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Global Production Planning with Fuzzy Exchange Rates

Jie Sun*
College of Mathematics and Computer Science, Hebei University, Baoding 071002, Hebei, China

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Abstract

This paper presents a new two-stage fuzzy optimization method for global production planning problem, in which exchange rates are characterized by fuzzy variables with known possibility distributions. For this purpose, we first introduce the measure generated by credibility distributions. Then we use the generated measure to define the Lebesgue-Stieltjes (L–S) integral of function of fuzzy variables. When demands are deterministic, we decompose the feasible regions into several subregions, and discuss the solution properties of the proposed optimization model in each subregion. According to the obtained theoretical results, we design a decomposition solution method for the proposed production planning problem. Finally, one numerical example is presented to demonstrate the developed new optimization method.

Keywords: production planning, fuzzy programming, L-S integral, domain decomposition

1 Introduction

Production planning is an overall plan in manufacturing systems. The traditional production planning is a simple material production process. With the continuous development of social economy and the increasingly fierce market competition, production planning is now a complex process, in which an enterprise should consider a variety of factors like market demands, cost information and capacity data that may affect the production planning. In recent years, the global production planning becomes highly conspicuous. Many enterprises increase their output by using favorable exchange rates, foreign economic policies and potential market demands. Due to the uncertain demands and exchange rates, the global production planning under uncertainty is an important issue for research. For example, Jucker et al. [7] considered the uncertain demand and price into their plant-location problem, they were the first to incorporate uncertain exchange rates in the context of an uncapacitated plant location problem. Hodder [5] introduced a capital market approach with an exchange rate mean-covariance objective function. Flaherty [3] considered the uncertain exchange rates and replaced the parameters by their expected values. Huchzermeier and Cohen [6] incorporated exchange rate into a global supply chain, and employed a recourse-after-capacity model in selecting the design of the supply chain. Rosenfield [15] addressed the planning capacity and facilities in the environment of uncertain exchange rates. Ding et al. [2] considered one production planning under uncertain exchange rate and demand with a risk and introduced a mean-variance utility function for risk aversion.

Since the pioneering work of Zadeh [18], fuzzy theory was being perfected and became a strong tool to deal with possibilistic uncertainty [8, 10, 19]. Since then a number of researchers have applied fuzzy theory into the production decision systems. For instance, Partha et al. [14] used fuzzy differential equation and fuzzy Riemann-integration to formulate a production inventory model. Halim et al. [4] considered a single-unit unreliable production system and developed two production planning models, they employed the graded mean integration representation methods to solve the problem. Sun et al. [16, 17] studied material procurement planning problems by using two-stage fuzzy optimization methods [11], they designed approximation-based heuristic algorithms to solve the proposed material procurement planning model. For recent development about two-stage fuzzy optimization methods, the interested reader may refer to [9, 12, 13].

Motivated by the work mentioned above, in the current development, we present a new two-stage fuzzy optimization method for global production planning problem, in which uncertain exchange rates are characterized by possibility distributions. Different from the expected value method in the literature, we employ

*Corresponding author. Email: sunjieweiwei@163.com (J. Sun).

L–S integral [1] to construct the objective function. The L–S integral is based on credibility distributions of fuzzy exchange rates. When demands are deterministic, we discussed the solution properties of our optimization model by decomposing the feasible region into several subregions. According to the obtained theoretical results, we give a decomposition method to solve our global production planning problem.

The rest of this paper is organized as follows. Section 2 gives the measure generated by credibility distribution and defines the L–S integral of function of fuzzy variables. In Section 3, we formulate a new two-stage optimization model for global production planning problem. Section 4 analyzes the solution properties of the proposed model by decomposing the feasible region into several subregions. Section 5 gives an application example to demonstrate the validity of the developed method. Finally, Section 6 gives the conclusions of this paper.

