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Optimization of Insuring Critical Path Problem with Uncertain Activity Duration Times

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Abstract

In fuzzy decision systems, we model insuring critical path problem by two-stage risk-aversion fuzzy programming. For the proposed two-stage insuring critical path problem, we first define three solution concepts, expected value solution, wait-and-see solution and here-and-now solution, then introduce the value of perfect information of value-at-risk (VaRVPI) and the value of the fuzzy solution of value-at-risk (VaRVFS). When activity duration times are continuous fuzzy variables, we adopt fuzzy simulation to turn the original insuring critical path problem into its approximate optimization problem. After that, we solve the approximate insuring critical path problem by hybrid algorithm, which combines dynamic programming method and genetic algorithm (GA). Finally, we report some computational results to illustrate the credibility of the developed modeling idea and the effectiveness of the designed algorithm.

Keywords: critical path, fuzzy programming, fuzzy simulation, genetic algorithm

1 Introduction

Due to the uncertainty involved in a complex decision system, the stochastic insuring critical paths problem has been studied widely in the field of critical path management. For example, Moehring [16] minimized the costs that a project would be completed by a given deadline if the duration for each activity is under uncertainty; Bowman [4] and Mitchell [15] treated mass uncertain information by heuristic-based and Monte Carlo simulation-based techniques, Shen [18] considered a class of two-stage stochastic optimization problems for insuring critical path and provided decomposition strategies to solve this problem, and Li et al. [8] developed a new risk averse two-stage stochastic insuring critical path problem, in which the minimum risk criterion was adopted in the first-stage objective function. On the other hand, Prade [17] first used fuzzy set theory to the project scheduling problem; Chen and Hsueh [5] developed a simple approach to critical path analysis in a project network with activity times being fuzzy numbers; Zammori [20] took additional critical fuzzy logic parameters into the factors for determining the critical path; Amiri and Golozari [1] introduced an algorithm which considers not only time factor but also cost, risk, and quality criteria to determine the critical path under fuzzy environment, and Zareei [21] gave a new approach for solving fuzzy critical path problem considering fuzzy duration.

In the current development, we study the insuring critical path problem by two-stage fuzzy optimization theory [12]. For this purpose, we develop a class of two-stage fuzzy insuring critical path models. In the proposed model, the first constraint function of the first-stage imposes the credibility of fuzzy event is more than the predetermined service level requirement. In the case that the uncertain activity duration times have continuous possibility distributions, our insuring critical path problem above become difficult to solve. To avoid this difficulty, we apply approximate approach (AA) [10] to uncertain activity duration times. The approximating insuring critical path problem is an integer programming problem with a number of logic constraints, which belongs to the class of NP-hard problems. In addition, the analytical expression of first-stage penalty function is unavailable, we cannot solve our insuring critical path problem by conventional optimization method or general purpose software. To overcome this difficulty, we employ a hybrid GA to compute critical path problem, and provide some numerical experiments to illustrate the efficiency of the method.

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The rest of paper is organized as follows. In Section 2, we present a new two-stage insuring critical path problem. Section 3 defines two fuzzy measures VaRVPI and VaRVFS for the proposed insuring critical path problem. In Section 4, we transform the original problem into its approximate programming model. In Section 5, a hybrid GA can be used to solve the approximate problem, in which the dynamic programming algorithm is used to find the critical path. In Section 6, we provide some numerical experiments to illustrate the efficiency of the designed algorithm. Finally, Section 7 draws our conclusions.

2 Formulation of Fuzzy Insuring Critical Path Problem

Generally, a complex project can be described by a directed graph. Let $G(N, A)$ be a directed graph representing a project, where $N = \{0, 1, \ldots, n\}$ is the set of nodes in the network, A is the set of arcs representing the activities of the project, and $(i, j) \in A$ is the arc from node i to node j.

