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Bipolar Aggregation Method for Fuzzy Nominal Classification Using Weighted Cardinal Fuzzy Measure (WCFM)

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Abstract

The issue of designing a procedure to assign objects (candidates, projects, decisions, options, etc.) characterized by multiple attributes or criteria to predefined classes characterized by fuzzyly defined multiple features, conditions or constraints, is considered in this paper. Such assignment problems are known in the literature as nominal or non ordered classification problems as opposed to ordinal classification in which case classes are ordered according to some desires of decision maker(s). Because of the importance of these problems in many domains such as social, economics, medical, engineering, mangement etc., there is a need to design sound and appropriate evaluation algorithms and methods to deal with them. In this paper we will consider an approach based on an evaluation strategy that consists in aggregating separately elements that act in the same sens (either contributing to the exlusion of a class from assignment or its consideration for inclusion given an object) that we refer to as bipolar analysis. Then, relying on the fact that elements to aggregate have synergetic relationships (they are complementary), we propose to use Choquet integral as the appropriate aggregation operator with a proposed fuzzy measure or capacity known as weighted cardinal fuzzy measure (WCFM) which tractability permits to overcome difficulties that dissuade the use of Choquet integral in practices. Furthermore, bipolar property results in evalaution by two degrees: classifiability measure that measures to what extent an object can be considered for inclusion in a class and rejectability measure, a degree that measures the extent to which one must avoid including an object to a class rendering final choice flexible as many classes may be qualified for inclusion of an object. Application of this approach to a real world problem in the domain of banking has shown a real potentiality.

Keywords: nominal classification, multi-attributes, multi-features, bipolar aggregation, Choquet integral, WCFM, classifiability, rejectability

1 Introduction

Many decision problems rising in different domains such as social, economics or engineering, among others, concern the assignment or classification of objects according to their scores for a certain number of criteria or attributes to classes that are characterized by some features. These problems constitute, therefore, multicriteria or multiattributes (attributes of the object to classify) and multi-objectives (multi-features classes to choose) decision making problems, a unified framework that is being developed (see [21, 22, 23]) to overcome the fact that the two paradigms (multicriteria and multi-objectives) have been almost always considered separately in the literature, see for instance [2, 3, 9, 12, 13, 14, 15, 16, 18, 24]. The majority of contributions to classification problems encountered in the literature concern mainly the ordered classification case, classes must be ordered, let say, from most/least desired class to least/most desired one, see for instance [4]. The purpose of classification methods or algorithms is then to establish a procedure that linearly rank classes and assign objects to them; one may notice that this is a relative decision making process as objects are finally compared with each other. But the case of non ordered classification where classes are just defined by some

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features, conditions or constraints over the attributes or criteria is of great importance in many domains. In finance and banking for instance, decision maker(s) face the problem of classifying customers for a credit or service into classes defined by entrance thresholds with regard to their performance in some attributes for instance, see [13]; in international finance or commerce, countries are often classified in different categories in terms of risk to which potential investors will be exposed in these countries (country risk classification) by using a certain number of attributes such as GDP per unit of energy use, telephone mainlines per 1000 people, human development index (HDI), percentage of military expenditure of the central government expenditure and others, see [25]; in medical domain, a medical practitioner classifies a patient as suffering a fever if its temperature is beyond a threshold and/or if it presents some other symptoms; in engineering a design must satisfy some objectives and constraints; in academic, a student will get his/her diploma or degree if his/her marks in some different disciplines are beyond some thresholds, etc..

Formally the nominal classification problems considered in this paper are defined by the following materials.

- An object u to be classified is characterized by a set of m attributes or criteria and the value (numeric or rendered numeric by a certain procedure) of attribute l is given by $\mathbf{x}_l$ so that this object can be designated by its attributes vector $\mathbf{x} \in \mathbb{R}^m$ (where $\mathbb{R}^m$ represents the set of vectors of dimension m with real entries). For instance in the case of financial portfolio management, an object u is a portfolio constituted by m assets and $\mathbf{x}_l$ is the amount of budget allocated to asset l.
- The former defined object must be assigned to one of the n classes of the set $\mathcal{C} = \left\{c^1,\ c^2,\ ...,\ c^n\right\}$; each class or category c^j is defined by n_j fuzzy features, conditions or constraints through scalar functions $\mathbf{f}_l^j(\mathbf{x})$, $l = 1, 2, ..., n_j$, of the attributes vector $\mathbf{x} \in \mathbb{R}^m$.

The aim of this paper is to derive a classification algorithm using the materials defined above. A number of multicriteria decision aid (MCDA) methods have been developed for nominal classification. They include multicriteria filtering [11], a method based on concordance and non-discordance principles; PROAFTN, see [1], a multicriteria fuzzy classification method; a method based on fuzzy integrals, see for instance [8]; TRINOMFC method [10] that computes local concordance; or the stochastic multicriteria acceptability analysis (SMAA) method that supports incomplete or inaccurate preference, see [26]. If these methods have been successfully applied in practice, many of them do have usability limitation (with regard to final users) such as complexity of how parameters must be specified by the users. The intention of this paper is to add a method to the panorama of existing methods that we hope will be easier (because of its flexibility) to use by the final users who in general are non specialists. In this paper we consider a method of nominal classification that is based, for a given object and a given class, on two measures corresponding respectively to what extent the object can be included in the class and to what extent it should be excluded, similar to satisficing games theory approach [19].

One of the contribution of this paper consists in a procedure that defines, for each feature $\mathbf{f}_l^j(\mathbf{x})$, two zones namely classification zone (that we refer to as **C zone**), rejectability zone (referred to as **R zone**) and an overlapping zone that we call doubtful zone with their respective membership functions $m_C^{j,l}\left(\mathbf{f}_k^j(\mathbf{x})\right)$ and $m_R^{j,l}\left(\mathbf{f}_k^j(\mathbf{x})\right)$. These memberships functions are then aggregated separately for each pair constituted by a class c^j and an object u using Choquet integral; with another contribution being the definition of a tractable fuzzy measure named weighted cardinal fuzzy measure (WCFM) based on the synergetic nature of attributes to aggregate to obtain classifiability and rejectability measures, $\psi_C^j(u)$ and $\psi_R^j(u)$ respectively. Main indices associated with a fuzzy measure namely interaction index I_{ij} between two elements i and j and Shapley index ϕ_i of element i are completely characterized for this WCFM. Then for each object u, the set $\mathcal{C}_q(u)$ is defined as the set of classes in which the object u can be included at the caution index q because the classifiability measure exceeds the rejectability measure multiplied by q. The final class in which it will be included can be chosen to optimize a certain ultimate criterion (maximization of the difference between the classifiability and the rejectability or maximization of caution index for instance). In terms of usability, one can notice that this approach can be totally transparent to the final user who may be concerned only in fuzzy parameters specification of classes and relative importance of feature within a class.

