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Multicriteria Entropy Bimatrix Goal Game: A Fuzzy Programming Approach

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Abstract

This paper analyzes the multicriteria bimatrix goal game under the light of entropy environment. In this approach, the entropy functions of the players are considered as objectives to the bimatrix game. The solution concepts behind this game are based on getting the probability to achieve some specified goals by determining G-goal security strategies (GGSS). We define the real coded Genetic Algorithm (GA) to obtain the bounds of the objectives of the proposed game. Then formulated model is solved by fuzzy programming technique. Finally a numerical example is included to illustrate the methodology.
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Keywords: bimatrix game, GGSS, entropy, fuzzy programming, genetic algorithm

1 Introduction

The concept of "entropy" is introduced to provide a quantitative measure of uncertainty. Entropy models are emerging as valuable tools in the study of various social and engineering problems.

Two-person zero-sum game models are accurate when stakes are small monetary amounts. But in reality sense, when the stakes are more complicated, as often in economic situations, it is not generally true that the interests of the two players are exactly opposed. Such type of game models are known as non-cooperative game model. In other words, such situations give rise to two-person non-zero sum game, called bimatrix game. A bimatrix game can be considered as a natural extension of the matrix game, to cover situations in which the outcome of a decision process does not necessarily dictate the verdict that what one player gains the other one has to lose.

The family of probability distributions of strategies of every players are consistent with given information for bimatrix game. We choose the distribution whose uncertainty or entropy is maximum. Each player is interested in making moves which will be as surprising and as uncertain to the other player as possible. For this reason, the players are involved in maximizing their entropies. Consequently, in the mathematical models of multicriteria bimatrix game with certain goals, incorporate an entropy function as one of their objectives. This model is known as multicriteria entropy bimatrix goal game model.

In conventional mathematical programming, the coefficients or parameters of the bimatrix game model are assumed to be deterministic and fixed. But, there are many situations where the parameters may not be exactly known i.e., the parameters may have some uncertainty in nature. Thus, the decision-making method under uncertainty is needed. From this point of view, the fuzzy programming has been incorporated in decision-making method. In fuzzy programming problem, the coefficients, constraints and the goals are viewed as fuzzy numbers or fuzzy sets. In decision-making process, the fuzzy set theory was initiated by Bellman and Zadeh [13] and later Zimmermann [14] showed that the classical algorithms could be used to solve multi-objective fuzzy linear programming problem.

In this paper, some references are presented including their work. Fernandez et al. [5] considered to solve the two-person multicriteria zero-sum games. As they have considered a multicriteria game, the solution

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concept is based on Pareto optimality. Finally they obtained the Pareto efficient solution for their proposed games. Roy [12] has presented the study of two different solution procedures for the two-person bimatrix game. The first solution procedure is applied to the game on getting the probability to achieve some specified goals along the player's strategy. The second specified goals along with the player's strategy by introducing the membership function of fuzzy programming defined on the pay-off matrix of the bimatrix game. In our recent paper [3], we have proposed a new solution concept by considering the entropy function as an objective of the players to bimatrix goal game and formulated some models, known as entropy bimatrix goal game models. Solutions are obtained by introducing the concept of Pareto-optimal security strategies(POSS). It is shown that the said models may have some risk factors in pay-offs for player with their measure of uncertainties in strategies. Also in our another recent paper [9], we have proposed a game model by considering entropy functions into the objectives of the players to the multicriteria goal game and named as multicriteria entropy goal game model. Solutions are obtained by introducing the concept of G-goal security strategies(GGSS). It includes as a part of solution with the probabilities of obtaining presanctified values of the pay-off functions when the players are wanted to maximize the information about their strategies. But, no studies have been made on multicriteria bimatrix goal game under the light of entropy environment.

2 Mathematical Model of a Bimatrix Game

A bimatrix game can be considered as a natural extension of the matrix game. A two-person non zero-sum game can be expressed by a bimatrix game, comprised of two $m \times n$ dimensional matrices, namely A and B, where

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{bmatrix}.$$

If player PI adopts the strategy "row i" and player PII adopts the strategy "column j", then a_{ij} denotes the expected payoff for player PI and b_{ij} denotes the expected payoff for player PII.

For two-person multicriteria non zero-sum game, multiple pair of $m \times n$ payoff matrices can be formulated as follows:

$$A^l = \begin{bmatrix} a^l_{11} & a^l_{12} & \dots & a^l_{1n} \\ a^l_{21} & a^l_{22} & \dots & a^l_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a^l_{m1} & a^l_{m2} & \dots & a^l_{mn} \end{bmatrix}, \; l = 1, \dots, n_1 \qquad B^l = \begin{bmatrix} b^l_{11} & b^l_{12} & \dots & b^l_{1n} \\ b^l_{21} & b^l_{22} & \dots & b^l_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b^l_{m1} & b^l_{m2} & \dots & b^l_{mn} \end{bmatrix}, \; l = 1, \dots, n_2$$

where the players PI and PII have n_1 and n_2 numbers of objectives respectively. Without any loss of generality, assuming that the players PI and PII both are maximized players.