For convenience, we denote fuzzy activity duration times vector in the project as $\xi = (d_{ij}(\gamma), g_{ij}(\gamma))$, where $d_{ij}(\gamma)$ is fuzzy uninsured task duration time from node i to node j, and $g_{ij}(\gamma)$ is fuzzy insured task duration time from node i to node j. In addition, we use $FS(i) = \{j|(i, j) \in A\}$ and $RS(i) = \{j|(j, i) \in A\}$ to represent the set of nodes adjacent from node i and adjacent to node i, respectively. $\sum_{(i,j)\in A} c_{ij}x_{ij}$ is the total cost of insuring arcs, where c_{ij} is the cost of insuring arc (i, j), and x_{ij} is 1 if arc (i, j) is insured, and 0 otherwise. $\Theta(Q(x, \xi))$ is the cost of penalizing, where Θ is the nondecreasing penalty function, $Q(x, \xi)$ is the critical path length, and x is a decision vector. The following constraint

$$\mathrm{Cr}\left\{\sum_{(i,j)\in A} c_{ij}x_{ij} \;+\; \Theta(Q(x, \xi(\gamma))) \le \varphi\right\} \ge 1 - \alpha$$

imposes the credibility of fuzzy event is more than the predetermined service level requirement.

Based on notations above, we construct the following two-stage fuzzy insuring critical path model

$$\begin{cases} \min & \varphi \\ \text{s.t.:} & \mathrm{Cr}\left\{\sum\limits_{(i,j)\in A} c_{ij}x_{ij} \;+\; \Theta(Q(x, \xi(\gamma))) \le \varphi\right\} \ge 1 - \alpha \\ & x_{ij} \in \{0, 1\}, \forall (i, j) \in A, \end{cases} \tag{1}$$

where $Q(x, \xi(\gamma))$ is the optimal value of the following programming model

$$\begin{cases} \max & \sum\limits_{(i,j)\in A} (d_{ij}(\gamma) - (d_{ij}(\gamma) - g_{ij}(\gamma))x_{ij})y_{ij} \\ \text{s.t.:} & \sum\limits_{j\in FS(0)} y_{0j} = 1 \\ & \sum\limits_{j\in FS(i)} y_{ij} - \sum\limits_{l\in RS(i)} y_{li} = 0, \forall i \in N\backslash\{0, n\} \\ & y_{ij} \in \{0, 1\}, \forall (i, j) \in A. \end{cases} \tag{2}$$

The objective of problem (2) is to maximize the sum of the insured task durations, where y_{ij} is equal to 1 if arc (i, j) is part of one identified critical path, and 0 otherwise. In addition, the first constraint imposes the single-assignment rule; the second constraint enforces flow-balance for critical path contiguity, and the third constraint bounds a binary decision variable.

Problem (1) is a two-stage purely integer programming. In addition, the analytical expression of Θ is unavailable, so it is difficult to compute the credibility constraint function in the first-stage. On the other hand, the second-stage problem (2) is a 0–1 integer programming, given first-stage decision x, we need to solve the second-stage an infinite number of times. As a consequence, when activity duration times have continuous possibility distribution, problem (1)–(2) becomes difficult to solve, we will employ the AA, dynamic programming method and genetic algorithm to find the optimal solutions. This issue will be addressed in the next section.

3 The VaRVPI and VaRVFS for Insuring Critical Path Problems

The measures about the expected value of perfect information ($EVPI$) and the value of the fuzzy solution (VFS) are based on the expected values, and they usually evaluate the risk-neutral fuzzy programming models [9, 19]. These measures cannot directly argue whether it is worth to solve our two-stage fuzzy insuring critical path problem. Therefore, we adapt these measures to risk-aversion fuzzy programming [11], and define some new measures to quantify the effect of fuzzy insuring critical path problem.

Let

$$f(x, \xi(\gamma)) = \sum_{(i,j)\in A} c_{ij}x_{ij} \ + \ \Theta(Q(x, \xi(\gamma))).$$

Then the wait-and-see solution is

$$VaRWS = \min \left\{ \varphi \mid \mathrm{Cr}\{\min_{x\in X} f(x, \xi(\gamma)) \le \varphi\} \ge 1 - \alpha \right\},$$

the here-and-now solution is

$$VaRRP = \min_{x\in X} \left\{ \varphi \mid \mathrm{Cr}\{f(x, \xi(\gamma)) \le \varphi\} \ge 1 - \alpha \right\},$$

and the expected value problem or mean value problem is

$$VaREV = \min \left\{ \varphi \mid \mathrm{Cr}\{f(\bar{x}(\bar{\xi}), \xi(\gamma)) \le \varphi\} \ge 1 - \alpha \right\},$$

where $\bar{\xi} = E(\xi)$ denotes the expected value of fuzzy vector ξ and $\bar{x}(\bar{\xi})$ is an optimal solution of $\min_{x\in X} f(x, \bar{\xi})$.

