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A New Heuristic Model for Fuzzy Transfer Point Location Problem

Hamed Kalantari *, Aghdas Badiee, Mehdi Ghazanfari
Department of Industrial Engineering, Iran University of Science and Technology, Narmak, Tehran, Iran

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Abstract

Transfer point location problem in which demand points are weighted and their coordinates are uniformly distributed for the case of minimax objective and planar topology has been introduced recently. In the real problems, such as disaster cases, different points of an area might be demand locations with different possibility degrees. Thus, developing a more applicable model is critical for these problems. In this paper, a new transfer point location problem is proposed in which demands are weighted in fuzzy form and have possibilistic coordinates. To formulate the problem, a new possibilistic model is developed which leads to construct the general model based on experts' viewpoints. The proposed model is formulated as a fuzzy unconstrained nonlinear programming in which decision variables are obtained as fuzzy numbers. Subject to the complexity of the developed model, a new Fuzzy Logic Controller (FLC) is designed based on the derived fuzzy decision variables to infer the optimum or near optimum values for all decision variables. Finally, a numerical example is applied to demonstrate the efficiency and efficacy of the developed FLC.
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Keywords: Transfer Point Location Problem, Fuzzy Inference System, fuzzy decision making in fuzzy environment, possibilistic programming, fuzzy decision variables

1 Introduction

The concept of Transfer Point Location Problem (TPLP) was first introduced by the pioneering works of Berman et al. [1]. The TPLP can be defined as follow: "serving n demand points by selecting a location for a new facility". This new location called "Transfer Point" can combine services as a hub center. Many applications of the TPLP can be found in the real world. Berman et al. [9] introduced the classic application of TPLP in which a hospital accepts the injured persons via a helicopter in transfer point. Other applications of TPLP are as follows: location of the transfer point for postal shipments collection or distribution, location of the transfer point in military logistic systems, location of the transfer point in disaster relief Logistic systems, location of the transfer point for agricultural and bestial crops collection, location of the transfer point in industrial product distribution systems and everywhere we concern with goods collection or distribution.

2 Related Works

In the literature, many problems are related to TPLP. The most relevant one are the hub and spoke location problems. Although, the network hub location problem was first addressed by Goldman [24], this was introduced by pioneering works of O'Kelly [15, 29, 30]. According to Campbell and O'Kelly [15] hub is a central facility that connects some interacting points like a switching point in network. Continuous hub location problem is related to locating hub facilities on a plane rather than on the network. For other studies, one can refer to Fernandes et al. [21], Ishfaq and Sox [26], Alumur [3], Contrerasp [18], O'Kelly [31], Cera et al. [16], Aykin and Brown [5], and Campbell [14]. Round-trip location problem is another relevant location problem to TPLP [19, 39] in which a facility is the start point of a service trip, the first customer is visited, trip continues by visiting other customer and then service trip returns to the start point. For more information one can refer to references [13, 23, 27, 32, 33].

* Corresponding author. Email: hamedkalantari@ind.iust.ac.ir (H. Kalantari).

Chien-Chang [17] proposed a Fuzzy Multiple Criteria Decision Making Method (FMCDM) to solve transshipment container port selection problem in fuzzy environment.

The collection depots location problem, introduced by Drezner and Wesolowsky [20], is another type of problem related to the TPLP. Berman et al. [7] investigated the properties of minimax and minisum versions of the collection depots location problem on network. As extensions of this problem, Berman and Huang [11] and Tamir and Halman [38] proposed the multiple facilities location problem on network.

The research on the one-center problem started with pioneering works of Sylvester [37]. A nondeterministic extension of this problem using random demand point weights was first studied by Frank [22] on the network topology. The same problem with probabilistic weights was addressed by Wesolowsky [40]. Berman et al. [12] investigated a minimax stochastic location problem in the plane topology. Averbakh and Bereg [4] studied interval data minimax regret one-center location problems in the plane topology. In their models and for the case of rectilinear distances, uncertainty was incorporated in the both weights and location coordinates of customers, while for the case of Euclidean distances, uncertainty was only considered in the weights. Foul [1] considered a one-center problem on the plane topology in which demand points were assumed to have a bivariate uniform distribution in a given rectangle.

Nowadays, an independent look to TPLP exists in location problems literature. Berman et al. [10] first generalized the TPLP and introduced the Multiple Transfer Points Location Problem (MTPLP). Also, in this model the single facility location was known. By network versions of minisum and minimax MTPLP, they proved that above models can be considered as a p+1-median problem and a p+1-center problem respectively, or as a p+q-median problem and a p+q-center problem, with a given facility. Also, two extended models were presented by Berman et al. [8] called Facility and Transfer Points Location Problem (FTPLP) and also Multiple Location of Transfer Points (MLTP). The problem is formulated on the network and in the plane with both minisum and minimax objectives. Instead of one facility location, the FTPLP models will lead to select the multiple facility locations. In MLTP, the facility location is proposed to be known and demand points are classified into some subsets such that each subset is served by a single transfer point. Recently, a flow-based formulation of minisum and minimax MLTP as a p-median and a p-center problem are presented by Sasaki et al. [35]. Hosseinijou and Bashiri [25] considered a situation in which demand points' location are stochastic variables with uniform distributions.

In this paper, we propose a new TPLP in which not only demand points are weighted, but also weights have possibilistic distributions. Also, possibilistic coordinates have been considered in this model. The problem is formulated as an unconstrained nonlinear programming. By solving the developed possibilistic model, optimum values of decision variables will be obtained in the form of fuzzy numbers. This means that based on the problem state (parameters realization) several points could be optimum location for the transfer point with different possibility degrees of occurrence.

Due to the complexity of the proposed model, especially for executives, a new type of Decision Support Systems (DSS) called Fuzzy Logic Controller (FLC) is designed. Since the FLCs are structured using the linguistic variables and If-then rules, they can be used conveniently by the executives. Therefore, this type of DSS is used to reach the optimum or near optimum values of the decision variables without solving of the original nonlinear programming problem directly. Other applications of this approach is investigated by Sadjadi et al. [34], and Abiri and Yousefli [2] in the pricing problem and the fuzzy location-allocation problems and Kalantari et al. [28], respectively. Table 1 compares the contributions of this paper with the related works on the TPLP.