The novelty brought by this paper to the field of nominal classification can be highlighted trough the following points:

- fuzzy characterization of classes: this characterization has a practical perspective as most of the time decision makers and/or experts may have difficulties characterizing classes by crisp features;

- possible indecision of decision makers between classes: indeed, because of the use of two measures in the classification process, it may happen that decision makers are not able to clearly decide the suitable class for inclusion of a given object so that this approach is more closed to how humans proceed in practice;

- homogeneuous aggregation: by clustering constraints to obtain homogenuous behavior of features (working toward classification or against), Choquet integral with a synergetic fuzzy measure named weighted cardinal fuzzy measure (wcfm) leads to a straightforward aggregation formula overcoming the traditional difficulty to use Choquet integral as an aggregation operator.

The remainder of this paper is organized as follows: in the second section, main contribution of this paper is presented going through fuzzyfication of features with regards to classes and the obtainment of the corresponding membership functions on a bipolar basis; definition of a tractable fuzzy measure in terms of WCFM based on synergetic behavior of features and determination of interaction index and Shapley index associated to this measure; derivation of a formula for related Choquet integral as the appropriate aggregation operator; definition of classifiability and rejectability measures; and finally presentation of a classification procedure; third section considers a real world application to show potential applicability of the approach developed so far and finally a conclusion is presented in the fourth section.

2 Bipolar Fuzzy Nominal Classification

Bipolarity is pervasive in human decision activities; indeed, cognitive psychologists observed for long time that humans usually evaluate alternatives by comparing their positive aspects and their negative aspects. Bipolarity has been already considered in multicriteria decision and classification problems; for instance concordance and discordance indices in ELECTRE approach (see [24] and references therein) falls in this framework though in this paper the final classification scheme differs from that of ELECTRE. In this section we will formulate a non ordered classification problem as defined in the introductory section using bipolar analysis. This approach, (see for instance [20, 21, 22, 23]), building on satisficing game theory [19], is showing promising application in various domains.

Formulating nominal classification problem considered in this paper in the framework of bipolar analysis return to establishing a procedure to compute classifiability measure $\psi_C^j(u)$ and rejectability measure $\psi_R^j(u)$ given an object u and a class c^j; in the following subsection, we will show how to obtain these parameters from problem specifications. Advantages of this approach are as the following: there is no need to normalize data, only the specification of parameters to define the doubtful zone for each class by experts counts; it is possible to classify using partial information, for instance if there is a total ignorance about a feature or condition of a class, one can disregard it in the classification process; information may arrive sequentially so that decision maker can revise its classification results at each information arrival.

2.1 Classifiability and Rejectability Measures Derivation Procedure

Bipolar reasoning is pervasive in decision analysis and constitutes a sort of divide to better apprehend paradigm. The stepping stones in bipolar analysis approach, for nominal classification, are the classifiability and rejectability measures $\psi_C^j(u)$ and $\psi_R^j(u)$ given a class c^j and an object u; so their derivation is an important step towards a sound classification algorithm. These measures must be established considering the performance of the considered object with regard to the considered class. As mentioned above, each feature k characterizing a class c^j is fuzzily described; as the characterization functions of features are scalar (or rendered scalar), this fuzzy description consists generally in four types. Thus, to consider that the object u characterized by vector $\mathbf{x}$ belongs to the class c^j if one were to decide only based on feature k, we consider the range of $\mathbf{f}_k^j(\mathbf{x})$ to be partitioned into two labels or zones: rejection zone (**R zone**), that is if $\mathbf{f}_k^j(\mathbf{x})$ belongs to this zone one should categorically exclude including object u in the corresponding class and classification zone (**C zone**) where if $\mathbf{f}_k^j(\mathbf{x})$ lays in, one should consider including the object u in this class; finally there is an overlapping zone where decision of including or excluding is not obvious that we refer to as doubtful zone. Let use define by $m_R^{j,k}\left(\mathbf{f}_k^j(\mathbf{x})\right)$ and $m_C^{j,k}\left(\mathbf{f}_k^j(\mathbf{x})\right)$ the membership degrees of these zones respectively, then description given above is illustrated by Figure 1.

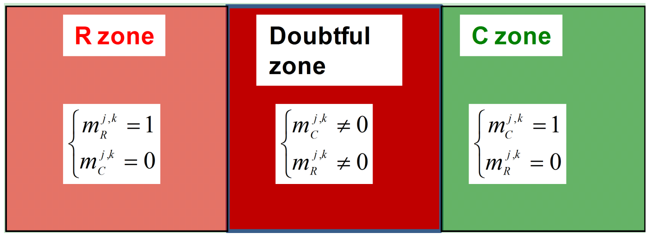

Figure 1: Characterization of R zone, C zone and doubtful zone of c^j with regrads to feature k

Remark 1: The partition of the range of $\mathbf{f}_k^j(\mathbf{x})$ in two zones is not necessary a fuzzy discretization, that is, it is not required to have in the doubtful zone the equality $m_R^{j,k}\left(\mathbf{f}_k^j(\mathbf{x})\right) + m_C^{j,k}\left(\mathbf{f}_k^j(\mathbf{x})\right) = 1$.

The membership function $m_\times^{j,k}\left(\mathbf{f}_k^j(\mathbf{x})\right)$ where $\times$ stands for R or C is a degree that measure the extent to which one should reject or consider classifying object u in the class c^j; given an object u and its attribute vector $\mathbf{x}$, these membership functions depends on the value of $\mathbf{f}_k^j(\mathbf{x})$; four main typologies characterize the behavior of $\mathbf{f}_k^j(\mathbf{x})$ that conditions the classification of object u in the class c^j.

- Above a threshold, that is to consider including the object u in the class c^j using only the feature k, $\mathbf{f}_k^j(\mathbf{x})$ must be greater than a given threshold. To determine $m_R^{j,k}\left(\mathbf{f}_k^j(\mathbf{x})\right)$ and $m_C^{j,k}\left(\mathbf{f}_k^j(\mathbf{x})\right)$ in this case, one must specify a value $\mathbf{f}_{k,\min_l}^j$ (l stands for lower) below which class c^j is definitely rejected (**R zone**) and the value $\mathbf{f}_{k,\min_u}^j$ (u stands for upper) above which one must definitely include the object u in the class c^j (**C zone**).

