nu_A=(\nu_A(x_1),\nu_A(x_2)...,\nu_A(x_n))$ and $\pi_A=(\pi_A(x_1),\pi_A(x_2),...,\pi_A(x_n))$.

For the IFS *B*, it is characterized by the degree of membership, $\mu_B(x_i)$, degree of non membership, $\nu_B(x_i)$ and degree of hesitation, $\pi_B(x_i)$ for $i=1,2,3,...,n$, which can be considered as vector representations with *n* elements: $\mu_B=(\mu_B(x_1),\mu_B(x_2),...,\mu_B(x_n)),\nu_B=(\nu_B(x_1),\nu_B(x_2),...,\nu_B(x_n))$ and $\pi_B=(\pi_B(x_1),\pi_B(x_2),...,\pi_B(x_n))$. Therefore, a novel cosine similarity measure between *A* and *B* is proposed as follows:

$$C_{IFS}(A,B)=\frac{1}{n}\frac{\sum_{i-1}^{n}\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)+\pi_A(x_i)\pi_B(x_i)}{\sqrt{\sum_{i=1}^{n}\mu_A^2(x_i)+\nu_A^2(x_i)+\pi_A^2(x_i)}\sqrt{\sum_{i=1}^{n}\mu_B^2(x_i)+\nu_B^2(x_i)+\pi_B^2(x_i)}}. \tag{6}$$

The cosine similarity measure of two IFS *A* and *B* satisfies the following properties:

1. $0 \le C_{IFS}(A,B) \le 1$;
2. $C_{IFS}(A,B) = C_{IFS}(B,A)$;
3. $C_{IFS}(A,B) = 1$ if $A = B$, i.e., $\mu_A(x_i)=\mu_B(x_i)$, $\nu_A(x_i)=\nu_B(x_i)$ and $\pi_A(x_i)=\pi_B(x_i)$ for $i=1,2,...,n$.

Proof:

1. It is obvious that the property is true according to cosine value for equation (6).
2. It is obvious that the property is true.

3. When $A = B$, there are $\mu_A(x_i) = \mu_B(x_i), \nu_A(x_i) = \nu_B(x_i)$, and $\pi_A(x_i) = \pi_B(x_i)$ for $i = 1, 2, ..., n$. So there is $C_{IFS}(A, B) = 1$. When $C_{IFS}(A, B) = 1$, there are $\mu_A(x_i) = \nu_A(x_i) = \mu_B(x_i) = \nu_B(x_i)$, and $\pi_A(x_i) = \pi_B(x_i)$ for $i = 1, 2, ..., n$. So there is $A = B$.

If we consider the weight of x_i, a weighted cosine similarity measure between IFSs A and B is proposed as follows:

$$C_{IFS}(A,B) = \sum_{i=1}^{n} w_i \frac{\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \pi_A(x_i)\pi_B(x_i)}{\sqrt{\mu_A^2(x_i) + \nu_A^2(x_i) + \pi_A^2(x_i)}\sqrt{\mu_B^2(x_i) + \nu_B^2(x_i) + \pi_B^2(x_i)}} \qquad (7)$$

where $w_i \in [0,1], i = 1, 2, ..., n$ and $\sum_{i=1}^{n} w_i = 1$. If $w_i = 1/n, i = 1, 2, ..., n$, then there is $C_{WIFS}(A, B) = C_{IFS}(A, B)$. Hesitation degree π for upper and lower boundaries are considered in the equation (6) and equation (7). The proposed methods are different from equation (4) and equation (5) based on the usage of vector representation in the equation. From the equation (6) and equation (7), it can be seen that the hesitation degree is also considered while in equation (4) and equation (5), there is no hesitation degree.

The weighted cosine similarity measure of two IFSs A and B also satisfies the following properties:

(i) $0 \le C_{WIFS}(A, B) \le 1$;

(ii) $C_{WIFS}(A, B) = C_{WIFS}(B, A)$;

(iii) $C_{IFS}(A, B) = 1$ if $A = B$, i.e., $\mu_A(x_i) = \mu_B$, (x_i) , $\nu_A(x_i) = \nu_B(x_i)$ and $\pi_A(x_i) = \pi_B(x_i)$ for $i = 1, 2, ..., n$.

Proof:

(i) It is obvious that the property is true according to cosine value for equation (7).

(ii) It is obvious that the property is true.