Table 1: Comparison among novelties of this paper and the related works in the literature

Papers	Objective function		Solution approach		Topology		Parameters			Uncertain decision variables	Decision aid system	Decision Support System
							Uncertain		Certain			
	Minimax	Minisum	Exact	Heuristic	Plane	Network	Stochastic	Fuzzy				
Berman et al. [9]	✓	✓	✓		✓	✓						
Berman et al. [10]	✓	✓		✓	✓	✓						
Berman et al. [8]		✓		✓		✓						
Sasaki et al. [35]		✓	✓			✓						
Hosseinijou and Bashiri [25]	✓		✓		✓		✓					
Kalantari et al. [28]		✓		✓	✓			✓	✓	✓	✓	✓
This paper		✓		✓	✓			✓		✓	✓	✓

Based on Table 1, unique novelties of this paper are as follows:

- The transfer point location problem is developed in the fuzzy environment.
- The transfer point location coordinates (decision variables of the TPLP) are considered in the form of fuzzy numbers. In other words, fuzzy decision making in the fuzzy environment concept is applied to the TPLP.
- All possible optimum values of the transfer point location coordinates, gathered in the form of fuzzy number, will be provided for the decision maker. It can give a wide vision of the possible situations and solutions to the decision maker and improve his/her knowledge over the problem.
- A new approach is developed using fuzzy rule based DSS to infer the optimum or near optimum values of the decision variables without solving the mathematical model of the TPLP directly.
- A new implication method is developed for fuzzy rule based inference system which is used to infer transfer point location.

The paper is organized as follows: Section 3 presents modeling and analysis of fuzzy transfer point location problem. Section 4 introduces the developed FLC. Section 5 demonstrates the proposed model implementation by using numerical example. Finally, section 6 discusses conclusion remarks and outlines some future research opportunities.

3 Fuzzy Transfer Point Location Problem

In this model, each demand point has a certain regional possibility distribution $\tilde{x}_i, \tilde{y}_i$. Also, assigned weight to each demand is defined with linguistic variables $\tilde{w}_i$. Since the demands' coordinates $(\tilde{x}_i, \tilde{y}_i)$ are supposed to be fuzzy numbers, subsequently the location of transfer point will be obtained in the form fuzzy number. The geometric representation of the developed model is shown in Figure 1.

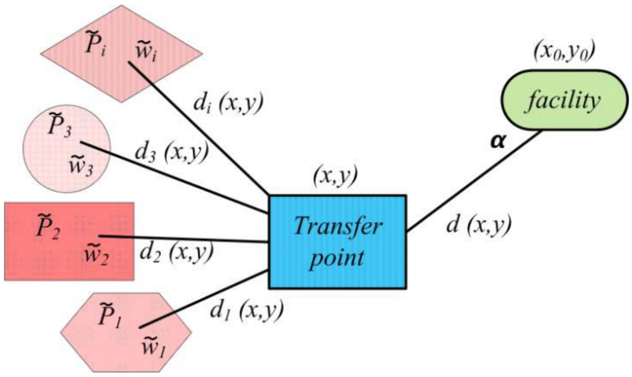


Figure 1: Geometric representation of fuzzy minisum TPLP model

A fuzzy TPLP model for the case of minisum objective is developed under following notations and assumptions:

Notations

n:	number of demand points.
α:	factor multiplying by the travel to the transfer point.
(x_0, y_0) :	location of the facility.
$\tilde{w}_i$:	possibilistic weight associated with demand point *i*.
$(\tilde{x}, \tilde{y})$:	fuzzy location of the transfer point.
$(\tilde{x}_i, \tilde{y}_i)$:	possibilistic coordinates of the demand point *i* $(\tilde{P}_i)$.
$d(x, y)$:	distance between the transfer point and the facility.
D_i:	distance between the demand point *i* and the facility.
$d_i(x, y)$:	distance between the demand point *i* and the transfer point.
d_{ij}:	distance between the demand points *i* and *j*.

Assumptions

I: Demand points Coordinates are uncertain and represented in the form of the triangular fuzzy numbers $(\tilde{x}_i, \tilde{y}_i) = \left((x_i^L, x_i^M, x_i^R), (y_i^L, y_i^M, y_i^R)\right)$;

II: The weights of demand points are uncertain and represented in the form of triangular fuzzy numbers $w_i = (w_i^L, w_i^M, w_i^R)$;

III: Distance measure between demand point and transfer point and between transfer point and the facility, are measured as $d_i(x,y) = (\tilde{x}_i - \tilde{x})^2 + (\tilde{y}_i - \tilde{y})^2$ and $d(x,y) = (\tilde{x} - x_0)^2 + (\tilde{y} - y_0)^2$ respectively.

Since the coordinates of the demand points are fuzzy in nature, subsequently the location of transfer point will be obtained in the form of fuzzy numbers. In fact, the aim is to obtain the possibility distributions of the decision variables $(\tilde{x}, \tilde{y})$ by (1). Thus, the minimum value of the objective function is derived.

$$F = min_{x,y}\left\{\sum_{i=1}^{n} \tilde{w}_i d_i(x,y) + \alpha\, d(x,y)\right\}$$
$$= min_{x,y}\{\textstyle\sum_{i=1}^{n} \tilde{w}_i[(\tilde{x}_i - \tilde{x})^2 + (\tilde{y}_i - \tilde{y})^2] + \alpha[(\tilde{x} - x_0)^2 + (\tilde{y} - y_0)^2]\} \quad (1)$$

All the distances of above model are convex and also the sum of convex functions set is convex. So, (1) is convex and if one point is a locally optimal point, it will be globally optimal too.

Now, to give a clarification for the applicability and develop a conceptual explanation of the model, it is essential to define the weight of demand point i (w_i) and the facility weight (α).

Weight of demand point i, w_i is defined as a function of three factors:

- The speed of transportation from demand area i to the transfer point, S_i.
- The unit cost of traveling from the demand area i to the transfer point, C_i.
- Preference of the demand point i, P_i.