- Below a threshold, that is to consider including the object u in the class c^j using only the feature k, $\mathbf{f}_k^j(\mathbf{x})$ must be less than a given threshold. Determination of $m_R^{j,k}\left(\mathbf{f}_k^j(\mathbf{x})\right)$ and $m_C^{j,k}\left(\mathbf{f}_k^j(\mathbf{x})\right)$ necessitates specifying a value $\mathbf{f}_{k,\max_l}^j$ below which class c^j is definitely considered for inclusion of object u (**C zone**) and the value $\mathbf{f}_{k,\max_u}^j$ above which one must exclude the class c^j from the inclusion of object u (**R zone**).

- In a range, to consider including the object u in the class c^j using only the feature k, $\mathbf{f}_k^j(\mathbf{x})$ must belongs to an interval; here four values must be specified to determine $m_R^{j,k}\left(\mathbf{f}_k^j(\mathbf{x})\right)$ and $m_C^{j,k}\left(\mathbf{f}_k^j(\mathbf{x})\right)$, $\mathbf{f}_{k,\min_l}^j$ the value below which and $\mathbf{f}_{k,\max_u}^j$ the value above which one must exclude the class c^j for the inclusion (**R zone**) of object u and $\mathbf{f}_{k,\min_u}^j$ and $\mathbf{f}_{k,\max_l}^j$ the values between which one must classify the considered object in the class c^j (**C zone**).

- Targeted or single value, to consider including the object u in the class c^j using only the feature k, $\mathbf{f}_k^j(\mathbf{x})$ must be equal to a given value; three parameters are necessary in order to determine $m_R^{j,k}\left(\mathbf{f}_k^j(\mathbf{x})\right)$ and $m_C^{j,k}\left(\mathbf{f}_k^j(\mathbf{x})\right)$, $\mathbf{f}_{k,\min}^j$ the value below which and $\mathbf{f}_{k,\max}^j$ the value above which one excludes the the class c^j (**R zone**) and the value $\mathbf{f}_{k,\min}^j$ for which one definitely classifies the object in the class c^j (**C zone**).

Description of the four typologies determining (**R zone**) and (**C zone**) memberships functions is illustrated on Figure 2.

Given an object u and a class c^j, the overall classifiability degree $\Psi_C^j(u)$ and the overall rejectability degree $\Psi_R^j(u)$ of the class c^j are obtained by aggregating the memberships degrees $m_C^{j,k}(.)$ and $m_R^{j,k}(.)$ for all features $k = 1, 2, ..., n_j$ as given by equations (1)-(2)

$$\Psi_C^j(u) = G_C\left(m_C^{j,1}(.), m_C^{j,2}(.), ..., m_C^{j,n_j}(.)\right), \tag{1}$$

$$\Psi_R^j(u) = G_R\left(m_R^{j,1}(.), m_R^{j,2}(.), ..., m_R^{j,n_j}(.)\right), \tag{2}$$

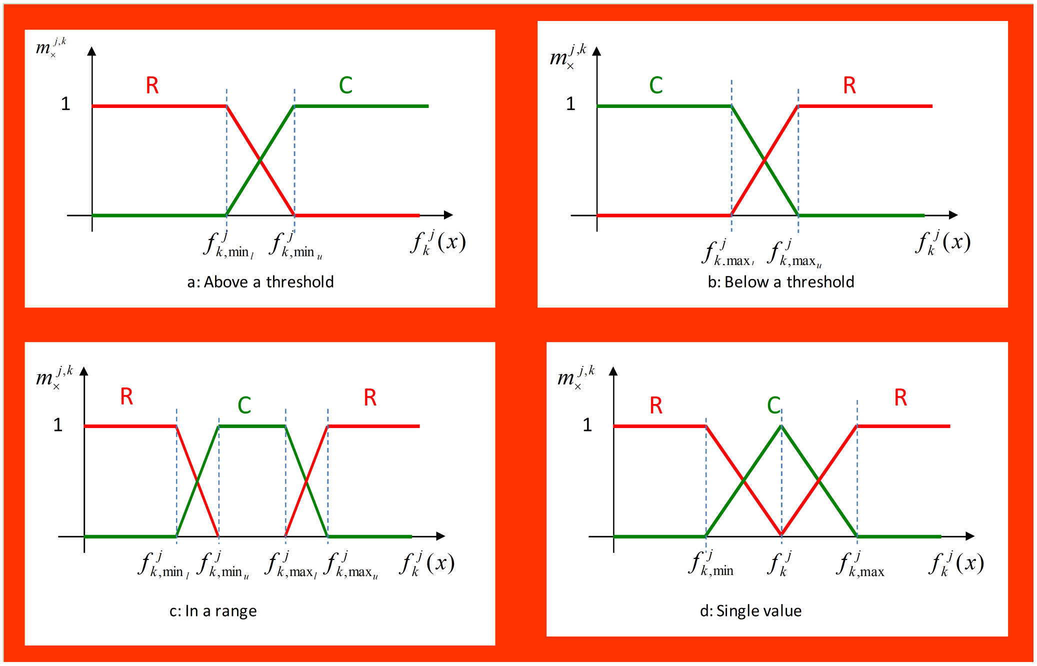

Figure 2: Characterization of the main typologies of a class feature

where G_C and G_R are some aggregation operators. Aggregation problem is an important subject in many domains where for a final decision purpose one has to aggregate many values. Many aggregations approach do exist and depend on the behavior of entities to aggregate and the perception of decision maker. In the following paragraph we will review some aggregation operators highlighting the Choquet integral that permits to take into account interactions between entities being aggregated.

2.1.1 Aggregation Operators

Let us consider a set of n numeric values θ_i, $i = 1, 2, ..., n$, to be aggregated and G an aggregation operator; many aggregation operators exist in the literature [5], the very basic one being the arithmetic mean; main aggregation operators used in decision analysis literature are recalled below.

- Arithmetic mean: the aggregated value is obtained by equation (3) below

$$G(\theta_1, \theta_2, ..., \theta_n) = \frac{\sum_{i=1}^{n} \theta_i}{n}. \tag{3}$$

- Quasi arithmetic means (geometric, harmonic, etc.): in this case, the aggregated value is given by equation (4)

$$G(\theta_1, \theta_2, ..., \theta_n) = f^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} f(\theta_i)\right), \tag{4}$$

  where f is any continuous strictly monotonic function; one can introduce relative importance weights with regards to elements to aggregate to obtain an aggregated value as given by equation (5) below

$$G(\theta_1, \theta_2, ..., \theta_n) = f^{-1}\left(\sum_{i=1}^{n} \omega_i f(\theta_i)\right) \tag{5}$$

  with $\omega_i \geq 0 \ \forall \ i$ and $\sum_{i=1}^{n} \omega_i = 1$.