(iii) When $A = B$, there are $\mu_A(x_i) = \mu_B(x_i)$, $\nu_A(x_i) = \nu_B(x_i)$ and $\pi_A(x_i) = \pi_B(x_i)$ for $i = 1, 2, ..., n$. So there is $C_{WIFS}(A, B) = 1$. When $C_{WIFS}(A, B) = 1$, there are $\mu_A(x_i) = \mu_B(x_i), \nu_A(x_i) = \nu_B(x_i)$ and $\pi_A(x_i) = \pi_B(x_i)$ for $i = 1, 2, ..., n$. So there is $A = B$.

As the new cosine similarity measure satisfies the similarity measure properties, the proposed weighted cosine similarity measure also satisfies the conditions.

4 Application of Pattern Recognition

In this section, the novel cosine similarity measure for IFSs is applied to pattern recognition to demonstrate the feasibility.

Example 1: The following example discusses the medical diagnosis problem retrieved from [11].

Let us consider a set of diagnosis Q = {Q_1(Viral fever), Q_2(Malaria), Q_3(Typhoid), Q_4(Stomach problem), Q_5(Chest problem)} and a set of symptoms S = {s_1(Temperature), s_2(Headache), s_3(Stomach pain), s_4(Cough), s_5(Chest pain)}. Suppose a patient with respect to all the symptoms can be represented by the following IFSs:

P(Patient) = {<s_1, 0.8, 0.1, 0.1>, < s_2, 0.6, 0.1, 0.3>, < s_3, 0.2, 0.8, 0>, < s_4, 0.6, 0.1, 0.3>, < s_5, 0.1, 0.6, 0.3>};

Then each diagnosis $Q_i (i = 1, 2, 3, 4, 5)$ can also be viewed as IFSs with respect to all the symptoms as follows:

Q_1 (Viral fever) = {<s_1, 0.4, 0, 0.6>, < s_2, 0.3, 0.5, 0.2>, < s_3, 0.1, 0.7, 0.2>, < s_4, 0.4, 0.3, 0.3>, < s_5, 0.1, 0.7, 0.2>};
Q_2 (Malaria) = {<s_1, 0.7, 0, 0.3>, < s_2, 0.2, 0.6, 0.2>, < s_3, 0, 0.9, 0.1>, < s_4, 0.7, 0, 0.3>, < s_5, 0.1, 0.8, 0.1>};
Q_3 (Typhoid) = {<s_1, 0.3, 0.3, 0.4>, < s_2, 0.6, 0.1, 0.3>, < s_3, 0.2, 0.7, 0.1>, < s_4, 0.2, 0.6, 0.2>, < s_5, 0.1, 0.9, 0>};
Q_4 (Stomach problem) = {<s_1, 0.1, 0.7, 0.2>, < s_2, 0.2, 0.4, 0.4>, < s_3, 0.8, 0, 0.2>, < s_4, 0.2, 0.7, 0.1>, < s_5, 0.2, 0.7, 0.1>};
Q_5 (Chest problem) = {<s_1, 0.1, 0.8, 0.1>, < s_2, 0, 0.8, 0.2>, < s_3, 0.2, 0.8, 0>, < s_4, 0.2, 0.8, 0>, < s_5, 0.8, 0.1, 0.1>}.

The aim is to classify pattern P to one of the classes Q_1, Q_2, Q_3, Q_4 and Q_5. Similarly, applying the equation (6) the following result is obtained:

$$C_{IFS}(P, Q_1) = 0.7953,\ C_{IFS}(P, Q_2) = 0.8766,\ C_{IFS}(P, Q_3) = 0.8147,\ C_{IFS}(P, Q_4) = 0.5185,\ C_{IFS}(P, Q_5) = 0.4348.$$

The above result shows that the degree of similarity between Q_2 and P is greater than others. Then, it can assign the patient to the diagnosis Q_2(Malaria) according to the recognition principle. In order to validate the results, two other methods from [12] and [15] are compared with the proposed method. A comparison result between the proposed method and other methods is discussed and listed in Table 1:

Table 1: Diagnosis of patient *P* symptom using different methods

Methods	Patient *P* symptom
Normalized Hamming distance [11]	Malaria
Symmetric discrimination measure for IFSs [12]	Viral fever
Cosine similarity measure [15]	Viral fever
The proposed cosine similarity measure	Malaria

From Table 1, it can be seen that the result from proposed method is consistent with the result from [11]. However, it differs from the result of [12] and cosine similarity measure without considering hesitation degree proposed by Ye [15].

Example 2: The following example discusses the pattern recognition problem about the classification of building materials retrieved from [13].