Definition 2.1 *(Mixed Strategies): The mixed strategy of the bimatrix game for player PI and PII are defined as follows:*

$$Y = \{\; y \in R^m; \; \sum_{i=1}^{m} y_i = 1; \; y_i \geq 0, \quad i = 1, 2, \dots, m \;\},$$

$$Z = \{\; z \in R^n; \; \sum_{j=1}^{n} z_j = 1; \; z_j \geq 0, \quad j = 1, 2, \dots, n \;\}.$$

Definition 2.2 *(Expected Payoffs of Multicriteria Bimatrix Game): For bimatrix game, if the player PI chooses the mixed strategy $y \in Y$ and the player PII chooses the mixed strategy $z \in Z$, then the l^{th} payoff for the player PI is represented by $S^l_1 = y^t A^l z, \quad l = 1, \dots, n_1$ and that of the k^{th} payoff for the player PII is represented by $S^k_2 = y^t B^k z, \quad k = 1, \dots, n_2$. Here the player PI chooses a mixed strategy y and the player PII chooses a mixed strategy z in a multicriteria bimatrix game $(A^l, B^k), l = 1, \dots, n_1; k = 1, \dots, n_2$.*

For simplicity, we have considered the number of alternatives are equal i.e, $n_1 = n_2$.

Definition 2.3 *(Expected Payoffs of Multicriteria Bimatrix Goal Game): The expected payoff* $v^l(y,z), l = 1,\dots,s$ *of the s-objective bimatrix game,* $A^l = (a^l_{ij})$ *and* $B^l = (b^l_{ij}), l = 1,\dots,s$ *with goals* $G^l = (G^l_1, G^l_2)$, *for each strategy pair* $y \in Y$ *and* $z \in Z$, *is defined as follows:*

$$v^l(y,z) = \left[v^l_1(y,z), v^l_2(y,z)\right], l = 1,\dots,s$$

where $v^l_1(y,z) = y^t A^l_G z,\ v^l_2(y,z) = y^t B^l_G z, l = 1,\dots,s, A^l_G = (\delta^1_{ij}(l)),\ i = 1,2,\dots,m,\ j = 1,2,\dots,n, l = 1,\dots,s,\ B^l_G = (\delta^2_{ij}(l)),\ i = 1,2,\dots,m,\ j = 1,2,\dots,n, l = 1,\dots,s,$ *where*

$$\delta^1_{ij}(l) = \begin{cases} 1 & \text{if } a^l_{ij} \geq G^l_1 \\ 0 & \text{otherwise} \end{cases} \quad l = 1,\dots,s \quad \text{and} \quad \delta^2_{ij}(l) = \begin{cases} 1 & \text{if } b^l_{ij} \geq G^l_2 \\ 0 & \text{otherwise} \end{cases} \quad l = 1,\dots,s.$$

Definition 2.4 *(G-goal Security Level): The G-goal security level for PI, for each* $y \in Y$, *is*

$$v^G(y)(l) = \left[v^G_1(y)(l), v^G_2(y)(l)\right],\ l = 1,\dots,s$$

where

$$v^G_1(y)(l) = \min_{z\in Z} v^l_1(y,z) = \min_{z\in Z} y^t A^l_G z = \min_{1\leq j\leq n} \sum_{i=1}^m y_i \delta^1_{ij}(l),$$

$$v^G_2(y)(l) = \min_{z\in Z} v^l_2(y,z) = \min_{z\in Z} y^t B^l_G z = \min_{1\leq j\leq n} \sum_{i=1}^m y_i \delta^2_{ij}(l).$$

Definition 2.5 (*G-goal Security Strategy): A strategy* $y^* \in Y$ *is a G-goal security strategy (GGSS) for PI if for each* $l = 1,\dots,s$ *there is no* $y \in Y$ *such that* $v^G(y^*)(l) \leq v^G(y)(l)$, $v^G(y^*)(l) \neq v^G(y)(l)$.