Based on the three solution concepts, two important indexes are introduced as follows.

Definition 1 *The expected value of perfect information of value-at-risk is defined as the difference between the wait-and-see and the here-and-now solution, namely,*

$$VaRVPI = VaRRP - VaRWS.$$

The value $VaRVPI$ measures the gain value-at-risk of perfect information based on the the credibility function. Thus, it quantifies the value of information based on the specified risk preferences.

For practical purposes, finding the wait-and-see solution or equivalently solving the distribution problem is still difficult to implement, so some people replace all fuzzy variables by their expected values.

Definition 2 *The value of the fuzzy solution for problem (1)–(2) is defined as*

$$VaRVFS = VaREV - VaRRP.$$

This quantity is the cost of ignoring information in choosing a decision. It measures the possible gain from solving the fuzzy insuring critical path model. Note that the high values of $VaRVFS$ would indicate the significant improvement in the solution quality by solving the two-stage fuzzy insuring critical path model.

In the following, we prove some basic relations between $VaRWS$, $VaRRP$ and $VaREV$, and state them in the following theorem.

Theorem 1 *For any two-stage fuzzy insuring critical path programming problem (1)–(2), the following relations hold true*

$$VaRWS \le VaRRP \le VaREV.$$

Thus, $VaRVPI \ge 0$ *and* $VaRVFS \ge 0$*.*

Proof. For every realization ξ, we have the relation

$$\min_{x\in X} f(x, \xi) \le f(x, \xi), \forall x.$$

Taking the possibility of both sides yields the following inequalities

$$\mathrm{Pos}\{\min_{x\in X} f(x, \xi) \le \varphi\} \ge \mathrm{Pos}\{f(x, \xi) \le \varphi\}, \forall x,$$

$$\mathrm{Pos}\{\min_{x\in X} f(x,\xi) > \varphi\} \leq \mathrm{Pos}\{f(x,\xi) > \varphi\}, \forall x.$$

From the definition of credibility measure, we get

$$\mathrm{Cr}\{\min_{x\in X} f(x,\xi) \leq \varphi\} \geq \mathrm{Cr}\{f(x,\xi) \leq \varphi\}, \forall x.$$

Furthermore, we obtain the first inequality

$$\min\left\{\varphi \mid \mathrm{Cr}\{\min_{x\in X} f(x,\xi(\gamma)) \leq \varphi\} \geq 1-\alpha\right\} \leq \min_{x\in X}\left\{\varphi \mid \mathrm{Cr}\{f(x,\xi(\gamma)) \leq \varphi\} \geq 1-\alpha\right\}.$$

i.e.,

$$VaRWS \leq VaRRP.$$

In addition, for every x, $\bar{x}(\bar{\xi})$ is just a feasible solution of the two-stage fuzzy insuring critical path problem. Therefore, the following inequality

$$VaRRP \leq VaREV$$

is true. The proof of the theorem is complete.

4 The Approximate Insuring Critical Path Model

In our problem, the fuzzy uninsured task duration times $d_{ij}(\gamma)$ are different for every arc (i,j), but they have some relations. The fuzzy insured task duration times $g_{ij}(\gamma)$ are related with the parameter $d_{ij}(\gamma)$. The parameter $d_{ij}(\gamma)$ can be described by the fuzzy parameter γ_1, and γ_2, i.e., $d_{ij}(\gamma) = o_{ij}\gamma_1 + p_{ij}\gamma_2 + q_{ij}$, where o_{ij}, p_{ij}, and q_{ij} are real numbers and they can be seen as weather, risk, and quality, respectively. The parameter $g_{ij}(\gamma)$ can be written similarly as $g_{ij}(\gamma) = r_{ij}\gamma_1 + s_{ij}\gamma_2 + t_{ij}$.

Now all the fuzzy variables turn into the functions of (γ_1,γ_2), we only need to deal with two fuzzy variables. We denote the two variables as a fuzzy vector and $\gamma = (\gamma_1,\gamma_2)$ has a triangular possibility distribution. When they are mutually independent fuzzy variables, we employ the AA [10] to approximate the original insuring critical path problem and refer to a sequence $\{\zeta_m\}$ of discrete fuzzy vector as the discretization of the fuzzy vector γ as follows.

