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Under-investment Problem with Adverse Selection in Uncertain Environment

Yinbo Lu[1], Zhiyong Huang[2,*], Jinwu Gao[2]

[1] *Hanqing Advanced Institute of Economics and Finance, Renmin University of China, Beijing 100872, China*

[2] *Department of mathematics, School of information, Renmin University of China, Beijing 100872, China*

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Abstract

This paper considers the under-investment problem with adverse selection in the framework of uncertain theory. We first construct a basic model which brings in uncertain distributions as the characteristics of projects' return. Then we consider two strategies, ex-ante monitoring to rate the borrowers' credit and peer-group formation with joint liability, to solve the under-investment problem. Our analysis show that in certain circumstance, these strategies are valid in the environment of uncertain theory.

Keywords: uncertain distribution, uncertain random variable, adverse selection

1 Introduction

The adverse selection models in the field of investment, proposed by Stiglitz and Weiss [9], have triggered hot discussion in academic community. It says that in an economy with both safe and risky borrowers, the interest rates are often prohibitively high as the banks are not able to distinguish different borrowers. As a result, the safe borrowers, the projects of which have higher probability to succeed, are always driven out of the credit market. Thus, only risky borrowers will exist in the credit market and both of the rate of investment in the economy and the repayment rate are very low. Such phenomena are common in poor areas of development countries and still have not been overcome by traditional commercial banks.

However, the recent success of the Grameen Bank in Bangladesh has raised great interest on explaining the reasons behind it and intense hope about popularizing such model in other poor areas. In practice, Grameen has some innovations, including the mechanism of peer group with joint liability and active monitoring in forms of weekly meetings. As a result, the repayment rate in Banladeshi rural areas is higher than 90%.

The existing literature trying to explain the Grameen's success mainly focus on the role of peer group formation. Armendariz and Gollier [1] present a model where the banks need to pay a verification cost when the loans cannot recover and a mechanism that borrowers form group of two with random matching and undertake joint liability when the other member defaults. The reduction in costs coming from verification can effectively reduce the interest rate. Ghatak [4] points out that with random matching to form joint liability, the borrowers' payoff actually remain the same as in the case of individual liability. Natarajan [8] puts forward a signalling mechanism from which the assortive matching can be available and as a result the safe borrowers' payoff is improved. Laffont and N'Guessan [6] declare that the collateral effect of group lending can only appear when the borrowers know each other from the aspect of maximizing social welfare under budget balance constraints for the banking sector.

Some other literature both motivate this paper's construction and extend the analysis framework of under-investment problems worth much attention. Ghatak and Guinnane [5] summarize that besides adverse selection, three other problems are of equal importance: the moral hazard, auditing costs and enforcement. And a well-structured liability contract can deal with all the four major problems by making use of local information of borrowers. Ghatak [3] models the projects' risk in a continuous range and thus different interest rates can result in different repayment rates. With local information, the borrowers in a self-selected group have projects with the same probability of success. Accordingly, the interest rates can be reduced and the

*Corresponding author. Email: Huangzhy@ruc.edu.cn (Z. Huang).

repayment rates can be largely improved. Chowdhury [2] creatively develops a model about moneylenders' monitoring and moral hazard. By active monitoring, the banks can enforce the borrowers to utilize the loan properly with a certain probability.

This paper reconsiders the adverse selection problems in the framework of uncertain theory [7]. For the best of our knowledge, no researchers have applied the uncertain theory to the adverse selection problems. The reason of using uncertain theory rather than probability theory in modeling involves some fundamental arguments. That is, when no sample are available, how can we estimate the probability distribution? Specifically, how can we estimate the probability of success of a project? When the banks only have part information about the borrowers, how to build a model? The advantage of using uncertain theory is that we can rationally deal with our belief degrees and reach appropriate results.

The reminder of this paper is structured as follows. Section 2 outlines the basic model. We give the economic environment in detail and assume that the return of projects have continuous uncertain distributions rather than binomial probability distributions. Section 3 pins down the under-investment problem with adverse selection in individual liability lending. In Section 4, we present a mechanism that by ex-ante monitoring, banks get some information about the borrowers and set up multiple interest rates according to different cases. Our analysis show that comparing to probability model, the uncertain model can better describe the acquired information about the borrowers' projects. As a result, safe borrowers are able to face lower interest rates. In Section 5, we briefly build the model of joint liability under uncertain theory. Our conclusion is similar to existing literature that the collateral effect is obvious with assortive matching. Section 6 gives a summary of this paper and looks forward to what need to be done in further research.