2.1 Determination of G-goal Security Strategies

The multicriteria bimatrix goal game model is represented as follows:

$$\begin{array}{rrcl} \text{Model 1} & \max & : & v^G_1(1),\dots,v^G_1(s) \\ & \max & : & v^G_2(1),\dots,v^G_2(s) \\ \text{subject to} & y^t A^l_G & \geq & [v^G_1(l),\dots,v^G_1(l)],\quad l = 1,\dots,s \\ & y^t B^l_G & \geq & [v^G_2(l)\dots,v^G_2(l)],\quad l = 1,\dots,s \\ & \sum_{i=1}^m y_i & = & 1;\quad y_i \geq 0,\quad i = 1,2,\dots,m \end{array}$$

Definition 2.6 *(Efficient Solution): A solution* $[v^{G*}(l); y^*],\ (l = 1,\dots,s)$ *is an efficient solution of above model (Model 1) if there does not exist any* $[v^G(l); y],\ (l = 1,\dots,s)$ *such that* $[v^G(l); y],\ (l = 1,\dots,s)$ *dominates* $[v^{G*}(l); y^*],\ (l = 1,\dots,s)$.

Theorem 2.1 *A strategy* $y^* \in Y$ *is a G-goal security strategies (GGSS) and* $v^{G*}(l) = v^G(y^*)(l) = \left[{v_1}^G(y^*)(l), {v_2}^G(y^*)(l)\right]$ $(l = 1,\dots,s)$ *is its G-goal security level iff* $[v^{G*}(1),\dots,v^{G*}(s); y^*]$ *is an efficient solution of Model 1.*

Proof: Let y^* be a GGSS. Then by definition there is no $y \in Y$ such that $v^G(y^*)(l) \leq v^G(y)(l)$, $v^G(y^*)(l) \neq v^G(y)(l)$ $(l = 1,\dots,s)$. This is an equivalent to

$$\left[\min_j(\sum_{i=1}^m y_i\delta^1_{ij}(l)), \min_j(\sum_{i=1}^m y_i\delta^2_{ij}(l))\right] \geq \left[\min_j(\sum_{i=1}^m y^*_i\delta^1_{ij}(l)), \min_j(\sum_{i=1}^m y^*_i\delta^2_{ij}(l))\right],$$

and

$$\left[\min_j(\sum_{i=1}^m y_i\delta^1_{ij}(l)), \min_j(\sum_{i=1}^m y_i\delta^2_{ij}(l))\right] \neq \left[\min_j(\sum_{i=1}^m y^*_i\delta^1_{ij}(l)), \min_j(\sum_{i=1}^m y^*_i\delta^2_{ij}(l))\right].$$

Hence $[v^{G*}(l);y^*]$, $(l=1,\dots,s)$ is an efficient solution of the problem

$$\max_{y\in Y}\left[\min_j(\sum_{i=1}^m y_i\delta^1_{ij}(l)),\min_j(\sum_{i=1}^m y_i\delta^2_{ij}(l))\right],\quad l=1,\dots,s.$$

Consequently the above solution is an equivalent to the solution of Model 1 i.e,

$$\begin{array}{llll}\text{Model 1} & \max & : & v_1^G(1),\dots,v_1^G(s)\\ & \max & : & v_2^G(1),\dots,v_2^G(s)\\ & \text{subject to} & & y^tA^l_G\ge[v_1^G(l),\dots,v_1^G(l)],\quad l=1,\dots,s\\ & & & y^tB^l_G\ge[v_2^G(l)\dots,v_2^G(l)],\quad l=1,\dots,s\\ & & & \sum_{i=1}^m y_i=1;\quad y_i\ge0,\quad i=1,\dots,m.\end{array}$$

Conversely, suppose that an efficient solution $[v^{G*}(l);y^*]$ $(l=1,\dots,s)$ of Model 1 and y^* is not a GGSS. Then, there exists $\overline{y}\in Y$ such that

$$\left[\min_j(\sum_{i=1}^m \overline{y_i}\delta^1_{ij}(l)),\min_j(\sum_{i=1}^m \overline{y_i}\delta^2_{ij}(l))\right]\ge\left[\min_j(\sum_{i=1}^m y_i^*\delta^1_{ij}(l)),\min_j(\sum_{i=1}^m y_i^*\delta^2_{ij}(l))\right],$$

and

$$\left[\min_j(\sum_{i=1}^m \overline{y_i}\delta^1_{ij}(l)),\min_j(\sum_{i=1}^m \overline{y_i}\delta^2_{ij}(l))\right]\ne\left[\min_j(\sum_{i=1}^m y_i^*\delta^1_{ij}(l)),\min_j(\sum_{i=1}^m y_i^*\delta^2_{ij}(l))\right].$$

Taking $\overline{v}(l)=[\overline{v}_1(l),\overline{v}_2(l)]$ $(l=1,\dots,s)$, where$\overline{v}_1(l)=\min_j(\sum_{i=1}^m\overline{y_i}\delta^1_{ij}(l))$, $\overline{v}_2(l)=\min_j(\sum_{i=1}^m\overline{y_i}\delta^2_{ij}(l))$, The vector $[\overline{v}(l);\overline{y}]$ $(l=1,\dots,s)$ is a feasible solution of Model 1 dominating $[v^{G*}(l);y^*]$, $(l=1,\dots,s)$. This is a contradiction. Hence the theorem is proved.

