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Projective Synchronization of Different Hyper-chaotic Systems by Active Nonlinear Control

Ayub Khan[1], Ram Pravesh Prasad[2,*]
[1]*Department of Mathematics, Zakir Husain Delhi College, University of Delhi, New Delhi -01, India*
[2]*Department of Mathematics, University of Delhi, New Delhi-07, India*

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Abstract

In this paper, we discuss projective synchronization of two identical and different hyper-chaotic systems using active nonlinear control method. The proposed method is applied to achieve projective synchronization of two identical hyper-chaotic Chen and Lu systems. Numerical simulations illustrate the effectiveness and feasibility of the proposed method.

Keywords: projective synchronization, hyper-chaotic Chen and Lu systems, active control

1 Introduction

The idea of chaos synchronization method was first reported by Ott [14], Pecora and Carroll [2, 15]. Chaos synchronization being an interesting phenomenon has been observed in mutually coupled, uni-directionally coupled, and even noise induced chaotic oscillators with wide applications in several fields such as physical systems, chemical systems, ecological systems, secure communications, biological systems, electronic circuits, etc.

In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. A particular chaotic system is called the master or drive system and another chaotic system is known as the slave or response system. The idea of chaos synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of master system asymptomatically. Since the seminal research by Pecora and Carroll on the synchronization of chaotic systems, a number of impressive approaches have been proposed for chaos synchronization such as OGY method, sample-data feedback method, time-delay feedback method, active control method, adaptive control method, back stepping method, sliding mode control method, etc.

So far, many types of chaos synchronization have been presented in the literature, such as complete synchronization [15], phase synchronization [5], generalized synchronization [18], anti-synchronization [4, 25], projective synchronization [8], generalized projective synchronization [11, 22], etc.

Complete synchronization characterized by the convergence of two chaotic trajectories has been observed in mutually coupled, unidirectionally coupled, and even noise induced chaotic oscillators [13, 17, 24]. Phase synchronization is characterized by the fact that the phase difference between two chaotic systems is locked within 2 pie and the amplitudes remains chaotic and uncorrelated [6, 13, 23, 24], generalized synchronization establishes the functional relation between master and slave systems [23]. In coupled partially linear systems, Mainieri and Rehacek investigated the synchronization of two identical systems (drive and response) which synchronize up to scaling factor. This type of chaotic synchronization is referred to as projective synchronization [20]. The phenomenon of anti-synchronization is noticeable in periodic chaotic systems [1, 3]. Recently, Kim et al. [9] and Hu et al. [7] have studied the anti-synchronization phenomena in mutually coupled and unidirectionally coupled identical chaotic systems. In fact the anti-synchronization of two different chaotic systems has greater potential applications than that of the two identical chaotic systems.

Projective synchronization in the form of chaos synchronization has been observed in partially linear chaotic system of four dimensions [12]. In projective synchronization, the phases are locked and the amplitudes of the two coupled systems synchronize up to a scaling factor. The scaling factor is a constant transformation between the

[*] Corresponding author. Email: ram.mbhu@gmail.com (R.P. Prasad).

synchronized variables of the master and slave systems. Projective synchronization is different from generalized synchronization [10, 16], for the slave system, projective synchronization is not asymptotically stable.

Projective synchronization can be identified as coupling of two identical copies of partially linear chaotic systems together. Let the master system drives the slave system through a coupling variable, in a manner that the response of the slave system can automatically track the response of the master system and move in the same dynamical pattern in a proportional scale. Mainieri and Rehacek [12] explained the mechanism of projective synchronization using cylindrical co-ordinates for three dimensional systems. In projective synchronization the time variation of the state vector ratio of the two systems becomes zero, there is the ratio tends to a fixed scaling factor. The scaling factor that characterizes the synchronized dynamics is, however, sensitive to initial conditions and defined on the chaotic variables. Thus, the outcome of the synchronized dynamics becomes unpredictable. The studies of [20, 19, 21] have revealed the manipulation of the scaling factor by introducing control scheme, through which the performance of the synchronized systems can be managed in a preferred way.

The manuscript is organised in seven sections. Section 2 deals with the system description. Section 3 covers the methodology while Section 4 and Section 5 deal with the projective synchronization of two identical hyper-chaotic Chen and Lu systems by active nonlinear control. In Section 6, the projective synchronization of two different hyper chaotic Chen and Lu systems by active nonlinear control has been explained while Section 7 is marked as conclusion.

