

Uncertain Core for Coalitional Game with Uncertain Payoffs

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Abstract

Coalitional game deals with situations that involve cooperations among the players. There are different solution concepts such as core, Shapley value and nucleolus. This paper investigates uncertain coalitional game in which the payoffs are uncertain variables. Within the framework of uncertainty theory, two definitions of uncertain core are proposed. Meanwhile, a sufficient and necessary condition is provided for finding the uncertain core. Finally, an example is provided for illustrating purpose.

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1 Introduction

Game theory is a collection of mathematical models that analyze conflict and cooperation between rational players. Since von Neumann and Morgenstern's seminal work [27], game theory has been used extensively in many fields like economics, sociology and politics. Coalitional game deals with situations that groups of players (coalitions) may enforce cooperative behavior. Hence, coalitional game is a competition between coalitions of players, rather than between individual players. In literature, many researchers introduced different solution concepts of coalitional game from different perspectives. While the Core by Aumann and Maschler [2] and Shapley value by Shapley and Shubik [25] are mostly used.

In real-world games, players are often lack of the information about the other players' (or even his own) payoffs [17]. Then, how to deal with such incomplete information of payoffs is a basic problem for further discussion of uncertain game. For scientists of probability theory, the payoffs like "about 10 thousands units" are assumed to be random variables, and the resulting random-payoff games were investigated by probabilistic methods [17, 7, 6, 3]. For scientists of fuzzy theory, the payoffs are regarded as fuzzy variables. Within the framework of credibility theory, Gao and his co-workers proposed a spectrum of credibilistic games including credibilistic strategic game [8, 9, 11, 15, 18], credibilistic coalitional game [12, 26, 28] and credibilistic extensive game [13]. Fuzzy cooperative game was also discussed by Aubin [1], Mareš [22], etc.

We note that, the reason why we use "about 10 thousands units" to estimate the payoff is that there are not enough history data (or even no) for probabilistic reasoning. So it is not appropriate to regard it as random variable. If the quantity is assumed to be a fuzzy variable, we can obtain that the payoff is "exactly 10 thousands units" with possibility measure 1 (or credibility measure 0.5). On the other hand, the opposite event "not exactly 10 thousands units" has the same possibility measure (or credibility measure). There is no doubt that nobody can accept this conclusion. This paradox shows that the quantity "about 10 thousands units" cannot be a fuzzy concept.

Yet, we know that such quantities behave neither like randomness nor like fuzziness. In order to formulate subjective uncertainties such as "about 10 thousands units", an uncertainty theory was founded by [19] and refined by [21]. Uncertainty theory is a branch of mathematics based on an axiomatic system, in which a quantity "about 10 thousands units" may be formulated as an uncertain variable with some uncertainty distribution. When the payoffs are characterized by uncertain variables, the players' objective functions are uncertain, too. In order to optimize their objectives, there are two ranking methods. One is the expected value criterion that corresponds to the decision situation in which the decision-maker wants to optimize the expected value of his uncertain objective. The other is the optimistic value criterion that models the situation that the decision maker would maximize the optimistic value of the uncertain objective at some given confidence

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level α . Recently, Gao [14] investigated uncertain bimatrix game in which the payoffs are uncertain variables under the framework of uncertainty theory.

In this work, the payoffs of the coalitional game are assumed to be uncertain variables according to an expert system, and we focus on the coalitional game with uncertain payoff as well as the solution concept of core. The rest of this paper is arranged as follows. In section 2, we briefly recall the core of coalitional game with transferable payoffs and introduce two uncertain ranking approaches to define the behavior types of players under uncertain situation. In section 3, two variations of core are proposed as the solutions of coalitional game with uncertain payoffs. One is the expected core, and the other is the α -optimistic core. Meanwhile, we gave a sufficient and necessary condition that provides a way to find the uncertain core. Lastly, we provide an example for illustrating purpose in Section 4.

2 Preliminaries

In this section, we first recall the notion of coalitional game with transferable payoff, and then give some basic results in uncertainty theory.