- Median: this is an ordinal operator where the aggregated value is given by the following equation (6)

$$G(\theta_1, \theta_2, ..., \theta_n) = \begin{cases} \theta_{\sigma(\frac{n+1}{2})} \text{ if } n \text{ is odd} \\ \frac{1}{2}\left(\theta_{\sigma(\frac{n}{2})} + \theta_{\sigma(\frac{n}{2}+1)}\right) \text{ if } n \text{ is even,} \end{cases} \tag{6}$$

  where σ is a permutation over the n values such that $\theta_{\sigma(1)} \leq \theta_{\sigma(2)} \leq \cdots \leq \theta_{\sigma(n)}$.

- Weighted minimum and weighted maximum: in this case θ_i are normalized so that they must belong to $\left[\begin{array}{cc} 0, & 1 \end{array}\right]$ and the aggregated values are given by equations (7)-(8)

$$G_{\min}(\theta_1, \theta_2, ..., \theta_n) = \min_i \left(\max\left(1 - \omega_i, \theta_i\right)\right), \tag{7}$$

$$G_{\max}(\theta_1, \theta_2, ..., \theta_n) = \max_i \left(\min\left(\omega_i, \theta_i\right)\right), \tag{8}$$

  where the weights ω_i are normalized such that $\max_i(\omega_i) = 1$.

- Ordered weighted averaging operators (OWA): this operator permits the possibility to aggregate with respect to vague statement such as "at least some criteria must be met" because the relative weights depend on the rank of the corresponding element and the aggregated value is obtained by equation (9)

$$G(\theta_1, \theta_2, ..., \theta_n) = \sum_{i=1}^{n} \omega_i \theta_{\sigma(i)} \tag{9}$$

  where σ is a permutation over the n values such that $\theta_{\sigma(1)} \leq \theta_{\sigma(2)} \leq \cdots \leq \theta_{\sigma(n)}$ and $\omega_i \geq 0 \ \forall \ i$, $\sum_{i=1}^{n} \omega_i = 1$;

These operators do have some drawbacks mainly when considering classification problems; they do not cope with interactions between criteria whereas this is common in practice. The aggregation operator known to cope with such interaction is the Choquet integral, see [7]. For this reason we will introduce it in the subsequent paragraphs.

2.1.2 Choquet Integral

When a set $N = \{1,\ 2,\ ...,\ n\}$ of attributes with numerical measures vector $\theta = \left[\begin{array}{cccc} \theta_1 & \theta_2 & ... & \theta_n \end{array}\right]$ must be aggregated using Choquet integral, the first and primary thing to do is to define a fuzzy measure or a capacity over the set N; definition of such fuzzy measure or capacity is given below.

Definition 1. [7] *Let $N = \{1,\ 2,\ ...,\ n\}$ be a set of n elements. A capacity or fuzzy measure over N is a set function $\mu : 2^N \to [0,1]$ verifying $\mu(\varnothing) = 0$, $\mu(N) = 1$, and $\mu(A) \leq \mu(B)$ whenever $A \subseteq B$.*

From a capacity μ over N one can determine the interaction index I_{ij} between two elements i and j as given by equation (10)

$$I_{ij} = \sum_{A \subseteq N - \{i,j\}} \left\{ \frac{(|N| - |A| - 2)!\,|A|!}{(|N| - 1)!} \left[\mu\left(A \cup \{i,j\}\right) - \mu\left(A \cup \{i\}\right) - \mu\left(A \cup \{j\}\right) + \mu\left(A\right)\right] \right\}, \tag{10}$$

where $|\Omega|$ is the cardinality of the discrete set Ω. This interaction index does have the following meanings.

- $I_{ij} > 0$ means that attributes i and j considered individually are not important whereas when considered together they become important; thus there is synergy or complementarity between them.

- $I_{ij} < 0$ means that attributes i and j are individually important but taken together the importance does not increase much more that is these attributes are substitutable, there is redundancy.

- $I_{ij} = 0$ means that attributes i and j are independent.

Another interesting index associated with a fuzzy measure and that measures the importance of a given element i is the so called Shapley index ϕ_i and is defined by equation (11), see [17],

$$\phi_i = \sum_{A \subseteq N-\{i\}} \left\{ \frac{(|N| - |A| - 1)!\,|A|!}{|N|!} \left[\mu\left(A \cup \{i\}\right) - \mu\left(A\right)\right] \right\}. \tag{11}$$

Given a capacity over N, the Choquet integral of numerically evaluated vector of attributes θ (that is the global score of the corresponding object) is given as the following definition.

Definition 2. [7] *Let μ be a capacity or fuzzy measure over N and θ the numerical values vector of elements of N. The Choquet integral of θ with regard to μ is given by equation (12)*

$$\mathcal{C}_\mu(\theta) = \sum_{i=1}^{n} \left(\theta_{\sigma(i)} - \theta_{\sigma(i-1)}\right) \mu(A_i), \tag{12}$$

where σ is a permutation over N such that the order of equation (13) is respected

$$\theta_{\sigma(1)} \leq \theta_{\sigma(2)} \leq \cdots \leq \theta_{\sigma(n)} \text{ and } \theta_{\sigma(0)} = 0, \tag{13}$$

and the subset A_i is given by equation (14)

$$A_i = \{\sigma(i),\ \sigma(i+1),\ ...,\ \sigma(n)\}. \tag{14}$$

One can see immediately that difficulty of using Choquet integral as an aggregation operator in practice comes from the necessity to define a fuzzy measure that necessitates specifying $2^{|N|}-2$ coefficients representing the measure of subsets of N other than $\varnothing$ and N. Thus, if the interaction nature (synergy, redundancy or independence) between elements to aggregate and their importance in terms of Shapley index, for instance, are known, this can guide fuzzy measure definition. Rightly, elements to aggregate in nominal classification are synergetic elements; indeed selection or rejection of a class by two features of an object is better than by a single feature; this observation will be considered in the following paragraph to define a tractable fuzzy measure named weighted cardinal fuzzy measure or WCFM in short.