2 The Basic Model

We consider a credit market where the borrowers are a continuum of size 1 households and lenders are banks. The number of banks is large so that each earns zero profit. We assume that the cost of 1 unit capital is ρ and the supply of capital is relatively abundant to the demand, so there is no need to ration the credit. Each household has one project in hand and needs one unit of capital and labor to enter it. The labor is self-committed but the capital has to be borrowed from the banks. We also assume that the households have no other wealth as collateral and the borrowed capital can only be payed back through the return of their projects.

There are two groups of households, according to the type of their projects. We index them as safe(s) and risky(r), respectively. The return of projects are uncertain variables and we use two uncertain distributions to characterize them. For the safe, the return ξ_s has a linear uncertain distribution, $L(a, b)$.

$$\Phi_s(x) = \begin{cases} 0, & if\, x \le a \\ (x-a)/(b-a), & if\, a \le x \le b \\ 1, & if\, x \ge b. \end{cases}$$

For the risky, the return ξ_r has a zigzag uncertain distribution, $Z(a, c, d)$.

$$\Phi_r(x) = \begin{cases} 0, & if\, x \le a \\ (x-a)/2(c-a), & if\, a \le x \le c \\ (x+d-2c)/2(d-c), & if\, c \le x \le d \\ 1, & if\, x \ge d. \end{cases}$$

We assume

$$a < c < (b+a)/2, d > b.$$

So that for the safe, they have 50% certainty that their projects will yield a return at least $(b+a)/2$, larger than that of the risky. For the risky, the maximum they can get from the projects is larger than that of the safe. As for the expected returns, we impose

$$E(\xi_s) = E(\xi_r) = \bar{R} > 1.$$

So that

$$(a+b)/2 = (a+2c+d)/4.$$

Thus in our assumptions, risky and safe households have the same mean return, but the risky projects have a greater spread than the safe projects, consistent with the suggestion of Stiglize & Weiss [9].

In the population, we suppose that the proportion of safe households is λ and others are risky households. Each household's project can be carried out independently. Each household also has an opportunity cost U to enter the project, where U is smaller than $\bar{R}$. That is, if they do not enter the project, they can provide their labor to other firms and get a return U. And both the banks and households are risk-neutral. The banks have no information of the households, thus cannot identify whether they are safe or risky. But the households know each other.

The moral risky is absent but to simplify our analysis, we impose the assumption that the households can always do worse than real outcomes. That means when they with certainty 1 feel that the return will be less than the opportunity cost U, they choose do nothing and default. The economic intuition behind is not hard to understand. For instance, the projects are planting vegetables to sell. Before harvest, the given market price of the certain vegetables are very low such that even if they do their best to get all sold, they cannot get as much as U. Thus they choose do nothing and just let the vegetables in the field and default. To emphasis, we formally state it as below.

Assumption 1. For a given interest rate, the households who borrowed the money either repay it fully when the return is larger than U or default when the return is smaller than U.

3 Individual Liability Lending

Now we consider how the credit market will reach equilibrium given the assumptions above. For the banks, they have to set the interest rate at head and the households are price-takers. With the zero profit condition(ZPC), the interest rate should ensure the banks to make both ends meet in equilibrium. If the banks have full information about the the households, they can set two different interest rates aiming at different groups of households. That is,

$$r_s(1-\Phi_s(U))=\rho, \tag{1}$$

$$r_r(1-\Phi_r(U))=\rho. \tag{2}$$

In equilibrium where both risky and safe households are borrowers, both r_s and r_r can not exceed U. Now we can prove that $r_s < r_r$, that is, the safe households enjoy a lower rate of interest as they are more likely to repay the loan.

Proposition 1 *If the banks have full information about the types of households, then we have $r_s < r_r$.*

Proof: We only need to prove that $\Phi_s(U) < \Phi_r(U)$. It is easily followed by

$$\Phi_s(U)-\Phi_r(U)=(U-a)\frac{2c-a-b}{2(b-a)(c-a)}<0.$$

QED.

However, the banks do not have the information to identify different households. So, they have to set an average interest rate to every households. The formula is given by

$$\bar{r}[\lambda(1-\Phi_s(U))+(1-\lambda)(1-\Phi_r(U))]=\rho. \tag{3}$$

Obviously, we find that $r_s < \bar{r} < r_r$. Under this situation, the safe face a higher interest rate than the case of full information as they have to partially undertake the risk of the risky projects. To ensure both groups of households to apply for the loan, two other constraints are necessary. That is, the pay-off of the households must be larger than their opportunity cost U. It follows that $E(\xi_s)-(1-\Phi_s(U))\bar{r} > U$ and $E(\xi_r)-(1-\Phi_r(U))\bar{r} > U$.

