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2-Vehicle Cost Varying Transportation Problem

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Abstract

In this paper we represent a two-vehicle cost varying transportation model. In this model the transportation cost is vary due to capacity of vehicles as well as amount of transport quantity. At first we propose an algorithm to determine unit transportation cost with initial allocation to the basic cells by North-west corner rule. Then solve it. The unit transportation cost vary during optimality test when allocations are changed. Numerical examples are presented to illustrate the two-vehicle cost varying transportation problem(TVCVTP). Finally, comparison is given to show better effective of this model.

Keywords: transportation problem, basic cell, non-basic cell, North-west corner rule

1 Introduction

Transportation problem is a special class of linear programming problem which deals with the distribution of single commodity from various sources of supply to various destination of demand in such a manner that the total transportation cost is minimized. In order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the model must be fixed at crisp values but in real life applications unit transportation cost may vary.

In transportation problem unit transportation cost is constant from each source to each destination. But in reality, it is not constant; it depends on amount of transport quantity and capacity of vehicles. If amount of quantity is small then small(capacity) vehicle is sufficient for deliver. Where as if amount of quantity is large then big(capacity) vehicle is needed. So, depend on amount of transport quantity and the capacity of vehicles, the unit transportation cost is not constant.

Here we present some transportation problem whose unit transportation cost is varied. This type of transportation problem is named as cost varying transportation problem. In our model we consider this type of transportation problem with two vehicles.

The basic transportation problem was originally developed by Hitchock [11]. Efficient methods of solution derived from the simplex algorithm were developed in 1947. The transportation problem can be modeled as a standard linear programming problem, which can be solved by simplex method. However, because of its very special mathematical structure, it was recognized early that the simplex method applied to transportation problem can be made quit efficient in terms of how to evaluate the necessary simplex-method information (variable to enter the basis, variable to leave the basis and optimality conditions).

In many real life situations, the commodity does vary in some characteristics according to its source and the final commodity mixture reaching at destinations, may then be required to have known specifications. TP with additional impurity restrictions was stated by Haley [10]. Chandra et al. [4] developed a method for solvig time minimizing TP with impurities. Interval transportation problem Pandian and Anuradha [16] (ITP) is a generalization of the TP in which input data are expressed as intervals instead of fixed values. This problem can arise when uncertainty exists in data problem and decision makers are more comfortable expressing it as intervals. Many researchers [1, 6, 12, 14, 15, 20] have proposed fuzzy and interval programming techniques

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for solving them. Chanas et al. [2] developed an algorithm determining the optimal integer solution of a more general fuzzy transportation problem. Das et al. [8] introduced a method, called fuzzy technique to solve ITP by considering the right bound and the midpoint of the interval. Sengupta and Pal [18] proposed a new fuzzy oriented method to solve ITP by considering the midpoint and width of the interval in the objective function. Singh and Saxena [19] proposed a method for solving multiobjective time TP with additional impurity restrictions. A procedure for finding an optimal solution to fully interval integer TP was presented by Pandian and Natarajan [17]. Dutta et al. [9] introduced a linear fractional programming method for solving a fuzzy TP with additional restrictions in which transportation costs are intervals. Pandian and Anuradha [16] have proposed a floating point method for solving TP with additional constraints.

In this paper, we present the 2-vehicle cost varying transportation problem which is a Bi-level Mathematical programming model. To solve this model, use north west corner rule for determining initial basic feasible solution and then set up unit transportation cost (which varies in each iteration) by proper choice of vehicles with our proposed algorithm. Apply optimality test for determining optimal solution. Comparison is made with single vehicle cost varying transportation model.

2 Mathematical Formulation

2.1 Preliminaries

A transportation problem can be stated in Model 1 as follows:
Model 1

$$\min \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij},$$

$$\text{subject to} \quad \sum_{i=1}^{m} x_{ij} = a_i, \quad i = 1,\dots,m \tag{1}$$

$$\sum_{j=1}^{n} x_{ij} = b_j, \quad j = 1,\dots,n \tag{2}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

$$x_{ij} \geq 0 \quad \forall i, \quad \forall j.$$

Table 1: Tabular representation of a multi-objective transportation problem

	D_1	D_2	...	D_n	*stock*
O_1	c_{11}	c_{12}	...	c_{1n}	a_1
O_2	c_{21}	c_{22}	...	c_{2n}	a_2
...	...	...	...	...	...
O_m	c_{n1}	c_{n2}	...	c_{nn}	a_m
Demand	b_1	b_2	...	b_n	

A transportation problem can be represent in Table 1. Where a_i is the quantity of material available at source $O_i, i = 1,\dots,m$, b_j is the quantity of material required at destination $D_j, j = 1,\dots,n$, c_{ij} is the unit cost of transportation from st source O_i to destination D_j.

The following terms are to be defined with reference to the transportation problems.

Definition 1. (Feasible Solution (F.S.)) *A set of non-negative allocations $x_{ij} \geq 0$ which satisfies* $(1), (2)$ *is known as feasible solution.*

Definition 2. (Basic Feasible Solution (B.F.S.)) *A feasible solution to a m-origin and n-destination problem is said to be basic feasible solution if number of positive allocations are* $(m + n - 1)$.

If the number of allocations in a basic feasible solutions are less than (m+n-1), it is called degenerate basic feasible solution (DBFS), otherwise non-degenerate basic feasible solution (NDBFS).

Definition 3. (Optimal Solution) *A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.*

Theorem 2.1. *The number of basic variables in a Transportation Problem(T.P.) is at most* $(m+n-1)$.