2 L–S Integral of Function of Fuzzy Variables

Suppose $\tilde{\xi}$ is a fuzzy variable, and its realized value is denoted by ξ. Let $\alpha(\xi) = \mathrm{Cr}\{\tilde{\xi} \le \xi\}$ be the credibility distribution of $\tilde{\xi}$. Then $\alpha(\xi)$ is a monotone increasing function. We define the α-measure of an interval I by the following formula

$$\mu_{\alpha(I)} = \begin{cases} \alpha(b^+) - \alpha(a^-), & \mathrm{I} = [a, b] \\ \alpha(b^+) - \alpha(a^+), & \mathrm{I} = (a, b] \\ \alpha(b^-) - \alpha(a^-), & \mathrm{I} = [a, b), \end{cases} \tag{1}$$

where $a \le b$, and if $a < b$, then $\mu_\alpha((a, b)) = \alpha(b^-) - \alpha(a^+)$. $I = (a, a)$ is an empty set and its measure is zero.

For multiple fuzzy variables $\tilde{\xi}_i$, $i = 1, 2, \ldots, n$, we denote $\alpha_i(\xi) = \mathrm{Cr}\{\tilde{\xi}_i \le \xi\}$ $(i = 1, 2, \ldots, n)$. The interval in $\Re^n$ is denoted as $I = I_1 \times I_2 \times \cdots \times I_n$. In this case, the α-measure of I is defined as

$$\mu_{\alpha_1 \times \cdots \times \alpha_n(I)} = \mu_{\alpha_1}(I_1) \times \cdots \times \mu_{\alpha_n}(I_n). \tag{2}$$

We next define the L–S integral of function of fuzzy variables $\tilde{\xi}_i$, $i = 1, 2, \ldots, n$. Let $g(\xi) : \Re^n \to \Re^+$ be a non-negative Borel measurable function. Then the L–S integral of fuzzy variable $g(\tilde{\xi}_1, \tilde{\xi}_2, \cdots, \tilde{\xi}_n)$ with respect to $\mu_{\alpha_1 \times \cdots \times \alpha_n}$ is defined as

$$\int_{\Re^n} g(\xi) d\mu_{\alpha_1 \times \cdots \times \alpha_n} = \lim_{n \to \infty} \sum_{k=0}^{\infty} \frac{k}{2^n} \mu_{\alpha_1 \times \cdots \times \alpha_n} \left\{ \xi \in \Re^n : \frac{k}{2^n} < g(\xi) \le \frac{k+1}{2^n} \right\}. \tag{3}$$

For general Borel measurable function $g(\xi)$, we define $g^+(\xi) = \max\{g(\xi), 0\}$ and $g^-(\xi) = \min\{g(\xi), 0\}$. Then the L–S integral of fuzzy variable $g(\tilde{\xi}_1, \tilde{\xi}_2, \cdots, \tilde{\xi}_n)$ is defined by

$$\int_{\Re^n} g(\xi) d\mu_{\alpha_1 \times \cdots \times \alpha_n} = \int_{\Re^n} g^+(\xi) d\mu_{\alpha_1 \times \cdots \times \alpha_n} - \int_{\Re^n} g^-(\xi) d\mu_{\alpha_1 \times \cdots \times \alpha_n}. \tag{4}$$

3 Formulation of Global Production Planning Problem

In this section, we give a two-stage optimization method for global production planning problem. The following notations are required in building our model:

- i: index representing the ith production, $i = 1, 2, ..., n$.
- x_i: the amount of product i.
- j: index representing the sale in country j, $j = 1, 2$.
- r_{ij}: unit revenue of product i from the market j.
- t_j: unit transportation cost between the producing country and market j.
- d_{ij}: the determistic demand of product i in country j.
- y_{ij}: the amount of product i shipped for sale to country j.

- $\tilde{\xi}_j$: a fuzzy variable representing uncertain exchange rate that converts foreign currency j to home country currency.
- c_i: unit cost of product i in the currency of the home country.

In the production planning problem, the decision-makers should first determine a product quantity. In other words, we denote x_i $i = 1, 2, \ldots, n$ as the first-stage decisions that must be taken before knowing the values of fuzzy variables. When we have known the values of fuzzy parameters, we must decide how to distribute the products. In this sense, y_{ij}, $i = 1, 2, \ldots, n; j = 1, 2$, are called the second-stage decisions.