2.2 Entropy Bimatrix Goal Game Model

Each player is interested in making moves which will be as surprising and as uncertain to the other player as possible. For this reason, the two players are involved in maximizing their entropies. The mathematical form of entropy are as follows:

$$H_1=-\sum_{i=1}^m y_i\ ln(y_i)\tag{1}$$

Without any loss of generality, we combined the Model 1 with the above entropy function (1) and we have formulated a new mathematical model namely Entropy Bimatrix Goal Game Model which is a multi-objective non-linear programming model and this model is defined for player PI as follows:

$$\begin{array}{llll}\text{Model 2} & \max & : & v_1^G(1),\dots,v_1^G(s)\\ & \max & : & v_2^G(1),\dots,v_2^G(s)\\ & \max & : & v_3^G=H_1\\ & \text{subject to} & & y^tA^l_G\ge[v_1^G(l),\dots,v_1^G(l)],\quad l=1,\dots,s\\ & & & y^tB^l_G\ge[v_2^G(l)\dots,v_2^G(l)],\quad l=1,\dots,s\\ & & & H_1=-\sum_{i=1}^m y_i\ ln(y_i)\\ & & & \sum_{i=1}^m y_i=1;\quad y_i\ge0,\quad i=1,2,\dots,m.\end{array}$$

Theorem 2.2 *A strategy $y^*\in Y$ is a G-goal security strategies (GGSS) and $v^{G*}(l)=v^G(y^*)(l)=\left[{v_1}^G(y^*)(l),{v_2}^G(y^*)(l)\right]$, $(l=1,\dots,s)$ and $v_3^G(y^*)$ is its G-goal security level iff $[v^{G*}(l);v_3^{G*};y^*]$ $(l=1,\dots,s)$*

is an efficient solution of Model 2.

Proof: Let y^* be a GGSS. Then by definition there is no $y \in Y$ such that $v^G(y^*)(l) \leq v^G(y)(l)$, $v^G(y^*)(l) \neq v^G(y)(l)$, $(l = 1, \ldots, s)$ and $v_3^G(y) \geq v_3^G(y^*)$. This is an equivalent to

$$\left[\min_j(\sum_{i=1}^m y_i\delta_{ij}^1(l)), \min_j(\sum_{i=1}^m y_i\delta_{ij}^2(l))\right] \geq \left[\min_j(\sum_{i=1}^m y_i^*\delta_{ij}^1(l)), \min_j(\sum_{i=1}^m y_i^*\delta_{ij}^2(l))\right],$$

and

$$\left[\min_j(\sum_{i=1}^m y_i\delta_{ij}^1(l)), \min_j(\sum_{i=1}^m y_i\delta_{ij}^2(l))\right] \neq \left[\min_j(\sum_{i=1}^m y_i^*\delta_{ij}^1(l)), \min_j(\sum_{i=1}^m y_i^*\delta_{ij}^2(l))\right],$$

and $H_1(y) \geq H_1(y^*)$. Hence $[v^{G*}(l); v_3^{G*}; y^*]$ $(l = 1, \ldots, s)$ is an efficient solution of the following problem

$$\max_{y \in Y}\left[\min_j(\sum_{i=1}^m y_i\delta_{ij}^1(l)), \min_j(\sum_{i=1}^m y_i\delta_{ij}^2(l))\right], \quad l = 1, \ldots, s.$$

$$\max_{y \in Y}[H_1(y)]$$

Consequently, the above solution is an equivalent to the solution of Model 2, i.e.,

$$\begin{array}{lll}
\text{Model 2} \quad \max & : & v_1^G(1), \ldots, v_1^G(s) \\
\max & : & v_2^G(1), \ldots, v_2^G(s) \\
\max & : & v_3^G = H_1 \\
\text{subject to} & & y^t A_G^l \geq [v_1^G(l), \ldots, v_1^G(l)], \quad l = 1, \ldots, s \\
& & y^t B_G^l \geq [v_2^G(l) \ldots, v_2^G(l)], \quad l = 1, \ldots, s \\
& & H_1 = -\sum_{i=1}^m y_i \; ln(y_i) \\
& & \sum_{i=1}^m y_i = 1; \quad y_i \geq 0, \quad i = 1, 2, \ldots, m.
\end{array}$$

