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Two-Stage Stochastic Programming Problems Involving Some Continuous Random Variables

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Abstract

In this paper, we proposed a solution procedure of a two-stage stochastic programming problem where the right hand side parameters follow either uniform or exponential or normal or log-normal distribution with known mean and variance. To establish the solution of the stated problem, we first convert the problem into an equivalent deterministic model. Then a standard linear/non-linear programming technique is applied to solve the transformed deterministic model. Illustrative numerical examples are provided to demonstrate the solution procedure of the developed methodology.

Keywords: two-stage stochastic programming, uniform distribution, exponential distribution, normal distribution, log-normal distribution, linear/non-linear programming technique

1 Introduction

Stochastic programming (SP) is an optimization framework for modelling problems that involve uncertainty. This implies that some of the parameters in the model coefficients are random variables with known probability distribution. Generally, SP is often used in several real-world decision making problems of management science, engineering, and technology. During the last five decades, SP has been applied in many areas such as, energy, finance, telecommunications, transportation, production control and scheduling, agriculture, military, environmental planning, etc.

When the optimal decision is not specified to the realization of future events, a static stochastic programming model is formulated, although in many contexts the decision maker has to make a decision before observing random events which influence the system he/she wants to control. Further, the optimum solution can be obtained after observation of the random events. For this case a special class of dynamic programming model has to be formulated known as two-stage stochastic programming problem, which is also known as the stochastic programming problem with recourse.

The formulation of two-stage stochastic programming problems was first introduced by Dantzig [7]. This model was further developed by Beale [4] and Dantzig and Madansky [8]. Subsequently, Wets [23] presented an equivalent convex program of a two stage stochastic programming under uncertainty, while Maarten [21] presented an additional bibliographical study of stochastic programming based on the study of nearly 351 research papers, from 1996-2007. Quite successfully, Maqsood et al. [19] proposed an interval-parameter fuzzy two-stage stochastic programming method for the planning of water-resources-management systems under uncertainty. This study was braced by Li et al. [13] who proposed an interval-parameter two-stage stochastic mixed integer programming technique for waste management under uncertainty.

Many studies have reinforced investigations of problem under uncertainty. For example, Huang et al. [10] proposed an inexact two-stage stochastic quadratic programming model for water resources management under uncertainty. Khor et al. [12] proposed a two-stage stochastic programming model with fixed recourse via scenario analysis with incorporation of risk management for an optimal midterm refinery planning that addresses three factors of uncertainties: prices of crude oil and saleable products (in the objective function),

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product demands (in the RHS coefficients), and product yields (in the LHS coefficients). Lu et al. [17] developed an inexact two-stage fuzzy-stochastic programming method for water resources management under uncertainty. Beraldi et al. [5] proposed a two-stage stochastic integer programming model for the integrated optimization of power production and trading which include a specific measure accounting for risk management. An interval-fuzzy two-stage stochastic linear programming method has developed by Li et al. [14] for planning waste-management systems under uncertainty. Guo and Huang [9] presented a two-stage fuzzy chance-constrained programming approach for water resources management under dual uncertainties Lu et al. [18] developed an inexact programming method based on two-stage stochastic programming and interval-parameter programming is developed to obtain optimal water-allocation strategies for agricultural irrigation systems. Li et al. [15] presented a two-stage stochastic programming model for chemical production planning optimization with management of purchase and inventory under economic uncertainties including prices of raw materials, product prices and demands, employing Monte Carlo sampling method. Chen et.al. [6] developed a two-stage inexact-stochastic programming method for planning carbon dioxide (CO_2) emission trading under uncertainty. Lin and Huang [16] presented an interval-parameter two-stage stochastic municipal energy systems planning model for supporting decisions of energy systems planning and GHG (greenhouse gases) emission management at a municipal level. Barik et al. [1] established a solution procedure for solving the two-stage stochastic linear programming problem considering both randomness and interval parameters in the problem formulation. They [2] also developed a solution procedure for the multiobjective two-stage stochastic linear programming problem considering some parameters of the linear constraints as interval type discrete random variables with known probability distribution. Bashiri and Rezaei [3] proposed an extended relocation model for warehouses configuration in a supply chain network, in which uncertainty is associated to operational costs, production capacity and demands.

After going through the above literatures studied by various researchers, which motivate us to introduce a new solution procedure for solving the two-stage stochastic programming problem where the right hand side parameters follow either uniform or exponential or normal or log-normal distribution with known mean and variance. The solution procedure has been illustrated with suitable numerical examples.