2 System Description

The Chen system is given by

$$\begin{aligned} \dot{x} &= a(y-x)+w \\ \dot{y} &= dx - xz + cy \\ \dot{z} &= xy - bz \\ \dot{w} &= yz + rw \end{aligned} \tag{2.1}$$

where x, y, z and w are state vectors and a>0, b>0, c>0, d>0 and r>0 are parameters of the system. When a=35, b=3, c=12, d=7 and r=0.5, the system is chaotic which is exhibited in Figure 1.

The Lu system is given by

$$\begin{aligned} \dot{x} &= a(y-x)+w \\ \dot{y} &= -xz + cy \\ \dot{z} &= xy - bz \\ \dot{w} &= xz + rw \end{aligned} \tag{2.2}$$

where x, y, z and w are state vectors and a, b, c and r are positive parameters of the system. The system displays the chaotic behaviour at a=36, b=3, c=20 and r=1.3 which is clear in Figure 2.

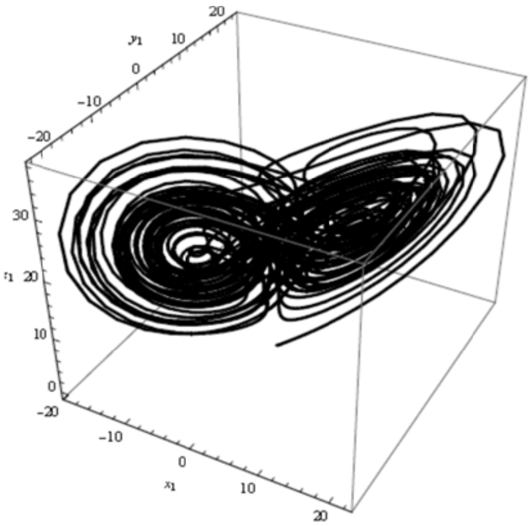

Figure 1: The chaotic behaviour of Chen system at a=35, b=3, c=12, d=7, and r=0.5

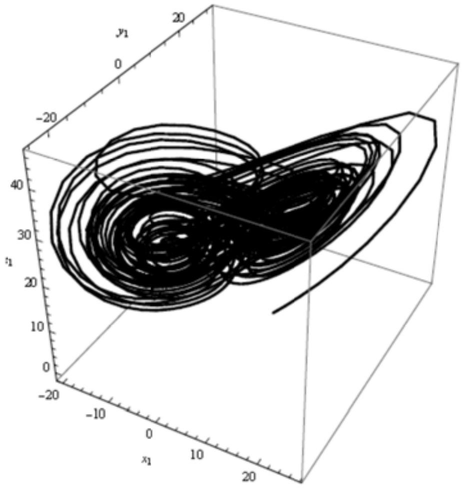

Figure 2: The chaotic behaviour of Lu system at a=36, b=3, c=20, and r=1.3

3 Methodology

In projective synchronization the master and slave vectors synchronize up to a scaling factor α, that is, the vectors become proportional. We define projective synchronization as follows.

Consider the chaotic systems:

$$\dot{x} = f(x) \tag{3.1}$$

where $x \in R^n$ is the state vector of the system and $f : R^n \to R^n$ is the continuous nonlinear vector function. System (3.1) representing the master system and the slave system is

$$\dot{y} = g(y) + u \tag{3.2}$$

where $y \in R^n$ is the state vector of the system (3.2), and $g : R^n \to R^n$ is the continuous non linear vector function, u is the vector controller.

If $f = g$, then x and y are the state vectors of two identical chaotic systems. If $f \neq g$, then x and y are the state vectors two different chaotic systems.

In the nonlinear feedback control approach, we design a feedback controller u, which synchronizes the states of system (3.1) and the slave system (3.2) for all initial conditions $x(0), y(0) \in R^n$ in the form of projective synchronization.

Defining the state error as

$$e = y - \alpha x\,. \tag{3.3}$$

The error dynamics is

$$\dot{e} = g(y) + u - \alpha f(x)\,. \tag{3.4}$$

Thus, the projective synchronization problem is essentially to find a feedback controller u so as to stabilize the error dynamics for all initial conditions $e(0) \in R^n$.

However, to achieve the projective synchronization of (3.1) and (3.2), we design a feedback controller u such that

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} \left\| y - \alpha x \right\| = 0 \text{ for all } e(0) \in R^n, \ \alpha \text{ is a scaling factor.} \tag{3.5}$$

Then the projective synchronization of the systems (3.1) and (3.2) is achieved.