Given four classes of building material each is represented by the intuitionistic fuzzy sets in the feature space and there is an unknown building material B:

A_1= {<x_1, 0.173, 0.524, 0.303>, <x_2, 0.102, 0.818, 0.080>, <x_3, 0.530, 0.326, 0.144>, <x_4, 0.965, 0.008, 0.027>, <x_5, 0.420, 0.351, 0.229>, <x_6, 0.008, 0.956, 0.036>, <x_7, 0.331, 0.512, 0.157>, <x_8, 1.000, 0.000, 0.000>, <x_9, 0.215, 0.625, 0.160>, <x_{10}, 0.432, 0.534, 0.034>, <x_{11}, 0.750, 0.126, 0.124>, <x_{12}, 0.432, 0.432, 0.136>};

A_2 = {<x_1, 0.510, 0.365, 0.125>, <x_2, 0.627, 0.125, 0.248>, <x_3, 1.000, 0.000, 0.000>,<x_4, 0.125, 0.648, 0.227>, <x_5, 0.026, 0.823, 0.151>, <x_6, 0.732, 0.153, 0.115>, <x_7, 0.556, 0.303, 0.141>, <x_8, 0.650, 0.267, 0.083>, <x_9, 1.000, 0.000, 0.000>, <x_{10}, 0.145, 0.762, 0.093>, <x_{11}, 0.047, 0.923, 0.030>, <x_{12}, 0.760, 0.231, 0.009>};

A_3 = {<x_1, 0.495, 0.387, 0.118>, <x_2, 0.603, 0298, 0.099>, <x_3, 0.987, 0.006, 0.007>,<x_4, 0.073, 0.849, 0.078>, <x_5, 0.037, 0.923, 0.040>, <x_6, 0.690, 0.268, 0.042>, <x_7, 0.147, 0.812, 0.041>, <x_8, 0.213, 0.653, 0.134>, <x_9, 0.501, 0.284, 0.215>, <x_{10}, 0.000, 1.000, 0.000>, <x_{11},0.324, 0.483, 0.193>, <x_{12}, 0.045, 0.912, 0.043>};

A_4 = {<x_1, 1.000, 0.000, 0.000>, <x_2, 1.000, 0.000, 0.000>, <x_3, 0.857, 0.123, 0.020>,<x_4, 0.734, 0.158, 0.108>, <x_5, 0.021, 0.896, 0.083>, <x_6, 0.076, 0.912, 0.012>, <x_7, 0.152, 0.712, 0.136>, <x_8, 0.113, 0.756, 0.131>, <x_9, 0.489, 0.389, 0.122>, <x_{10}, 1.000, 0.000, 0.000>, <x_{11}, 0.386, 0.485, 0.129>, <x_{12}, 0.028, 0.912, 0.06>};

B = {<x_1, 0.978, 0.003, 0.019>, <x_2, 0.980, 0.012, 0.008>, <x_3, 0.798, 0.132, 0.070>,<x_4, 0.693, 0.213, 0.094>, <x_5, 0.051, 0.876, 0.073>, <x_6, 0.123, 0.756, 0.121>, <x_7, 0.152, 0.721, 0.127>, <x_8, 0.113, 0.732, 0.155>, <x_9, 0.494, 0.368, 0.138>, <x_{10}, 0.987, 0.000, 0.013>, <x_{11}, 0.376, 0.423, 0.201>, <x_{12}, 0.012, 0.897, 0.091>}.

Our aim is to justify which class the unknown pattern *B* belongs to. Similarly, applying the equation (6) the following result obtained:

$$C_{IFS}(B, A_1) = 0.6337,\ \ C_{IFS}(B, A_2) = 0.6532,\ C_{IFS}(B, A_3) = 0.7570,\ \ C_{IFS}(B, A_4) = 0.9977.$$

The above result shows that the degree of similarity between A_4 and *B* is greater than others. Then, it can assign the unknown building material *B* to the building material A_4 according to the recognition principle. In order to validate the results, two other methods from [5] and [14] are compared with the proposed method. A comparison result between the proposed method and other methods is discussed and listed in Table 2:

Table 2: Classification of unknown building material using different methods

Method	Unknown Pattern *B* Belonging
Principle of minimum degree of difference between IFSs [13]	A_4
Similarity measure of IFSs based on L_p metric [5]	A_4
Entropy measure for IVIFSs [14]	A_4
The proposed cosine similarity measure	A_4

From Table 2, it can be seen that the result obtained from the proposed method is consistent with the other methods. The unknown pattern *B* belongs to the building material A_4.