2 Stochastic Programming Problem

Optimization problems involving some random parameters in the model coefficients can be modelled as stochastic programming problems. Mathematically, a stochastic programming problem can be stated as:

$$\min : z = \sum_{j=1}^{n} c_j x_j \tag{2.1}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, i = 1, 2, \ldots, m \tag{2.2}$$

$$\sum_{j=1}^{n} r_{sj} x_j \geq h_s, s = 1, 2, \ldots, l \tag{2.3}$$

$$x_j \geq 0, j = 1, 2, \ldots, n \tag{2.4}$$

where $x_j, j = 1, 2, \ldots, n$ are the decision variables, $a_{ij}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$ are the coefficients of the technological matrix, $c_j, j = 1, 2, \ldots, n$ are the coefficients associated with the objective function. Only the right hand side parameters $b_i, i = 1, 2, \ldots, m$ are considered as random variables which follow either uniform, or exponential, or normal or log-normal distribution with finite mean and variance.

2.1 Two-Stage Stochastic Programming Problem and Its Deterministic Model

Basically, two-stage stochastic programming problems are formulated to optimize the decisions which are made in two different stages. The first-stage decisions are made before the realization of the random events and the second-stage decisions are made after they have been realized. Mathematically, two-stage stochastic programming [22, 11] problem with simple recourse can be stated as:

$$\min : \bar{z} = \sum_{j=1}^{n} c_j x_j + E(\sum_{i=1}^{m} p_i |y_i|) \tag{2.5}$$

subject to

$$y_i = b_i - \sum_{j=1}^{n} a_{ij} x_j, i = 1, 2, \ldots, m \tag{2.6}$$

$$\sum_{j=1}^{n} r_{sj} x_j \geq h_s, s = 1, 2, \ldots, l \tag{2.7}$$

$$y_i \geq 0, x_j \geq 0, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \tag{2.8}$$

where it is assumed that the first stage decision variables x_j, $j = 1, 2, \ldots, n$ and second stage decision variables y_i, $i = 1, 2, \ldots, m$ are deterministic in the problem, p_i , $i = 1, 2, \ldots, m$ are the penalty cost associated with the discrepancy between $\sum_{j=1}^{n} a_{ij}$ and b_i and E is used to represent the expected value associated with the random variables $b_i, i = 1, 2, \ldots, m$.

The deterministic models of the two-stage stochastic programming problem when the right hand side parameter b_i follows some continuous distribution can be established as follows:

2.1.1 Case-I: b_i Follows Uniform Distribution

It is assumed that b_i, $i = 1, 2, \ldots, m$ are independent uniform random variables with

$$\mu_{b_i} = E(b_i) = \frac{L_i + U_i}{2}, i = 1, 2, 3, \ldots, m, \tag{2.9}$$

$$\sigma_{b_i}^2 = Var(b_i) = \frac{(U_i - L_i)^2}{12}, i = 1, 2, 3, \ldots, m. \tag{2.10}$$

The probability density function(pdf) of the i-th uniform random variable b_i is given by

$$f(b_i) = \begin{cases} \frac{1}{U_i - L_i}, & L_i \leq b_i \leq U_i \\ 0, & b_i < L_i, b_i > U_i \end{cases} \tag{2.11}$$

where L_i and U_i are the minimum and maximum values of the i-th uniform random variable b_i.

We compute $E(p_i|y_i|) = p_i E(|b_i - g_i|), i = 1, 2, \ldots, m$ by using the pdf (2.11) of the i-th uniform random variable b_i, where $g_i = \sum_{j=1}^{n} a_{ij} x_j, i = 1, 2, \ldots, m$ and $g_i \geq 0$

$$E(|b_i - g_i|) = \int_{L_i}^{U_i} |b_i - g_i| \frac{1}{U_i - L_i} db_i, i = 1, 2, \ldots, m. \tag{2.12}$$

Integrating (2.12), we obtain

$$E(|b_i - g_i|) = \frac{1}{U_i - L_i} \left[U_i^2 + L_i^2 - g_i(U_i + L_i) \right], i = 1, 2, \ldots, m. \tag{2.13}$$

Hence,

$$E(\sum_{i=1}^{m} p_i |y_i|) = \sum_{i=1}^{m} p_i \frac{1}{U_i - L_i} \left[U_i^2 + L_i^2 - g_i(U_i + L_i) \right]. \tag{2.14}$$

Using (2.14) in the two-stage stochastic programming model (2.5)-(2.8), we establish the deterministic model as:

$$\min : \bar{z} = \sum_{j=1}^{n} c_j x_j + \sum_{i=1}^{m} p_i \frac{1}{U_i - L_i} \left[U_i^2 + L_i^2 - g_i(U_i + L_i) \right] \tag{2.15}$$

subject to

$$\sum_{j=1}^{n} r_{sj} x_j \geq h_s, s = 1, 2, \ldots, l \tag{2.16}$$

$$x_j \geq 0, j = 1, 2, \ldots, n \tag{2.17}$$

where $g_i = \sum_{j=1}^{n} a_{ij} x_j, i = 1, 2, \ldots, m$.