Theorem 2.2. *There exits a F.S. in each Transportation Problem (T.P.).*

Theorem 2.3. *In each T.P. there exits at least one B.F.S. which makes the objective function a minimum.*

Theorem 2.4. *The solution of a T.P. is never unbounded.*

Definition 4. (Loop) *In the Transportation table, a sequence of cells is said to form a loop, if*
(i) each adjacent pair of cells either lies in the same column or in the same row;
(ii) not more than two consecutive cells in the sequence lie in the same row or in the same column;
(iii) the first and the last cells in the sequence lie either in the same row or in the same column;
(iv) the sequence must involve at least two rows or two columns of the table.

Theorem 2.5. *A sub-set of the columns of the coefficient matrix of a T.P. are linearly dependent, iff, the corresponding cells or a sub-set of them can be sequenced to form a loop.*

2.2 North-West Corner Rule

Step 1. Compute min (a_1, b_1). If $a_1 < b_1$, min $(a_1, b_1) = a_1$ and if $a_1 > b_1$, min $(a_1, b_1) = b_1$. Select $x_{11} = min(a_1, b_1)$ allocate the value of x_{11} in the cell $(1, 1)$.

Step 2. If $a_1 < b_1$, compute min $(a_2, b_1 - a_1)$. Select $x_{21} =$ min $(a_2, b_1 - a_1)$ and allocate the value of x_{21} in the cell $(2, 1)$.

If $a_1 > b_1$, compute min $(a_1 - b_1, b_2)$. Select $x_{12} =$ min $(a_1 - b_1, b_2)$ and allocate the value of x_{12} in the cell $(1, 2)$.

Let us now make an assumption that $a_1 - b_1 < b_2$. With this assumption the next cell for which some allocation is to made, is the cell $(2, 2)$.

If $a_1 = b_1$, then allocate 0 only in one of two cells $(2, 1)$ or $(1, 2)$. The next allocation is to be made cell $(2, 2)$.

In general, if an allocation is made in the cell $(i + 1, j)$ in the current step, the next allocation will be made either in cell (i, j) or $(i, j + 1)$.

The feasible solution obtained by this away is always a B.F.S..

2.3 Optimality Test:

In order to test for optimality we should follow the procedure as given bellow:

Step 1. Start with B.F.S. consisting of $m + n - 1$ allocation in independent positions.

Step 2. Determine a set of $m + n$ numbers $u_i, i = 1, \dots, m$ and $v_j, j = 1, \dots, n$ such that in each cell (i, j) $c_{ij} = u_i + v_j$.

Step 3. Calculate cell evaluations (unit cost difference) d_{ij} for each empty cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$.

Step 4. Examine the matrix of cell evaluation d_{ij} for negative entries and conclude that
(i) If all $d_{ij} > 0$, then solution is optimal and unique.
(ii) If all $d_{ij} \geq 0$ and at least one $d_{ij} = 0$, then solution is optimal and alternative solution also exists.
(iii) If at least one $d_{ij} < 0$, then solution is not optimal.

If it is so, further improvement is required by repeating the above process after Step 5 and onwards.

Step 5. (i) See the most negative cell in the matrix $[d_{ij}]$.
(ii) Allocate θ to this empty cell in the final allocation table. Subtract and add the amount of this allocation to other corners of the loop in order to restore feasibility.
(iii) This value of θ, in general is obtained by equating to zero the minimum of the allocations containing $-\theta$

(not $+\theta$) only at the corners of the closed loop.
(iv) Substitute the value of θ and find a fresh allocation table.

Step 6. Again, apply the above test for optimality till we find $d_{ij} \geq 0$.

2.4 2-Vehicle Cost Varying Transportation Problem

Suppose there are two types off vehicles V_1, V_2 from each source to each destination. Let C_1 and $C_2(> C_1)$ be the capacities(in unit) of the vehicles V_1 and V_2 respectively. $R_{ij} = (R^1_{ij}, R^2_{ij})$ represent transportation cost for each cell (i, j), where R^1_{ij} is the transportation cost from source $O_i, i = 1, \ldots, m$ to the destination $D_j, j = 1, \ldots, n$ by the vehicle V_1, and R^2_{ij} is the transportation cost from source $O_i, i = 1, \ldots, m$ to the destination $D_j, j = 1, \ldots, n$ by the vehicle V_2. So, cost varying transportation problem can be represent in the following tabulated form.

Table 2: Tabular representation of cost varying transportation problem

	D_1	D_2	...	D_n	*stock*
O_1	R^1_{11}, R^2_{11}	R^1_{12}, R^2_{12}	...	R^1_{1n}, R^2_{1n}	a_1
O_2	R^1_{21}, R^2_{21}	R^1_{22}, R^2_{22}	...	R^1_{2n}, R^2_{2n}	a_2
...	...	...	...	...	...
O_m	R^1_{m1}, R^2_{m1}	R^1_{m2}, R^2_{m2}	...	R^1_{mn}, R^2_{mn}	a_m
Demand	b_1	b_2	...	b_n	

2.5 Solution Procedure of 2-Vehicle Cost Varying Transportation Problem

2.5.1 Determination of c_{ij}

To solve this problem, apply our proposed Algorithms stated as follows:
Case 1:

$$\max_i \ a_i \leq C_2.$$

2.5.2 Algorithm (TP1)

Step 1. Since unit cost is not determined (because it depends on quantity of transport), North-west corner rule (because it does not depend on unit transportation cost) is applicable to allocate initial B.F.S..