4 Projective Synchronization of Two Identical Hyper-chaotic Chen Systems

We consider the two identical hyper chaotic Chen systems described as the drive system with subscript 1 and the response system with subscript 2:

$$\begin{aligned}\dot{x}_1 &= a(y_1 - x_1) + w_1\\ \dot{y}_1 &= dx_1 - x_1 z_1 + cy_1\\ \dot{z}_1 &= x_1 y_1 - bz_1\\ \dot{w}_1 &= y_1 z_1 + rw_1,\end{aligned} \tag{4.1}$$

$$\begin{aligned}\dot{x}_2 &= a(y_2 - x_2) + w_2 + u_1\\ \dot{y}_2 &= dx_2 - x_2 z_2 + cy_2 + u_2\\ \dot{z}_2 &= x_2 y_2 - bz_2 + u_3\\ \dot{w}_2 &= y_2 z_2 + rw_2 + u_4.\end{aligned} \tag{4.2}$$

Four control functions $u_i, i = 1,2,3,4$ can synchronize two identical systems in the sense of projective synchronization. Here, we define the state errors as $e_1 = x_2 - \alpha x_1, e_2 = y_2 - \alpha y_1, e_3 = z_2 - \alpha z_1, e_4 = w_2 - \alpha w_1$. In order to observe the projective synchronization between systems (4.1) and (4.2). The errors dynamics is developed as

$$\begin{aligned}\dot{e}_1 &= a(e_2 - e_1) + e_4 + u_1\\ \dot{e}_2 &= de_1 + ce_2 - x_2 z_2 + \alpha x_1 z_1 + u_2\\ \dot{e}_3 &= x_2 y_2 - be_3 - \alpha x_1 y_1 + u_3\\ \dot{e}_4 &= y_2 z_2 + re_4 - \alpha y_1 z_1 + u_4.\end{aligned} \tag{4.3}$$

Further, we chose the active control functions as

$$\begin{aligned}u_1 &= V_1\\ u_2 &= x_2 z_2 - \alpha x_1 z_1 + V_2\\ u_3 &= -x_2 y_2 + \alpha y_1 x_1 + V_3\\ u_4 &= -y_2 z_2 + \alpha y_1 z_1 + V_4.\end{aligned} \tag{4.4}$$

Substituting (4.4) and (4.3), we get

$$\begin{aligned}\dot{e}_1 &= a(e_2 - e_1) + e_4 + V_1\\ \dot{e}_2 &= de_1 + ce_2 + V_2\\ \dot{e}_3 &= -be_3 + V_3\\ \dot{e}_4 &= re_4 + V_4.\end{aligned} \tag{4.5}$$

The system (4.5) to be controlled must be a linear system with the control input function $V = [V_1, V_2, V_3, V_4]^T$ as the functions of the error states e_1, e_2, e_3 and e_4. When the error system (4.5) is stabilized by the feedback *V*, the error will converge to zero as $t \to \infty$ which implies that the systems (4.1) and (4.2) are globally synchronized. Choosing *V* as

$$(V_1, V_2, V_3, V_4)^T = A(e_1, e_2, e_3, e_4)^T$$

where A is a 4×4 constant matrix. For the error system (4.5) to be asymptotically stable, the elements of the matrix *A* are chosen so as the error system (4.5) will have all eigen values with negative real parts. Various choices of *A* are possible. For though there are various choices in the selection of matrix, the best possible choice is

$$A = \begin{bmatrix} 0 & -a & 0 & -1\\ -d & -c-1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -r-1 \end{bmatrix},$$

$$\begin{aligned}\dot{e}_1 &= -ae_1\\ \dot{e}_2 &= -e_2\\ \dot{e}_3 &= -be_3\\ \dot{e}_4 &= -e_4.\end{aligned} \tag{4.6}$$

With the choice of matrix *A*, the error system (4.5) becomes linear and has eigen values $-a$, -1, $-b$, -1. This ensures that the error states e_1, e_2, e_3 and e_4 converge to zero as t tends to infinity and this implies the projective synchronization between two identical hyper chaotic Chen systems.

Numerical Results.