S_i and C_i are the factors which can be changed by transportation vehicle and route specifications in movement between demand area i and the transfer point. Also, the factor of P_i can be a function of population number and potential of occurrence of emergency need in the demand area i. So, there is $w_i = f\ (S_i, C_i, P_i)$ which will increase by growing in P_i and C_i but will decrease by growing in S_i.

Let define the weight of the facility (α) as a function of three factors:

- Travel speed of from the transfer point to the facility, TS_c.
- The unit cost of transportation from transfer point to the facility, C_c.
- Service level risk, SL_c.

TS_c and C_c are the factors which can be changed by transportation vehicle and route specifications in movement between the transfer point and the facility. Also, the factor of SL_c can be defined as a function of frequency of occurrence of emergency need in demand areas and availability of service vehicle which travels between transfer point and the facility. It is obvious when it is busy any other emergency service not will be performed. Hence, there is $\alpha = f(C_c, TS_c, SL_c)$ which will increase by growing in C_c and SL_c but will decrease by growing in TS_c. Growing in α value, attract transfer point to reduce vehicle's travel time and cost for increasing its availability which can be achieved by locating transfer point near the facility. The optimal location for transfer point with respect to different α levels when (x_0, y_0) =(35,18), are shown in Figure 2.

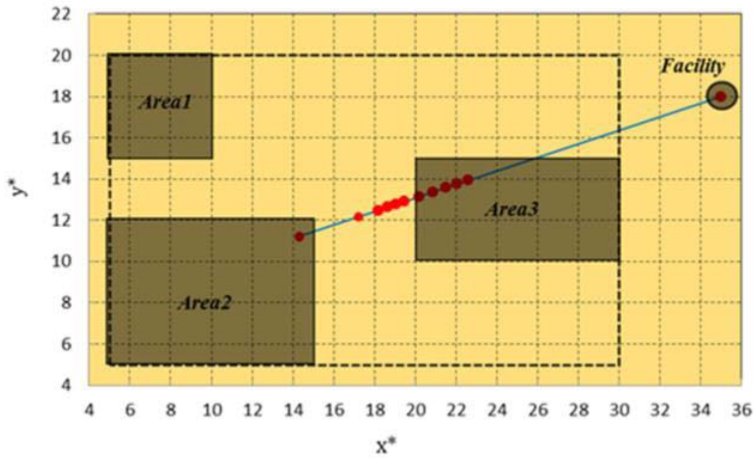


Figure 2: The optimal location for transfer point with different α levels

The optimal location of transfer point (x, y) is placed among the demand areas, depends on their weights. Also, the value of the objective function increases by increasing in the α value. Figure 3 shows the trend of the objective function.

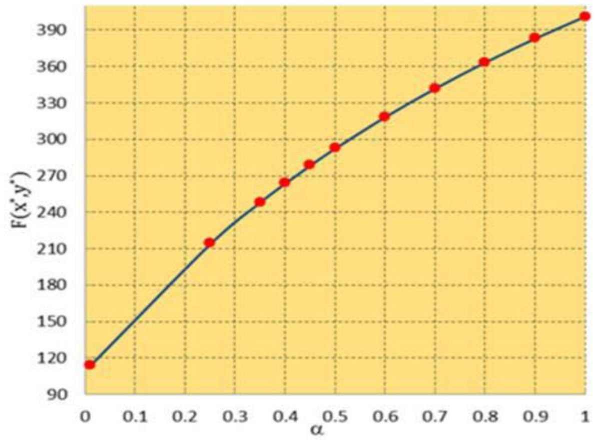


Figure 3: Trend of the objective value with different α values

According to whatever said before, the novel TPLP model is considered as an unconstrained nonlinear optimization problem which has a complexity in solving it by appling standard mathematical programming [2]. Thus, a new FLC with developed implication method for solving the model is designed to achieve the optimum or near optimum value of decision variables. Next section describes the new designed FLC.

4 A New Heuristic Model

Fuzzy logic controller is one type of decision support system that helps decision maker to make optimum or near optimum decision. In the FLCs, rules are constructed to model processes in a simple way [6]. There are two common implemented methods for fuzzy modeling in which experts' knowledge and historical data of system's behavior is applied, respectively. In the first one called direct approach, knowledge is achieved as If-Then rules with fuzzy predictions that create relations between relevant system variables [41]. Making a proper prediction of relations between antecedents and consequents is the most significant problem in this approach. However, availability of experts and reliability of their opinions are not permanent assumptions. By use of the presented method, we can overcome this problem and also provide a new approach for solving optimization problems based on fuzzy rule base. Mamdani's controller is developed to determine the values of the decision variables. The steps of designed FLC are as follows:

I. Assign linguistic terms to the fuzzy input parameters. In the developed Fuzzy TPLP, both of the coordinates $\tilde{x}_i$ and $\tilde{y}_i$ and also weights can be described with three linguistic variables as Figure 4.

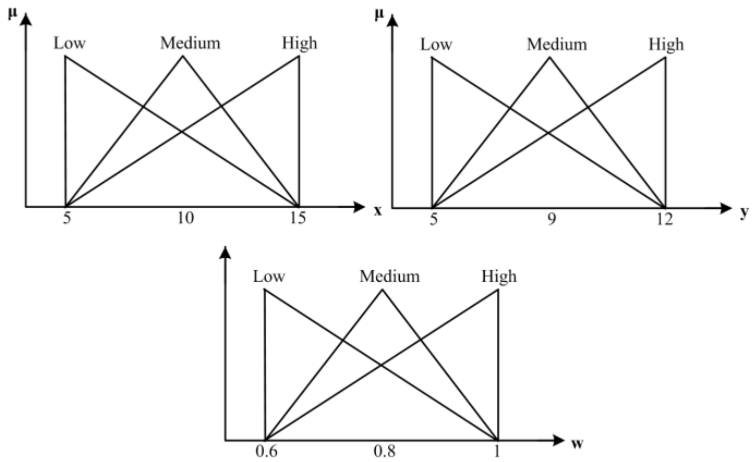


Figure 4: Fuzzy values for one set of input parameters $\tilde{x}_i$, $\tilde{y}_i$ and $\tilde{w}_i$

II. Consequents calculation. As mentioned above, in conventional fuzzy controllers a decision maker defined consequents in terms of linguistic variables identical with the antecedents. Thus, there was not any contribution in the consequents calculation. Hence, in this paper, a new method based on the optimum knowledge is developed to calculate the consequents. The following steps describe the calculation of the consequents:

Step 1: Rule base simulation. In this step, in order to calculate the consequent of each rule, all compositions of the linguistic variables of the antecedents must be built. Then, for each rule, the bellow steps must be applied.