Step 2. After the allocate x_{ij} by North-west corner rule, for basic cell we determine c_{ij} (unit transportation cost from source O_i to destination D_j) as

$$c_{ij} = \begin{cases} \frac{R1_{ij}}{x_{ij}} & \text{if} \quad x_{ij} \leq C_1 \\ \frac{R2_{ij}}{x_{ij}} & \text{if} \ \ C_1 < x_{ij} \leq C_2 \\ 0 & \text{if} \ \ x_{ij} = 0. \end{cases} \tag{3}$$

Step 3. For non-basic cell (i, j) possible allocation is the minimum of allocations in i^{th} row and j^{th} column (for possible loop). If possible allocation is x_{ij}, then non-basic cell c_{ij} (unit transportation cost from source O_i to destination D_j) as

$$c_{ij} = \begin{cases} \frac{R1_{ij}}{x_{ij}} & \text{if} \quad x_{ij} \leq C_1 \\ \frac{R2_{ij}}{x_{ij}} & \text{if} \ \ C_1 < x_{ij} \leq C_2 \\ 0 & \text{if} \ \ x_{ij} = 0. \end{cases} \tag{4}$$

In this manner we convert cost varying transportation problem to a usual transportation problem but c_{ij} is not fixed, it may be changed (when this allocation will not serve optimal value) during optimality test.

Step 4. During optimality test some basic cell changes to non-basic cell and some non-basic cell changes to basic cell, depends on running basic cell we first fix c_{ij} by Step 2 and for non-basic we fix c_{ij} by Step 3.

Step 5. Repeat Step 2 to Step 4 until we obtain optimal solution.

Case 2:

$$\max_i \ a_i \leq C_1 + C_2.$$

To solve this problem, apply our proposed algorithm stated as follows:

2.5.3 Algorithm (TP2)

Step 1. Since unit cost is not determined (because it depends on quantity of transport), North-west corner rule (because it does not depend on unit transportation cost) is applicable to allocate initial B.F.S..

Step 2. After the allocate x_{ij} by North-west corner rule, for basic cell we determine c_{ij} unit transportation cost from source O_i to destination D_j) as

$$c_{ij} = \begin{cases} \frac{R1_{ij}}{x_{ij}} & \text{if} \quad x_{ij} \leq C_1 \\ \frac{R2_{ij}}{x_{ij}} & \text{if} \ \ C_1 < x_{ij} \leq C_2 \\ \frac{R1_{ij}+R2_{ij}}{x_{ij}} & \text{if} \ \ C_2 < x_{ij} \leq C_1 + C_2 \\ 0 & \text{if} \ \ x_{ij} = 0. \end{cases} \tag{5}$$

Step 3. For non-basic cell (i, j) possible allocation is the minimum of allocations in i^{th} row and j^{th} column (for possible loop). If possible allocation is x_{ij}, then non-basic cell c_{ij} (unit transportation cost from source O_i to destination D_j) as

$$c_{ij} = \begin{cases} \frac{R1_{ij}}{x_{ij}} & \text{if} \quad x_{ij} \leq C_1 \\ \frac{R2_{ij}}{x_{ij}} & \text{if} \ \ C_1 < x_{ij} \leq C_2 \\ \frac{R1_{ij}+R2_{ij}}{x_{ij}} & \text{if} \ \ C_2 < x_{ij} \leq C_1 + C_2 \\ 0 & \text{if} \ \ x_{ij} = 0. \end{cases} \tag{6}$$

In this manner we convert cost varying transportation problem to a usual transportation problem but c_{ij} is not fixed, it may be changed (when this allocation will not serve optimal value) during optimality test.

Step 4. During optimality test some basic cell changes to non-basic cell and some non-basic cell changes to basic cell, depends on running basic cell we first fix c_{ij} by Step 2 and for non-basic we fix c_{ij} by Step 3.

Step 5. Repeat Step 2 to Step 4 until we obtain optimal solution.

2.5.4 Algorithm (TP3)

Step 1. Since unit cost is not determined (because it depends on quantity of transport), North-west corner rule (because it does not depend on unit transportation cost) is applicable to allocate initial B.F.S..

Step 2. After the allocate x_{ij} by North-west corner rule, for basic cell we determine c_{ij}^r (unit transportation cost from source O_i to destination D_j) as

$$c_{ij} \quad = \quad \begin{cases} \frac{p1_{ij}R1_{ij}+p2_{ij}R2_{ij}}{x_{ij}}, & \text{if} \ \ x_{ij} \neq 0 \\ 0 & \text{if} \ \ x_{ij} = 0, \end{cases} \tag{7}$$

where $p1_{ij},\ p2_{ij}, i = 1,\dots,m; j = 1,\dots,n$ are integer solutions of

$$\begin{array}{ll} \min & p1_{ij}R1_{ij} + p2_{ij}R2_{ij} \\ \text{s.t.} & x_{ij} \le p1_{ij}C_1 + p2_{ij}C_2. \end{array}$$

Step 3. For non-basic cell (i, j) possible allocation is the minimum of allocations in i^{th} row and j^{th} column (for possible loop). If possible allocation is x_{ij}, then non-basic cell c_{ij} (unit transportation cost from source O_i to destination D_j) as

$$c_{ij} = \begin{cases} \frac{p1_{ij}R1_{ij}+p2_{ij}R2_{ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0, \end{cases} \quad (8)$$

where $p1_{ij},\ p2_{ij}, i = 1,\dots,m; j = 1,\dots,n$ are integer solutions of

$$\begin{array}{ll} \min & p1_{ij}R1_{ij} + p2_{ij}R2_{ij} \\ \text{s.t.} & x_{ij} \le p1_{ij}C_1 + p2_{ij}C_2. \end{array}$$

In this manner we convert cost varying transportation problem to a usual transportation problem but c_{ij} is not fixed, it may be changed (when this allocation will not serve optimal value) during optimality test.

Step 4. During optimality test some basic cell changes to non-basic cell and some non-basic cell changes to basic cell, depends on running basic cell we first fix c_{ij} by Step 2 and for non-basic we fix c_{ij} by Step 3.