Numerical results are presented to demonstrate the effectiveness of the proposed method. The system becomes hyper chaotic when the parameters values are taken as a=35, b=3, c=12, d=7 and r=0.5. The initial conditions of the master and slave systems are $x_1(0) = -0.1, y_1(0) = 0.2, z_1(0) = -0.6, w_1(0) = 0.4,$ $x_2(0) = -1,$ $y_2(0) = 0.4,$ $z_2(0) = -0.2,$ $w_2(0) = 1,$ respectively. Moreover, the scaling factor is taken as $\alpha = 3$. Figure 3 displays the dynamics of the synchronization errors for the master and slave systems. This indicates that two identical hyper-chaotic Chen systems are synchronized in the sense of projective synchronization.

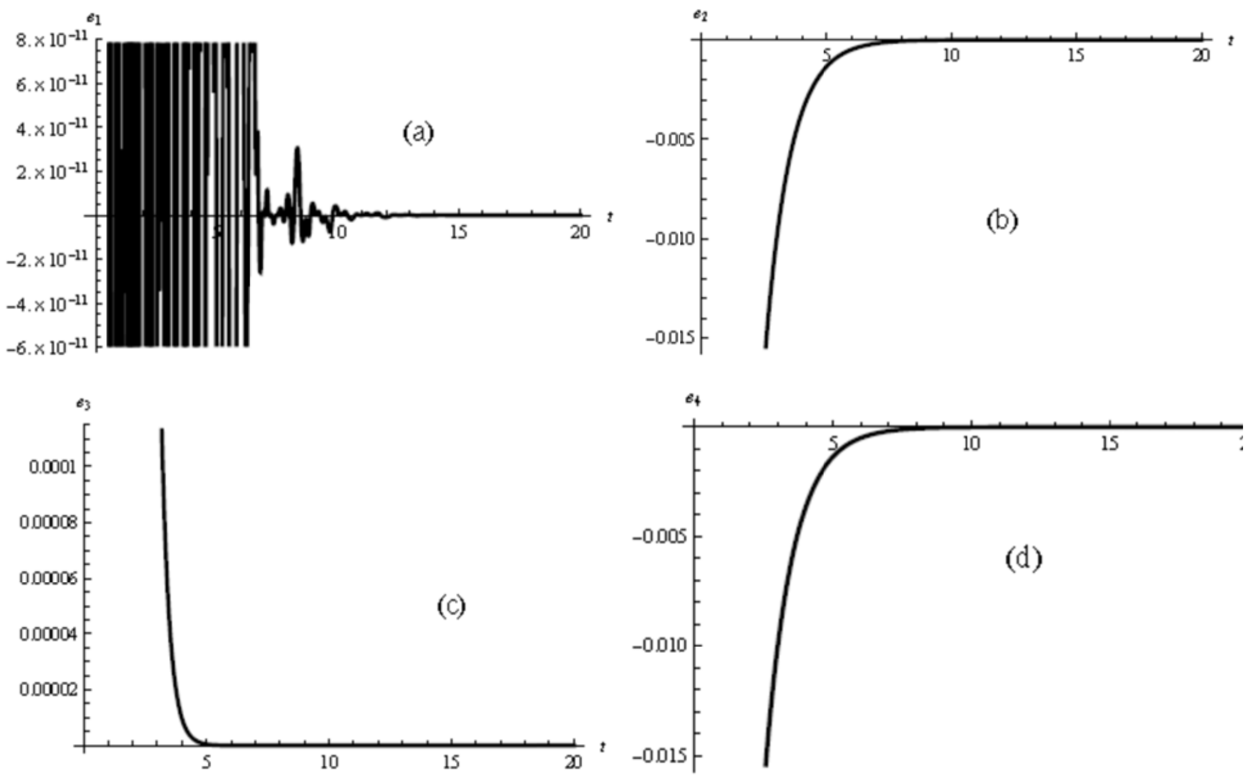

Figure 3: Synchronization errors of (a) $e_1 - t$ (b) $e_2 - t$ (c) $e_3 - t$ (d) $e_4 - t$