In the allocation process, because of the influence of uncertain exchange rates the amount of product i shipped to each country may less than the demand. We express this situation in constraints (5)

$$y_{ij} \le d_{ij} \quad \forall\ i = 1, 2, \ldots, n, j = 1, 2. \tag{5}$$

In the production process, because of the influence of income and freight, the firm may not serve a market completely, which is represented as constraints (6)

$$y_{i1} + y_{i2} \le x_i, i = 1, 2, \ldots, n. \tag{6}$$

Our goal is to maximize the profits in the production process. So using the above notations, we formally build a two-stage model as follows

$$\begin{array}{ll} \max z(x) = & \sum\limits_{i=1}^{n}(-c_i x_i + \iint\limits_{R^2}(r_{i1}\xi_1 - t_1)y_{i1} + (r_{i2}\xi_2 - t_2)y_{i2})d\alpha_1 \times \alpha_2(\xi_1, \xi_2) \\ \text{s.t.:} & y_{ij} \le d_{ij}, i = 1, 2, ..., n, j = 1, 2, \\ & y_{i1} + y_{i2} \le x_i, i = 1, 2, \ldots, n, \\ & x_i \ge 0, y_{i1}, y_{i2} \ge 0, i = 1, 2, \ldots, n, \end{array} \tag{7}$$

where $\alpha_1(\xi_1) = \mathrm{Cr}\{\tilde{\xi}_1 \le \xi_1\}$ and $\alpha_2(\xi_2) = \mathrm{Cr}\{\tilde{\xi}_2 \le \xi_2\}$.

Since the objective is separable, we can solve problem (7) indirectly by solving its subproblems. In the next section, we will address this issue in detail.

4 Theoretical Results and Solution Method

4.1 Theoretical Results

For convenience, we denote the second-stage value function as $Q\left(x_i|(\xi_1, \xi_2)\right)$, i.e.,

$$Q\left(x_i|(\xi_1, \xi_2)\right) = \max(r_{i1}\xi_1 - t_1)y_{i1} + (r_{i2}\xi_2 - t_2)y_{i2}. \tag{8}$$

Based on the values of exchange rates ξ_1 and ξ_2, the feasible region is decomposed into five subregions,

$$\begin{array}{ll} A: & \left\{\frac{t_1}{r_{i1}} < \xi_1 < \infty, 0 \le \xi_2 \le \frac{t_2}{r_{i2}}\right\}; \\ B: & \left\{\frac{t_1}{r_{i1}} < \xi_1 < \infty, \frac{t_2}{r_{i2}} < \xi_2 < \frac{r_{i1}\xi_1 - t_1 + t_2}{r_{i2}}\right\}; \\ C: & \left\{\frac{t_1}{r_{i1}} < \xi_1 < \infty, \frac{r_{i1}\xi_1 - t_1 + t_2}{r_{i2}} < \xi_2 < \infty\right\}; \\ D: & \left\{0 \le \xi_1 \le \frac{t_1}{r_{i1}}, \frac{t_2}{r_{i2}} < \xi_2 < \infty\right\}; \\ E: & \left\{0 \le \xi_1 \le \frac{t_1}{r_{i1}}, 0 \le \xi_2 \le \frac{t_2}{r_{i2}}\right\}. \end{array}$$

By the properties of the second-stage programming model, the second value function $Q\left(x_i|(\xi_1, \xi_2)\right)$ is piecewise linear with break points $\min(d_{i1}, d_{i2})$, $\max(d_{i1}, d_{i2})$, and $(d_{i1} + d_{i2})$. Therefore, we can design a feasible domain decomposition method to solve model (7). When demands d_{i1} and d_{i2} are deterministic, the real line is decomposed into the following four subintervals $[0, \min(d_{i1}, d_{i2})]$, $[\min(d_{i1}, d_{i2}), \max(d_{i1}, d_{i2})]$, $[\max(d_{i1}, d_{i2}), d_{i1} + d_{i2}]$, and $[d_{i1} + d_{i2}, \infty)$. After decomposing the feasible region, we can find the local optimal solutions on each subregion.

In the following results, we assume that uncertain exchange rates ξ_1, ξ_2 have general probability distributions.