2.1.3 Weighted Cardinal Fuzzy Measure

As stated in previous paragraph, the difficulty of computing Choquet integral is to define a fuzzy measure over the set N that necessitates obtaining $2^{|N|} - 2$ coefficients that represent the measure of subsets of N other than $\varnothing$ and N. This can be done by experts if the set N is not too large, otherwise by some practical considerations, such as k-additive fuzzy measure, one can obtain this integral with less computational effort through interaction indices, see [6]. Given the observation made in the previous paragraph concerning the nature of interaction between elements for classification in or rejection of a class, the fuzzy measure of a subset of features should be proportional to the length or cardinality of that subset. A capacity or fuzzy measure μ over N is said to be cardinal if and only if the following conditions of equation (15) are verified

$$\mu(A) = \mu(B) \text{ if } |A| = |B|\,,\ \forall\ A,\ B \subseteq N; \tag{15}$$

by so doing only the cardinality of a subset matters and not its content; in addition, this measure is additive. To overcome this inconvenient and the difficulty to obtain a general fuzzy measure for a large set N, we introduce the notion of weighted cardinal measure where the cardinality of the subset is modulated by the relative importance of its elements; indeed some elements may be more or less important for the classification or rejection action. Let us suppose that a relative importance weight ω_i (with the condition $\sum_{i \in N} \omega_i = 1$) with regards to classification goal is obtained for each element i of N using approaches such as analytic hierarchy process (AHP), see [15]; then we propose to define a fuzzy measure that we refer to as weighted cardinal fuzzy measure (WCFM) by the following equation (16),

$$\mu(A) = \frac{|A|}{|N|} \left(\sum_{i \in A} \omega_i \right),\ \forall\ A \subseteq N. \tag{16}$$

This measure has characteristics of the following theorem that are proved below.

Theorem 1. *The measure defined by equation (16) has the following properties.*

i) It is a non additive fuzzy measure.

ii) It is a synergetic fuzzy measure where the interaction index I_{ij} between two elements i and j associated to it is proportional to the sum $\omega_i + \omega_j$ of their relative importance weights and is given by equation (17)

$$I_{ij} = \frac{\omega_i + \omega_j}{|N|}. \tag{17}$$

iii) The Shapley index of element i associated with measure (16) is linear in the relative importance degree ω_i of that element and is given by equation (18)

$$\phi_i = \frac{|N|\,\omega_i + 1}{2\,|N|} = \frac{\omega_i}{2} + \frac{1}{2\,|N|}. \tag{18}$$

Proof. i) It is obvious that relations of equation (19) are verified by measure μ defined in equation (16)

$$\mu(\varnothing) = 0 \text{ and } \mu(N) = 1; \tag{19}$$

now let us suppose that $A \subset B \subseteq N$, then it means that there exists a non empty subset $C \subset N$ such that $B = A \cup C$ and $A \cap C = \varnothing$. So we have result of equation (21) for $\mu(B)$

$$\mu(B) = \frac{|A| + |C|}{|N|} \left(\sum_{i \in A} \omega_i + \sum_{i \in C} \omega_i \right) \tag{20}$$

$$= \mu(A) + \mu(C) + \frac{|C|}{|N|} \left(\sum_{i \in A} \omega_i \right) + \frac{|A|}{|N|} \left(\sum_{i \in C} \omega_i \right), \tag{21}$$

meaning that conditions of equation (22) are valid

$$\mu(B) > \mu(A) + \mu(C), \tag{22}$$

because as $C \neq \varnothing$, we have condition of (23)

$$\frac{|C|}{|N|} \left(\sum_{i \in A} \omega_i \right) + \frac{|A|}{|N|} \left(\sum_{i \in C} \omega_i \right) > 0. \tag{23}$$

Conditions of equations (19) and (22) imply that μ really defines a non additive fuzzy measure.

ii) One can easily verify result of equation (24)

$$\mu\left(A \cup \{i, j\}\right) - \mu\left(A \cup \{i\}\right) - \mu\left(A \cup \{j\}\right) + \mu\left(A\right) = \frac{\omega_i + \omega_j}{|N|}, \tag{24}$$

so that the interaction index I_{ij} as defined by equation (10) is given by equation (26)

$$I_{ij} = \sum_{A \subseteq N - \{i,j\}} \left\{ \frac{(|N| - |A| - 2)!\,|A|!}{(|N| - 1)!} \left[\frac{\omega_i + \omega_j}{|N|} \right] \right\} \tag{25}$$

$$= \left(\frac{\omega_i + \omega_j}{|N|} \right) \left(\sum_{A \subseteq N - \{i,j\}} \left\{ \frac{(|N| - |A| - 2)!\,|A|!}{(|N| - 1)!} \right\} \right). \tag{26}$$

But there are $(|N| - 2)!/(|N| - |A| - 2)!\,|A|!$ subsets of $N - \{i, j\}$ containing $|A|$ elements, thus we have following results of equations (27)-(28)

$$\sum_{A \subseteq N - \{i,j\}} \left\{ \frac{(|N| - |A| - 2)!\,|A|!}{(|N| - 1)!} \right\} = \sum_{|A|=0}^{|N|-2} \left\{ \left(\frac{(|N| - 2)!}{(|N| - |A| - 2)!\,|A|!} \right) \left(\frac{(|N| - |A| - 2)!\,|A|!}{(|N| - 1)!} \right) \right\} \tag{27}$$

$$= \sum_{|A|=0}^{|N|-2} \left\{ \left(\frac{(|N| - 2)!}{(|N| - 1)!} \right) \right\} = \frac{1}{(|N| - 1)} \sum_{|A|=0}^{|N|-2} \{1\} = 1, \tag{28}$$

which proves point ii) of the theorem.

iii) One can verify easily equation (29)

$$\mu\left(A\cup\{i\}\right)-\mu\left(A\right)=\frac{\omega_i}{|N|}+\frac{|A|}{|N|}\omega_i+\frac{\sum_{k\in A}\left(\omega_k\right)}{|N|}, \tag{29}$$

so that the Shapley index as given by equation (11) is reduced to (37)