Example 3: The following example also discusses the pattern recognition problem about the classification of hybrid mineral retrieved from [13].

Given five kinds of mineral fields, each is featured by the content of six minerals and has one kind of typical hybrid mineral. The five kinds of typical hybrid mineral can be express by five IFSs C_1, C_2, C_3, C_4 and C_5 in the feature space X={x_1,x_2, …., x_6} and there is unknown hybrid mineral B:

C_1= {<x_1, 0.739, 0.125, 0.136>, <x_2, 0.033, 0.818, 0.149>, <x_3, 0.188, 0.626, 0.186>, <x_4, 0.492, 0.358, 0.150>, <x_5, 0.020, 0.628, 0.352>, <x_6, 0.739, 0.125, 0.136>};
C_2= {<x_1, 0.124, 0.665, 0.211>, <x_2, 0.030, 0.825, 0.145>, <x_3, 0.048, 0.800, 0.152>, <x_4, 0.136, 0.648, 0.216>, <x_5, 0.019, 0.823, 0.158>, <x_6, 0.393, 0.553, 0.054>};
C_3= {<x_1, 0.449, 0.387, 0.164>, <x_2, 0.662, 0.298, 0.040>, <x_3, 1.000, 0.000, 0.000>, <x_4, 1.000, 0.000, 0.000>, <x_5, 1.000, 0.000, 0.000>, <x_6, 1.000, 0.000, 0.000>};
C_4= {<x_1, 0.280, 0.715, 0.005>, <x_2, 0.521, 0.368, 0.111>, <x_3, 0.470, 0.423, 0.107>, <x_4, 0.295, 0.658, 0.047>, <x_5, 0.188, 0.806, 0.006>, <x_6, 0.735, 0.118, 0.147>};
C_5= {<x_1, 0.326, 0.452, 0.222>, <x_2, 1.000, 0.000, 0.000>, <x_3, 0.182, 0.725, 0.093>, <x_4, 0.156, 0.765, 0.079>, <x_5, 0.049, 0.986, 0.055>, <x_6, 0.675, 0.263, 0.062>};
B = {<x_1, 0.629, 0.003, 0.068>, <x_2, 0.524, 0.356, 0.120>, <x_3, 0.210, 0.689, 0.101>, <x_4, 0.218, 0.753, 0.029>, <x_5, 0.069, 0.876, 0.055>, <x_6, 0.658, 0.256, 0.086>}.

Our aim is to justify which kind of mineral the unknown hybrid mineral B belongs to. Assume the weights of x_1, x_2,x_3, x_4,x_5 and x_6 are 1/6. By applying equation (7), the following result obtained:

$$C_{WIFS}(C_1,B)=0.8685,$$
$$C_{WIFS}(C_2,B)=0.8256,$$
$$C_{WIFS}(C_3,B)=0.5831,$$
$$C_{WIFS}(C_4,B)=0.9236,$$
$$C_{WIFS}(C_5,B)=0.9432.$$

The above result shows that the degree of similarity between C_5 and B is the largest. Therefore, it is clear that hybrid mineral B should be classified to C_5. This result is in agreement with the ones obtained from [13].

Example 4: In order to demonstrate the applications of the proposed weighted cosine similarity measures for IFSs to pattern recognition, the problem retrieved from [6] is discussed.

There are three known patterns A_1, A_2 and A_3, respectively. The patterns are represented by the following IFSs in the given finite universe $X=\{x_1,x_2,x_3\}$:

A_1 = {<x_1, 1, 0, 0>, < x_2, 0.8, 0, 0.2>, <x_3, 0.7, 0.1, 0.2>};
A_2 = {<x_1, 0.8, 0.1, 0.1>, < x_2, 1, 0, 0 >, <x_3, 0.9, 0, 0.1>};
A_3 = {<x_1, 0.6, 0.2, 0.2>, < x_2, 0.8, 0, 0.2>, <x_3, 1, 1, 0>}.

Given an unknown pattern Q which is represented by the IFS:

Q = {<x_1, 0.5, 0.3, 0.2>, < x_2, 0.6, 0.2, 0.2>, <x_3, 0.8, 0.1, 0.1}>}.