2.1 The Core

In the early 1950s, Gillies [16] introduced the notion of the core as a tool to study stable sets. Later, Shapley and Shubik [25] developed it as a solution concept of coalition game. Now we give the definition of coalition game with transferable payoff.

Definition 2.1 ([16]) *A coalitional game $\langle N, v \rangle$ with transferable payoff consists of:*

- 1) *a finite set N (the set of the players);*
- 2) *v is function from the set of coalition S to the real number field where every nonempty subset S of N .*

For each coalition S , the payoff $v(S)$ is the total payoff that is available for the division among the members of S .

An assumption of the this model is that

$$v(U \cup T) \geq v(S) + v(T)$$

for all the S and T with $S \cap T = \emptyset$. That is, the payoff of a coalition must more than sum of the payoff that each player could receive if he or she does not join the coalition. This condition is called superadditivity.

Definition 2.2 ([23]) *A coalitional game $\langle N, v \rangle$ with transferable payoff is cohesive, if*

$$v(N) \geq \sum_{i=1}^k v(S_i)$$

for each partition $\{S_1, S_2, \dots, S_k\}$ of N .

This is a special case of the condition of superadditivity, and ensures that it is optimal that the coalition N of all players form.

In a coalitional game $\langle N, v \rangle$ with transferable payoff, where $N = \{1, 2, \dots, n\}$, we call $\mathbf{x} = (x_1, x_2, \dots, x_n)$ is called an allocation of N if it satisfies $\sum_{i=1}^n x_i = v(N)$ and $x_i \geq v(\{i\})$ for $i \in N$. Let $x(S) = \sum_{i \in S} x_i$, a vector $(x_i)_{i \in N}$ is a S -feasible payoff if $x(S) \leq v(S)$. And a N -feasible payoff vector is a feasible payoff profile.

Definition 2.3 ([16]) *The core of the coalitional game $\langle N, v \rangle$ with transferable payoff is the set of feasible payoff profiles $(x_i)_{i \in N}$, for which there is no coalition S and S -feasible payoff vector $(y_i)_{i \in N}$ for which $y_i > x_i$ for all $i \in S$.*

From this definition, it is inferred that the core is the set of feasible payoff vector $(x_i)_{i \in N}$ such that $x(S) \geq v(S)$, $\forall S \in 2^N$.

2.2 Uncertainty Theory

Uncertain measure \mathcal{M} is a real-valued set-function on a σ -algebra \mathcal{L} over a nonempty set Γ satisfying normality, duality, subadditivity and product axioms. The triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space.

Definition 2.4 ([19]) *An uncertain variable is a function from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that for any Borel set B of real numbers, the set*

$$\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}$$

is an event.

In order to describe uncertain variables in practice, uncertainty distribution $\Phi : \mathfrak{R} \rightarrow [0, 1]$ of an uncertain variable ξ is defined as

$$\Phi(x) = \mathcal{M}\{\xi \leq x\}.$$

An uncertain variable ξ is called zigzag if it has a uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leq a \\ (x - a)/2(b - a), & \text{if } a \leq x \leq b \\ (x + c - 2b)/2(c - b), & \text{if } b \leq x \leq c \\ 1, & \text{if } x \geq c, \end{cases}$$

denoted by $\mathcal{Z}(a, b, c)$ where a, b, c are real numbers with $a < b < c$.

An uncertainty distribution $\Phi(\cdot)$ is said to be regular if its inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0, 1)$. And $\Phi^{-1}(\cdot)$ is called the inverse uncertainty distribution of ξ . In this paper, we assume that all the payoffs are characterized by regular uncertain variables.

Therefore, the inverse uncertainty distribution of zigzag uncertain variable $\mathcal{Z}(a, b, c)$ is

$$\Phi^{-1}(\alpha) = \begin{cases} (1 - 2\alpha)a + 2\alpha b, & \text{if } \alpha < 0.5 \\ (2 - 2\alpha)b + (2\alpha - 1)c, & \text{if } \alpha \geq 0.5. \end{cases}$$

Definition 2.5 ([20]) *The uncertain variables $\xi_1, \xi_2, \dots, \xi_m$ are said to be independent if*

$$\mathcal{M}\left\{\bigcap_{i=1}^m \{\xi_i \in B_i\}\right\} = \min_{1 \leq i \leq m} \mathcal{M}\{\xi_i \in B_i\}$$

for any Borel sets B_1, B_2, \dots, B_m of real numbers.