2.1.2 Case-II: b_i Follows Exponential Distribution

It is assumed that b_i, $i = 1, 2, \ldots, m$ are independent exponential random variables with

$$\mu_{b_i} = E(b_i) = \frac{1}{\lambda_i}, i = 1, 2, 3, \ldots, m, \tag{2.18}$$

$$\sigma_{b_i}^2 = Var(b_i) = \frac{1}{\lambda_i^2}, i = 1, 2, 3, \ldots, m. \tag{2.19}$$

The probability density function(pdf) of the i-th normal random variable b_i is given by

$$f(b_i) = \begin{cases} \lambda_i e^{-\lambda_i b_i}, & 0 \le b_i < \infty \\ 0, & b_i < 0 \end{cases} \tag{2.20}$$

where $\lambda_i > 0$ is the parameter of the distribution, often called the rate parameter.

We compute $E(p_i|y_i|) = p_i E(|b_i - g_i|), i = 1, 2, \ldots, m$ by using the pdf (2.20) of the i-th normal random variable b_i, where $g_i = \sum_{j=1}^n a_{ij} x_j, i = 1, 2, \ldots, m$ and $g_i \ge 0$

$$E(|b_i - g_i|) = \int_0^\infty |b_i - g_i| \lambda_i e^{-\lambda_i b_i} db_i, i = 1, 2, \ldots, m. \tag{2.21}$$

Integrating (2.21), we obtain

$$E(|b_i - g_i|) = \left(\frac{2}{\lambda_i}\right) e^{-\lambda_i g_i} + g_i - \frac{1}{\lambda_i}, i = 1, 2, \ldots, m. \tag{2.22}$$

Hence,

$$E(\sum_{i=1}^m p_i|y_i|) = \sum_{i=1}^m p_i \left[\left(\frac{2}{\lambda_i}\right) e^{-\lambda_i g_i} + g_i - \frac{1}{\lambda_i}\right]. \tag{2.23}$$

Using (3.84) in the two-stage stochastic programming model (2.5)-(2.8), we establish the deterministic model as:

$$\min : \bar{z} = \sum_{j=1}^n c_j x_j + \sum_{i=1}^m p_i \left[\left(\frac{2}{\lambda_i}\right) e^{-\lambda_i g_i} + g_i - \frac{1}{\lambda_i}\right] \tag{2.24}$$

subject to

$$\sum_{j=1}^n r_{sj} x_j \ge h_s, s = 1, 2, \ldots, l \tag{2.25}$$

$$x_j \ge 0, j = 1, 2, \ldots, n \tag{2.26}$$

where $g_i = \sum_{j=1}^n a_{ij} x_j, i = 1, 2, \ldots, m$.

2.1.3 Case-III: b_i Follows Normal Distribution

It is assumed that b_i, $i = 1, 2, \ldots, m$ are independent normal random variables with
$E(b_i) = \mu_i$ and $Var(b_i) = \sigma_i^2$, $i = 1, 2, \ldots, m$.

Let the probability density function(pdf) of the i-th normal random variable b_i is given by

$$f(b_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\frac{1}{2}\left(\frac{b_i - \mu_i}{\sigma_i}\right)^2}, \quad -\infty < b_i < \infty, \sigma_i > 0 \tag{2.27}$$

where $E(b_i) = \mu_i$ and $Var(b_i) = \sigma_i^2$ be the mean and variance of the i-th normal random variable b_i.

We compute $E(p_i|y_i|) = p_i E(|b_i - g_i|), i = 1, 2, \ldots, m$ by using the pdf (2.27) of the i-th normal random variable b_i, where $g_i = \sum_{j=1}^n a_{ij} x_j, i = 1, 2, \ldots, m$ and $g_i \ge 0$

$$E(|b_i - g_i|) = \int_{-\infty}^\infty |b_i - g_i| \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\frac{1}{2}\left(\frac{b_i - \mu_i}{\sigma_i}\right)^2} db_i, i = 1, 2, \ldots, m. \tag{2.28}$$

Integrating (2.28), we obtain

$$E(|b_i - g_i|) = \mu_i - g_i + 2(g_i - \mu_i)\Phi\left(\frac{g_i - \mu_i}{\sigma_i}\right) + \frac{2\sigma_i}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{g_i-\mu_i}{\sigma_i}\right)^2}, i = 1, 2, \ldots, m. \tag{2.29}$$

Hence,

$$E(\sum_{i=1}^{m} p_i |y_i|) = \sum_{i=1}^{m} p_i \left[\mu_i - g_i + 2(g_i - \mu_i)\Phi\left(\frac{g_i - \mu_i}{\sigma_i}\right)\right] + \sum_{i=1}^{m} p_i \left[\frac{2\sigma_i}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{g_i-\mu_i}{\sigma_i}\right)^2}\right]. \tag{2.30}$$

Using (2.30) in the two-stage stochastic programming model (2.5)-(2.8), we establish the deterministic model as:

$$\min : \bar{z} = \sum_{j=1}^{n} c_j x_j + \sum_{i=1}^{m} p_i \left[\mu_i - g_i + 2(g_i - \mu_i)\Phi\left(\frac{g_i - \mu_i}{\sigma_i}\right)\right] + \sum_{i=1}^{m} p_i \left[\frac{2\sigma_i}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{g_i-\mu_i}{\sigma_i}\right)^2}\right] \tag{2.31}$$

subject to

$$\sum_{j=1}^{n} r_{sj} x_j \geq h_s, s = 1, 2, \ldots, l \tag{2.32}$$

$$x_j \geq 0, j = 1, 2, \ldots, n \tag{2.33}$$

where $g_i = \sum_{j=1}^{n} a_{ij} x_j, i = 1, 2, \ldots, m$.