Step 5. Repeat Step 2. to Step 4 until we obtain optimal solution.

2.5.5 Bi-level Mathematical Programming for 2-Vehicle Cost Varying Transportation Problem

The bi-level mathematical programming for 2-vehicle cost varying transportation problem is formulated in Model 2 as follows:
Model 2

$$\min \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}, \quad (9)$$

where

$$c_{ij} = \begin{cases} \frac{p1_{ij}R1_{ij}+p2_{ij}R2_{ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0, \end{cases}$$

c_{ij} is determined by following mathematical programming, $p1_{ij},\ p2_{ij}, i = 1,\dots,m; j = 1,\dots,n$ are integer solutions of

$$\begin{array}{ll} \min & p1_{ij}R1_{ij} + p2_{ij}R2_{ij} \\ \text{s. t.} & x_{ij} \le p1_{ij}C_1 + p2_{ij}C_2 \\ & \sum_{i=1}^{m} x_{ij} = a_i, \quad i = 1,\dots,m \\ & \sum_{j=1}^{n} x_{ij} = b_j, \quad j = 1,\dots,n \\ & \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \\ & x_{ij} \ge 0 \ \ \forall i, \ \ \forall j. \end{array} \quad (10)$$

3 Numerical Example

Example 1: Consider a cost varying transportation problem as

	D_1	D_2	D_3	*stock*
O_1	$5, 10$	$8, 12$	$6, 9$	15
O_2	$6, 8$	$12, 15$	$15, 18$	12
O_3	$4, 6$	$8, 16$	$5, 10$	3
Demand	10	10	10	

The capacities of vehicles of V_1 and V_2 are respectively, $C_1 = 10$ and $C_2 = 20$.

Step 1. By North-west corner rule, initial B.F.S. is

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 10$ $5, 10$	$x_{12} = 5$ $8, 12$	$6, 9$	15
O_2	$6, 8$	$x_{22} = 5$ $12, 15$	$x_{23} = 7$ $15, 18$	12
O_3	$4, 6$	$8, 16$	$x_{33} = 3$ $5, 10$	3
Demand	10	10	10	

Step 2. Using (3), we determine $c_{11} = 5/10$, $c_{12} = 8/5$, $c_{22} = 12/5$, $c_{23} = 15/7$, $c_{33} = 5/3$.

Step 3. Using (4), we determine $c_{13} = 6/5$, $c_{21} = 6/5$, $c_{31} = 4/3$, $c_{32} = 8/3$.

With these c_{ij} the transportation problem converted to

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 10,\ c_{11} = \frac{5}{10}$ $5, 10$	$x_{12} = 5,\ c_{12} = \frac{8}{5}$ $8, 12$	$c_{13} = \frac{6}{5}$ $6, 9$	15
O_2	$c_{21} = \frac{6}{5}$ $6, 8$	$x_{22} = 5,\ c_{22} = \frac{12}{5}$ $12, 15$	$x_{23} = 7,\ c_{23} = \frac{15}{7}$ $15, 18$	12
O_3	$c_{31} = \frac{4}{3}$ $4, 6$	$c_{32} = \frac{8}{3}$ $8, 16$	$x_{33} = 3,\ c_{33} = \frac{5}{3}$ $5, 10$	3
Demand	10	10	10	

optimality test

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$, each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$.

So the tabular representation of $u_i, i = 1, 2, 3,\ \ v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table.

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 10,\ c_{11} = \frac{5}{10}$ $5, 10$	$x_{12} = 5,\ c_{12} = \frac{8}{5}$ $8, 12$	$c_{13} = \frac{6}{5}$ $6, 9\quad d_{13} = -\frac{1}{7}$	0
O_2	$c_{21} = \frac{6}{5}$ $6, 8\quad d_{21} = -\frac{1}{10}$	$x_{22} = 5,\ c_{22} = \frac{12}{5}$ $12, 15$	$x_{23} = 7,\ c_{23} = \frac{15}{7}$ $15, 18$	$\frac{4}{5}$
O_3	$c_{31} = \frac{4}{3}$ $4, 6\quad d_{31} = \frac{107}{210}$	$c_{32} = \frac{8}{3}$ $8, 16\quad d_{32} = \frac{78}{105}$	$x_{33} = 3,\ c_{33} = \frac{5}{3}$ $5, 10$	$\frac{34}{105}$
v_j	$\frac{5}{10}$	$\frac{8}{5}$	$\frac{47}{35}$	

Since $d_{13} = -1/7 < 0$ and $d_{21} = -1/10 < 0$, solution is not optimal. So a loop occurred in cells (1,1), (1,2), (2,1), (2,2) and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*stock*
O_1	$x_{11}=5,\ c_{11}=\frac{5}{5}$ $5,10$	$x_{12}=10,\ c_{12}=\frac{8}{10}$ $8,12$	$c_{13}=\frac{6}{5}$ $6,9$	15
O_2	$x_{21}=5,\ c_{21}=\frac{6}{5}$ $6,8$	$c_{22}=\frac{12}{5}$ $12,15$	$x_{23}=7,\ c_{23}=\frac{15}{7}$ $15,18$	12
O_3	$c_{31}=\frac{4}{3}$ $4,6$	$c_{32}=\frac{8}{3}$ $8,16$	$x_{33}=3,\ c_{33}=\frac{5}{3}$ $5,10$	3
Demand	10	10	10	

Determine a set of 6 numbers $u_i, i=1,2,3$ and $v_j, j=1,2,3$ such that in each cell basic (i,j) $c_{ij}=u_i+v_j$, each non-basic cell (i,j) by using formula $d_{ij}=c_{ij}-(u_i+v_j.)$

So the tabular representation of $u_i, i=1,2,3,\ \ v_j, j=1,2,3$ and d_{ij} non-basic cell (i,j) is given in the following table.