Conversely, suppose that an efficient solution $[v^{G*}(l); v_3^{G*}; y^*]$ $(l = 1, \ldots, s)$ of Model 2 and y^* is not a GGSS. Then, there exists $\overline{y} \in Y$ such that

$$\left[\min_j(\sum_{i=1}^m \overline{y_i}\delta_{ij}^1(l)), \min_j(\sum_{i=1}^m \overline{y_i}\delta_{ij}^2(l))\right] \geq \left[\min_j(\sum_{i=1}^m y_i^*\delta_{ij}^1(l)), \min_j(\sum_{i=1}^m y_i^*\delta_{ij}^2(l))\right],$$

and

$$\left[\min_j(\sum_{i=1}^m \overline{y_i}\delta_{ij}^1(l)), \min_j(\sum_{i=1}^m \overline{y_i}\delta_{ij}^2(l))\right] \neq \left[\min_j(\sum_{i=1}^m y_i^*\delta_{ij}^1(l)), \min_j(\sum_{i=1}^m y_i^*\delta_{ij}^2(l))\right]$$

and $H_1(\overline{y}) \geq H_1(\overline{y}^*)$. Taking $\overline{v}(l) = [\overline{v}_1(l), \overline{v}_2(l)]$ $(l = 1, \ldots, s)$ and $\overline{v}_3$ where

$$\overline{v}_1(l) = \min_j(\sum_{i=1}^m \overline{y_i}\delta_{ij}^1(l)), \quad \overline{v}_2(l) = \min_j(\sum_{i=1}^m \overline{y_i}\delta_{ij}^2(l)),$$

$\overline{v}_3 = \overline{H_1} = \max \; H_1(\overline{y})$. The vector $[\overline{v}(l), ; \overline{v}_3; \overline{y}]$ $(l = 1, \ldots, s)$ is a feasible solution of Model 2 dominating $[v^{G*}(l); v_3^{G*}; y^*]$ $(l = 1, \ldots, s)$. This is a contradiction. Hence the theorem is proved.

3 Solution Procedure

3.1 Fuzzy Programming

In fuzzy programming, first we construct the membership function for each objective function from Model 2. Let $\mu_l(v_1^G(l)), \mu_2(v_2^G(l))$, $l = 1, \ldots, s$, $\mu_3(H_1)$ be the membership functions for objectives respectively and they are defined as follows:

$$\mu_1(v_1^G(l)) = \begin{cases} 0 & \text{if } v_1^G(l) \le v_1^{G-}(l) \\ \frac{v_1^G(l)-v_1^{G-}(l)}{v_1^{G+}(l)-v_1^{G-}(l)} & \text{if } v_1^{G-}(l) \le v_1^G(l) \le v_1^{G+}(l) \\ 1 & \text{if } v_1^G(l) \ge v_1^{G+}(l), \end{cases} \tag{2}$$

$$\mu_2(v_2^G(l)) = \begin{cases} 0 & \text{if } v_2^G(l) \le v_2^{G-}(l) \\ \frac{v_2^G(l)-v_2^{G-}(l)}{v_2^{G+}(l)-v_2^{G-}(l)} & \text{if } v_2^{G-}(l) \le v_2^G(l) \le v_2^{G+}(l) \\ 1 & \text{if } v_2^G(l) \ge v_2^{G+}(l), \end{cases} \tag{3}$$

and

$$\mu_3(H_1) = \begin{cases} 0 & \text{if } H_1 \le H_1^- \\ \frac{H_1-H_1^-}{H_1^+-H_1^-} & \text{if } H_1^- \le H_1 \le H_1^+ \\ 1 & \text{if } H_1 \ge H_1^+ \end{cases} \tag{4}$$

where $v_1^{G+}(l), v_1^{G-}(l)$ $(l = 1,\dots,s)$ respectively, represent maximum and minimum values of $v_1^G(l)$ $(l = 1,\dots,s)$; $v_2^{G+}(l), v_2^{G-}(l)$ $(l = 1,\dots,s)$ respectively, represent maximum and minimum values of $v_2^G(l)$ $(l = 1,\dots,s)$ and H_1^+, H_1^- respectively, represent maximum and minimum values of H_1, for player PI.