Step 2: Define ε-levels for the antecedents. Let define the ε-level of the antecedents $\tilde{x}_i$, $\tilde{y}_i$ and $\tilde{w}_i$ as follows:

$$x_{i_\varepsilon} = \left[x_{i_\varepsilon}^L, x_{i_\varepsilon}^U\right] = \left[min\{x \in S(\tilde{x}) | \mu_{\tilde{x}}(x) \geq \varepsilon\}, max\{x \in S(\tilde{x}) | \mu_{\tilde{x}}(x) \geq \varepsilon\}\right] \quad (2)$$

$$y_{i_\varepsilon} = \left[y_{i_\varepsilon}^L, y_{i_\varepsilon}^U\right] = \left[\min\{y \in S(\tilde{y}) | \mu_{\tilde{y}}(y) \geq \varepsilon\}, \max\{y \in S(\tilde{y}) | \mu_{\tilde{y}}(y) \geq \varepsilon\}\right] \quad (3)$$

$$w_{i_\varepsilon} = \left[w_{i_\varepsilon}^L, w_{i_\varepsilon}^U\right] = \left[min\{w \in S(\tilde{w}) | \mu_{\tilde{w}}(w) \geq \varepsilon\}, max\{w \in S(\tilde{w}) | \mu_{\tilde{w}}(w) \geq \varepsilon\}\right] \quad (4)$$

Step 3: Calculate the decision variables $\left(x_\varepsilon^{*\zeta}, y_\varepsilon^{*\zeta}\right)$ and the objective function $\left(F_\varepsilon^{*\zeta}\right)$ at different ε-levels by (5), (6) and (7). It is essential to know that, the more ε-levels are considered, the better possibility distributions of the consequents will be obtained. In this step, "ζ" is the state, i.e., the "Lower" or the "Upper" bound, of the antecedents and the consequents at the ε-level. To construct the consequents, let define the conditions as follows:

- If "ζ" be the "Lower" bound of the antecedents, then solve (5), (6) and (7) by the antecedents $\left(x_{i_\varepsilon}^L, y_{i_\varepsilon}^L\right)$ and $w_{i_\varepsilon}^L$ to obtain the decision variables $\left(x_\varepsilon^{*L}, y_\varepsilon^{*L}\right)$ and the objective function F_ε^{*L} at ε-level.
- If "ζ" be the "Upper" bound of the antecedents, put antecedents $\left(x_{i_\varepsilon}^U, y_{i_\varepsilon}^U\right)$ and $w_{i_\varepsilon}^U$ in (5), (6) and (7) to obtain the decision variables $\left(x_\varepsilon^{*U}, y_\varepsilon^{*U}\right)$ and the objective function F_ε^{*U} at ε-level.

$$x_\varepsilon^{*\zeta} = \frac{\alpha x_0 + \sum_{i=1}^n w_{i_\varepsilon}^\zeta x_{i_\varepsilon}^\zeta}{\alpha + \sum_{=1}^n w_{i_\varepsilon}^\zeta} \quad (5)$$

$$y_\varepsilon^{*\zeta} = \frac{\alpha y_0 + \sum_{i=1}^n w_{i_\varepsilon}^\zeta y_{i_\varepsilon}^\zeta}{\alpha + \sum_{i=1}^n w_{i_\varepsilon}^\zeta} \quad (6)$$

$$F_\varepsilon^{*\zeta} = \sum_{i=1}^n w_{i_\varepsilon}^\zeta \left[\left(x_{i_\varepsilon}^\zeta - x_\varepsilon^{*\zeta}\right)^2 + \left(y_{i_\varepsilon}^\zeta - y_\varepsilon^{*\zeta}\right)^2\right] + \alpha\left[\left(x_\varepsilon^{*\zeta} - x_0\right)^2 + \left(y_\varepsilon^{*\zeta} - y_0\right)^2\right] \quad (7)$$

For the particular ε-level, the resulted objective function $F_\varepsilon^{*\zeta}$ and the decision variables $\left(x_\varepsilon^{*\zeta}, y_\varepsilon^{*\zeta}\right)$ have the possibility degree ε for the occurrence, when all of the parameters $\tilde{x}_i$, $\tilde{y}_i$ and $\tilde{w}_i$ in (5), (6) and (7) occur in the "*Lower*" or the "*Upper*" bounds of the ε-level.

Step 4: Collecting the "*Lower*" and "*Upper*" bounds of the decision variables $\left(x_\varepsilon^{*\zeta}, y_\varepsilon^{*\zeta}\right)$ and the objective function $F_\varepsilon^{*\zeta}$ for all values of the ε-level derived from step 3. Now, the possibility distributions of the consequents are derived.

III. Rule base construction. According to Step 2, to obtain the consequents the possibilistic TPLP must be solved initially. Now, the fuzzy rule base must be designed using fuzzy parameters as the antecedents and the fuzzy decision variables as the consequents.

IV. Implication. So far, Mamdani's controller has used triangular norms as implication method. But, in this paper, a new implication method is developed named Correlation Coefficient – based Implication (CCI) method. The steps of proposed method are as follows:

Step 1: Randomly generate $a_i \in Supp\,(x_i)$ and $b_i \in Supp\,(y_i)$ and $c_i \in Supp\,(w_i)$, then derive optimum location (x^*, y^*) of the decision variables for N times in which N is a sufficient large number.

Step 2: Using values obtained through previous step, calculate ρ_x^i ; $i = 1, \dots, n$ as the correlation coefficient between the antecedents x_i *and* w_i; $i = 1, \dots, n$ and the consequent x, respectively. Also, calculate ρ_y^i; $i = 1, \dots, n$ as the correlation coefficient between each antecedent y_i *and* w_i; $i = 1, \dots, n$ and the consequent y, respectively like ρ_x^i. Then, put $\rho_x^* = \max_i\left|\rho_x^i\right|$ and $\rho_y^* = \max_i\left|\rho_y^i\right|$ as the candidate antecedents named $\tilde{x}_0$ and $\tilde{y}_0$, respectively.