As $(1-\Phi_s(U)) > (1-\Phi_r(U))$, we can conclude that if the safe households are in the credit market, then so do the risky. But there exists a situation where there is only risky households in the credit market. We define the maximum interest rate at which the safe household will give up the loan as r^*. Formally,

$$r^*=(E(\xi_s)-U)/(1-\Phi_s(U)).$$

Clearly, if $\bar{r} > r^*$, $\bar{r}$ cannot result in an equilibrium. In this case, safe borrowers will exit the market and the equilibrium interest rate would be

$$r_r = \rho/(1 - \Phi_r(U)).$$

and the repayment rate is equal to $(1 - \Phi_r(U))$. This situation is referred to as the under-invest problem in credit markets with adverse selection.

4 Individual Liability with Ex-ante Monitoring

Now we consider a mechanism that the banks do some ex-ante monitoring to get some information about the types of households and set interest rates according to the results. We analyze two models about the mechanism. Firstly, with some monitoring cost, the banks can have a probability get full information about the households. Secondly, the banks identify the borrower as a certain type with some certainty, and thus give them a credit rating and set various interest rates.

4.1 Ex-ante Monitoring in the Framework of Probability

Let us describe the monitoring process.

STEP1. The households apply for the loan, and the banks spend an amount of $\theta m^2/2$ to obtain information regarding the identity of the projects with probability $min(m, 1)$. Here $\theta > 0$ is constant and indicates the level of difficulty of obtaining information. m is a choice variable (Here the functional form of money spent and information obtained comes from [2]).
STEP2. If the banks successfully identify the type of projects, the rate of interest would be r_r or r_s defined in equation (1)-(2). Otherwise, the interest rate would be the average rate $\bar{r}$ defined in equation (3).
STEP3. The applicants decide whether to accept the loan.

Thus for the banks, this strategy can be described as a combination $(r_r, r_s, \bar{r}, m)$. The only variable parameter to be determined is m by the constraint of ZPC. For the households, the safe and risky face different expected interest rates, since their projects both have a probability $min(m, 1)$ to be identified by the banks. If in equilibrium both types apply for the loan, the formula to determine m would be

$$\lambda[mr_s + (1-m)\bar{r}](1 - \Phi_s(U)) + (1-\lambda)[mr_r + (1-m)\bar{r}](1 - \Phi_r(U)) = \rho + \theta m^2/2. \quad (4)$$

Note that we have replaced the $min(m, 1)$ with m. That is because m is at most 1 to be helpful to the banks. We can prove that there is a unique solution in equation (4).

Proposition 2 *The unique m in $[0, 1]$ of the solution of equation (4) is zero.*

Proof: Let $f(m) = RHS - LHS$, where $RHS(LHS)$ is the right(left) side of equation (4). Obviously, LHS is a combination of equation (1)-(3). Thus LHS $=\rho$ and $f(m) = \theta m^2/2$. The only solution is $m = 0$. QED.

Intuitively, since the banks pay a monitoring cost, it has some probability to know whether the household is safe or risky. So defining different interest rates to different monitoring results is reasonable. However, our analysis show that the only practical method is to pay no monitoring cost and give every households the same interest rates with ZPC.

The solution to such a dilemma is to transfer the monitoring cost to either the borrowers or the government. For instance, if the banks successfully identify the type of projects as risky, then set the interest rate as $r_r + \epsilon$, where ϵ is a positive number. As long as the expected interest rate faced by the safe borrowers are lower than that in the case of individual liability, such a strategy is valid. In practice, the governments can afford the monitoring cost instead of the banks and then the safe households would face a much lower interest rate.

4.2 Ex-ante Monitoring in the Framework of Uncertainty

In reality, the ex-ante monitoring may only give the banks a certain belief that whether the projects are safe or risky. In the framework of uncertainty, we can describe such intuition easily. Now let us describe again the monitoring process.