For each integer m, we define the discrete fuzzy vector $\zeta_m = (\zeta_{m,1},\zeta_{m,2})$ as follows

$$\zeta_m = h_m(\gamma) = (h_{m,1}(\gamma_1), h_{m,2}(\gamma_2)),$$

where the fuzzy variables $\zeta_{m,1} = h_{m,1}(\gamma_1)$, $\zeta_{m,2} = h_{m,2}(\gamma_2)$,

$$h_{m,i}(u) = \begin{cases} a_i, & u_i \in [a_i, a_i+\frac{1}{m}], \\ \sup\{\frac{k_i}{m} \mid k_i \in Z \text{ such that } \frac{k_i}{m} \leq u\}, & u_i \in [a_i+\frac{1}{m}, b_i], \end{cases}$$

and Z is the set of all integer.

Let $v_{m,1}$ and $v_{m,2}$ denote the possibility distribution of the fuzzy variable $\gamma_{1,m}$ and $\gamma_{2,m}$, respectively. Then we have

$$v_{m,1}(\frac{k_1}{m}) = \mathrm{Pos}\{\frac{k_1}{m} \leq \gamma_{1,m} < \frac{k_1+1}{m}\},$$
$$v_{m,2}(\frac{k_2}{m}) = \mathrm{Pos}\{\frac{k_2}{m} \leq \gamma_{2,m} < \frac{k_2+1}{m}\}.$$

When they are mutually independent triangular fuzzy variables, we can obtain the possibility distribution of the fuzzy vector ζ_m as follows

$$\begin{aligned} v_m(\tfrac{k_1}{m},\tfrac{k_2}{m}) &= \mathrm{Pos}\{\tfrac{k_1}{m} \leq \gamma_{1,m} < \tfrac{k_1+1}{m}, \tfrac{k_2}{m} \leq \gamma_{2,m} < \tfrac{k_2+1}{m}\} \\ &= v_{m,1}(\tfrac{k_1}{m}) \bigwedge v_{m,2}(\tfrac{k_2}{m}). \end{aligned}$$

Now we deal with the following credibility level constraints

$$\mathrm{Cr}\left\{\sum_{(i,j)\in A} c_{ij}x_{ij} \ + \ \Theta(Q(x,\xi(\gamma))) \leq \varphi\right\} \geq 1-\alpha, \forall (i,j) \in A.$$

First, assume $(h_{m,1}(\gamma_1), h_{m,2}(\gamma_2))$ has the following discrete possibility distribution

$$\begin{pmatrix} (\hat{\gamma}^1_{1,m}, \hat{\gamma}^1_{2,m}) & (\hat{\gamma}^2_{1,m}, \hat{\gamma}^2_{2,m}) & \cdots & (\hat{\gamma}^K_{1,m}, \hat{\gamma}^K_{2,m}) \\ v^1_m & v^2_m & \cdots & v^K_m \end{pmatrix}.$$

Then possibility distribution of fuzzy vector $\xi = (d_{ij}, g_{ij}), \forall (i,j) \in A$ is as follows

$$\begin{pmatrix} (\hat{d}^1_{ij,m}, \hat{g}^1_{ij,m}) & (\hat{d}^2_{ij,m}, \hat{g}^2_{ij,m}) & \cdots & (\hat{d}^K_{ij,m}, \hat{g}^K_{ij,m}) \\ v^1_m & v^2_m & \cdots & v^K_m \end{pmatrix},$$

where $v^k_m \geq 0$, and $\max\limits_{1\leq k\leq K} v^k_m = 1, k = 1, 2, \ldots, K$.

By introducing a "big enough" constant M, the credibility level constraints can be turned to

$$\sum_{(i,j)\in A} c_{ij}x_{ij} \ + \ \Theta(Q(x, \hat{\zeta}^k_m(\gamma))) - M \leq \varphi, k = 1, 2, \ldots, K, \forall (i,j) \in A.$$

In addition, we introduce a binary vector z whose components $z_k, k = 1, 2, \ldots, K$, take 0 if the corresponding set of constraints has to be satisfied and 1 otherwise.