$$\begin{aligned}
\phi_i &= \sum_{A\subseteq N-\{i\}}\left\{\left(\frac{(|N|-|A|-1)!\,|A|!}{|N|!}\right)\left(\frac{\omega_i}{|N|}+\frac{|A|}{|N|}\omega_i+\frac{\sum_{k\in A}\left(\omega_k\right)}{|N|}\right)\right\} && (30)\\
&= \frac{\omega_i}{|N|}\sum_{A\subseteq N-\{i\}}\left\{\left(\frac{(|N|-|A|-1)!\,|A|!}{|N|!}\right)\right\}+\frac{\omega_i}{|N|}\sum_{A\subseteq N-\{i\}}\left\{\left(\frac{(|N|-|A|-1)!\,|A|!}{|N|!}\right)|A|\right\}\\
&\quad+\sum_{A\subseteq N-\{i\}}\left\{\left(\frac{(|N|-|A|-1)!\,|A|!}{|N|!}\right)\left(\frac{\sum_{k\in A}\left(\omega_k\right)}{|N|}\right)\right\} && (31)\\
&= \frac{\omega_i}{|N|}\sum_{|A|=0}^{|N|-1}\left\{\left(\frac{(|N|-|A|-1)!\,|A|!}{|N|!}\right)\left(\frac{(|N|-1)!}{|A|!\,(|N|-1-|A|)!}\right)\right\}\\
&\quad+\frac{\omega_i}{|N|}\sum_{|A|=0}^{|N|-1}\left\{\left(\frac{(|N|-|A|-1)!\,|A|!}{|N|!}\right)\left(\frac{(|N|-1)!}{|A|!\,(|N|-1-|A|)!}\right)|A|\right\}\\
&\quad+\sum_{A\subseteq N-\{i\}}\left\{\left(\frac{(|N|-|A|-1)!\,|A|!}{|N|!}\right)\left(\frac{\sum_{k\in A}\left(\omega_k\right)}{|N|}\right)\right\} && (32)\\
&= \frac{\omega_i}{|N|}+\frac{\omega_i}{|N|}\sum_{|A|=0}^{|N|-1}\left\{\frac{|A|}{|N|}\right\}+\sum_{A\subseteq N-\{i\}}\left\{\left(\frac{(|N|-|A|-1)!\,|A|!}{|N|!}\right)\left(\frac{\sum_{k\in A}\left(\omega_k\right)}{|N|}\right)\right\} && (33)\\
&= \frac{\omega_i}{|N|}+\frac{\omega_i}{|N|}\left(\frac{|N|\,(|N|-1)}{2\,|N|}\right)+\sum_{A\subseteq N-\{i\}}\left\{\left(\frac{(|N|-|A|-1)!\,|A|!}{|N|!}\right)\left(\frac{\sum_{k\in A}\left(\omega_k\right)}{|N|}\right)\right\} && (34)\\
&= \frac{\omega_i\,(|N|+1)}{2\,|N|}+\sum_{A\subseteq N-\{i\}}\left\{\left(\frac{(|N|-|A|-1)!\,|A|!}{|N|!}\right)\left(\frac{\sum_{k\in A}\left(\omega_k\right)}{|N|}\right)\right\}. && (35)
\end{aligned}$$

By rearranging the second term of equation (35) considering the fact that each weight ω_k will appear $\left(\frac{|A|}{|N|}\right)\left(\frac{(|N|-1)!}{|A|!(|N|-1-|A|)!}\right)$ times in subsets of dimension $|A|$ we obtain the following equations (36)-(37)

$$\begin{aligned}
\phi_i &= \frac{\omega_i\,(|N|+1)}{2\,|N|}+\\
&\quad\sum_{|A|=0}^{|N|-1}\left\{\left(\frac{(|N|-|A|-1)!\,|A|!}{|N|!}\right)\left(\frac{|A|}{|N|-1}\right)\left(\frac{(|N|-1)!}{|A|!\,(|N|-1-|A|)!}\right)\left(\frac{\sum_{k\in N-\{i\}}\left(\omega_k\right)}{|N|}\right)\right\} && (36)\\
&= \frac{\omega_i\,(|N|+1)}{2\,|N|}+\sum_{|A|=0}^{|N|-1}\left\{\left(\frac{|A|}{|N|\,(|N|-1)}\right)\left(\frac{\sum_{k\in N-\{i\}}\left(\omega_k\right)}{|N|}\right)\right\}. && (37)
\end{aligned}$$

But we have $\sum_{k\in N-\{i\}}\left(\omega_k\right)=1-\omega_i$ so that ϕ_i is given by equation (41) which concludes the proof of point iii).

$$\phi_i = \frac{\omega_i\left(|N|+1\right)}{2\,|N|} + \frac{1-\omega_i}{|N|}\sum_{|A|=0}^{|N|-1}\left\{\left(\frac{|A|}{|N|\left(|N|-1\right)}\right)\right\} \tag{38}$$

$$= \frac{\omega_i\left(|N|+1\right)}{2\,|N|} + \frac{1-\omega_i}{|N|}\left\{\left(\frac{1}{|N|\left(|N|-1\right)}\right)\left(\frac{|N|\left(|N|-1\right)}{2}\right)\right\} \tag{39}$$

$$= \frac{\omega_i\left(|N|+1\right)}{2\,|N|} + \frac{1-\omega_i}{2\,|N|} \tag{40}$$

$$\phi_i = \frac{\omega_i\,|N|+1}{2\,|N|} = \frac{\omega_i}{2} + \frac{1}{2\,|N|} \tag{41}$$

It is straightforward that this index fulfills one of the important point of Shapley index axioms that is we have result of equation (42)

$$\sum_{i=1}^{|N|}\phi_i = \sum_{i=1}^{|N|}\left(\frac{\omega_i\,|N|+1}{2\,|N|}\right) = \frac{|N|+|N|}{2\,|N|} = 1. \tag{42}$$

Furthermore, we see that the Shapley index (a sort of absolute importance) is not null even for an element with a null relative importance, this is due to synergetic property.

□

Calculating the Choquet integral with a weighted cardinal fuzzy measure is straightforward and depend only on the relative weight vector ω and the numerical n dimension vector θ of values to aggregate; denoting this integral by $\mathcal{C}_{\omega}^{wcfm}(\theta)$, it is therefore given by equation (43)

$$\mathcal{C}_{\omega}^{wcfm}(\theta) = \sum_{k=1}^{n}\left\{\left\{\left(\frac{n-(k-1)}{n}\right)\left(\sum_{j\in A_k}\omega_j\right)\right\}\left(\theta_{\sigma(k)}-\theta_{\sigma(k-1)}\right)\right\}, \tag{43}$$

where σ is a permutation over N such that the order of equation (13) is realized and A_k is defined by in equation (14).