Our aim is to classify the pattern Q to one of the classes A_1, A_2 and A_3. Assume the weights of x_1, x_2 and x_3 are 0.5, 0.3 and 0.2. By applying equation (7), the following result obtained:

$$C_{WIFS}(A_1,Q)=0.8884,\ C_{WIFS}(A_2,Q)=0.9191,\ C_{WIFS}(A_3,Q)=0.9713.$$

The above result shows that the degree of similarity between A_3 and Q is the largest. Therefore, it is clear that pattern Q should be classified to A_3. This result is in agreement with the ones obtained from [6, 8, 12, 15, 17]. A comparison result between the proposed method and the other methods is listed in Table 3. From Table 3, it can be seen that the result from the proposed method is consistent with other methods.

Table 3: Classification of unknown pattern B using different methods

Methods	Unknown Pattern B Belonging
Degree of similarity between IFSs [6]	A_3
Modified Dengfeng-Chuntian similarity measure [8]	A_3
Symmetric discrimination measure for IFSs [12]	A_3
Cross entropy of IVIFSs [17]	A_3
Cosine similarity measure [15]	A_3
The proposed weighted cosine similarity measure	A_3

5 Conclusion

In this paper, the knowledge of degree of membership, non membership and degree of hesitation of IFSs are considered concurrently as the vector representations in vector multiplication. The new cosine similarity measure and weighted cosine similarity measure for IFSs was proposed. It is important to note that the presence of degree of hesitation in the similarity measure has become a new contribution in this paper. Finally, the numerical examples have successfully demonstrated the feasibility of the proposed cosine similarity measure and weighted cosine similarity measure in pattern recognition.

References

[1] Atanassov, K., Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, vol.20, no.1, pp.87–96, 1986.

[2] Bhattacharya, A., On a measure of divergence of two multinomial populations, *The Indian Journal of Statistics*, vol.7, no.4, pp.401–406, 1946.

[3] Chaira, T., and A.K. Ray, A new measure using intuitionistic fuzzy set theory and its application to edge detection, *Applied Soft Computing*, vol.8, no.2, pp.919–927, 2008.

[4] Hung, W.L., and M.S. Yang, Similarity measures of intuitionistic fuzzy sets based on Hausdorff distance, *Pattern Recognition Letters*, vol.25, no.14, pp.1603–1611, 2004.

[5] Hung, W.L., and M.S. Yang, Similarity measures of intuitionistic fuzzy sets based on Lp metric, *International Journal of Approximate Reasoning*, vol.46, no.1, pp.120–136, 2007.

[6] Li, D., and C. Cheng, New similarity measures of intuitionistic fuzzy sets and application to pattern recognition, *Pattern Recognition Letters*, vol.23, nos.1-3, pp.221–225, 2002.

[7] Liang, Z., and P. Shi, Similarity measures on intuitionistic fuzzy sets, *Pattern Recognition Letters*, vol.24, no.15, pp.2687–2693, 2003.

[8] Mitchell, H.B., On the Dengfeng-Chuntian similarity measure and its application to pattern recognition, *Pattern Recognition Letters*, vol.24, no.16, pp.3101–3103, 2003.

[9] Salton, G., and M.J. McGill, *Introduction to Modern Information Retrieval*, McGraw-Hill, Auckland, 1983.

[10] Szmidt, E., and J. Kacprzyk, Entropy for intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, vol.118, no.3, pp.467–477, 2001.

[11] Szmidt, E., and J. Kacprzyk, Intuitionistic fuzzy sets in some medical applications, *Fifth International Conference on Intuitionistic Fuzzy Sets*, vol.4, pp.58–64, 2001.

[12] Vlachos, I.K., and G.D. Sergiadis, Intuitionistic fuzzy information-application to pattern recognition, *Pattern Recognitions Letters*, vol.28, no.2, pp.197–206, 2007.

[13] Wang, W., and X. Xin, Distance measure between intuitionistic fuzzy sets, *Pattern Recognition Letters*, vol.26, no.13, pp.2063–2069, 2005.

[14] Wei, C.P., Wang, P., and Y.Z. Zhang, Entropy, similarity measure of interval-valued intuitionistic fuzzy sets and their applications, *Information Sciences*, vol.181, no.19, pp.4273–4286, 2011.

[15] Ye, J., Cosine similarity measures for intuitionistic fuzzy sets and their applications, *Mathematical and Computer Modelling*, vol.53, nos.1-2, pp.91–97, 2011.

[16] Zadeh, L.A., Fuzzy sets, *Information Control*, vol.8, no.3, pp.338–352, 1965.

[17] Zhang, Q.S., Jiang, S., Jia, B., and S. Luo, Some information measures for interval-valued intuitionistic fuzzy sets, *Information Sciences*, vol.180, no.24, pp.5130–5145, 2010.