Proposition 1. *Let* $0 \le x_i \le \min(d_{i1}, d_{i2})$*, and* $E_{i1} = \int_{(\frac{t_1}{r_{i1}},\infty)} \int_{[0,\frac{r_{i1}e_1-t_1+t_2}{r_{i2}})} (r_{i1}\xi_1 - t_1) d\alpha_1 \times \alpha_2(\xi_1, \xi_2) + \int_{[0,\frac{r_{i2}e_2-t_2+t_1}{r_{i1}})} \int_{(\frac{t_2}{r_{i2}},\infty)} (r_{i2}\xi_2 - t_2) d\alpha_1 \times \alpha_2(\xi_1, \xi_2)$*. If* $E_{i1} \le c_i$*, then the local optimal production quantity* $x_i = 0$*. If* $E_{i1} > c_i$*, then the local optimal production quantity* $x_i = \min(d_{i1}, d_{i2})$*.*

Proof. When $0 \le x_i \le \min(d_{i1}, d_{i2})$, we define the following two sets

$$A_1 = I_1 \times I_2 : (\frac{t_1}{r_{i1}}, \infty) \times [0, \frac{r_{i1}\xi_1 - t_1 + t_2}{r_{i2}}), A_2 = I_3 \times I_4 : [0, \frac{r_{i2}\xi_2 - t_2 + t_1}{r_{i1}}) \times (\frac{t_2}{r_{i2}}, \infty).$$

Then the objective function is calculated as

$$\begin{aligned} z(x_i) =& - c_i x_i + \int_{R^2}\int Q\left(x_i|(\xi_1,\xi_2)\right) d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ =& - c_i x_i + x_i(\int_{A_1}\int (r_{i1}\xi_1 - t_1) d\alpha_1 \times \alpha_2(\xi_1,\xi_2) + \int_{A_2}\int (r_{i2}\xi_2 - t_2) d\alpha_1 \times \alpha_2(\xi_1,\xi_2)) \\ =& (-c_i + E_{i1}) x_i. \end{aligned}$$

It is evident that $z(x_i)$ is a linear function of x_i. If $E_{i1} \le c_i$, then $z(x_i)$ is monotone decreasing, which implies the local optimal production quantity $x_i = 0$. On the other hand, if $E_{i1} > c_i$, then the local optimal production quantity $x_i = \min(d_{i1}, d_{i2})$. □

Proposition 2. *Let* $\min(d_{i1}, d_{i2}) \le x \le \max(d_{i1}, d_{i2})$ *with* $d_{i1} < d_{i2}$*, and* $E_{i2} = \int_B \int (r_{i2}\xi_2 - t_2) d\alpha_1 \times \alpha_2(\xi_1, \xi_2) + \int_C \int (r_{i2}\xi_2 - t_2) d\alpha_1 \times \alpha_2(\xi_1, \xi_2) + \int_D \int (r_{i2}\xi_2 - t_2) d\alpha_1 \times \alpha_2(\xi_1, \xi_2)$*. If* $E_{i2} > c_i$*, then the optimal production quantity* $x_i = d_{2i}$*. If* $E_{i2} \le c_i$*, then the optimal production quantity* $x_i = d_{i1}$*.*

Proof. When $d_{i1} \le x_i \le d_{i2}$, the objective function is computed as

$$\begin{aligned} z(x_i) =& - c_i x_i + \int_{R^2}\int Q\left(x_i|(\xi_1,\xi_2)\right) d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ =& - c_i x_i + \int_A\int (r_1\xi_1 - t_1) d_{i1} d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ &+ \int_B\int (r_{i1}\xi_1 - t_1) d_{i1} + (r_{i2}\xi_2 - t_2)(x_i - d_{i1}) d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ &+ \int_C\int (r_{i2}\xi_2 - t_2) x_i d\alpha_1 \times \alpha_2(\xi_1,\xi_2) + \int_D\int (r_{i2}\xi_2 - t_2) x_i d\alpha_1 \times \alpha_2(\xi_1,\xi_2). \end{aligned}$$