2.1.4 Case-IV: b_i Follows Log-Normal Distribution

It is assumed that b_i, $i = 1, 2, \ldots, m$ are independent log-normal random variables with mean and variance as given by

$$M_i = E(b_i) = e^{\mu_i + \frac{\sigma_i^2}{2}}, i = 1, 2, 3, \ldots, m, \tag{2.34}$$

$$S_i^2 = Var(b_i) = (e^{\sigma_i^2} - 1)e^{2\mu_i + \sigma_i^2}, i = 1, 2, 3, \ldots, m \tag{2.35}$$

where μ_i and σ_i, $i = 1, 2, \ldots, m$are the expected value and standard deviation of the variables natural logarithm (i.e. $\ln b_i$ is normally distributed).

Using (2.34) and (2.35), the parameter μ_i and σ_i can be calculated as:

$$\mu_i = \ln(M_i) - 0.5\ln(1 + \frac{S_i^2}{M_i^2}), i = 1, 2, 3, \ldots, m, \tag{2.36}$$

$$\sigma_i^2 = \ln(1 + \frac{S_i^2}{M_i^2}), i = 1, 2, 3, \ldots, m. \tag{2.37}$$

The probability density function(pdf) of the i-th log-normal random variable b_i is given by

$$f(b_i) = \frac{1}{b_i\sqrt{2\pi\sigma_i^2}} e^{-\frac{1}{2}\left(\frac{\ln b_i - \mu_i}{\sigma_i}\right)^2}, \quad 0 < b_i < \infty, \sigma_i > 0. \tag{2.38}$$

We compute $E(p_i|y_i|) = p_i E(|b_i - g_i|), i = 1, 2, \ldots, m$ by using the pdf (2.38) of the i-th normal random variable b_i, where $g_i = \sum_{j=1}^{n} a_{ij} x_j, i = 1, 2, \ldots, m$ and $g_i \geq 0$.

$$E(|b_i - g_i|) = \int_0^\infty |b_i - g_i| \frac{1}{b_i\sqrt{2\pi\sigma_i^2}} e^{-\frac{1}{2}\left(\frac{\ln b_i - \mu_i}{\sigma_i}\right)^2} db_i, i = 1, 2, \ldots, m. \tag{2.39}$$

Integrating (2.39), we get

$$\begin{aligned} E(|b_i - g_i|) = {} & e^{\left(\mu_i + \frac{\sigma_i^2}{2}\right)} - g_i + 2g_i\Phi\left(\frac{\ln g_i - \mu_i}{\sigma_i}\right) \\ & -2e^{\left(\mu_i + \frac{\sigma_i^2}{2}\right)}\Phi\left(\frac{\ln g_i - \mu_i - \sigma_i^2}{\sigma_i}\right), i = 1, 2, \ldots, m. \end{aligned} \tag{2.40}$$

Hence,

$$E(\sum_{i=1}^{m} p_i|y_i|) = \sum_{i=1}^{m} p_i \left[e^{\left(\mu_i + \frac{\sigma_i^2}{2}\right)} + 2g_i \Phi\left(\frac{\ln g_i - \mu_i}{\sigma_i}\right) \right] - \sum_{i=1}^{m} p_i \left[g_i + 2e^{\left(\mu_i + \frac{\sigma_i^2}{2}\right)} \Phi\left(\frac{\ln g_i - \mu_i - \sigma_i^2}{\sigma_i}\right) \right]. \tag{2.41}$$

Using (2.41) in the two-stage stochastic programming model (2.5)-(2.8), we establish the deterministic model as:

$$\min : \bar{z} = \sum_{j=1}^{n} c_j x_j + \sum_{i=1}^{m} p_i \left[e^{\left(\mu_i + \frac{\sigma_i^2}{2}\right)} + 2g_i \Phi\left(\frac{\ln g_i - \mu_i}{\sigma_i}\right) \right] - \sum_{i=1}^{m} p_i \left[g_i + 2e^{\left(\mu_i + \frac{\sigma_i^2}{2}\right)} \Phi\left(\frac{\ln g_i - \mu_i - \sigma_i^2}{\sigma_i}\right) \right] \tag{2.42}$$

subject to

$$\sum_{j=1}^{n} r_{sj} x_j \geq h_s, s = 1, 2, \ldots, l \tag{2.43}$$

$$x_j \geq 0, j = 1, 2, \ldots, n \tag{2.44}$$

where $g_i = \sum_{j=1}^{n} a_{ij} x_j, i = 1, 2, \ldots, m$.