	D_1	D_2	D_3	u_i
O_1	$x_{11}=5,\ c_{11}=\frac{5}{5}$ $5,10$	$x_{12}=10,\ c_{12}=\frac{8}{10}$ $8,12$	$c_{13}=\frac{6}{5}$ $6,9\quad d_{13}=-\frac{32}{35}$	0
O_2	$x_{21}=5,\ c_{21}=\frac{6}{5}$ $6,8$	$c_{22}=\frac{12}{5}$ $12,15\quad d_{22}=\frac{7}{5}$	$x_{23}=7,\ c_{23}=\frac{15}{7}$ $15,18$	$\frac{1}{5}$
O_3	$c_{31}=\frac{4}{3}$ $4,6\quad d_{31}=\frac{82}{105}$	$c_{32}=\frac{8}{3}$ $8,16\quad d_{32}=\frac{143}{105}$	$x_{33}=3,\ c_{33}=\frac{5}{3}$ $5,10$	$-\frac{47}{105}$
v_j	1	$\frac{8}{10}$	$\frac{74}{35}$	

Since $d_{13}=-32/35<0$, solution is not optimal. So a loop occurred in cells (1,1), (1,3), (2,1), (2,3) and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*Stock*
O_1	$c_{11}=\frac{5}{5}$ $5,10$	$x_{12}=10,\ c_{12}=\frac{8}{10}$ $8,12$	$x_{13}=5,\ c_{13}=\frac{6}{5}$ $6,9$	15
O_2	$x_{21}=10,\ c_{21}=\frac{6}{10}$ $6,8$	$c_{22}=\frac{12}{2}$ $12,15$	$x_{23}=2,\ c_{23}=\frac{15}{2}$ $15,18$	12
O_3	$c_{31}=\frac{4}{3}$ $4,6$	$c_{32}=\frac{8}{3}$ $8,16$	$x_{33}=3,\ c_{33}=\frac{5}{3}$ $5,10$	3
Demand	10	10	10	

Determine a set of 6 numbers $u_i, i=1,2,3$ and $v_j, j=1,2,3$ such that in each cell basic (i,j) $c_{ij}=u_i+v_j$, each non-basic cell (i,j) by using formula $d_{ij}=c_{ij}-(u_i+v_j)$.

So the tabular representation of $u_i, i=1,2,3,\ \ v_j, j=1,2,3$ and d_{ij} non-basic cell (i,j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$c_{11}=\frac{5}{5}$ $5,10\quad d_{11}>0$	$x_{12}=10,\ c_{12}=\frac{8}{10}$ $8,12$	$x_{13}=5,\ c_{13}=\frac{6}{5}$ $6,9$	$\frac{6}{5}$
O_2	$x_{21}=10,\ c_{21}=\frac{6}{10}$ $6,8$	$c_{22}=\frac{12}{2}$ $12,15\quad d_{22}<0$	$x_{23}=2,\ c_{23}=\frac{15}{2}$ $15,18$	$\frac{15}{2}$
O_3	$c_{31}=\frac{4}{3}$ $4,6\quad d_{31}>0$	$c_{32}=\frac{8}{3}$ $8,16\quad d_{32}>0$	$x_{33}=3,\ c_{33}=\frac{5}{3}$ $5,10$	$\frac{5}{3}$
v_j	$-\frac{69}{10}$	$-\frac{2}{5}$	0	

Since $d_{22}<0$, solution is not optimal. So a loop occurred in cells$(1,2),(2,2),(1,3),(2,3)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*Stock*
O_1	$c_{11} = \frac{5}{5}$ $5, 10$	$x_{12} = 8,\ c_{12} = \frac{8}{8}$ $8, 12$	$x_{13} = 7,\ c_{13} = \frac{6}{7}$ $6, 9$	15
O_2	$x_{21} = 10,\ c_{21} = \frac{6}{10}$ $6, 8$	$x_{22} = 2\ c_{22} = \frac{12}{2}$ $12, 15$	$c_{23} = \frac{15}{2}$ $15, 18$	12
O_3	$c_{31} = \frac{4}{3}$ $4, 6$	$c_{32} = \frac{8}{3}$ $8, 16$	$x_{33} = 3,\ c_{33} = \frac{5}{3}$ $5, 10$	3
Demand	10	10	10	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$, each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$.

So the tabular representation of $u_i, i = 1, 2, 3,\ \ v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$c_{11} = \frac{5}{5}$ $5, 10\quad d_{11} > 0$	$x_{12} = 8,\ c_{12} = \frac{8}{8}$ $8, 12$	$x_{13} = 7,\ c_{13} = \frac{6}{7}$ $6, 9$	-5
O_2	$x_{21} = 10,\ c_{21} = \frac{6}{10}$ $6, 8$	$x_{22} = 2,\ c_{22} = \frac{12}{2}$ $12, 15$	$c_{23} = \frac{15}{2}$ $15, 18\quad d_{23} > 0$	0
O_3	$c_{31} = \frac{4}{3}$ $4, 6\quad d_{31} > 0$	$c_{32} = \frac{8}{3}$ $8, 16\quad d_{32} > 0$	$x_{33} = 3,\ c_{33} = \frac{5}{3}$ $5, 10$	$-\frac{88}{21}$
v_j	$\frac{6}{10}$	6	$\frac{41}{7}$	

Since all $d_{ij} > 0$ for all non-basic cell so the table give optimal solution. $x_{12} = 8, x_{13} = 7, x_{21} = 10$, $x_{22} = 2$, $x_{33} = 3$. Minimum cost $Z^* = 8 + 6 + 6 + 10 + 5 = 25$ unit(Rs.)