To convert in a single objective non-linear model from multi-objective non-linear model, we have introduced the concept of fuzzy programming technique [i.e, using the equations (2),(3),(4)] and with the help of Model 2, then we have formulated the following single objective non-linear model and this model is denoted by Model 3 as

$$\begin{aligned} \text{Model 3} \quad \max \quad : \quad & \lambda \\ \text{subject to} \quad & \lambda \le \frac{v_1^G(l)-v_1^{G-}(l)}{v_1^{G+}(l)-v_1^{G-}(l)}, \quad l = 1,\dots,s \\ & \lambda \le \frac{v_2^G(l)-v_2^{G-}(l)}{v_2^{G+}(l)-v_2^{G-}(l)}, \quad l = 1,\dots,s \\ & \lambda \le \frac{H_1-H_1^-}{H_1^+-H_1^-} \\ & y^t A_G^l \ge [v_1^G(l),\dots,v_1^G(l)], \quad l = 1,\dots,s \\ & y^t B_G^l \ge [v_2^G(l)\dots,v_2^G(l)], \quad l = 1,\dots,s \\ & H_1 = -\sum_{i=1}^{m} y_i \; ln(y_i) \\ & \sum_{i=1}^{m} y_i = 1; \quad y_i \ge 0, \quad i = 1,2,\dots,m. \end{aligned}$$

Now to solve the Model 3, we have required the values of $v_1^{G+}(l), v_1^{G-}(l), v_2^{G+}(l), v_2^{G-}(l)$, $(l = 1,\dots,s)$ H_1^+, H_1^-. These values are determined by our proposed GA which is defined in the next subsection.

3.2 Genetic Algorithm(GA)

Now, we developed an algorithm for determining the $v_1^{G+}(l), v_2^{G-}(l)$, $(l = 1,\dots,s)$ and H_1^+, H_1^-. The following steps of GA are shown as follows:

Step 1: Initialize the parameters of GA of the proposed Entropy Bimatrix Goal Game model.
Step 2: $t = 0$ (t represents the number of current generation).
Step 3: Initialize $P(t)$ [$P(t)$ represents the population at the t-th generation].
Step 4: Evaluate $P(t)$.

Step 5: Find optimal result from $P(t)$.
Step 6: $t = t + 1$.
Step 7: If ($t >$ maximum generation number) go to Step 13.
Step 8: Alter $P(t)$ by mutation.
Step 9: Evaluate $P(t)$.
Step 10: Find optimal result from $P(t)$.
Step 11: Compare optimal results of $P(t)$ and $P(t-1)$ and store better one.
Step 12: Go to Step 6.
Step 13: Print optimal result.
Step 14: Stop.

To implement the above GA for the proposed model, the following basic components are considered: (i) Parameters of GA (ii) Chromosome Representation (iii) Initialization (iv) Evaluation Function (v) Selection Process (vi) Genetic Operators (crossover and mutation) which are defined as follows:

• Parameters of GA : GA depends on different parameters like population size(POPSIZE), probability of crossover(PCROS), probability of mutation(PMUTE) and maximum number of generation (MAXGEN). In this study, we have taken the value of these parameters as follows:
POPSIZE= 25 PCROS= 0 PMUTE=0.6 MAXGEN= 80

• Chromosome Representation
The chromosome is defined as $(y_1^a, y_2^a, y_3^a, \ldots, y_m^a)$ where $y_i^a \in Y, i = 1, 2, 3, \ldots, m$.

• Initialization
In this study; y_1^a, y_2^a, $\ldots, y_{m-1}^a, y_m^a$ are randomly given values and the chromosomes must satisfy that $y_1^a + y_2^a + y_3^a + \ldots + y_m^a = 1$. This process is randomly generating each element in $(y_1^a, y_2^a, y_3^a, \ldots, y_m^a)$ and $y_1^a + y_2^a + y_3^a + \ldots + y_m^a = 1$; Moreover the number of chromosome is limited to 25 when each new run begins.

• Evaluation function
Once $(y_1^a, y_2^a, y_3^a, \ldots, y_m^a)$ is determined, the corresponding $v_1^{Ga}(l), v_2^{Ga}(l)$, $(l = 1, 2, \ldots, s)$ can be computed by Model 1 and H_1^a can be computed by (1).

• Optimum 1
For 25 chromosomes, we get 25 set of values of $v_1^{Ga}(l), v_2^{Ga}(l)$, $(l = 1, 2, \ldots, s)$ and H_1^a. For each $l = 1, 2, \ldots, s$, among these 25 values of $v_1^{Ga}(l)$, we have stored maximum and minimum values in $v_1^{Ga+}(l)$ and $v_1^{Ga-}(l)$, respectively. Similarly, for each $l = 1, 2, \ldots, s$, among these values of $v_2^{Ga}(l), l = 1, 2, \ldots, s$ we have stored maximum and minimum values in $v_2^{Ga+}(l)$ and $v_2^{Ga-}(l)$, respectively. In each iteration, these maximum and minimum values are globally stored in $VMAX1(l), VMIN1(l), VMAX2(l), VMIN2(l)$, $(l = 1, 2, \ldots, s)$ respectively. Similarly, among 25 values of H_1^a we stored maximum value in H_1^{a+} and minimum value in H_1^{a-} and they are also globally stored in another locations $HMAX1$ and $HMIN1$ respectively, in each iteration.