Step 2: If a_0 and b_0 be the inputs of the candidate antecedents $\tilde{x}_0$ and $\tilde{y}_0$, then put $\alpha = \mu_{\tilde{x}_0}(a_0)$ and $\beta = \mu_{\tilde{y}_0}(b_0)$.

Step 3: Calculate x_R and x_L as the upper and lower bounds of $\mu_{\tilde{x}}^{-1}(\alpha_0)$ respectively and also y_R and y_L respectively as the upper and lower bounds of $\mu_{\tilde{y}}^{-1}(\beta_0)$.

Step 4: Based on the sign (negative or positive) of the correlation between the antecedent $\tilde{x}_0$ and the consequent x, one of the cases 1 or 2 will be fired to derive x, and also y will be obtained through the cases 3 or 4, subject to the sign of the correlation between the antecedent $\tilde{y}_0$ and consequent y , according to the following algorithm:

- $Case1\text{: } Correlation(\tilde{x}_0,\ \tilde{x}) > 0$
  $If\ a_0 > x_0^M, then\ x = x_R, Else\ x = x_L.$
- $Case2\text{: } Correlation(\tilde{x}_0,\ \tilde{x}) < 0$
  $If\ a_0 > x_0^M, then\ x = x_L, Else\ x = x_R.$
- $Case3\text{: } Correlation(\tilde{y}_0,\ \tilde{y}) > 0$
  $If\ b_0 > y_0^M, then\ y = y_R, Else\ y = y_L.$
- $Case4\text{: } Correlation(\tilde{y}_0,\ \tilde{y}) < 0$
  $If\ b_0 > y_0^M, then\ y = y_L, Else\ y = y_R.$

Developed CCI algorithm is shown in Figure 5. This figure shows the function of the CCI method for a typical rule. In part "a" the typical rule is depicted, and in part "b" the candidate antecedent $\tilde{x}_0$ and its related consequent x are prepared for implementing CCI method. Putting the random value of the $\tilde{x}_0$, the membership function of it will be derived as "α". This "α" value is the fulfillment degree of the consequent x which make two crisp value for consequent named x_L and x_R. Finally, based on the correlation coefficient sign of the $\tilde{x}_0$ and x, one of these two values will be chosen.

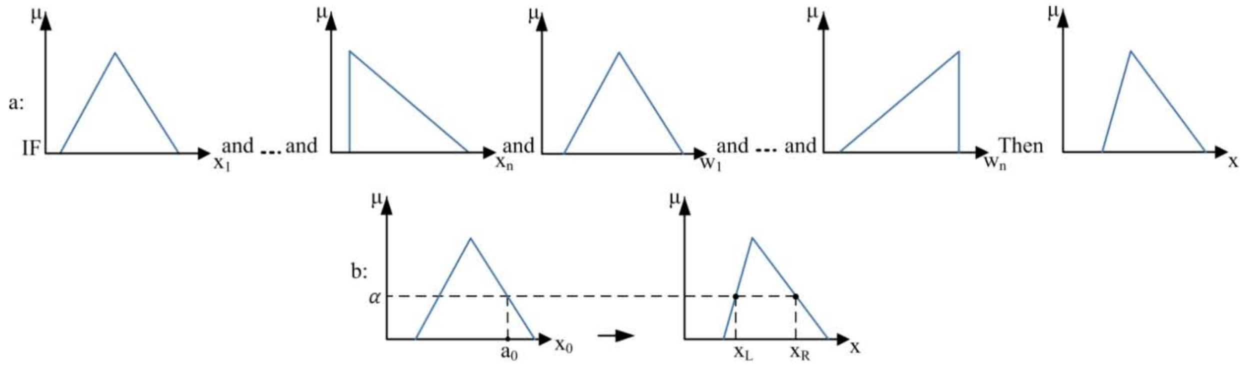


Figure 5: a: Representation of a rule from the fuzzy rule base; b: CCI method

It must be mentioned that the above algorithm in the rule base must be applied for all rules in the rule base. As a result, only one value for each rule will be obtained. Now, it is time to aggregate the derived values into one value for each consequent.

V. Aggregation. The aggregated outputs $\hat{x}$ and $\hat{y}$ (decision variables in Fuzzy TPLP) are calculated by use of the weighted average as (8) and (9).

$$\hat{x} = \frac{\sum_{i=1}^{k} \alpha_i x_i}{\sum_{i=1}^{k} \alpha_i} \tag{8}$$

$$\hat{y} = \frac{\sum_{i=1}^{k} \beta_i y_i}{\sum_{i=1}^{k} \beta_i} \tag{9}$$

In the above, k indicates the number of rules in the rule base and α_i and β_i are activation degrees of the i^{th} rule which obtained from above algorithm. Meanwhile, x_i and y_i are the values derived from pervious stage of the algorithm for i^{th} rule.

It is essential to mention that no defuzzification method has been used in the above algorithm. The implication of the developed FLC model is demonstrated by use of a numerical example in the next section.

5 Numerical Example

In this section, the Fuzzy Minisum TPLP problem is designed to demonstrate the applicability of the proposed model and also the efficiency of the designed FLC with three fuzzy demand areas as follow:

$$F = \min_{x,y} \left\{ \sum_{i=1}^{3} \tilde{w}_i d_i^2(x,y) + \alpha\, d^2(x,y) \right\} \tag{10}$$

As said, each demand coordinates have a possibility distribution $(\tilde{x}_i, \tilde{y}_i)$ and weights $\widetilde{w}_i$ are in form of linguistic terms created by the fuzzy numbers. Consider the location of the facility is (x_0, y_0) =(35,18) and other parameters are as Table 2.