Thus the first-order derivative of $z(x_i)$ is

$$\begin{aligned} \frac{dz(x_i)}{dx_i} =& - c_i + \int_B\int (r_{i2}\xi_2 - t_2) d\alpha_1 \times \alpha_2(\xi_1,\xi_2) + \int_C\int (r_{i2}\xi_2 - t_2) d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ &+ \int_D\int (r_{i2}\xi_2 - t_2) d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ =& - c_i + E_{i2}. \end{aligned}$$

Due to the convexity of $z(x_i)$, the first-order derivative is sufficient to determine the optimal behavior of x_i. Solving the equation

$$\frac{dz(x_i)}{dx_i} = 0,$$

we get $E_{i2} = c_i$. So, when $E_{i2} > c_i$, we have $x_i = d_{i1}$; otherwise, the local optimal solution $x_i = d_{i2}$. □

Proposition 3. *Let* $\max(d_{i1}, d_{i2}) \le x_i \le d_{i1} + d_{i2}$*, and* $E_{i3} = \int_B \int (r_{i2}\xi_2 - t_2) d\alpha_1 \times \alpha_2(\xi_1, \xi_2) + \int_C \int (r_{i1}\xi_1 - t_1) d\alpha_1 \times \alpha_2(\xi_1, \xi_2)$*. If* $E_{i3} > c_i$*, then the local optimal production quantity is* $d_{i1} + d_{i2}$*. If* $E_{i3} \le c_i$*, then the local optimal production quantity is* $\max(d_{i1}, d_{i2})$*.*

Proof. By the suppositon of propositon, the objective function is calculated as

$$\begin{aligned} z(x_i) =& - c_i x_i + \int_{R^2}\int Q\left(x_i|(\xi_1,\xi_2)\right) d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ =& - c_i x_i + \int_A\int (r_{i1}\xi_1 - t_1)d_{i1}d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ &+ \int_B\int (r_{i1}\xi_1 - t_1)d_{i1} + (r_{i2}\xi_2 - t_2)(x_i - d_{i1})d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ &+ \int_C\int (r_{i1}\xi_1 - t_1)(x_i - d_{i2}) + (r_{i2}\xi_2 - t_2)d_{i2}d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ &+ \int_D\int (r_{i2}\xi_2 - t_2)d_{i2}d\alpha_1 \times \alpha_2(\xi_1,\xi_2). \end{aligned}$$

Thus the first-order derivative of $z(x_i)$ is

$$\frac{dz(x_i)}{dx_i} = - c_i + \int_B\int (r_{i2}\xi_2 - t_2)d\alpha_1 \times \alpha_2(\xi_1,\xi_2) + \int_C\int (r_{i1}\xi_1 - t_1)d\alpha_1 \times \alpha_2(\xi_1,\xi_2) = -c_i + E_{i3}.$$

Solving the equation

$$\frac{dz(x_i)}{dx_i} = 0,$$

we get $E_{i3} = c_i$. So, when $E_{i3} \le c_i$, we have $x_i = \max(d_{i1}, d_{i2})$; when $E_{i3} > c_i$, we have $x_i = d_{i1} + d_{i2}$. □

Proposition 4. *If $d_{i1} + d_{i2} \le x_i < \infty$, then the local optimal production quantity $x_i = d_{i1} + d_{i2}$.*

Proof. Because the demand is deterministic, the production quantity cannot be greater than aggregate demand, i.e. $x_i \le d_{i1} + d_{i2}$. Therefore, we can simplify the solution process in this region and obtain the local optimal solution $x_i = d_{i1} + d_{i2}$. □

4.2 Solution Method

Based on the theoretical results obtained in the above section, we can find the local optimal solutions on each subregion. We next provide a general solution procedure to find the global optimal solution and the optimal value. We take the product i as an example. Suppose $d_{i1} < d_{i2}$, and the case $d_{i1} \ge d_{i2}$ can be treated similarly. We first divide the feasible region into several subregions; then, we find the local optimal solution on each subregion and calculate the corresponding objective value. Finally, we compare the obtained objective values on each subregion and get the global optimal solution. The procedure is summarized as follows.