According to the definition of credibility measure [13], the credibility level constraints can be equivalently rewritten as

$$\begin{gathered} \sum_{(i,j)\in A} c_{ij}x_{ij} \ + \ \Theta(Q(x, \hat{\zeta}^k_m(\gamma))) - Mz_k \leq \varphi, k = 1, 2, \ldots, K, \forall (i,j) \in A, \\ \max_{1\leq k\leq K} v^k_m(1 - z_k) - \max_{1\leq k\leq K} v^k_m z_k \geq 1 - 2\alpha, k = 1, 2, \ldots, K, \\ z_k \in \{0, 1\}, k = 1, 2, \ldots, K, \forall (i,j) \in A. \end{gathered} \tag{3}$$

If the credibility level $0 < 1 - \alpha \leq 0.5$, then inequality (3) is equivalent to

$$\max_{1\leq k\leq K} v^k_m(1 - z_k) \geq 2(1 - \alpha), k = 1, 2, \ldots, K,$$

If the credibility level $0.5 < 1 - \alpha \leq 1$, then inequality (3) can be turned to the following equivalent form

$$\max_{1\leq k\leq K} v^k_m z_k \leq 2\alpha, k = 1, 2, \ldots, K.$$

As a consequence, when $0.5 < 1 - \alpha \leq 1$, the original problem is approximated by the following programming problem

$$\begin{cases} \min & \varphi \\ \text{s.t.:} & \sum\limits_{(i,j)\in A} c_{ij}x_{ij} \ + \ \Theta(Q(x, \hat{\zeta}^k_m(\gamma))) - Mz_k \leq \varphi, k = 1, 2, \ldots, K \\ & \max\limits_{1\leq k\leq K} v^k_m z_k \leq 2\alpha, k = 1, 2, \ldots, K, \\ & z^k \in \{0, 1\}, k = 1, 2, \ldots, K \\ & x_{ij} \in \{0, 1\}, \forall (i,j) \in A, \end{cases} \tag{4}$$

where $Q(x, \hat{\zeta}^k_m(\gamma))$ is the optimal value of the following programming problem:

$$\begin{cases} \max & \sum\limits_{(i,j)\in A} (d^k_{ij,m}(\gamma) - (d^k_{ij,m}(\gamma) - g^k_{ij,m}(\gamma))x_{ij})y_{ij} \\ \text{s.t.:} & \sum\limits_{j\in FS(0)} y_{0j} = 1 \\ & \sum\limits_{j\in FS(i)} y_{ij} - \sum\limits_{l\in RS(i)} y_{li} = 0, \forall i \in N\backslash\{0, n\} \\ & y_{ij} \in \{0, 1\}, \forall (i,j) \in A. \end{cases} \tag{5}$$

Now we turn the original infinite-dimensional optimization problem (1)–(2) into an approximating finite-dimensional one. The convergence of the AA is stated in the following theorem.

Theorem 2 *Consider fuzzy insuring critical path problem (1)–(2). Suppose the activity times ξ is a continuous and bounded fuzzy vector, and the sequence $\{\zeta_m\}$ of fuzzy vector is the discretization of ξ, then for every feasible solution x, we have*

$$\lim_{m\to\infty} \Theta(Q(x,\zeta_m)) = \Theta(Q(x,\xi)).$$

Proof. Since for any feasible solution x and every realization $\xi(\gamma)$ of the fuzzy activity times ξ, the functions $Q(x,\xi)$, which together with the suppositions of the theorem satisfy the condition of Theorem 2 in Ref. [10], which proves the assertion of the theorem.

The approximate problem (4)–(5) is an integer programming model with logic constraints, which belongs to the class of NP-hard problems. In addition, the analytical expression of the first-stage penalty function $\Theta(Q(x,\hat{\zeta}_m^k(\gamma)))$ is unavailable, we cannot solve the two-stage problem by conventional optimization method or general purpose software. In the next section, we will design a heuristic solution method to solve the approximate problem (4)–(5).