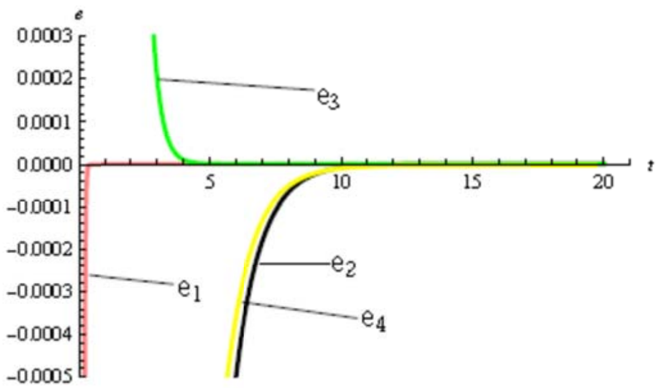

Figure 4: Synchronization error of e_1, e_2, e_3 and e_4

5 Projective Synchronization of Two Identical Hyper-Chaotic Lu Systems

In this section we study projective synchronization between two identical hyper- chaotic Lu system by active nonlinear control. We denote the master system with subscript 1 and slave system with subscript 2 in their following representations:

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) + w_1 \\ \dot{y}_1 &= -x_1 z_1 + c y_1 \\ \dot{z}_1 &= x_1 y_1 - b z_1 \\ \dot{w}_1 &= x_1 z_1 + r w_1, \end{aligned} \tag{5.1}$$

$$\begin{aligned} \dot{x}_2 &= a(y_2 - x_2) + w_2 + u_1 \\ \dot{y}_2 &= -x_2 z_2 + c y_2 + u_2 \\ \dot{z}_2 &= x_2 y_2 - b z_2 + u_3 \\ \dot{w}_2 &= x_2 z_2 + r w_2 + u_4. \end{aligned} \tag{5.2}$$

Four control functions $u_i, i = 1,2,3,4$ are introduced to synchronize the two identical systems in the form of projective synchronization.

We define the state errors as $e_1 = x_2 - \alpha x_1, e_2 = y_2 - \alpha y_1, e_3 = z_2 - \alpha z_1, e_4 = w_2 - \alpha w_1$.

In order to observe the projective synchronization between systems (5.1) and (5.2), we obtain the errors dynamics

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + e_4 + u_1 \\ \dot{e}_2 &= -x_2 z_2 + c e_2 + \alpha x_1 z_1 + u_2 \\ \dot{e}_3 &= x_2 y_2 - b e_3 - \alpha x_1 y_1 + u_3 \\ \dot{e}_4 &= x_2 z_2 + r e_4 - \alpha x_1 z_1 + u_4. \end{aligned} \tag{5.3}$$

Choosing the active control functions

$$\begin{aligned} u_1 &= V_1 \\ u_2 &= x_2 z_2 - \alpha x_1 z_1 + V_2 \\ u_3 &= -x_2 y_2 + \alpha x_1 y_1 + V_3 \\ u_4 &= -x_2 z_2 + \alpha x_1 z_1 + V_4. \end{aligned} \tag{5.4}$$

Substituting (5.4) and (5.3), we get

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + e_4 + V_1 \\ \dot{e}_2 &= c e_2 + V_2 \\ \dot{e}_3 &= -b e_3 + V_3 \\ \dot{e}_4 &= r e_4 + V_4. \end{aligned} \tag{5.5}$$

Thus, the system (5.5) to be controlled is a linear system with the control input function $V = [V_1, V_2, V_3, V_4]^T$ being the functions of the error states e_1, e_2, e_3 and e_4. When the error system (5.5) is stabilized by the feedback *V*, the error will converge to zero as $t \to \infty$ which implies that the systems (5.1) and (5.2) are globally synchronized. We choose *V* as follows:

$$(V_1, V_2, V_3, V_4)^T = A(e_1, e_2, e_3, e_4)^T$$

where A is a 4×4 constant matrix. For the error system (5.5) to be asymptotically stable, the elements of the matrix *A* are chosen so as the error system (5.5) will have all eigen values with negative real parts. From the various possible choices of A, the most adequate choice is

$$A = \begin{bmatrix} 0 & -a & 0 & -1 \\ 0 & -c-1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -r-1 \end{bmatrix}.$$

$$\begin{aligned} \dot{e}_1 &= -ae_1 \\ \dot{e}_2 &= -e_2 \\ \dot{e}_3 &= -be_3 \\ \dot{e}_4 &= -e_4 \end{aligned} \tag{5.6}$$

From system (5.6) it is obvious that the eigen values are -*a*, -1,-*b*, -1. This ensures that the error states e_1, e_2, e_3 and e_4 converge to zero as t tends to infinity and this indicates that the projective synchronization between two identical hyper chaotic Lu systems is achieved.

Numerical Results.