Step 1: Divide the feasible region $[0, \infty)$ of the product i into four subregion, $[0, \min(d_{i1}, d_{i2})]$, $[\min(d_{i1}, d_{i2}), \max(d_{i1}, d_{i2})]$, $[\max(d_{i1}, d_{i2}), d_{i1} + d_{i2}]$, $[d_{i1} + d_{i2}, \infty)$.

Step 2: Find the local optimal solution in each subregion and calculate the corresponding objective value.
According to Proposition 1, we can find the local optimal solution in $[0, \min(d_{i1}, d_{i2})]$:
If $E_{i1} \le c_i$, we get the maximum objective value 0 at 0; if $E_{i1} > c_i$ we obtain the maximum objective value $(-c_i + E_{i1})d_{i1}$ at $x_i = d_{i1}$.
According to Proposition 2, we can find the local optimal solution in $[\min(d_{i1}, d_{i2}), \max(d_{i1}, d_{i2})]$:
If $E_{i2} \le c_i$, the local optimal solution is $x_i = d_{i1}$ with objective value $(-c_i + E_{i1})d_{i1}$;
If $E_{i2} > c_i$ we reach maximum at point $x_i = d_{i2}$ and the value is
$-c_i d_{i2} + \int_A\int (r_{i1}\xi_1 - t_1)d_{i1}d\alpha_1 \times \alpha_2(\xi_1,\xi_2) + \int_B\int (r_{i1}\xi_1 - t_1)d_{i1} + (r_{i2}\xi_2 - t_2)(d_{i2} - d_{i1})d\alpha_1 \times \alpha_2(\xi_1,\xi_2) + \int_C\int (r_{i2}\xi_2 - t_2)d_{i2}d\alpha_1 \times \alpha_2(\xi_1,\xi_2) + \int_D\int (r_{i2}\xi_2 - t_2)d_{i2}d\alpha_1 \times \alpha_2(\xi_1,\xi_2).$ (∗)
According to Proposition 3, we can find the local optimal solution in $[\max(d_{i1}, d_{i2}), d_{i1} + d_{i2}]$:
If $E_{i3} > c_i$, at the point $x_i = d_{i1} + d_{i2}$ we get the maximum value $(-c_i + E_{i1})(d_{i1} + d_{i2})$;
If $E_{i3} \le c_i$, then the objective value is the same as the quantity defined in (∗).
In the subregion $[d_{i1} + d_{i2}, \infty)$, Proposition 4 implies $x_i = d_{i1} + d_{i2}$ and the value is
$(-c_i + E_{i1})(d_{i1} + d_{i2})$.

Step 3: Comparing the objective values on each subregion, we can obtain the global optimal solution.

In the next section, we will provide a numerical example to demonstrate the validity of the proposed decomposition method.

5 Numerical Experiments

In order to demonstrate the developed two-stage optimization method, we next present a numerical example, and solve the instance by the proposed decomposition method.

5.1 Problem Description

Consider an electronics production planning problem that includes three markets, one is domestic that produces the electronic products, the other two are abroad markets that required the produced products in the domestic market. Three kinds of electronic products are produced in domestic firm, i=1 represents USB flash disk, i=2 indicates MP5 and i=3 shows removable drives. The unit transport cost t_{ij}, the unit revenue r_{ij} and the demand d_{ij} in each country are provided in Table 1. The exchange rates ξ_1, ξ_2 are triangular fuzzy variables (8,9,10) and (4,5,6), respectively. Their credibility distribution functions are $\alpha_1(\xi_1)$, $\alpha_2(\xi_2)$ and calculated by

$$\alpha_1(\xi_1) = \mathrm{Cr}\{\tilde{\xi}_1 \le \xi_1\} = \begin{cases} 0, & \xi_1 \le 8 \\ \frac{\xi_1-8}{2}, & 8 \le \xi_1 \le 9 \\ \frac{\xi_1-8}{2}, & 9 \le \xi_1 \le 10 \\ 1, & \xi_1 \ge 10, \end{cases} \tag{9}$$

and

$$\alpha_2(\xi_2) = \mathrm{Cr}\{\tilde{\xi}_2 \le \xi_2\} = \begin{cases} 0, & \xi_2 \le 4 \\ \frac{\xi_2-4}{2}, & 4 \le \xi_1 \le 5 \\ \frac{\xi_2-4}{2}, & 5 \le \xi_1 \le 6 \\ 1, & \xi_2 \ge 6. \end{cases} \tag{10}$$