5 Solution Method

5.1 Computing Critical Paths

To solve the approximate critical path model, it is required to compute critical path effectively. According to the characteristics of network structure and the optimality principle of dynamic programming [3], we employ the following calculation formula to update the longest path of the network

$$f(j) = \max_{(i,j)\in A}\{f(i) + (d[i][j] - (d[i][j] - g[i][j])x[i][j])\}, \tag{6}$$

where $f(j)$ represents the longest path of node j from the project starting point, $f(i)$ represents the longest path of node i from the project starting point, $d[i][j]$ represents an uninsured task duration of arc $(i,j)\in A$, $g[i][j]$ represents an insured task duration of arc $(i,j)\in A$, and $x[i][j]$ represents whether arc (i,j) is insured, i.e., 1 if arc (i,j) is insured, and 0 otherwise.

5.2 A Hybrid Genetic Algorithm

The GA was proposed by Holland [6] in 1975. Since the GA does not require the specific mathematical analysis of optimization problems, it is more likely to find the global optimal solutions of complex optimization problem. In recent years, the GA has been well developed and has led to some of the most successful applications in the literature [2, 7, 14]. We use the dynamic programming method to compute critical paths in the second-stage programming, and embed the method into a GA to produce a hybrid solution method. For the sake of clarity, we briefly discuss the solution representation, initialization, crossover process and mutation process in our hybrid GA.

Solution Representation. In our algorithm, a solution $x = (x_{ij})_{1\times m}$ is represented by the chromosome $C = (ch_{ij})_{1\times m}$, where each gene ch_{ij} is restricted in 0s and 1s, where 1 represents that arc (i,j) is insured, and 0 represents that arc (i,j) is not insured. This representation indicates that infeasibility cannot occur, and each new chromosome remains a feasible solution to problem (4).

Initialization. For the first-stage of the approximate model (4), we randomly generate a point $C = (ch_{ij})_{1\times m}$ from the $\{0,1\}^m$. If the point is feasible, then we take it as an initial chromosome. From the set of arcs A, the InsuredArray is randomly generated for the initial population generation step. One number is randomly generated from $(0,1)$, if it is bigger than 0.5 or equal to 0.5, then we set it to 1; otherwise, we set it to 0. Using this method, we can obtain initial feasible chromosomes. Repeating this process *pop_size* times, the *pop_size* feasible chromosomes $C_1, C_2, \ldots, C_{pop\_size}$ can be produced.

Crossover Process. In crossover operation, a probability parameter P_c for the selection of parents to crossover is predetermined. We randomly generate a real number from $(0,1)$, if it is less than P_c at the kth selection, then the chromosomes C_k is selected as a parent. We apply single-point crossover operation on InsuredArray of the input strings by considering the same crossover point selected at random. The offsprings are generated by combining the left and the right parts.

Mutation Process. In mutation operator, we predetermine a parameter $P_m \in (0,1)$ to represent the probability of mutation for the InsuredArray. We randomly generate a number from $(0,1)$. If the number is bigger than P_m, then the operation is performed. Next we randomly generate an integer l from $(1, pop\_size-1)$, then the following three gene values from the lth gene value to the $(l+2)th$ gene value are changed, i.e., 0 is replaced with 1, and 1 is replaced with 0.

Based on the discussion above, the process of the hybrid GA for solving the approximate critical path problem (4)–(5) is as follows.

Algorithm 1 (Hybrid GA)

Step 0: Set parameters P_c and P_m.

Step 1: Initialize *pop_size* feasible chromosomes.

Step 2: Solve the second-stage model (5) according to formula (6).

Step 3: Compute the fitness of each chromosome.

Step 4: Select the chromosomes by spinning the roulette wheel. The selection process is fitness-proportional.

Step 5: Update the chromosomes by crossover and mutation operations.

Step 6: Repeat Step 2 to Step 5 for a given number of cycles.

Step 7: Report the best chromosome as the approximate optimal solution.

6 Computational Results

In this section, we consider a project for fuzzy insuring critical path problem as shown in Figure 1. The triangular fuzzy uninsured task duration times and the triangular fuzzy insured task duration times are given in Table 1, where $\gamma_1 = (100, 200, 300), \gamma_2 = (79, 109, 139)$ are mutually independent triangular fuzzy variables. We employ the AA with 1000 sample points. For each arc $(i, j) \in A$, we generate the cost c_{ij} to insure arc $(i, j) \in A$ from a uniform distribution over the interval [80,100]. Finally, we round the values of c_{ij} to the nearest integer values. In addition, let $M = 10^6$, and we suppose that the penalty function in our model is defined as

$$\Theta(t) = \begin{cases} 0, & 0 < t \le 1000, \\ 400 + \sqrt{t - 1000}, & 1000 < t \le 1150, \\ 500 + (t - 1150), & 1150 < t \le 1250, \\ 700 + (t - 1250)^2, & t > 1250. \end{cases}$$