Table 2: Parameters of the example

P_i	$\tilde{x}_i$	$\tilde{y}_i$	$\widetilde{w}_i$
1	(5, 8, 10)	(15, 18, 20)	(0.01, 0.25, 0.5)
2	(5, 10, 15)	(5, 9, 12)	(0.6, 0.8, 1)
3	(20, 25, 30)	(10, 12, 15)	(0.3, 0.5, 0.7)

The optimum possibility distributions of the transfer point coordinates will be derived by solving the (10). Using (5), (6) and (7), the transfer points' coordinates are achieved as $(\tilde{x}, \tilde{y})$ = ((18.24, 17.03, 21.42), (10.48, 13.44, 15.32)) and the objective function value is calculated as (266.22, 272.59, 278).

Based on the fuzzy parameters and the decision variables $\tilde{x}$ and $\tilde{y}$ and also the objective function $\tilde{F}$, following If-Then rule could be structured:

If $\tilde{P}_1$ = ((5, 10, 10), (5, 8, 10)) and $\tilde{P}_2$ = (5, 5, 15), (5, 10, 15)) and $\tilde{P}_3$ = ((20, 20, 30), (20, 30, 30)) and $\widetilde{w}_1$ = (0.01, 0.25, 0.5) and $\widetilde{w}_2$ = (0.6, 0.6, 1) and $\widetilde{w}_3$ = (0.3, 0.3, 0.7) Then $\tilde{P}$ = ((18.24, 17.03, 21.42), (10.48, 13.44, 15.32)) and $\tilde{F}$ = (266.22, 272.59, 278).

In order to construct the fuzzy rule base as a DSS for this model, uncertain parameters must be described in the terms of linguistic variables. For this purpose, $\tilde{P}_1, \tilde{P}_2, \tilde{P}_3$ and their weights, $\widetilde{w}_1, \widetilde{w}_2, \widetilde{w}_3$, are described with three linguistic variables *Low*, *Medium* and *High* and also are quantified in the form of triangular fuzzy numbers as exposed in Table 3.

Table 3: Triangular fuzzy numbers of the coordinates $\tilde{P}_i$ and the weights $\widetilde{w}_i$

$\tilde{P}_i$	$\tilde{x}_i$			$\tilde{y}_i$			$\widetilde{w}_i$		
	Low	*Medium*	*High*	*Low*	*Medium*	*High*	*Low*	*Medium*	*High*
$\tilde{P}_1$	(5,5,10)	(5,8,10)	(5,10,10)	(15,15,20)	(15,18,20)	(15,20,20)	(0.01,0.01,0.5)	(0.01,0.25,0.5)	(0.01,0.5,0.5)
$\tilde{P}_2$	(5,5,15)	(5,10,15)	(5,15,15)	(5,5,12)	(5,9,12)	(5,12,12)	(0.6,0.6,1)	(0.6,0.8,1)	(0.6,1,1)
$\tilde{P}_3$	(20,6,30)	(20,25,30)	(20,30,30)	(10,10,15)	(10,12,15)	(10,15,15)	(0.3,0.3,0.7)	(0.3,0.5,0.7)	(0.3,0.7,0.7)

As an example, fuzzy values of the uncertain coordinates of the first demand area $(\tilde{x}_1, \tilde{y}_1)$ are shown in Figure 6.

A Decision Support System in the form of fuzzy rule based on the several values of uncertain parameters and related values of decision variables is designed. A part of the resulted FLC construction is shown in Table 4. In this table, five rules are designed and the possibility distribution of the objective value $(\tilde{F})$ and the decision variable $(\tilde{x}, \tilde{y})$ are derived consequently.