Using the notations above, the production planning about product 1 is build as the following mathematical programming model

$$\begin{array}{ll} \max & z(x_1) = -30x_1 + \int_{(4,\infty)} \int_{[0,\frac{2\xi_1+2}{3})} (10\xi_1 - 40) d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ & \qquad + \int_{(0,\frac{3\xi_2-2}{2})} \int_{[\frac{10}{3},\infty)} (10\xi_1 - 40) d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ \text{s.t.:} & 0 \le x_1 \le 200 \\ & y_{11} + y_{12} \le x_1 \\ & 0 \le y_{11} \le 200 \\ & 0 \le y_{12} \le 300. \end{array} \tag{11}$$

When $200 \le x_1 \le 300$, the objective function becomes

$$\begin{array}{rl} \max\ z(x_1) & = -30x_1 + \int_{(4,\infty)} \int_{[0,\frac{10}{3})} 200(10\xi_1 - 40) d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ & + \int_{(4,\infty)} \int_{[\frac{10}{3},\frac{2\xi_1+2}{3})} 200(10\xi_1 - 40) + (x_1 - 200)(15\xi_2 - 50) d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ & + \int_{(4,\infty)} \int_{[\frac{2\xi_1+2}{3},\infty)} (15\xi_2 - 50) x_1 d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ & + \int_{[0,4]} \int_{[\frac{10}{3},\infty)} (15\xi_2 - 50) x_1 d\alpha_1 \times \alpha_2(\xi_1,\xi_2), \end{array} \tag{12}$$

and the other constraints are same as in model (11).

When $300 \le x_1 \le 500$, the objective function reads

$$\begin{array}{rl} \max\ z(x_1) & = -30x_1 + \int_{(4,\infty)} \int_{[0,\frac{10}{3})} 200(10\xi_1 - 40) d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ & + \int_{(4,\infty)} \int_{[\frac{10}{3},\frac{2\xi_1+2}{3})} 200(10\xi_1 - 40) + (x_1 - 200)(15\xi_2 - 50) d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ & + \int_{(4,\infty)} \int_{[\frac{2\xi_1+2}{3},\infty)} (10\xi_1 - 40)(x_1 - 300) + 300(15\xi_2 - 50) d\alpha_1 \times \alpha_2(\xi_1,\xi_2) \\ & + \int_{[0,4]} \int_{[\frac{10}{3},\infty)} 300(15\xi_2 - 50) d\alpha_1 \times \alpha_2(\xi_1,\xi_2), \end{array} \tag{13}$$

and the other constraints are same as in model (11).

5.2 Computational Results

In this section, we solve models (11), (12) and (13) by our decomposition method, and get the following results. In subregion $[0, 20]$, the local optimal solution is 200 with objective value 4000. In subregion $[200, 300]$, the

Table 1: Problem data (cost, freight, price, demand)

Product	$c_i(RMB)$	$t_{ij}(RMB)$	$r_{ij}(GBP, AUD)$	d_{ij}
1	30	40,50	10,15	200,300
2	110	50,60	30,70	150,200
3	460	70,80	65,110	50,100

Table 2: Solution results of production planning problem

Product	*Subregion*	$Solution_{opt}$	$Value_{opt}$	$Global\ Solution_{opt}$	$Global\ Value_{opt}$
	$[0, 200]$	200	4000		
1	$[200, 300]$	200	4000	200	4000
	$[300, 500]$	300	3500		
	$[0, 150]$	150	27080		
2	$[150, 200]$	200	36080	350	37125
	$[200, 350]$	350	37125		
	$[0, 50]$	50	6950		
3	$[50, 100]$	100	7350	100	7350
	$[100, 150]$	100	7350		

local optimal solution is 200 with objective value 4000. In subregion $[300, 500]$, the local optimal solution is 300 with objective value 3500. Comparing the obtained objective values on each subregion, we obtain the global optimal solution is 200, and the optimal value is 4000. Using the similar method we can find the optimal production plan for products 2 and 3, and report the computational results in Table 2.