Example 2: Consider a cost varying transportation problem as

	D_1	D_2	D_3	*stock*
O_1	$6, 9$	$10, 15$	$8, 12$	25
O_2	$4, 6$	$12, 18$	$6, 9$	17
O_3	$14, 21$	$8, 12$	$4, 6$	8
Demand	23	15	12	

The capacities of vehicles of V_1 and V_2 are respectively, $C_1 = 10$ and $C_2 = 20$.

Step 1. By North-west corner rule, initial B.F.S. is

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 23$ $6, 9$	$x_{12} = 2$ $10, 15$	$8, 12$	25
O_2	$4, 6$	$x_{22} = 13$ $12, 18$	$x_{23} = 4$ $6, 9$	17
O_3	$14, 21$	$8, 12$	$x_{33} = 8$ $4, 6$	8
Demand	23	15	12	

Step 2. Using (5), we determine $c_{11} = 15/23$, $c_{12} = 10/2$, $c_{22} = 18/13$, $c_{23} = 6/4$, $c_{33} = 4/8$.

Step 3. Using (6), we determine $c_{13} = 8/2$, $c_{21} = 6/13$, $c_{31} = 14/8$, $c_{32} = 8/8$.

With these c_{ij} the transportation problem converted to

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 23,\ c_{11} = \frac{15}{23}$ $6, 9$	$x_{12} = 2,\ c_{12} = \frac{10}{2}$ $10, 15$	$c_{13} = \frac{8}{2}$ $8, 12$	25
O_2	$c_{21} = \frac{6}{13}$ $4, 6$	$x_{22} = 13,\ c_{22} = \frac{18}{13}$ $12, 18$	$x_{23} = 4,\ c_{23} = \frac{6}{4}$ $6, 9$	17
O_3	$c_{31} = \frac{14}{8}$ $14, 21$	$c_{32} = \frac{8}{8}$ $8, 12$	$x_{33} = 4,\ c_{33} = \frac{4}{8}$ $4, 6$	8
Demand	23	15	12	

optimality test

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$, each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$.

So the tabular representation of $u_i, i = 1, 2, 3,\ \ v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table.

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 23,\ c_{11} = \frac{15}{23}$ $6, 9$	$x_{12} = 2,\ c_{12} = \frac{10}{2}$ $10, 15$	$c_{13} = \frac{8}{2}$ $8, 12\quad d_{13} < 0$	0
O_2	$c_{21} = \frac{6}{13}$ $4, 6\quad d_{21} > 0$	$x_{22} = 13,\ c_{22} = \frac{18}{13}$ $12, 18$	$x_{23} = 4,\ c_{23} = \frac{6}{4}$ $6, 9$	$-\frac{47}{13}$
O_3	$c_{31} = \frac{14}{8}$ $14, 21\quad d_{31} > 0$	$c_{32} = \frac{8}{8}$ $8, 12\quad d_{32} > 0$	$x_{33} = 4,\ c_{33} = \frac{4}{8}$ $4, 6$	$-\frac{60}{13}$
v_j	$\frac{15}{23}$	5	$\frac{133}{26}$	

Since $d_{13} < 0$, solution is not optimal. So a loop occurred in cells$(1, 3), (1, 2), (2, 2), (2, 3), (1, 3)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 23,\ c_{11} = \frac{15}{23}$ $6, 9$	$c_{12} = \frac{10}{2}$ $10, 15$	$x_{13} = 2,\ c_{13} = \frac{8}{2}$ $8, 12$	25
O_2	$c_{21} = \frac{6}{13}$ $4, 6$	$x_{22} = 15,\ c_{22} = \frac{18}{15}$ $12, 18$	$x_{23} = 2,\ c_{23} = \frac{6}{2}$ $6, 9$	17
O_3	$c_{31} = \frac{14}{8}$ $14, 21$	$c_{32} = \frac{8}{8}$ $8, 12$	$x_{33} = 4,\ c_{33} = \frac{4}{8}$ $4, 6$	8
Demand	23	15	12	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$, each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$.

So the tabular representation of $u_i, i = 1, 2, 3,\ \ v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table.

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 23,\ c_{11} = \frac{15}{23}$ $6, 9$	$c_{12} = \frac{10}{2}$ $10, 15\ d_{12} > 0$	$x_{13} = 2,\ c_{13} = \frac{8}{2}$ $8, 12$	4
O_2	$c_{21} = \frac{6}{13}$ $4, 6\quad d_{21} > 0$	$x_{22} = 15,\ c_{22} = \frac{18}{15}$ $12, 18$	$x_{23} = 2,\ c_{23} = \frac{6}{2}$ $6, 9$	3
O_3	$c_{31} = \frac{14}{8}$ $14, 21\quad d_{31} > 0$	$c_{32} = \frac{8}{8}$ $8, 12\quad d_{32} > 0$	$x_{33} = 4,\ c_{33} = \frac{4}{8}$ $4, 6$	$\frac{1}{2}$
v_j	$-\frac{15}{4}$	$-\frac{9}{5}$	0	

Since all $d_{ij} > 0$ for all non-basic cell, the table give optimal solution. $x_{11} = 23, x_{13} = 2, x_{22} = 15, x_{23} = 2$, $x_{33} = 8$. Minimum cost $Z^* = 15 + 8 + 18 + 6 + 4 = 51$ unit(Rs.).

Example 3: Consider a cost varying transportation problem as

	D_1	D_2	D_3	*stock*
O_1	$4, 8$	$5, 10$	$10, 20$	48
O_2	$2, 3$	$8, 16$	$6, 12$	52
O_3	$7, 14$	$3, 6$	$9, 18$	25
Demand	75	30	20	

The capacities of vehicles of V_1 and V_2 are respectively, $C_1 = 6$ and $C_2 = 18$.