Definition 2.6 ([19]) *Let ξ be an uncertain variable. Then the expected value of ξ is defined by*

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \geq r\}dr - \int_{-\infty}^0 \mathcal{M}\{\xi \leq r\}dr$$

provided that at least one of the two integrals is finite.

If ξ is a regular variable with uncertainty distribution $\Phi(\cdot)$, then the expected value may be calculated by

$$E[\xi] = \int_0^{+\infty} (1 - \Phi(x))dx - \int_{-\infty}^0 \Phi(x)dx = \int_0^1 \Phi^{-1}(\alpha)d\alpha.$$

Definition 2.7 ([19]) *Let ξ be a regular uncertain variable, and $\alpha \in (0, 1]$. Then*

$$\xi_{\text{sup}}(\alpha) = \sup\{r \mid \mathcal{M}\{\xi \geq r\} \geq \alpha\} = \Phi^{-1}(1 - \alpha)$$

is called the α -optimistic value to ξ .

Lemma 2.1 ([19]) *Let ξ and η be independent regular uncertain variables, and $\alpha \in (0, 1]$. Then for any nonnegative real numbers a and b , we have*

$$(a\xi + b\eta)_{\text{sup}}(\alpha) = a\xi_{\text{sup}}(\alpha) + b\eta_{\text{sup}}(\alpha).$$

Let ξ , η be two independent regular uncertain variables, $\alpha \in (0, 1]$, and r be a real number. We have the following uncertain ranking methods:

Expected Value Criterion:

$$\xi \geq \eta \quad \text{iff} \quad E[\xi] \geq E[\eta].$$

Optimistic Value Criterion:

$$\xi \geq \eta \quad \text{iff} \quad \xi_{\text{sup}}(\alpha) \geq \eta_{\text{sup}}(\alpha).$$

3 Two Uncertain Cores

In this section, we assume the payoffs in coalitional game to be uncertain variables, and give an analysis of the uncertain coalitional game. Firstly, we define the uncertain coalitional game.

Definition 3.1 *An uncertain coalitional game $\langle N, \tilde{v} \rangle$ with transferable payoff consists of:*

- 1) *a finite set N (the set of the players);*
- 2) *an uncertain variable $\tilde{v}(S)$ which is an uncertain payoff for every nonempty subset S of N (a coalition).*

As the traditional definition of transferable payoff, the uncertain payoff $\tilde{v}(S)$ is the total payoff that is available for the division among the members of S , for each coalition S .

3.1 The Expected Core

When the players' goals are to maximize the expected value of their uncertain objectives, we will use the expected value criterion and present the expected core as follows.

Definition 3.2 *An uncertain coalitional game $\langle N, \tilde{v} \rangle$ with transferable payoff is expected cohesive, if*

$$E[\tilde{v}(N)] \geq \sum_{i=1}^k E[\tilde{v}(S_i)]$$

for each partition $\{S_1, S_2, \dots, S_k\}$ of N .

Definition 3.3 *For any coalition S of an uncertain coalitional game $\langle N, \tilde{v} \rangle$, a vector $(x_i)_{i \in N}$ is called an expected feasible payoff vector if $x(S) \leq E[\tilde{v}(S)]$.*

Definition 3.4 *The expected core of an uncertain coalitional game $\langle N, \tilde{v} \rangle$, called an EC, is the set of expected feasible payoff profiles $(x_i)_{i \in N}$, for which there is no coalition S and expected feasible payoff vector $(y_i)_{i \in N}$ such that $y_i > x_i$ for all $i \in S$.*

From this definition, it is inferred that the expected core is the set of feasible payoff vector $(x_i)_{i \in N}$ such that $x(S) \geq E[\tilde{v}(S)]$, $\forall S \in 2^N$.