From Table 2, we observe that the optimal production planning for products 1, 2 and 3 are 200, 350 and 100, respectively, whose maximum profit is 48475.

6 Conclusions

In this paper, we presented a new two-stage fuzzy optimization method for global production planning problem. We first generated an additive measure by using credibility distribution, and used the obtained measure to define the L–S integral of function of fuzzy variables. Then, taking the L–S integral as our research tool, we built a two-stage programming model for global production planning problem, in which the exchange rates are described by possibility distributions. When demands are deterministic, we decomposed the feasible region into several subregions and found the local optimal solution on each subregion. To demonstrate our new modeling idea, we presented a numerical example and solved the instance by our decomposition solution method.

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References

[1] Carter, M., and B. van Brunt, *The Lebesgue-Stieltjes Integral*, Springer-Verlag, New York, 2000.

[2] Ding, Q., Dong, L., and P. Kouvelis, On the integration of production and financial hedging decisions in global markets, *Operations Research*, vol.55, no.3, pp.470–489, 2007.

[3] Flaherty, T., *Global Operations Management*, McGraw Hill, New York, 1996.

[4] Halim, K.A., Giri, B.C., and K.S. Chaudhuri, Fuzzy production planning models for an unreliable production system with fuzzy production rate and stochastic/fuzzy demand rate, *International Journal of Industrial Engineering Computations*, vol.2, no.1, pp.179–192, 2011.

[5] Hodder, J.E., Financial market approaches to facility location under uncertainty, *Operations Research*, vol.32, no.6, pp.1374–1380, 1984.

[6] Huchzermeier, A., and M. Cohen, Valuing operational flexibility under exchange rate risk, *Operations Research*, vol.44, no.1, pp.100–113, 1996.

[7] Jucker, J.V., and R.C. Carlson, The simple plant-location problem under uncertainty, *Operarions Research*, vol.24, no.6, pp.1045–1055, 1976.

[8] Klir, G., On fuzzy-set interpretation of possibility theory, *Fuzzy Sets and Systems*, vol.108, no.3, pp.263–273, 1999.

[9] Li, Z., and X. Dai, Optimization of insuring critical path problem with uncertain activity duration times, *Journal of Uncertain Systems*, vol.7, no.1, pp.72–80, 2013.

[10] Liu, B., *Uncertainty Theory*, Springer-Verlag, Berlin, 2004.

[11] Liu, Y., Fuzzy programming with recourse, *International Journal of Uncertainty, Fuzziness and Knowledge-based Systems*, vol.13, no.4, pp.381–413, 2005.

[12] Liu, Y., and X. Bai, Studying interconnections between two classes of two-stage fuzzy optimization problems, *Soft Computing*, vol.17, no.4, pp.569–578, 2013.

[13] Liu, Y., Chen, Y., Liu, Y., and R. Qin, *Fuzzy Optimization Methods with Applications*, Science Press, Beijing, 2013.

[14] Partha, G., Manas, K.M., and M. Manoranjan, A production inventory model with fuzzy production and demand using fuzzy differential equation: an interval compared genetic algorithm approach, *Engineering Applications of Artificial Intelligence*, vol.26, no.2, pp.766–778, 2013.

[15] Rosenfield, D.B., Global and variable cost manufacturing systems, *European Journal of Operational Research*, vol.95, no.2, pp.325–343, 1996.

[16] Sun, G., Liu, Y., and Y. Lan, Optimizing material procurement planning problem by two-stage fuzzy programming, *Computers & Industrial Engineering*, vol.58, no.1, pp.97–107, 2010.

[17] Sun, G., Liu, Y., and Y. Lan, Fuzzy two-stage material procurement planning problem, *Journal of Intelligent Manufacturing*, vol.22, no.2, pp.319–331, 2011.

[18] Zadeh, L.A., Fuzzy sets, *Information and Control*, vol.8, no.3, pp.338–353, 1965.

[19] Zadeh, L.A., Fuzzy sets as a basis for a theory of possibility, *Fuzzy Sets and Systems*, vol.1, no.1, pp.3–28, 1978.