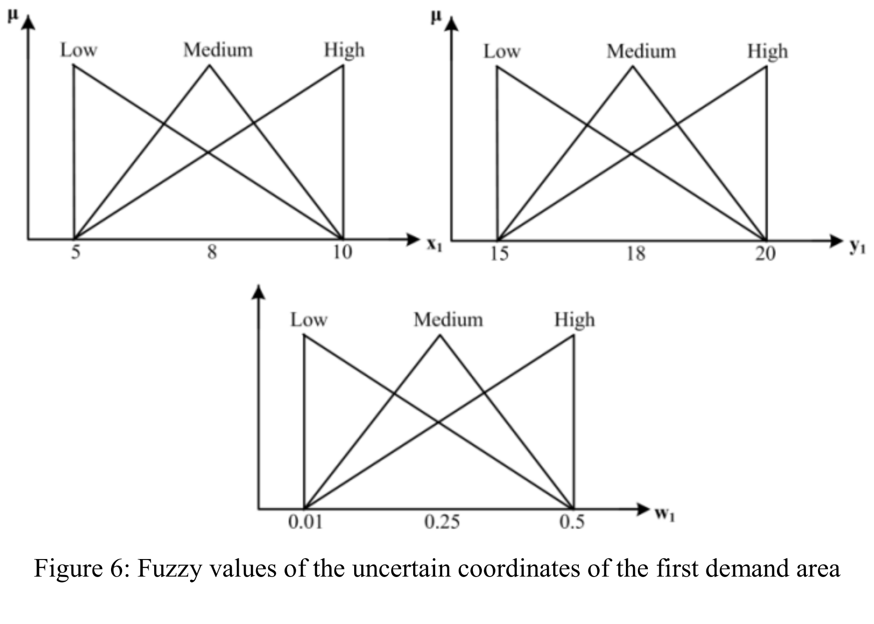


Figure 6: Fuzzy values of the uncertain coordinates of the first demand area

Table 4: Fuzzy control rule base

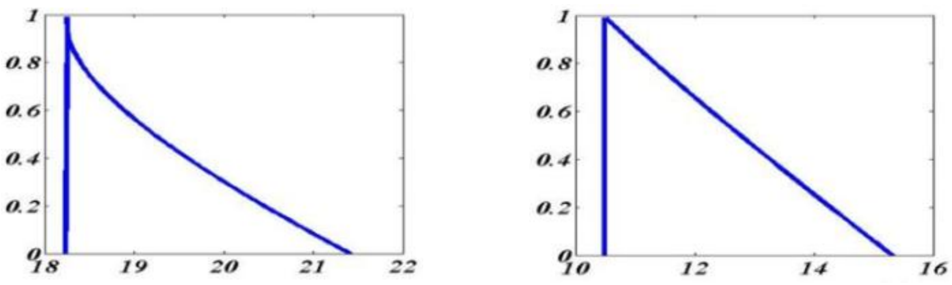
$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{w}_1$	$\tilde{w}_2$	$\tilde{w}_3$	$(\tilde{x}, \tilde{y})$	$\tilde{F}$
L	L	L	L	L	L	L	L	L	((18.24,18.21,21.421),(10.48,10.56,15.32))	(266.22,277.63,278)
H	L	L	M	M	H	M	L	L	((18.24, 17.03, 21.42),(10.48, 13.44, 15.32))	(266.22,272.59,278)
M	L	L	L	L	L	L	L	L	((18.24,18.25,21.42),(10.48,10.56,15.32))	(266.22,276.2755,278)
H	L	L	L	M	H	M	L	L	((18.24,17.08,21.42),(10.489,13.62,15.32))	(266.22,268.51,278)
L	L	L	L	L	M	L	L	L	((18.24,18.21,21.42),(10.48,10.91,15.32))	(266.22,277.3402,278)

Now, it is time to describe how the possibility distributions of the objective value ($\tilde{F}$) and the decision variable ($\tilde{x}$,$\tilde{y}$) are derived in detail. Using (5), (6) and (7), the upper and the lower bounds of the objective function ($\tilde{F}$) and the decision variable ($\tilde{x}$,$\tilde{y}$) will be obtained at 101 different ε levels (ε = 0, 0.01, 0.02, …, 0.99, 1). ε is the possibility degree and with decrease of the ε value, the degree of possibility is reduced and vice versa. In other words, when $\varepsilon = 0$, uncertain parameters are placed in their upper or lower bounds. This fact shows the inexpertness of a person who estimates the parameters. On the other hand, when $\varepsilon = 1$, the uncertain parameters are in their mean values. This condition, points to the expertness of the parameters estimator. The possibility distributions of the objective value ($\tilde{F}$) and the decision variable ($\tilde{x}$,$\tilde{y}$) at 11 different ε levels, when $(\tilde{x}_1, \tilde{y}_1)$, $(\tilde{x}_2, \tilde{y}_2)$, $(\tilde{x}_3, \tilde{y}_3)$ and weights, $\tilde{w}_1, \tilde{w}_2$ and $\tilde{w}_3$, are (*Medium, Low*), (*Low, Low*), (*Low, Low*), *Low*, *Low* and *Low*, respectively, are shown in Table 5.

Table 5: Generation of ($\tilde{F}$) and ($\tilde{x}$,$\tilde{y}$) when $\alpha = 0.2$

ε levels	x_L	x_U	y_L	y_U	F_L	F_U
0.01	18.23	21.413	10.478	15.321	266.221	278.004
0.1	18.237	20.972	10.478	14.853	260.913	277.933
0.2	18.239	20.501	10.478	14.336	256.972	277.856
0.3	18.241	20.055	10.478	13.826	254.930	277.781
0.4	18.243	19.639	10.478	13.321	254.614	277.708
0.5	18.246	19.260	10.478	12.824	255.836	277.636
0.6	18.248	18.923	10.478	12.335	258.392	277.567
0.7	18.251	18.639	10.478	11.857	262.053	277.499
0.8	18.253	18.421	10.478	11.393	266.558	277.432
0.9	18.255	18.284	10.478	10.945	271.596	277.368
1	18.257	18.253	10.478	10.519	276.789	277.305

Also, the possibility distributions of $\tilde{x}$ and $\tilde{y}$ are shown in Figure 7.



Optimum fuzzy coordinate ($\tilde{x}$) Optimum fuzzy coordinate ($\tilde{y}$)

Figure 7: The generation of possibility distributions of x̃ and ỹ using FLC

The developed FLC will be able to solve the problem proficiently in the form of (10) with deterministic parameters. 100 samples of the demand weights and coordinates exist in three demand areas are generated randomly and solved by the developed FLC ($F(x^*, y^*)$). Then, outputs are compared with the optimum solutions ($F^*(x, y)$). Table 6 presents some of these examples.

The results of the comparison between values of developed FLC and optimum solution for 100 randomly generated examples are shown in Table 7.

As a result of this table, average deviation of x, y and $F(x, y)$ are 6.02%, 3.86% and 9.84%, respectively. To demonstrate the robustness of the value of the average deviation for 100 different samples and also to show the pattern of the deviations dispersion, the cdf curve of the $F(x, y)$ deviations for 100 samples are provided as Figure 8.

Table 6: Some examples when (x^*, y^*) = (10.34, 14.07) and $\alpha = 0.2$

E.g.	$(\tilde{x}_1, \tilde{y}_1)$	$(\tilde{x}_2, \tilde{y}_2)$	$(\tilde{x}_3, \tilde{y}_3)$	$\tilde{w}_1$	$\tilde{w}_2$	$\tilde{w}_3$	$F(x^*,y^*)$	$F^*(x, y)$	Error
1	(5.26,17.76)	(9.42,7.2)	(23.86,10.54)	0.07	0.91	0.49	259.56	246.23	13.34
2	(5.26,15.72)	(14.08,11.72)	(22.24,12.3)	0.25	0.68	0.5	214.21	182.51	31.69
3	(7.42,20)	(5.76,8.5)	(25.38,14.32)	0.39	0.71	0.57	314.76	337.04	22.29
4	(7.86,18.68)	(6.74,10.72)	(27.3,13.94)	0.36	0.6	0.46	293.60	288.05	5.54
5	(6.8,17.82)	(14.38,5.42)	(25.34,13.48)	0.26	0.77	0.65	209.42	234.70	25.29
6	(9.58,15.66)	(9,9.72)	(28.64,11.06)	0.16	0.91	0.47	262.47	272.80	10.33
7	(8.68,17.7)	(7.6,10.18)	(20.42,12.54)	0.18	0.76	0.6	278.46	236.09	42.37
8	(5.78,19.4)	(8.76,9.26)	(29.84,11.92)	0.04	0.64	0.4	264.19	235.32	28.87
9	(6.8,17.36)	(8.88,5.38)	(24.12,14.64)	0.25	0.65	0.47	263.30	271.20	7.90
10	(5.02,17.56)	(14.14,7.74)	(22.66,13.76)	0.33	0.78	0.54	213.81	231.29	17.47
11	(8.02,19.62)	(10.02,8.9)	(21.24,14.2)	0.28	0.78	0.33	256.58	231.12	25.45
12	(5.4,19.32)	(5.86,5.4)	(24.18,14.82)	0.13	0.84	0.34	312.23	327.76	15.53
13	(8.44,15.38)	(10.5,8.94)	(22.52,13.1)	0.42	0.93	0.6	253.10	245.12	7.98
14	(5.56,16.4)	(14.24,9.1)	(23.94,12.8)	0.24	0.87	0.42	212.23	200.88	11.36
15	(6.12,15.6)	(14.94,5.1)	(28.6,11.56)	0.24	0.64	0.67	200.01	235.39	35.38