3 Numerical Examples

In this Section, we have considered two numerical examples to verify the solution procedure of the above two-stage stochastic programming (TSP) models as follows:

3.1 Example 1:

Formulating the four two-stage stochastic programming (TSP) models with simple unit recourse cost considering only the right hand side parameter b_i as uniform, exponential, normal, and log-normal random variables can be formulated as follows:

Case 1: Uniform Random Variables

$$\min : \bar{z} = 20x_1 + 60x_2 + 40x_3 + E(|y_1|) + E(|y_2|) + E(|y_3|) \tag{3.1}$$

subject to

$$y_1 = b_1 - (4x_1 + 6x_2 + 4x_3) \tag{3.2}$$

$$y_2 = b_2 - (10x_1 + 3x_2 + 3x_3) \tag{3.3}$$

$$y_3 = b_3 - (5x_1 + 2x_2 + 5x_3) \tag{3.4}$$

$$16x_1 + 22x_2 + 20x_3 \geq 56 \tag{3.5}$$

$$80x_1 + 60x_2 + 75x_3 \geq 210 \tag{3.6}$$

$$80x_1 + 60x_2 + 75x_3 \leq 250 \tag{3.7}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, y_1 \geq 0, y_2 \geq 0, y_3 \geq 0 \tag{3.8}$$

where it is assume that b_1, b_2, and b_3 are independent uniform random variables with given means and variances as:

$$E(b_1) = 15, E(b_2) = 20, E(b_3) = 14$$

and

$$Var(b_1) = 25, Var(b_2) = 36, Var(b_3) = 16.$$

The above model (3.1)-(3.8) can be simplified to:

$$\min : \bar{z} = 20x_1 + 60x_2 + 40x_3 + E(|b_1 - g_1|) + E(|b_2 - g_2|) + E(|b_3 - g_3|) \tag{3.9}$$

subject to

$$16x_1 + 22x_2 + 20x_3 \geq 56 \tag{3.10}$$

$$80x_1 + 60x_2 + 75x_3 \geq 210 \tag{3.11}$$

$$80x_1 + 60x_2 + 75x_3 \leq 250 \tag{3.12}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \tag{3.13}$$

where $g_1 = 4x_1 + 6x_2 + 4x_3$, $g_2 = 10x_1 + 3x_2 + 3x_3$, $g_3 = 5x_1 + 2x_2 + 5x_3$.

Using the equation (2.9) and (2.10), the minimum and maximum values of $b_i, i = 1, 2, 3$ are calculated as:

$$L_1 = 6.34, U_1 = 23.66, L_2 = 9.6, U_2 = 30.4, L_3 = 7.07, U_3 = 20.93.$$

Thus using the minimum and maximum values of $b_i, i = 1, 2, 3$, the desired equivalent deterministic model can be established as:

$$\min : \bar{z} = 20x_1 + 60x_2 + 40x_3 - 2.117647g_1 - 0.923077g_2 - 1.010101g_3 + 36.328409 \tag{3.14}$$

subject to

$$16x_1 + 22x_2 + 20x_3 \geq 56 \tag{3.15}$$

$$80x_1 + 60x_2 + 75x_3 \geq 210 \tag{3.16}$$

$$80x_1 + 60x_2 + 75x_3 \leq 250 \tag{3.17}$$

$$x_j \geq 0, j = 1, 2, 3 \tag{3.18}$$

where $g_1 = 4x_1 + 6x_2 + 4x_3$, $g_2 = 10x_1 + 3x_2 + 3x_3$, $g_3 = 5x_1 + 2x_2 + 5x_3$

Case 2: Exponential Random Variables

$$\min : \bar{z} = 20x_1 + 60x_2 + 40x_3 + E(|y_1|) + E(|y_2|) + E(|y_3|) \tag{3.19}$$

subject to

$$y_1 = b_1 - (4x_1 + 6x_2 + 4x_3) \tag{3.20}$$

$$y_2 = b_2 - (10x_1 + 3x_2 + 3x_3) \tag{3.21}$$

$$y_3 = b_3 - (5x_1 + 2x_2 + 5x_3) \tag{3.22}$$

$$16x_1 + 22x_2 + 20x_3 \geq 56 \tag{3.23}$$

$$80x_1 + 60x_2 + 75x_3 \geq 210 \tag{3.24}$$

$$80x_1 + 60x_2 + 75x_3 \leq 250 \tag{3.25}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, y_1 \geq 0, y_2 \geq 0, y_3 \geq 0 \tag{3.26}$$

where it is assume that b_1, b_2, and b_3 are independent exponential random variables with

$$E(b_1) = 15, E(b_2) = 20, E(b_3) = 14, Var(b_1) = 25, Var(b_2) = 36, Var(b_3) = 16.$$

Then the above model (3.19)-(3.26) can be simplified as:

$$\min : \bar{z} = 20x_1 + 60x_2 + 40x_3 + E(|b_1 - g_1|) + E(|b_2 - g_2|) + E(|b_3 - g_3|) \tag{3.27}$$

subject to

$$16x_1 + 22x_2 + 20x_3 \geq 56 \tag{3.28}$$

$$80x_1 + 60x_2 + 75x_3 \geq 210 \tag{3.29}$$

$$80x_1 + 60x_2 + 75x_3 \leq 250 \tag{3.30}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \tag{3.31}$$

where $g_1 = 4x_1 + 6x_2 + 4x_3$, $g_2 = 10x_1 + 3x_2 + 3x_3$, $g_3 = 5x_1 + 2x_2 + 5x_3$.