Definition 3.5 *The set $B = \{S | S \in 2^N, S \neq \emptyset\}$ is called a balanced collection, if there exists real numbers $\lambda_S \in [0, 1]$ such that*

$$\sum_{S \in B} \lambda_S \mathbf{1}_S = \mathbf{1}_N, \quad \lambda_S \in [0, 1]$$

where column vector $\mathbf{1}_S \in \mathbb{R}^N$,

$$(\mathbf{1}_S)_i = \begin{cases} 1, & i \in S, \\ 0, & \text{others.} \end{cases}$$

In this case, $(\lambda_S)_{S \in B}$ are called weights of balanced collection B .

For an example, let $N = \{1, 2, 3, 4\}$, so $B = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3, 4\}\}$ is a balanced collection with weights $1/3, 1/3, 1/3, 2/3$.

Definition 3.6 An uncertain coalitional game $\langle N, \tilde{v} \rangle$ is a balanced game, if for any balanced collection B and λ_S , we have

$$\sum_{S \in B} \lambda_S E[\tilde{v}(S)] \leq E[\tilde{v}(N)].$$

Base on the definitions give above, we give the theorem of non-emptiness of the core.

Theorem 3.1 The expected core of an uncertain coalitional game $\langle N, \tilde{v} \rangle$ with transferable payoff is nonempty if and only if $\langle N, \tilde{v} \rangle$ is balanced.

Proof: First, assuming that $(x_i)_{i \in N}$ is a expected payoff profile in the core of $\langle N, \tilde{v} \rangle$ and $(\lambda_S)_{S \in B}$ are weights of balanced collection B , we prove that $\sum_{S \in B} \lambda_S E[\tilde{v}(S)] \leq E[\tilde{v}(N)]$. It follows from the definition of EC that

$$E[\tilde{v}(S)] \leq x(S) = \sum_{i \in S} x_i.$$

Then, we have

$$\lambda_S E[\tilde{v}(S)] \leq \lambda_S \left(\sum_{i \in S} x_i \right).$$

So,

$$\begin{aligned} \sum_{S \in B} \lambda_S E[\tilde{v}(S)] &\leq \sum_{S \in B} \lambda_S \left(\sum_{i \in S} x_i \right) \\ &= \sum_{S \in B} \lambda_S (\mathbf{1}_S)^T (x_1, x_2, \dots, x_n)^T \\ &= (\mathbf{1}_N)^T (x_1, x_2, \dots, x_n)^T \\ &= x(N) \leq E[\tilde{v}(N)]. \end{aligned}$$

The necessary condition is proved.

Second, now assume that $\langle N, \tilde{v} \rangle$ is balanced. Then there is no balanced collection $(\lambda_S)_{S \in B}$ of weights for which $\sum_{S \in B} \lambda_S E[\tilde{v}(S)] > E[\tilde{v}(N)]$. Therefore the convex set

$$\mathbf{V} = \{y = ((\mathbf{1}_N)^T, E[\tilde{v}(N)] + \varepsilon) \in \mathfrak{R}^{n+1}, \varepsilon > 0\}$$

is disjoint from the convex cone set

$$\mathbf{C} = \left\{ z = \sum_{S \in B} \lambda_S ((\mathbf{1}_S)^T, E[\tilde{v}(S)]) \in \mathfrak{R}^{n+1} \right\},$$

$\forall S \in B \text{ and } \lambda_S > 0$

since if not then $\mathbf{1}_N = \sum_{S \in B} \lambda_S \mathbf{1}_S$, so that $(\lambda_S)_{S \in B}$ is balanced collection of weights and $\sum_{S \in B} \lambda_S E[\tilde{v}(S)] > E[\tilde{v}(N)]$. Then by the separating hyperplane theorem, there is a nonzero vector $(\alpha_N, \alpha) \in \mathfrak{R}^{n+1}$, such that for all y and z ,

$$(\alpha_N, \alpha) z^T \geq 0 > (\alpha_N, \alpha) y^T.$$

Since $\langle N, \tilde{v} \rangle$ is balanced, $((\mathbf{1}_N)^T, E[\tilde{v}(N)])$ is in C . Thus,