• Selection
Selection procedure is omitted because here objectives are more than one so we can not choose the weaker chromosome that serve worst value for all objectives.

• Crossover
Since it is not easy to design a crossover between chromosomes for satisfying that $y_1^a + y_2^a + y_3^a + \ldots + y_m^a = 1$, therefore no crossover is applied in this study.

• Mutation
It is applied to single chromosome. It is designed as an order of elements in $(y_1^a, y_2^a, y_3^a, \ldots, y_m^a)$ by randomly determined cut-point. Consider an example: if the original chromosome is $(y_1^a, y_2^a, y_3^a, \ldots, y_m^a)$ and cut-point is randomly determined between the string: y_1^a and y_2^a, y_3^a,$\ldots, y_m^a$, then moreover newly mutated chromosome $(y_1^{'}, y_2^{'}, y_3^{'}, \ldots, y_m^{'})$ is $(y_2^a, y_3^a, \ldots, y_m^a, y_1^a)$.
In each iteration the (POPSIZE * PMUTE) number of chromosome are chosen for mutation.

• Iteration
The number of iteration is set to 80 runs, each of which begins with the different random seed.

• Optimum 2
After completing all the iterations, we determine $v_1^{G+}(l)$, $(l = 1, 2, \ldots, s)$ as the maximum among all $VMAX1(l), l = 1, 2, \ldots, s$ and $v_1^{G-}(l)$, $(l = 1, 2, \ldots, s)$ as the minimum among all $VMIN1(l)$, $(l = 1, 2, \ldots, s)$. Similarly, we determine $v_2^{G+}(l)$, $(l = 1, 2, \ldots, s)$ as the maximum among all $VMAX2(l), l = 1, 2, \ldots, s$ and $v_2^{G-}(l)$, $(l = 1, 2, \ldots, s)$ as the minimum among all $VMIN2(l)$, $(l = 1, 2, \ldots, s)$. Also, H_1^+ is the maximum among all $HMAX1$ and H_1^- is the minimum among all $HMIN1$ are determined.

4 Numerical Example

Example : Consider the following bimatrix game

$$A(1) = \begin{bmatrix} 7 & 4 & 3 \\ 5 & 6 & 2 \\ 2 & 5 & 4 \end{bmatrix}, \quad B(1) = \begin{bmatrix} 6 & 3 & 5 \\ 7 & 6 & 2 \\ 3 & 8 & 4 \end{bmatrix}, \tag{5}$$

$$A(2) = \begin{bmatrix} 7 & 4 & 3 \\ 3 & 6 & 2 \\ 5 & 3 & 3 \end{bmatrix}, \quad B(2) = \begin{bmatrix} 6 & 4 & 3 \\ 3 & 2 & 7 \\ 5 & 3 & 8 \end{bmatrix}. \tag{6}$$

Let $G_1^1 = 5, G_1^2 = 4, G_2^1 = 4, G_2^2 = 5$, be the goals specified by PI. Then

$$A_G^1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B_G^1 = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \tag{7}$$

$$A_G^2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad B_G^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}. \tag{8}$$

The maximum and minimum values of $v_1^G(l)$ and $v_2^G(l)$ $(l = 1, 2)$ are summarized in Table 1 which is computed by genetic algorithm as follows:

Table 1: Maximum and minimun values of objectives

	maximum value	*minimum value*
$v_1^G(1)$	$v_1^{G+}(1) = 0.94715$	$v_1^{G-}(1) = 0.0238$
$v_1^G(2)$	$v_1^{G+}(2) = 0.995$	$v_1^{G-}(2) = 0.5092$
$v_2^G(1)$	$v_2^{G+}(1) = 0.6459$	$v_2^{G-}(1) = 0.0238$
$v_2^G(2)$	$v_2^{G+}(2) = 0.4908$	$v_2^{G-}(2) = 0.005$
H_1	$H_1^+ = 1.097652$	$H_1^- = 0.126435$