Step 1. By North-west corner rule, initial B.F.S. is

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 48$ $4, 8$	$5, 10$	$10, 20$	48
O_2	$x_{21} = 27$ $2, 3$	$x_{22} = 25$ $8, 16$	$6, 12$	52
O_3	$7, 14$	$x_{32} = 5$ $3, 6$	$x_{33} = 20$ $9, 18$	25
Demand	75	30	20	

Step 2. Using (5), we determine $c_{11} = 24/48$, $c_{21} = 6/27$, $c_{22} = 32/25$, $c_{32} = 3/5$, $c_{33} = 27/20$.

Step 3. Using (6), we determine $c_{12} = 20/25$, $c_{23} = 18/20$, $c_{13} = 30/20$, $c_{31} = 7/5$.

With these c_{ij} the transportation problem converted to

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 48,\ c_{11} = \frac{24}{48}$ $4, 8$	$c_{12} = \frac{20}{25}$ $5, 10$	$c_{13} = \frac{30}{20}$ $10, 20$	48
O_2	$x_{21} = 27,\ c_{21} = \frac{6}{27}$ $2, 3$	$x_{22} = 25,\ c_{22} = \frac{32}{25}$ $8, 16$	$c_{23} = \frac{18}{20}$ $6, 12$	52
O_3	$c_{31} = \frac{7}{5}$ $7, 14$	$x_{32} = 5,\ c_{32} = \frac{3}{5}$ $3, 6$	$x_{33} = 20,\ c_{33} = \frac{27}{20}$ $9, 18$	25
Demand	75	30	20	

optimality test

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$, each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$.

So the tabular representation of $u_i, i = 1, 2, 3,\ \ v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table.

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 48,\ c_{11} = \frac{24}{48}$ $4, 8$	$c_{12} = \frac{20}{25}$ $5, 10\quad d_{12} < 0$	$c_{13} = \frac{30}{20}$ $10, 20\quad d_{13} > 0$	0
O_2	$x_{21} = 27,\ c_{21} = \frac{6}{27}$ $2, 3$	$x_{22} = 25,\ c_{22} = \frac{32}{25}$ $8, 16$	$c_{23} = \frac{18}{20}$ $6, 12\quad d_{23} > 0$	$-\frac{5}{18}$
O_3	$c_{31} = \frac{7}{5}$ $7, 14\quad d_{31} < 0$	$x_{32} = 5,\ c_{32} = \frac{3}{5}$ $3, 6$	$x_{33} = 20,\ c_{33} = \frac{27}{20}$ $9, 18$	$\frac{2077}{900}$
v_j	$\frac{1}{2}$	$\frac{701}{450}$	$-\frac{431}{450}$	

Since $d_{12} < 0$, solution is not optimal. So a loop occurred in cells$(1, 1), (1, 2), (2, 2), (2, 1), (1, 1)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 23,\ c_{11} = \frac{12}{23}$ $4, 8$	$x_{12} = 25,\ c_{12} = \frac{20}{25}$ $5, 10$	$c_{13} = \frac{30}{20}$ $10, 20$	48
O_2	$x_{21} = 52,\ c_{21} = \frac{9}{52}$ $2, 3$	$c_{22} = \frac{32}{25}$ $8, 16$	$c_{23} = \frac{18}{20}$ $6, 12$	52
O_3	$c_{31} = \frac{7}{5}$ $7, 14$	$x_{32} = 5,\ c_{32} = \frac{3}{5}$ $3, 6$	$x_{33} = 20,\ c_{33} = \frac{27}{20}$ $9, 18$	25
Demand	75	30	20	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$, each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$.

So the tabular representation of $u_i, i = 1, 2, 3,\ \ v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table.

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 23,\ c_{11} = \frac{12}{23}$ $4, 8$	$x_{12} = 25,\ c_{12} = \frac{20}{25}$ $5, 10$	$c_{13} = \frac{30}{20}$ $10, 20 \quad d_{13} < 0$	0
O_2	$x_{21} = 52,\ c_{21} = \frac{9}{52}$ $2, 3$	$c_{22} = \frac{32}{25}$ $8, 16 \quad d_{22} > 0$	$c_{23} = \frac{18}{20}$ $6, 12 \quad d_{23} < 0$	$-\frac{417}{1196}$
O_3	$c_{31} = \frac{7}{5}$ $7, 14 \quad d_{31} > 0$	$x_{32} = 5,\ c_{32} = \frac{3}{5}$ $3, 6$	$x_{33} = 20,\ c_{33} = \frac{27}{20}$ $9, 18$	$-\frac{1}{5}$
v_j	$\frac{12}{23}$	$\frac{20}{25}$	$\frac{31}{20}$	

Since $d_{13} < 0$ (i.e, most negative), solution is not optimal. So a loop occurred in cells (1,2), (3,2), (3,3), (1,3), (1,2) and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 23\ c_{11} = \frac{12}{23}$ $4, 8$	$x_{12} = 5,\ c_{12} = \frac{5}{5}$ $5, 10$	$x_{13} = 20,\ c_{13} = \frac{30}{20}$ $10, 20$	48
O_2	$x_{21} = 52,\ c_{21} = \frac{9}{52}$ $2, 3$	$c_{22} = \frac{8}{5}$ $8, 16$	$c_{23} = \frac{18}{20}$ $6, 12$	52
O_3	$c_{31} = \frac{7}{5}$ $7, 14$	$x_{32} = 25,\ c_{32} = \frac{6}{25}$ $3, 6$	$x_{33} = 20,\ c_{33} = \frac{27}{20}$ $9, 18$	25
Demand	75	30	20	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$, each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$.

So the tabular representation of $u_i, i = 1, 2, 3,\ \ v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table.