Using the equation (2.18) and (2.19), the parameter values of $b_i, i = 1, 2, 3$ are calculated as:

$$\lambda_1 = 0.111, \lambda_2 = 0.167, \lambda_3 = 0.143.$$

Thus using the parameter values of $b_i, i = 1, 2, 3$, the desired equivalent deterministic model can be established as:

$$\begin{aligned} \min : \bar{z} = 20x_1 + 60x_2 + 40x_3 + 30e^{-0.066667g_1} + 40e^{-0.0.05g_2} \\ +28e^{-0.071429g_3} + g_1 + g_2 + g_3 - 49 \end{aligned} \tag{3.32}$$

subject to

$$16x_1 + 22x_2 + 20x_3 \geq 56 \tag{3.33}$$

$$80x_1 + 60x_2 + 75x_3 \geq 210 \tag{3.34}$$

$$80x_1 + 60x_2 + 75x_3 \leq 250 \tag{3.35}$$

$$x_j \geq 0, j = 1, 2, 3 \tag{3.36}$$

where $g_1 = 4x_1 + 6x_2 + 4x_3$, $g_2 = 10x_1 + 3x_2 + 3x_3$, $g_3 = 5x_1 + 2x_2 + 5x_3$.

Case 3: Normal Random Variables

$$\min : \bar{z} = 20x_1 + 60x_2 + 40x_3 + E(|y_1|) + E(|y_2|) + E(|y_3|) \tag{3.37}$$

subject to

$$y_1 = b_1 - (4x_1 + 6x_2 + 4x_3) \tag{3.38}$$

$$y_2 = b_2 - (10x_1 + 3x_2 + 3x_3) \tag{3.39}$$

$$y_3 = b_3 - (5x_1 + 2x_2 + 5x_3) \tag{3.40}$$

$$16x_1 + 22x_2 + 20x_3 \geq 56 \tag{3.41}$$

$$80x_1 + 60x_2 + 75x_3 \geq 210 \tag{3.42}$$

$$80x_1 + 60x_2 + 75x_3 \leq 250 \tag{3.43}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, y_1 \geq 0, y_2 \geq 0, y_3 \geq 0 \tag{3.44}$$

where it is assume that b_1, b_2, and b_3 are independent normal random variables with

$$E(b_1) = \mu_1 = 15, E(b_2) = \mu_2 = 20, E(b_3) = \mu_3 = 14$$

and

$$Var(b_1) = \sigma_1^2 = 25, Var(b_2) = \sigma_2^2 = 36, Var(b_3) = \sigma_3^2 = 16.$$

Then the above model (3.37)-(3.44) can be simplified as:

$$\min : \bar{z} = 20x_1 + 60x_2 + 40x_3 + E(|b_1 - g_1|) + E(|b_2 - g_2|) + E(|b_3 - g_3|) \tag{3.45}$$

subject to

$$16x_1 + 22x_2 + 20x_3 \geq 56 \tag{3.46}$$

$$80x_1 + 60x_2 + 75x_3 \geq 210 \tag{3.47}$$

$$80x_1 + 60x_2 + 75x_3 \leq 250 \tag{3.48}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \tag{3.49}$$

where $g_1 = 4x_1 + 6x_2 + 4x_3$, $g_2 = 10x_1 + 3x_2 + 3x_3$, $g_3 = 5x_1 + 2x_2 + 5x_3$.

Using the given mean and variance values of the normal random variables $b_i, i = 1, 2, 3,$ the desired equivalent deterministic model can be established as:

$$\begin{aligned}\min : \bar{z} = {} & 20x_1 + 60x_2 + 40x_3 - g_1 - g_2 - g_3 \\ & +(2g_1 - 30)\Phi\left(\frac{g_1 - 15}{5}\right) + (2g_2 - 40)\Phi\left(\frac{g_2 - 20}{6}\right) \\ & +(2g_3 - 28)\Phi\left(\frac{g_3 - 14}{4}\right) + \frac{10}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{g_1-15}{5}\right)^2} \\ & + \frac{12}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{g_2-20}{6}\right)^2} + \frac{8}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{g_3-14}{4}\right)^2} + 49 \end{aligned} \tag{3.50}$$

subject to

$$16x_1 + 22x_2 + 20x_3 \geq 56 \tag{3.51}$$

$$80x_1 + 60x_2 + 75x_3 \geq 210 \tag{3.52}$$

$$80x_1 + 60x_2 + 75x_3 \leq 250 \tag{3.53}$$

$$x_j \geq 0, j = 1, 2, 3 \tag{3.54}$$

where $g_1 = 4x_1 + 6x_2 + 4x_3$, $g_2 = 10x_1 + 3x_2 + 3x_3$, $g_3 = 5x_1 + 2x_2 + 5x_3$.