2.1.4 Classifiability and Rejectability Measures

Based on results of equation (43), given an object u characterized by its attributes vector and a class c_j with its features vector $\mathbf{f}_k^j(\mathbf{x})$ for $k = 1 : n_j$ and their relative importance vector ω^j one first determines classifiability and rejectability membership functions vectors $m_C^j(.)$ and $m_R^j(.)$ as defined previously and compute the aggregated classifiability index $\Psi_C^j(u)$ and the aggregated rejectability index $\Psi_R^j(u)$ as Choquet integral of $m_C^j(.)$ and $m_R^j(.)$ based on a WCFM represented by relative weight vector ω^j as given by the following equations (44)-(45):

$$\Psi_C^j(u) = \mathcal{C}_{\omega^j}^{wcfm}(m_C^j(.)) = \sum_{k=1}^{n_j}\left\{\left\{\left(\frac{n_j-(k-1)}{n_j}\right)\left(\sum_{l\in A_k}\omega_l\right)\right\}\left(m_C^{j,\sigma(k)}(.) - m_C^{j,\sigma(k-1)}(.)\right)\right\}, \tag{44}$$

$$\Psi_R^j(u) = \mathcal{C}_{\omega^j}^{wcfm}(m_R^j(.)) = \sum_{k=1}^{n_j}\left\{\left\{\left(\frac{n_j-(k-1)}{n_j}\right)\left(\sum_{l\in A_k}\omega_l\right)\right\}\left(m_R^{j,\sigma(k)}(.) - m_R^{j,\sigma(k-1)}(.)\right)\right\}. \tag{45}$$

But the classifiability or rejectability is a relative operation so that one must ultimately establish a sort of inter classes adequacy measures. To this end the ultimate classifiability measure $\psi_C^j(u)$ and rejectability measure $\psi_R^j(u)$ of an object u with regard to the class c_j are given by equation (46)

$$\psi_C^j(u) = \frac{\Psi_C^j(u)}{\sum_{k=1}^{n}\left\{\Psi_C^j(u)\right\}} \text{ and } \psi_R^j(u) = \frac{\Psi_R^j(u)}{\sum_{k=1}^{n}\left\{\Psi_R^j(u)\right\}}. \tag{46}$$

In the following subsection, we will sketch some classification procedures, based on measures $\psi_C^j(u)$ and $\psi_R^j(u)$, that may be used for final assignment purpose.

2.2 Classification Procedure

Given an object u, possible classes where it can be included are those for which the classifiability measure exceeds the rejectability measure to some extents. To ensure some security when including an object, one may introduce a caution index q so that one can consider including an object u in a class c^j if and only if the classifiability measure $\psi_C^j(u)$ exceeds the rejectability measure $\psi_R^j(u)$ multiplied by the index of caution q; thus the possible inclusion set $\mathcal{C}_q(u)$ for the object u at the caution index q is given by equation (47)

$$\mathcal{C}_q(u) = \left\{c_j \in \mathcal{C} : \psi_C^j(u) \geq q\psi_R^j(u)\right\}. \tag{47}$$

Remark 2: In fact the index q may be an increasing function of rejectability measure $\psi_R^j(u)$ in order to take into account misclassification aversion of decision maker who may want to penalize large rejectability measure.

For final class selection, one may consider optimizing some performance index. For instance if decision makers are able to specify the caution index q, then the final optimal class $c^*(u)$ for a given object u can be considered to be the **maximum discriminant** one that is given by equation (48)

$$c^*(u) = \arg\left\{\max_{c^j \in \mathcal{C}_q(u)} \left\{\psi_C^j(u) - q\psi_R^j(u)\right\}\right\}. \tag{48}$$

On the contrary, when there is no information about the desired caution index, the assigned class $c^*(u)$ can naturally be chosen to maximize this index (**maximum index of caution**) that in return minimize risk of misclassification; in this case the optimal inclusion class $c^*(u)$ for a given object u is given by the following equation (49)

$$c^*(u) = \arg\left\{\max_{c^j \in \mathcal{U}} \left\{\frac{\psi_C^j(u)}{\psi_R^j(u)}\right\}\right\}. \tag{49}$$

In the next section we will consider an application in the domain of banking to show potential applicability of the approach developed in this paper.

3 Application

This application is extracted from [13] and concerns the problem of assigning retailers that use EFTPoS (Electronic Fund Transfer at Point of Sale) service of a bank to some classes in order for the bank manager to consider their appropriate strategic treatment. Any retailer is characterized by 13 attributes or criteria, see [13] for their significance and there are four classes which are characterized by conditions in the form of above a threshold on each attribute in terms of parameter $\mathbf{b}_j$. Raw data of this application is shown on the following Table 1.

To fit our model we consider the doubtful zone of each attribute to range from δ% of the threshold to the threshold so that the the parameter δ can be used to do a sensitivity analysis. Let denote by X the 20×13 matrix representing the characterization of units to classify as shown on the previous table. From this table classifiability and rejectability membership functions data X_C and X_R are obtained by equation (50)

$$\begin{cases} m_C^{j,k}(u) = 0 \text{ and } m_R^{j,k}(u) = 1 \text{ if } X(u,j) \leq \frac{\delta}{100}\mathbf{b}^k(j) \\ m_C^{j,k}(u) = 1 \text{ and } m_R^{j,k}(u) = 0 \text{ if } X(u,j) \geq \mathbf{b}^k(j) \\ m_C^{j,k}(u) = \left(\frac{1}{\mathbf{b}^k(j) - \frac{\delta}{100}\mathbf{b}^k(j)}\right)\left(X(u,j) - \frac{\delta}{100}\mathbf{b}^k(j)\right) \text{ if } \frac{\delta}{100}\mathbf{b}^k(j) < X(u,j) < \mathbf{b}^k(j) \\ m_R^{j,k}(u) = -\left(\frac{1}{\mathbf{b}^k(j) - \frac{\delta}{100}\mathbf{b}^k(j)}\right)\left(X(u,j) - \mathbf{b}^k(j)\right) \text{ if } \frac{\delta}{100}\mathbf{b}^k(j) < X(u,j) < \mathbf{b}^k(j). \end{cases} \tag{50}$$

Classifiability and rejectability indices are then given by equation (51)

$$\psi_C^j(u) = \frac{\mathcal{C}_\omega^{wcfm}\left(m_C^j(u)\right)}{\sum_l \left\{\mathcal{C}_\omega^{wcfm}\left(m_C^l(u)\right)\right\}} \text{ and } \psi_R^j(u) = \frac{\mathcal{C}_\omega^{wcfm}\left(m_R^j(u)\right)}{\sum_l \left\{\mathcal{C}_\omega^{wcfm}\left(m_R^l(u)\right)\right\}}, \tag{51}$$

where the weighted cardinal fuzzy measure μ is related to weighting vector ω obtained by normalization of original weighting vector as given by equation (52)