$$(\alpha_N, \alpha) \begin{pmatrix} \mathbf{1}_N \\ E[\tilde{v}(N)] \end{pmatrix} > (\alpha_N, \alpha) \begin{pmatrix} \mathbf{1}_N \\ E[\tilde{v}(N)] + \varepsilon \end{pmatrix}$$

and then $\alpha < 0$. Let

$$\mathbf{x} = -\frac{\alpha_N}{\alpha},$$

we have

$$\left(-\frac{\alpha N}{\alpha}, -1\right) z^T \geq 0.$$

When $\mathbf{x} \geq 0$, $\mathbf{x} \in \mathfrak{R}^n$, $((\mathbf{1}_S)^T, E[\tilde{v}(S)]) \in \mathbf{C}$, $\forall S \in B$,

$$(\mathbf{x}, -1) \begin{pmatrix} \mathbf{1}_S \\ E[\tilde{v}(S)] \end{pmatrix} \geq 0.$$

Hence,

$$x(S) \geq E[\tilde{v}(S)] \quad \text{for } \forall S \in B.$$

On the other hand,

$$0 > \left(-\frac{\alpha N}{\alpha}, -1\right) \begin{pmatrix} \mathbf{1}_N \\ E[\tilde{v}(N)] + \varepsilon \end{pmatrix}.$$

So,

$$E[\tilde{v}(N)] \geq x(N).$$

That is, \mathbf{x} is a payoff profile. The theorem is proved. \square

3.2 The α -Optimistic Core

When the players' goals are to maximize the optimistic profits under a predetermined confidence level α , we will use the α -optimistic criterion and present the α -optimistic core as follows.

Definition 3.7 An uncertain coalitional game $\langle N, \tilde{v} \rangle$ with transferable payoff is α -optimistic cohesive, if

$$(\tilde{v}(N))_{sup}(\alpha) \geq \sum_{i=1}^k (\tilde{v}(S_i))_{sup}(\alpha)$$

for each partition $\{S_1, S_2, \dots, S_k\}$ of N and all $\alpha \in (0, 1)$.

Definition 3.8 A vector $(x_i)_{i \in N}$ is an optimistic feasible payoff vector if for any coalition S , $x(S) \leq (\tilde{v}(S))_{sup}(\alpha) = \Phi_S^{-1}(1 - \alpha)$ at the given confidence $\alpha \in [0, 1]$, where $\Phi_S(\cdot)$ is an uncertainty distribution of regular uncertain variable $\tilde{v}(S)$.

Definition 3.9 The α -optimistic core of an uncertain coalitional game $\langle N, \tilde{v} \rangle$ with transferable payoff, called an OC, is the set of optimistic feasible payoff profiles $(x_i)_{i \in N}$, for which there is no coalition S and optimistic feasible payoff vector $(y_i)_{i \in N}$ such that $y_i > x_i$ for all $i \in S$.

Definition 3.10 An uncertain coalitional game $\langle N, \tilde{v} \rangle$ is a balanced game, if for any balanced collection B and λ_S , we have

$$\sum_{S \in B} \lambda_S (\tilde{v}(S))_{sup}(\alpha) \leq (\tilde{v}(N))_{sup}(\alpha).$$

Obviously, the core is the set of optimistic feasible payoff vector $(x_i)_{i \in N}$ such that $x(S) \geq \Phi_S^{-1}(1 - \alpha)$ for all $S \in 2^N$. So, we have the following theorem of non-emptiness of the α -optimistic core.

Theorem 3.2 The α -optimistic core of an uncertain coalitional game $\langle N, \tilde{v} \rangle$ with transferable payoff is nonempty if and only if $\langle N, \tilde{v} \rangle$ is balanced.

The Proof is similar to 3.1. So we omit it here.

4 An Example

In this section, we give an example to calculate the expected and α -optimistic core of uncertain coalitional game.

Suppose that there is a three-player majority game, noted $N = \{1, 2, 3\}$, i.e., three players can obtain payoff, any two of them can obtain payoff independently of the actions of the third, and each player alone can obtain nothing, independently of the actions of the remaining two players.