Case 4: Log-Normal Random Variables

$$\min : \bar{z} = 20x_1 + 60x_2 + 40x_3 + E(|y_1|) + E(|y_2|) + E(|y_3|) \tag{3.55}$$

subject to

$$y_1 = b_1 - (4x_1 + 6x_2 + 4x_3) \tag{3.56}$$

$$y_2 = b_2 - (10x_1 + 3x_2 + 3x_3) \tag{3.57}$$

$$y_3 = b_3 - (5x_1 + 2x_2 + 5x_3) \tag{3.58}$$

$$16x_1 + 22x_2 + 20x_3 \geq 56 \tag{3.59}$$

$$80x_1 + 60x_2 + 75x_3 \geq 210 \tag{3.60}$$

$$80x_1 + 60x_2 + 75x_3 \leq 250 \tag{3.61}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, y_1 \geq 0, y_2 \geq 0, y_3 \geq 0 \tag{3.62}$$

where it is assume that b_1, b_2, and b_3 are independent log-normal random variables with

$$E(b_1) = M_1 = 15, E(b_2) = M_2 = 20, E(b_3) = M_3 = 14$$

and

$$Var(b_1) = S_1^2 = 25, Var(b_2) = S_2^2 = 36, Var(b_3) = S_3^2 = 16.$$

Using (2.36) and (2.37), we calculate the values of μ_i and σ_i, $i = 1, 2, 3$ as:

$$\mu_1 = 2.65537, \mu_2 = 2.952643, \mu_3 = 2.599821$$

and

$$\sigma_1^2 = 0.105361, \sigma_2^2 = 0.086178, \sigma_3^2 = 0.078472.$$

Then the above model (3.55)-(3.62) can be simplified as:

$$\min : \bar{z} = 20x_1 + 60x_2 + 40x_3 + E(|b_1 - g_1|) + E(|b_2 - g_2|) + E(|b_3 - g_3|) \tag{3.63}$$

subject to

$$16x_1 + 22x_2 + 20x_3 \geq 56 \tag{3.64}$$

$$80x_1 + 60x_2 + 75x_3 \geq 210 \tag{3.65}$$

$$80x_1 + 60x_2 + 75x_3 \leq 250 \tag{3.66}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \tag{3.67}$$

where $g_1 = 4x_1 + 6x_2 + 4x_3$, $g_2 = 10x_1 + 3x_2 + 3x_3$, $g_3 = 5x_1 + 2x_2 + 5x_3$.

Now, the desired equivalent deterministic model can be established as:

$$\begin{aligned}\min: \bar{z} = {} & 20x_1 + 60x_2 + 40x_3 - g_1 - g_2 - g_3 \\ & +2g_1\Phi\left(\frac{\ln g_1 - 2.65537}{0.324592}\right) + 2g_2\Phi\left(\frac{\ln g_2 - 2.952643}{0.293561}\right) \\ & +2g_3\Phi\left(\frac{\ln g_3 - 2.599821}{0.280129}\right) - 30.000008\Phi\left(\frac{\ln g_1 - 2.760731}{0.324592}\right) \\ & -39.99999\Phi\left(\frac{\ln g_2 - 3.038821}{0.293561}\right) - 27.99999\left(\frac{\ln g_3 - 2.678293}{0.280129}\right) + 48.999994\end{aligned} \tag{3.68}$$

subject to

$$16x_1 + 22x_2 + 20x_3 \geq 56 \tag{3.69}$$

$$80x_1 + 60x_2 + 75x_3 \geq 210 \tag{3.70}$$

$$80x_1 + 60x_2 + 75x_3 \leq 250 \tag{3.71}$$

$$x_j \geq 0, j = 1, 2, 3 \tag{3.72}$$

where $g_1 = 4x_1 + 6x_2 + 4x_3$, $g_2 = 10x_1 + 3x_2 + 3x_3$, $g_3 = 5x_1 + 2x_2 + 5x_3$.