$$\omega(j) = \frac{\omega(j)}{\sum_{j=1}^{13} \omega(j)}. \tag{52}$$

Table 1: Raw data

	$\mathbf{a_{01}}$	$\mathbf{a_{02}}$	$\mathbf{a_{03}}$	$\mathbf{a_{04}}$	$\mathbf{a_{05}}$	$\mathbf{a_{06}}$	$\mathbf{a_{07}}$	$\mathbf{a_{08}}$	$\mathbf{a_{09}}$	$\mathbf{a_{10}}$	$\mathbf{a_{11}}$	$\mathbf{a_{12}}$	$\mathbf{a_{13}}$
$\mathbf{u_{01}}$	29	22	28	25	69	25	61	52	25	39	58	61	68
$\mathbf{u_{02}}$	80	78	88	69	59	30	50	45	48	42	22	15	27
$\mathbf{u_{03}}$	77	90	88	61	63	28	35	33	51	33	22	28	33
$\mathbf{u_{04}}$	16	39	26	25	55	25	50	51	43	65	37	38	73
$\mathbf{u_{05}}$	28	56	51	21	34	8	37	61	30	37	55	66	98
$\mathbf{u_{06}}$	79	75	80	65	60	25	30	34	22	19	22	18	21
$\mathbf{u_{07}}$	50	6	54	25	38	21	47	41	40	57	65	65	88
$\mathbf{u_{08}}$	44	19	31	55	49	29	80	70	73	55	48	29	45
$\mathbf{u_{09}}$	49	43	28	29	61	22	67	42	25	39	51	62	55
$\mathbf{u_{10}}$	30	25	30	51	55	44	82	84	90	74	32	15	32
$\mathbf{u_{11}}$	30	29	32	87	86	80	77	46	28	49	25	29	33
$\mathbf{u_{12}}$	49	17	54	25	37	21	47	39	42	54	65	55	98
$\mathbf{u_{13}}$	42	14	27	51	43	22	74	67	69	53	40	25	92
$\mathbf{u_{14}}$	25	19	26	90	81	79	70	44	32	45	28	24	30
$\mathbf{u_{15}}$	42	14	27	51	56	46	81	78	82	53	40	25	33
$\mathbf{u_{16}}$	80	77	79	69	65	22	31	37	28	22	19	21	29
$\mathbf{u_{17}}$	21	15	22	86	79	83	68	40	30	41	20	19	25
$\mathbf{u_{18}}$	18	12	25	82	81	79	64	38	29	39	19	15	27
$\mathbf{u_{19}}$	22	18	26	49	51	41	80	80	86	69	24	11	26
$\mathbf{u_{20}}$	41	35	44	29	34	21	47	61	50	57	62	61	98
ω	10	12	4	13	13	8	10	4	4	8	4	8	2
$\mathbf{b}^1$	75	70	75	60	55	20	25	35	20	15	15	10	20
$\mathbf{b}^2$	15	10	20	75	70	75	60	30	25	35	15	10	20
$\mathbf{b}^3$	15	10	20	45	45	40	75	70	75	60	15	10	20
$\mathbf{b}^4$	55	10	20	15	10	20	35	30	40	70	75	60	55

The two classification approaches in terms of equation (48) with a caution index $q = 1$ and equation (49) lead to the same results given on the following Table 2; on this table are also reported results obtained by the classification approach developed in [13] and named **NeXClass** as well as original heuristic used by managers before approach developed in [13].

When comparing to the heuristic originally used by mangers as a benchmark, we see that our approach performs better than the NexClass algorithm developed by [13] with in general less than 3 misclassification whereas results of [13] present 3 misclassifications. Though, the main goal of this paper is not to propose a numerically superior algorithm for nominal classification, (it rather proposes a structured framework to capture practical aspects of near human ways to classify), this application nevertheless shows that the proposed approach performs well numerically.

Table 2: Results of application

	Our procedure results					Results from [13]	
	$\delta = 0$	$\delta = 10$	$\delta = 50$	$\delta = 90$	$\delta = 100$	**NeXClass**	**Orig. heur.**
$\mathbf{u_{01}}$	c_2	c_2	c_2	c_2	c_4	c_4	c_3
$\mathbf{u_{02}}$	c_1	c_1	c_1	c_1	c_1	c_1	c_1
$\mathbf{u_{03}}$	c_1	c_1	c_1	c_1	c_1	c_1	c_1
$\mathbf{u_{04}}$	c_4	c_4	c_4	c_4	c_4	c_3	c_4
$\mathbf{u_{05}}$	c_4	c_4	c_4	c_4	c_4	c_4	c_4
$\mathbf{u_{06}}$	c_1	c_1	c_1	c_1	c_1	c_1	c_1
$\mathbf{u_{07}}$	c_4	c_4	c_4	c_4	c_4	c_4	c_4
$\mathbf{u_{08}}$	c_3	c_3	c_3	c_3	c_3	c_3	c_3
$\mathbf{u_{09}}$	c_4	c_4	c_4	c_2	c_4	c_4	c_4
$\mathbf{u_{10}}$	c_3	c_3	c_3	c_3	c_3	c_3	c_3
$\mathbf{u_{11}}$	c_2	c_2	c_2	c_2	c_2	c_2	c_2
$\mathbf{u_{12}}$	c_4	c_4	c_4	c_4	c_4	c_4	c_4
$\mathbf{u_{13}}$	c_3	c_3	c_3	c_2	c_2	c_4	c_3
$\mathbf{u_{14}}$	c_2	c_2	c_2	c_2	c_2	c_2	c_2
$\mathbf{u_{15}}$	c_3	c_3	c_3	c_3	c_3	c_3	c_3
$\mathbf{u_{16}}$	c_1	c_1	c_1	c_1	c_1	c_1	c_1
$\mathbf{u_{17}}$	c_2	c_2	c_2	c_2	c_2	c_2	c_2
$\mathbf{u_{18}}$	c_2	c_2	c_2	c_2	c_2	c_2	c_2
$\mathbf{u_{19}}$	c_3	c_3	c_3	c_3	c_3	c_3	c_3
$\mathbf{u_{20}}$	c_4	c_4	c_4	c_4	c_4	c_4	c_4

4 Conclusion

The problem of fuzzy nominal classification, that consists in an assignment of objects characterized by many attributes to predefined classes characterized by fuzzy features or conditions, has been considered in this paper. The method considered to formulate the classification model highlights the bipolarity that exist between the realization of a feature and the inclusion of the considered object in a class. As humans proceed in practice by balancing "pros" and "cons" during a decision process, this approach proposes, for a pair constituted by an object to be classified and a class, derivation of two measures: the *classifiability* that is a degree measuring to what extent this object can be included in that class and the *rejectability* that measures the degree to which one should avoid including the considered object into the specified class. As these measures are obtained using Choquet integral as an aggregation operator and based on the fact that elements to aggregate behave in synergetic way, an easy to compute fuzzy measure known as weighted cardinal fuzzy measure (WCFM) has been proposed; interaction index and Shapley index associated to this measure have been completely characterized. The application of this approach to a real world problem has shown real potentiality.

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