Numerical results are presented to demonstrate the effectiveness of the proposed method. The system becomes hyper chaotic when the parameters values are taken as *a*=36, *b*=3, c=20 and *r*=1.3 also the initial conditions of the master and slave systems are $x_1(0)=5, y_1(0)=8, z_1(0)=-1, w_1(0)=-3,$ $x_2(0)=-2, y_2(0)=-3, z_2(0)=8, w_2(0)=10,$ respectively. Moreover, the scaling factor is taken as $\alpha = 2$. Figure 4 displays the dynamics of the synchronization errors for the master and slave systems. This shows that two identical hyper-chaotic Lu systems can be synchronized in the sense of projective synchronization.

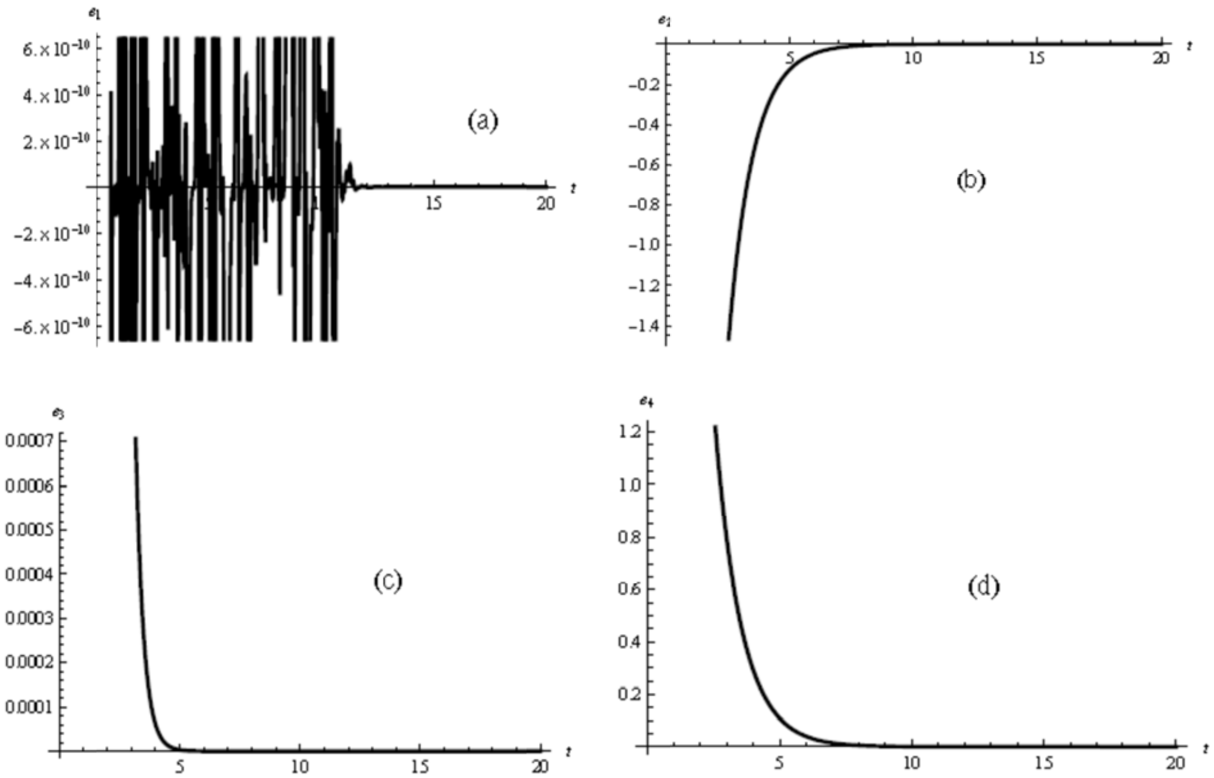

Figure 5: Synchronization errors of (a) $e_1 - t$ (b) $e_2 - t$ (c) $e_3 - t$ (d) $e_4 - t$

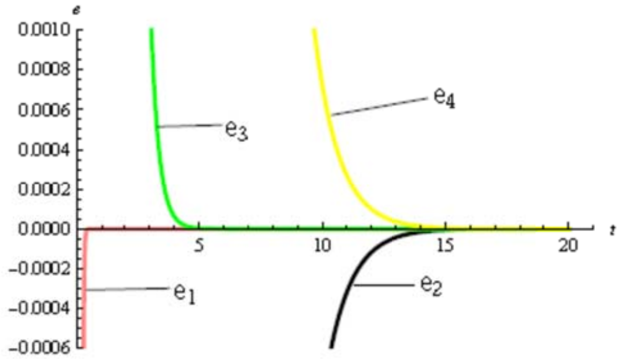

Figure 6: Synchronization error of e_1, e_2, e_3 and e_4

6 Projective Synchronization of Two Different Hyper-chaotic Systems

In this section we study projective synchronization between hyper-chaotic Chen and hyper chaotic Lu systems by active nonlinear control method. Here, we consider the Chen system (master system) and Lu system (slave system) as