The above linear programming model (3.14)-(3.18) and non-linear programming models (3.32)-(3.36), (3.50)-(3.54), and (3.68)-(3.72) are solved by using MAPLE 12.0 and LINGO 11.0[20] package and the optimal solutions are given in Table 1 as follows:

Table 1: Optimal solutions of the model

Problem Types	Optimal decision variables	Value of the objective function
Uniform Distribution	$x_1^* = 2.675, x_2^* = 0.6, x_3^* = 0.0$	$\bar{z}^* = 99.09854$
Exponential Distribution	$x_1^* = 2.0, x_2^* = 0.0, x_3^* = 1.2$	$\bar{z}^* = 125.4002$
Normal Distribution	$x_1^* = 2.0, x_2^* = 0.0, x_3^* = 1.2$	$\bar{z}^* = 101.5759$
Log-normal Distribution	$x_1^* = 2.0, x_2^* = 0.0, x_3^* = 1.2$	$\bar{z}^* = 101.5782$

3.2 Example 2:

Further, we formulate a two-stage stochastic programming (TSP) model with simple unit recourse cost considering only the right hand side parameter b_i as uniform, exponential and normal random variables as follows:

$$\min: \bar{z} = 165x_1 + 130x_2 + 140x_3 + E(|y_1|) + E(|y_2|) + E(|y_3|) \tag{3.73}$$

subject to

$$y_1 = b_1 - (10x_1 + 7x_2 + 8x_3) \tag{3.74}$$

$$y_2 = b_2 - (x_1 + 10x_2 + 15x_3) \tag{3.75}$$

$$y_3 = b_3 - (7x_1 + 8x_2 + 5x_3) \tag{3.76}$$

$$15x_1 + 20x_2 + 18x_3 \geq 87 \tag{3.77}$$

$$43x_1 + 80x_2 + 90x_3 \leq 250 \tag{3.78}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, y_1 \geq 0, y_2 \geq 0, y_3 \geq 0 \tag{3.79}$$

where it is assume that b_1 is an uniform random variable with known mean and variance

$$E(b_1) = 15, Var(b_1) = 3.$$

Similarly, the mean of an exponential random variable b_2 is given by:

$$E(b_2) = 18.$$

Further, the mean and variance of the normal random variable b_3 is given by:

$$E(b_3) = \mu = 20,\ Var(b_3) = \sigma^2 = 4.$$

The above model (3.73)-(3.79) can be simplified to:

$$\min : \bar{z} = 165x_1 + 130x_2 + 140x_3 + E(|b_1 - g_1|) + E(|b_2 - g_2|) + E(|b_3 - g_3|) \tag{3.80}$$

subject to

$$15x_1 + 20x_2 + 18x_3 \geq 87 \tag{3.81}$$

$$43x_1 + 80x_2 + 90x_3 \leq 250 \tag{3.82}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \tag{3.83}$$

where $g_1 = 10x_1 + 7x_2 + 8x_3$, $g_2 = x_1 + 10x_2 + 15x_3$, $g_3 = 7x_1 + 8x_2 + 5x_3$.

Using the equation (2.9) and (2.10), the minimum and maximum values of b_1 are calculated as:

$$L = 12, U = 18.$$

Similarly, using equation (2.18), the parameter value of b_2 is calculated as: $\lambda = 0.05556$.

Thus using the minimum and maximum values of b_1, the parameter value of b_2, and the mean and variance of b_3, the required equivalent deterministic model can be established as:

$$\begin{aligned} \min : \bar{z} = 165x_1 + 130x_2 + 140x_3 + 5g_1 - g_2 + g_3 - 36e^{0.05556g_2} \\ -2(g_3 - 20)\Phi\Big(\frac{g_3 - 20}{2}\Big) - \frac{4}{\sqrt{2\pi}}e^{0.5\left(\frac{g_3-20}{2}\right)^2} - 80 \end{aligned} \tag{3.84}$$

subject to

$$15x_1 + 20x_2 + 18x_3 \geq 87 \tag{3.85}$$

$$43x_1 + 80x_2 + 90x_3 \leq 250 \tag{3.86}$$

$$x_j \geq 0, j = 1, 2, 3 \tag{3.87}$$

where $g_1 = 10x_1 + 7x_2 + 8x_3$, $g_2 = x_1 + 10x_2 + 15x_3$, $g_3 = 7x_1 + 8x_2 + 5x_3$.

The above non-linear programming model (3.84)-(3.87) is solved by using MAPLE 12.0 and LINGO 11.0[20] package and the optimal solutions are given in Table 2 as follows:

Table 2: Optimal solutions of the model

Problem Types	Optimal decision variables	Value of the objective function
Mixed Type	$x_1^* = 5.764706, x_2^* = 0.02647059, x_3^* = 0.0$	$\bar{z}^* = 131.1472$

4 Conclusion

In this paper, we proposed a solution procedure of a two-stage stochastic programming problem by considering only the right hand side parameter as either uniform, or exponential, or normal or log-normal random variable with finite mean and variance. All other parameters in the model are assumed to be deterministic. We have considered two numerical examples, in the first example, we have established four different deterministic models for four random variables. Among the four deterministic models, first one is linear model associated with uniform random variable and other three are non-linear programming models which are associated with exponential, normal, and log-normal random variables. A standard linear/non-linear programming technique is used to solve these models and the result shown in Table 1.

In the second example, we have stated one two-stage stochastic programming problem with right hand side parameter as mixed type of random variables. This leads to a non-linear programming problem and is solved and the result shown in Table 2. From the two different types of numerical examples, we noticed that our method has verified successfully.

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