STEP1. The households apply for the loan, and the banks spend an amount of $\theta m^2/2$ to obtain information regarding the projects. Specifically, the return with 50 percentage certainty. According to our assumption, the safe have a project with 50 percentage certainty at least $(a+b)/2$ and that of the risky is c, smaller than$(a+b)/2$. We use $\xi(\eta, \xi_s^b, \xi_r^b)$ as the uncertain random variable to characterize it. Where η is a random variable indicating the true type of projects, and ξ_s^b, ξ_r^b are uncertain variables symbolling the banks' information about the projects' return with 50 percentage certainty, respectively. And the superscript b means banks.
STEP2. According to ξ, the banks set three different interest rates. As ξ may be values that only a certain type of project can reach or that both types of projects can reach.
STEP3. The applicants decide whether to accept the loan.

Now let us specify the uncertain random variable ξ. To simplify the question, we restrict its value in $[c, (a+b)/2]$. When no monitoring cost is spent, the banks would expect every project' return with 50 percentage certainty falls in $[c, (a+b)/2]$ uniformly. So $L(c, (a+b)/2)$ is an appropriate uncertain distribution when the cost is zero. When monitoring cost is a positive number, we would expect that for different type of projects, the banks' monitoring results would have different uncertain distributions. Thus we model ξ as follows:

$$\xi(\eta, \xi_s^b, \xi_r^b) = I_{(\eta=safe)}\xi_s^b + I_{\eta=risky}\xi_r^b, \tag{5}$$

$$\xi_s^b = L(c+m, (a+b)/2), \tag{6}$$

$$\xi_r^b = L(c, (a+b)/2 - m), \tag{7}$$

where $I_{(\cdot)}$ is the indicator function.

Now let us consider the contract of interest rate r. When $\xi \in [c, c+m]$, η can only be risky; when $\xi \in [(a+b)/2 - m, (a+b)/2]$, η can only be safe. So we define the formula of r as follows

$$r = \begin{cases} r_1, & if\, c \le \xi \le c+m \\ r_2, & if\, c+m \le \xi \le (a+b)/2 - m \\ r_3, & if\, (a+b)/2 - m \le \xi \le (a+b)/2. \end{cases}$$

To determine the values of (r_1, r_2, r_3), we apply the ZPC. Thus

$$\begin{aligned} & r_1 \cdot \nu_\xi\{c \le \xi \le c+m\} \cdot \nu_{\xi_r}\{\xi_r \ge U\} + r_3 \cdot \nu_\xi\{(a+b)/2 - m \le \xi \le (a+b)/2\} \cdot \nu_{\xi_s}\{\xi_s \ge U\} \\ & \quad + r_2 \cdot \nu_\xi\{c+m \le \xi \le (a+b)/2 - m\} \cdot \{Pr(\eta = safe) \cdot \nu_{\xi_s^b}\{\xi_s^b \ge c+m\}\} \\ & \quad\quad + r_2 \cdot \nu_\xi\{c+m \le \xi \le (a+b)/2 - m\} \cdot \{Pr(\eta = risky) \cdot \nu_{\xi_r^b}\{\xi_r^b \le (a+b)/2 - m\}\} \\ & \quad\quad\quad = \rho + \theta m^2/2 \end{aligned} \tag{8}$$

where ν symbol the uncertain measure of uncertain variables or uncertain random variables.

To see how the under-investment problem is solved in such a mechanism, we give the following proposition.

Proposition 3 *In equation (8), there are three free parameters. And to get the safe type of households in the market in equilibrium and simplify the analysis, we can increase the* r_1 *and at the same time make* r_2 *and* r_3 *relatively small, specifically, smaller than* r^**, the maximum interest rate at which the safe household would give up the loan.*

5 Joint Liability with Assortive Matching

Now we consider the strategy that banks lend capital on the basis of joint liability mechanism. Borrowers form groups of two assortive and if one member of the group defaults, the other have to assume an extra liability. Each type of households will search the same type of households to form a group and thus only $(safe, safe)$ and $(risky, risky)$ will become the borrowers. The contract of loans becomes a pair (r, c), where r is the interest rate and c is the amount of liability when the other defaults. We can think of c as collateral.

To be consistent with assumption 1, we still assume that each household either pay back all the liability or defaults. That is, if the projects have non-zero return which are not enough to pay the liability(including the possible extra liability caused by the partner's default), the households will always give up the return and leave default records. We state that as follows.

Assumption 2. In the assortive matching, there is three possible results about the loans' repaying. First, both repay r and r is smaller than U. Second, one pays $r+c$ and the other pays zero, as the other member feel that his project can only get a return smaller than r. Third, both defaults. Reasons can be that both members can only get a return smaller than r or that one can only get a return smaller than r and the other get a return smaller than $r+c$.