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) + w_1 \\ \dot{y}_1 &= dx_1 - x_1 z_1 + cy_1 \\ \dot{z}_1 &= x_1 y_1 - bz_1 \\ \dot{w}_1 &= y_1 z_1 + rw_1, \end{aligned} \tag{6.1}$$

$$\begin{aligned} \dot{x}_2 &= a(y_2 - x_2) + w_2 + u_1 \\ \dot{y}_2 &= -x_2 z_2 + cy_2 + u_2 \\ \dot{z}_2 &= x_2 y_2 - bz_2 + u_3 \\ \dot{w}_2 &= x_2 z_2 + rw_2 + u_4. \end{aligned} \tag{6.2}$$

The control functions $u_i\,(i = 1,2,3,4)$ can synchronize two different hyper-chaotic Chen and Lu systems in the sense of projective synchronization.

We define the state errors $e_1 = x_2 - \alpha x_1, e_2 = y_2 - \alpha y_1, e_3 = z_2 - \alpha z_1, e_4 = w_2 - \alpha w_1$ and then we obtain the errors dynamics as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + e_4 + u_1 \\ \dot{e}_2 &= -x_2 z_2 + ce_2 - \alpha dx_1 + \alpha x_1 z_1 + u_2 \\ \dot{e}_3 &= x_2 y_2 - be_3 - \alpha x_1 y_1 + u_3 \\ \dot{e}_4 &= x_2 z_2 + re_4 - \alpha y_1 z_1 + u_4. \end{aligned} \tag{6.3}$$

Choosing the active control functions

$$\begin{aligned} u_1 &= V_1 \\ u_2 &= x_2 z_2 + \alpha dx_1 - \alpha x_1 z_1 + V_2 \\ u_3 &= -x_2 y_2 + \alpha x_1 y_1 + V_3 \\ u_4 &= -x_2 z_2 + \alpha y_1 z_1 + V_4. \end{aligned} \tag{6.4}$$

Substituting (6.4) in (6.3), we get

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + e_4 + V_1 \\ \dot{e}_2 &= ce_2 + V_2 \\ \dot{e}_3 &= -be_3 + V_3 \\ \dot{e}_4 &= re_4 + V_4. \end{aligned} \tag{6.5}$$

Thus, the system (6.5) to be controlled is a linear system with the control input function $V = [V_1, V_2, V_3, V_4]^T$ and the functions of the error states e_1, e_2, e_3 and e_4. When the error system (6.5) is stabilized by the feedback *V*, the error will converge to zero as $t \to \infty$ which implies that the systems (6.1) and (6.2) are globally synchronized. We choose *V* such that

$$(V_1, V_2, V_3, V_4)^T = A(e_1, e_2, e_3, e_4)^T$$

where *A* is a 4×4 constant matrix. For the error system (6.5) to be asymptotically stable, the elements of the matrix *A* are chosen so that the error system (6.5) will have all eigen values with negative real parts. Out of various choices of *A* the best choice is

$$A = \begin{bmatrix} 0 & -a & 0 & -1 \\ 0 & -c-1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -r-1 \end{bmatrix}.$$

Hence the error system (6.5) becomes into

$$\begin{aligned} \dot{e}_1 &= -ae_1 \\ \dot{e}_2 &= -e_2 \\ \dot{e}_3 &= -be_3 \\ \dot{e}_4 &= -e_4. \end{aligned} \tag{6.6}$$

For this the error system (6.5) becomes linear and has eigen values –*a*, -1, -*b*, -1. This ensures that the error states e_1, e_2, e_3 and e_4 converge to zero as t tends to infinity and this implies that the projective synchronization between two different hyper chaotic Chen and Lu systems is achieved.

Numerical Results.