Table 7: Results of comparison between values of the developed FLC and optimum solution

Deviation from optimum solution (%)	*x*	*y*	*F(x, y)*
Minimum deviation	0.056	0.038	0.231
Maximum deviation	19.043	12.11	25.275
Average deviation	6.200	3.859	9.842

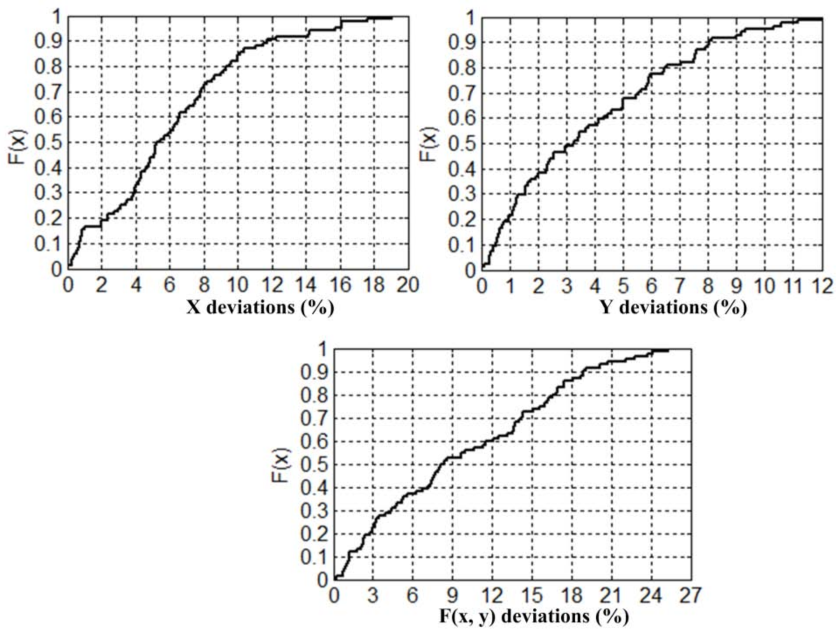


Figure 8: Cdf curve of the x, y and $F(x, y)$ deviations obtained from the developed FLC

This figure depicts that about 90% of the errors are less than 20% and over 60% are less than 12% which demonstrates the effectiveness of the developed FLC. Also, contour lines of the 100 samples when $\alpha = 0.2$ are plotted in Figure 9. In this figure, the output of the developed FLC (model solution) is pictured by a white star shape and the actual solution is shown by a black star shape.

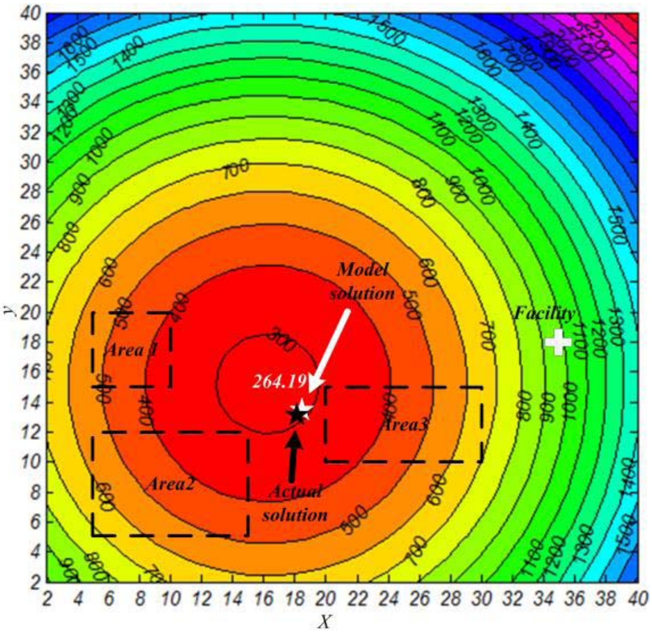


Figure 9: Contour lines for validation of example when $\alpha = 0.2$

It can be observed that the developed FLC contour is extremely close to actual solution contour. Hence, the developed FLC result is near optimal with respect to fuzzy nature of the problem.

It is so clear from the above issues that the developed model is so reliable and can be used as a powerful DSS by decision makers. It must be considered that a lot of scenarios were studied for getting these results.

6 Conclusion Remarks and Future Research

In this paper, a new transfer point location problem has been proposed in which demands have fuzzy coordinates and also demand weights are fuzzy in nature. A possibilistic unconstrained nonlinear programming was used to formulate the model. Our contributions of this paper are summarized as bellow: To give a clarification for the applicability of the model, conceptual justifications are designed in this paper. Due to complexity in solving the model, a new Fuzzy Logic Controller has been designed to reach the optimum or near optimum values of decision variables. In this new Fuzzy Logic Controller, consequents were derived based on optimum knowledge where in Mamdani's inference system they were presented by Decision Maker. Also, a new implication method was developed named Correlation Coefficient – based Implication function in order to implement the consequents in the Inference System. The implication of the developed FLC model is demonstrated the numerical example and providing validation tests considering fuzzy nature of the problem. The results demonstrated that the proposed model with developed FLC is so reliable and could be used as a powerful Decision Support System by decision makers. More studies on the problem with probability distributions of the demand coordinates, network version of fuzzy minisum TPLP or fuzzy version of the MTPLP can be mentioned as future researches.

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