With the help of above values from Table-1 and Model 3, we have formulated the model (Model 4) as follows:

$$\begin{aligned} \text{Model 4} \quad \max \quad &: \lambda \\ \text{subject to} \quad & \lambda \leq \frac{v_1^G(1) - 0.0238}{0.94715 - 0.0238} \\ & \lambda \leq \frac{v_1^G(2) - 0.5092}{0.995 - 0.5092} \\ & \lambda \leq \frac{v_2^G(1) - 0.0238}{0.6459 - 0.0238} \\ & \lambda \leq \frac{v_2^G(2) - 0.005}{0.4908 - 0.005} \\ & \lambda \leq \frac{H_1 - 0.126435}{1.097652 - 0.126435} \\ & y^t A_G^l \geq [v_1^G(l), \ldots, v_1^G(l)], \quad l = 1, 2, 3 \\ & y^t B_G^l \geq [v_2^G(l) \ldots, v_2^G(l)], \quad l = 1, 2, 3 \\ & H_1 = -\sum_{i=1}^{3} y_i \; ln(y_i) \\ & \sum_{i=1}^{3} y_i = 1; \quad y_i \geq 0, \quad i = 1, 2, 3. \end{aligned}$$

The aspiration level λ^* with an objective is determined from Model 4 by the help of Lingo package and the efficient solution is represented in the following Table 2.

Table 2: Results of Model 4

aspiration level	*efficient solution*		
	probability of goal	*entropy*	*GGSS*
$\lambda^* = 0.3924114$	$v_1^{G*}(1) = 0.6003331$, $v_1^{G*}(2) = 0.6998335$, $v_2^{G*}(1) = 0.4821191$, $v_2^{G*}(2) = 0.1956335$	$H_1^* = 1.088996$	$y^* = (0.3996670, 0.3001665, 0.3001665)$

Thus when player PI is interested to maximize the measure of uncertainty to apply his/her strategies then it is seen that if PI plays his/her strategy $(0.399667, 0.3001665, 0.3001665)$ then he/she gets at least $G_1 = 5$ with a probability 0.6003331 and at least $G_2 = 4$ with a probability 0.6998335 in first criteria. And for second criteria PI plays his/her strategy $(0.3996669, 0.3001665, 0.3001665)$ then he/she gets at least $G_1 = 5$ with a probability 0.4821191 and at least $G_2 = 4$ with a probability 0.1956335 With the help of Table 1, we considered the objectives without entropy function in Model 3 and we formulated the Model 5 which is as follows:

$$\begin{aligned}
\text{Model 5} \quad \max \quad : \quad & \delta \\
\text{subject to} \quad & \delta \leq \frac{v_1^G - 0.000153}{0.5000 - 0.000153} \\
& \delta \leq \frac{v_2^G - 0.000153}{0.5000 - 0.000153} \\
& y_1 + y_2 \geq v_1^G \\
& y_1 + y_2 + y_3 \geq v_1^G \\
& y_1 \geq v_2^G \\
& y_1 + y_2 + y_3 \geq v_2^G \\
& y_3 \geq v_2^G \\
& \sum_{i=1}^{3} y_i = 1; \quad y_i \geq 0, \quad i = 1, 2, 3.
\end{aligned}$$

The aspiration level δ^* for the objective is determined from Model 5 by the help of Lingo package and the efficient solution is represented in the following Table 3.

Table 3: Results of Model 5

aspiration level	*efficient solution*	
	probability of goal	*GGSS*
$\delta^* = 0.3924114$	$v_1^{G*}(1) = 0.6003331$, $v_1^{G*}(2) = 0.6998335$, $v_2^{G*}(1) = 0.4821191$, $v_2^{G*}(2) = 0.1956335$	$y^* = (0.399667, 0.3001665, 0.3001665)$

Thus when player PI is interested to maximize the measure of uncertainty with applying his/her strategies then it is seen that PI plays his/her strategy $(0.399667, 0.3001665, 0.3001665)$ and he/she gets at least $G_1 = 5$ with a probability 0.6003331 and at least $G_2 = 4$ with a probability 0.6998335 in first criteria. And for second criteria PI plays his/her strategy $(0.3996669, 0.3001665, 0.3001665)$ then he/she gets at least $G_1 = 5$ with a probability 0.4821191 and at least $G_2 = 4$ with a probability 0.1956335.

From Tables 2 and 3 for both the models (Models 4 and 5) have identical results.

5 Conclusion

This paper presented the study of multicriteria bimatrix goal game under the light of entropy environment. In practical problems, when conflict situations are more complicated like players have many criterions, then the single criteria entropy bimatrix goal game model is not applicable. In this situation, multicriteria entropy bimatrix goal game model is highly applicable to handle the problem. Using the goal, we considered the solution not only the strategy played by the players, but also the probabilities of getting at least goal values of the players. Therefore, with this approach, each player has gained the information about the probability of achieving the possible outcomes of the multicriteria entropy bimatrix goal game.

To obtain the GGSS, we applied the fuzzy based genetic algorithm to multicriteria entropy bimatrix goal game model. We have shown that all these strategies together with their associated probabilities can be obtained as an efficient solution of a particular non-linear model. The model with entropy is highly significant related to the real-life practical problem on multicriteria entropy bimatrix goal game.

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