Numerical results are presented to demonstrate the effectiveness of the proposed method. The two systems becomes hyper chaotic when the parameters values are taken as a = 35, b = 3, c = 12, d = 7, r=0.5, and a=36, b=3, c=20, r=1.3, respectively, and the initial conditions of the master and slave systems are $x_1(0) = -1,\ y_1(0) = 0.4,$ $z_1(0) = -0.2, w_1(0) = 1, x_2(0) = 6, y_2(0) = 0.5, z_2(0) = 4, w_2(0) = -9,$ respectively. Moreover, the scaling factor is taken as $\alpha = 2$. Figure 5 displays the dynamics of the synchronization errors for the master and slave systems. This shows that two different hyper-chaotic Chen and Lu systems are synchronized in the sense of projective synchronization.

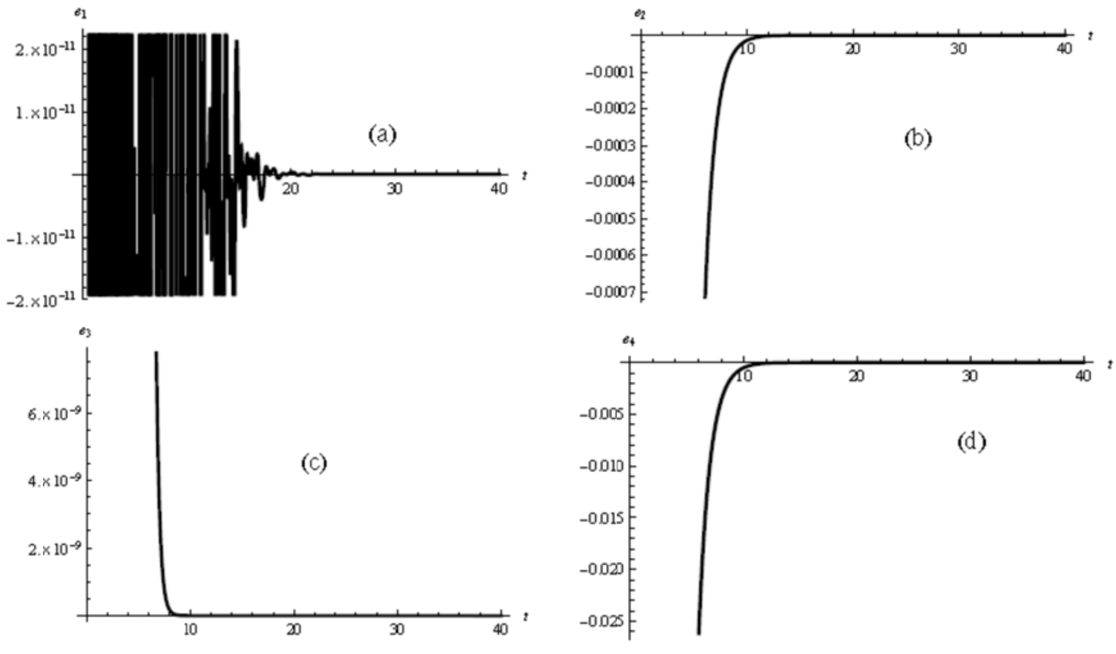

Figure 7: Synchronization errors of (a) $e_1 - t$ (b) $e_2 - t$ (c) $e_3 - t$ (d) $e_4 - t$

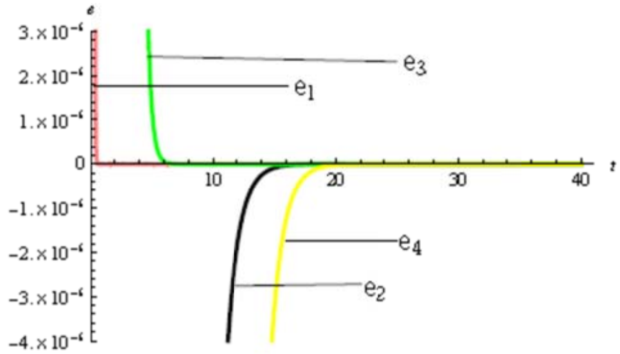

Figure 8: Synchronization error of e_1, e_2, e_3 and e_4

7 Conclusion

The projective synchronization between two identical hyper-chaotic systems and two different hyper-chaotic systems via active nonlinear control method has been achieved. The designed controller is applied to synchronize two indentical hyper-chaotic systems. Moreover, the active controller is applied to obtain the synchronization between hyper-chaotic Chen and Lu systems in the sense of projective synchronization. Numerical simulations show the effectively and feasibility of the proposed method.

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