The payoff of coalitions are zigzag uncertain variables. There are following:

$$\begin{aligned} \tilde{v}(\{1\}) &= \tilde{v}(\{2\}) = \tilde{v}(\{3\}) = 0, \\ \tilde{v}(\{1, 2\}) &= (110, 120, 150), \tilde{v}(\{1, 3\}) = (100, 130, 140), \\ \tilde{v}(\{2, 3\}) &= (95, 125, 135), \tilde{v}(\{1, 2, 3\}) = (130, 205, 260). \end{aligned}$$

First, we assume that the players all adopt the expected criterion, then the expected payoffs of the coalitions are given as follows:

$$\begin{aligned} E[\tilde{v}(\{1\})] &= E[\tilde{v}(\{2\})] = E[\tilde{v}(\{3\})] = 0, \\ E[\tilde{v}(\{1, 2\})] &= 125, E[\tilde{v}(\{1, 3\})] = 125, \\ E[\tilde{v}(\{2, 3\})] &= 120, E[\tilde{v}(\{1, 2, 3\})] = 200. \end{aligned}$$

The balanced collection is $B = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$, then the weights of B are $1/2, 1/2, 1/2$. Hence,

$$\sum_{S \in B} \lambda_S E[\tilde{v}(S)] = 185 < E[\tilde{v}(N)] = 200.$$

That is, (N, \tilde{v}) is balanced. Therefore, the expected core is nonempty by Theorem 3.1, and the payoff profiles $(125, 122.5, 122.5)$ is in the core.

Second, we assume that the players all want to get as much profit as possible with a confidence level 0.7. That is,

$$\begin{aligned} (\tilde{v}(\{1\}))_{sup}(0.7) &= (\tilde{v}(\{2\}))_{sup}(0.7) = (\tilde{v}(\{3\}))_{sup}(0.7) = 0, \\ (\tilde{v}(\{1, 2\}))_{sup}(0.7) &= 116, \quad (\tilde{v}(\{1, 3\}))_{sup}(0.7) = 118, \\ (\tilde{v}(\{2, 3\}))_{sup}(0.7) &= 113, \quad (\tilde{v}(\{1, 2, 3\}))_{sup}(0.7) = 175. \end{aligned}$$

We can get the 0.7-optimistic core is nonempty and payoff profiles $(117, 114.5, 115.5)$ is in the core. If we change the confidence level from 0.5 to 0.7, we get that core is still nonempty but the payoffs different. When the confidence level is 0.8 and 0.9, the core is empty. We can calculate the core is empty when confidence level above 0.71875. And the computational results is arranged in Table 1.

Table 1: Payoff profiles at different confidence levels

No.	α	$(\tilde{v}(\{1, 2\}))_{sup}(\alpha)$	$(\tilde{v}(\{1, 3\}))_{sup}(\alpha)$	$(\tilde{v}(\{2, 3\}))_{sup}(\alpha)$	$(\tilde{v}(\{1, 2, 3\}))_{sup}(\alpha)$	Payoff profile
1	0.5	120	130	125	205	(125, 122.5, 127.5)
2	0.6	118	124	119	190	(121, 118.5, 121.5)
3	0.7	116	118	113	175	(117, 114.5, 115.5)
4	0.71875	115.625	116.875	111.875	172.075	—
5	0.8	114	112	107	160	—
6	0.9	112	106	101	145	—

Comparing the conclusions in the Table 1, the difference of core reflects the difference of players' preference: some player have more confidence on the coalition in the game and choose lower confidence level; others do not prefer to afford the risk of coalition and hope to keep stable payoffs under high confidence level. So when the confidence above some value (i.e. 0.71875), the core is empty. From the table, it is obvious that the profile is linear to α . So, the uncertain cores have the same characters which provide a stable method to solve uncertain coalitional game.

5 Conclusion

In this paper, we investigated an uncertain coalitional game with transferable payoff. Based on the uncertainty theory, we proposed the two definition of uncertain cores. Moreover, we gave a sufficient and necessary condition that provides a way to find the non-emptiness of the uncertain core. Finally, an example is provided for illustrating purpose.

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