Table 1: Fuzzy duration times of tasks

Arc	d_{ij}	g_{ij}	Arc	d_{ij}	g_{ij}
(0,1)	(100,200,300)	(80,180,280)	(4,7)	(25, 75,125)	(30,70,110)
(0,2)	(200,400,600)	(180,380,580)	(4,8)	(100,200,300)	(80,180,280)
(0,3)	(158,218,278)	(160,220,280)	(5,8)	(129,209,289)	(80,180,280)
(1,4)	(160,360,560)	(155,355,555)	(6,8)	(79,109,139)	(80,110,140)
(2,4)	(80,180,280)	(115,215,315)	(6,9)	(75,175,275)	(80,180,280)
(2,5)	(50,100,150)	(47,97,147)	(7,10)	(179,309,439)	(178,308,438)
(2,6)	(75,140,215)	(79,109,139)	(8,10)	(75,150,225)	(80,180,280)
(3,6)	(170,270,370)	(150,280,410)	(9,10)	(158,218,278)	(130,190,250)

Our hybrid GA was coded in the C++ programming language. The numerical experiments were carried out on a personal computer (Lenovo with Intel Pentium(R) Dual-Core E5700 3.00GHZ CPU and RAM 2.00GB), using the Microsoft Windows 7 operating system. For different parameters population size *pop_size*, crossover probability P_c, mutation probability P_m, and the number of generation Gen, the computational results of the total costs are reported in Table 2. To compare the results of objective values, we define the relative error as

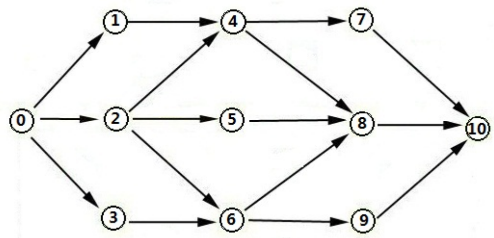

Figure 1: A project

(optimal value-actual value)/optimal value $\times 100\%$. It can be seen from Table 2 that the relative errors do not exceed 5% when we set different parameters, which implies that the hybrid GA is robust to parameters and effective for solving problem (4)–(5).

Furthermore, for $\alpha = 0.8$, we obtain the computational results $VaRWS = 290$, and $VaREV = 900$. Then $VaRVPI = 716 - 290 = 426$, $VaRVFS = 900 - 716 = 184$. That is, for our insuring critical path problem, the value of perfect information is 426, and the gain from solving our insuring problem is 184.

Table 2: Computational results for numerical experiments($\alpha = 0.8, Gen = 1000$)

pop_size	P_c	P_m	Objective value	Relative error(%)
30	0.2	0.4	716	0
30	0.4	0.2	736	2.23%
35	0.1	0.4	749	4.61%
35	0.2	0.2	725	1.26%
40	0.1	0.2	725	1.26%
40	0.3	0.4	736	2.23%
45	0.3	0.3	736	2.23%
45	0.4	0.2	724	1.12%
50	0.3	0.3	748	4.47%
50	0.3	0.4	716	0

7 Conclusions

On the basis of two-stage fuzzy optimization theory, we have studied the insuring critical path problem. The major new results of the paper are summarized as follows.

(i) We presented a two-stage fuzzy insuring critical path problem, in which we adopted the credibility criterion and a penalty function in the objective of the first-stage, and the penalty function is nondecreasing and may be convex or nonconvex. In addition, the task durations in the second-stage problem are characterized by continuous fuzzy variables.

(ii) We introduced two fuzzy measures VaRVPI and VaRVFS for the proposed fuzzy insuring critical path problem.

(iii) For general task duration distributions, we used the AA to transform the original fuzzy insuring critical path problem into its approximate programming problem. Furthermore, we designed a hybrid GA to solve the approximate critical path problem.

(iv) To demonstrate the developed modeling idea and the effectiveness of our hybrid GA, a number of numerical experiments has been performed. The computational results showed that the designed solution method is feasible and effective.

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