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 23,\ c_{11} = \frac{12}{23}$ $4, 8$	$x_{12} = 5,\ c_{12} = \frac{5}{5}$ $5, 10$	$x_{13} = 20,\ c_{13} = \frac{30}{20}$ $10, 20$	0
O_2	$x_{21} = 52,\ c_{21} = \frac{9}{52}$ $2, 3$	$c_{22} = \frac{8}{5}$ $8, 16 \quad d_{22} > 0$	$c_{23} = \frac{18}{20}$ $6, 12 \quad d_{23} < 0$	$-\frac{417}{1196}$
O_3	$c_{31} = \frac{7}{5}$ $7, 14 \quad d_{31} > 0$	$x_{32} = 25,\ c_{32} = \frac{6}{25}$ $3, 6$	$c_{33} = \frac{27}{20}$ $9, 18 \quad d_{33} > 0$	$-\frac{19}{25}$
v_j	$\frac{12}{23}$	$\frac{25}{25}$	$\frac{3}{2}$	

Since $d_{23} < 0$ (i.e, most negative), solution is not optimal. So a loop occurred in cells (1,1), (1,3), (2,3), (2,1), (1,1) and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 43,\ c_{11} = \frac{28}{43}$ $4, 8$	$x_{12} = 5,\ c_{12} = \frac{5}{5}$ $5, 10$	$c_{13} = \frac{30}{20}$ $10, 20$	48
O_2	$x_{21} = 32,\ c_{21} = \frac{6}{32}$ $2, 3$	$c_{22} = \frac{8}{5}$ $8, 16$	$x_{23} = 20,\ c_{23} = \frac{18}{20}$ $6, 12$	52
O_3	$c_{31} = \frac{28}{25}$ $7, 14$	$x_{32} = 25,\ c_{32} = \frac{6}{25}$ $3, 6$	$c_{33} = \frac{27}{20}$ $9, 18$	25
Demand	75	30	20	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$, each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$.

So the tabular representation of $u_i, i = 1, 2, 3,\ \ v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table.

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 43,\ c_{11} = \frac{28}{43}$ $4, 8$	$x_{12} = 5,\ c_{12} = \frac{5}{5}$ $5, 10$	$c_{13} = \frac{30}{20}$ $10, 20 \quad d_{13} > 0$	0
O_2	$x_{21} = 32,\ c_{21} = \frac{6}{32}$ $2, 3$	$c_{22} = \frac{8}{5}$ $8, 16 \quad d_{22} > 0$	$x_{23} = 20,\ c_{23} = \frac{18}{20}$ $6, 12$	$-\frac{319}{688}$
O_3	$c_{31} = \frac{28}{25}$ $7, 14 \quad d_{31} > 0$	$x_{32} = 25,\ c_{32} = \frac{6}{25}$ $3, 6$	$c_{33} = \frac{27}{20}$ $9, 18 \quad d_{33} > 0$	$-\frac{19}{25}$
v_j	$\frac{28}{43}$	$\frac{25}{25}$	1.364	

Since all $d_{ij} > 0$ for all non-basic cell so the table give optimal solution. $x_{11} = 43, x_{12} = 5, x_{21} = 32$, $x_{23} = 20$, $x_{32} = 25$. Minimum cost $Z^* = 28 + 5 + 6 + 18 + 6 = 61$ unit(Rs.).

If the Examples 1, 2 and 3 are solved by considering only single vehicle either V_1 or V_2 then minimum transportation cost is increased. The results of Examples 1, 2 and 3 for V_1, for V_2 and for both V_1, V_2 are given in the following Table 3.

Table 3: The computational results of Examples 1, 2 and 3

Problems	*Single Vehicle CVTP*		*Two-vehicle CVTP*
	V_1	V_2	V_1, V_2
Example 1	$Z^* = 25$ $x_{12} = 8, x_{13} = 7$ $x_{21} = 10, x_{22} = 2$ $x_{33} = 3$	$Z^* = 54$ $x_{11} = 5, x_{12} = 10$ $x_{21} = 2, x_{23} = 10$ $x_{31} = 3$	$Z^* = 25$ $x_{12} = 8, x_{13} = 7$ $x_{21} = 10, x_{22} = 2$ $x_{33} = 3$
Example 2	$Z^* = 60$ $x_{11} = 23, x_{13} = 2$ $x_{22} = 15, x_{23} = 6$ $x_{33} = 4$	$Z^* = 63$ $x_{11} = 23, x_{13} = 2$ $x_{22} = 15, x_{23} = 6$ $x_{33} = 8$	$Z^* = 51$ $x_{11} = 23, x_{13} = 2$ $x_{22} = 15, x_{23} = 2$ $x_{33} = 8$
Example 3	$Z^* = 88$ $x_{11} = 43, x_{12} = 5$ $x_{21} = 32, x_{23} = 20$ $x_{32} = 25$	$Z^* = 87$ $x_{11} = 23, x_{12} = 25$ $x_{22} = 52$ $x_{32} = 5, x_{33} = 20$	$Z^* = 61$ $x_{11} = 43, x_{12} = 5$ $x_{21} = 32, x_{23} = 20$ $x_{32} = 25$

It is seen from Table 3 that two-vehicle cost varying transportation model give more efficient result than a single vehicle cost varying transportation model.

4 Conclusion

In this paper we have developed two-vehicle cost varying transportation problem. We transfer this cost varying transportation problem to usual transportation problem by North-west corner rule and by proposed algorithm. Then apply optimality test where unit transportation cost vary from one table to another table. Finally, achieve optimal solution. Comparing numerically, it is seen that two-vehicle cost varying transportation model gives more efficient result than single objective cost varying transportation problem. This problem is more real life problem than usual transportation problem.

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