Now given the contract structure and the additional assumptions, we can easily write the ZPC. Without loss of generality, we first analysis the situations facing the group $(safe, safe)$, marking the two members as 1 and 2.

Case 1. Both pay r. $\nu\{case1\} = \nu\{\xi_1 \geq U \wedge \xi_2 \geq U\} = \nu\{\xi_1 \geq U\} \wedge \nu\{\xi_2 \geq U\} = 1 - \Phi_{safe}(U)$.
Case 2. Member 1 pays $r+c$ and member 2 defaults. Denote $\omega = maximum(r+c, U)$,

$$\nu\{case2\} = \nu\{\xi_1 \geq \omega \wedge \xi_2 \leq U\} = \nu\{\xi_1 \geq \omega\} \wedge \nu\{\xi_2 \leq U\}.$$

If we further assume that $(r+c) \leq \bar{R}$, we can simplify the equation above as $\nu\{case2\} = \nu\{\xi_2 \leq U\} = \Phi_{safe}(U)$.
Case 3. Member 1 defaults and member 2 pays $r+c$. Similar to case 2, we have $\nu\{case3\} = \nu\{case2\}$.
Case 4. Both default.

$$\begin{aligned}\nu\{case4\} &= \nu\{(\xi_1 \leq U \wedge \xi_2 \leq U) \cup (\xi_1 \leq \omega \wedge \xi_2 \leq U) \cup (\xi_1 \leq U \wedge \xi_2 \leq \omega)\} \\ &= \nu\{(\xi_1 \leq \omega \wedge \xi_2 \leq U) \cup (\xi_1 \leq U \wedge \xi_2 \leq \omega)\} \\ &\leq \nu\{\xi_1 \leq \omega \wedge \xi_2 \leq U\} + \nu\{\xi_1 \leq U \wedge \xi_2 \leq \omega\} \\ &= 2\Phi_{safe}(U).\end{aligned}$$

The analysis of the group $(risky, risky)$ are similar. So the ZPC now becomes:

$$\lambda\{\nu_{s,s}\{case1\} \cdot r + \nu_{s,s}\{case2\} \cdot (r+c)\} + (1-\lambda)\{\nu_{r,r}\{case1\} \cdot r + \nu_{r,r}\{case2\} \cdot (r+c)\} = \rho. \quad (9)$$

Proposition 4 *If we have the assumption that* $(r+c) \leq \bar{R}$*, then equation (9) becomes*

$$\lambda c \Phi_{safe}(U) + (1-\lambda) c \Phi_{risky}(U) = \rho.$$

That means ZPC has nothing to do with r *if condition* $(r+c) \leq \bar{R}$ *is satisfied. We can have a small* r *and a relatively large* c *to attract all the potential borrowers into the credit market.*

Proof: It is obvious with the analysis of case1-4. QED.
The intuition behind is that the safe borrowers have a higher chance to pay back the loan, thus they hope a low interest rate and a high collateral when they can organize themselves into the group with another safe borrower. On the contrast, the risky borrowers have a higher chance to default and thus they hope a high interest rate and a low collateral. With the mechanism of assortive matching, the banks can choose appropriate combination of r and c to make sure both types of borrowers would exsit in the credit market in equilibrium.

6 Concluding Remarks

In this paper, we reconsider the under-investment problem with adverse selection under the framework of uncertain theory. To pin down the adverse selection problem, we first rebuild the basic model which explains reasons of adverse selection. We apply two types of uncertain distributions to describe the return of different projects. And to simplify the analysis, we assume that borrowers will pay back the loans only when the return is larger than the opportunity cost. Such an assumption though has its economic explanation in reality but is not necessary. In further research, we can remove it and try to give similar results with more complex mathematical expression.

Given the basic model, the remaining analysis is not hard to go through. With ex-ante monitoring, we can either assume that banks can get full information about the borrowers with a certain probability, or assume that banks can obtain part information about each borrower. Given the latter assumption, the banks can set three different interest rates and ensure that safe borrowers will exist in the credit market. With joint liability, the difference between uncertain theory and probability theory appears when considering the measure of both losing the ability to pay back the loans. However, the conclusion is similar. The collateral effect exists when the peer group is self-selected with assortive matching.

Our framework could be extended in various aspects. The borrowers may not know each other and thus the group formation is random, or the borrowers may have to pay some cost to get the information about potential partners. The verification cost could also be added when the borrowers default. Furthermore, in poor areas policy makers are interested in improving the investment rate and thus the goal of maximizing the social welfare is of much importance. We can build a model based on maximizing the social welfare with policy subsidies.

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