Journal of Uncertain Systems
Vol.7, No.1, pp.13-21, 2013
Online at: www.jus.org.uk

Fuzzy Multi-objective Optimization of a Synthesis Unit Utilizing Uncertain Data

Harish Garg*, S.P. Sharma
Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee 247667, India

Received 8 June 2011; Revised 6 May 2012

Abstract

In any manufacturing system, the cost is considered to be the most significant factor and the reliability of any industrial system is also an increasingly important issue. On the other hand the available information about the constituent components of the system is most of the time imprecise, incomplete, vague and conflicting and the management decisions are based on experience. So it is very difficult to analyze their behavior and to predict their failure pattern. In such situations the decision making is difficult and the presence of multi-objectives gives rise to multi-objective optimization problem which leads to Pareto optimal solutions instead of single optimal solution. The objective of this paper is to quantify the uncertainties that makes the decision more realistic, generic and extendable to application domain. This paper deals the fuzzy multi-objective reliability optimization problem for synthesis unit of a complex configuration urea fertilizer plant where reliability and cost of the system are involved as conflicting objectives. By taking into account the DM/system expert preferences in the form of weights and the objectives' membership functions, the problem is formulated as a single objective optimization problem and then solved with the particle swarm optimization.

Keywords: uncertain data, fuzzy optimization, reliability, particle swarm optimization

1 Introduction

Reliability is one of the vital attributes of performance in arriving at the optimal design of a system because it directly and significantly influences the system's performance. Conventionally reliability is defined as the probability that a component will not fail to perform within specified limits in a given time while working in a stated environment. In traditional reliability analysis, the survivor probabilities of the components of a system are considered as exact values. That is, that every probability involved is perfectly determinable. But this precision is not true in the real system while there also exits some uncertainty in the value of the survivor probabilities obtained from experiment. The main aspects of uncertainty are: randomness and fuzziness. Uncertainty arising from variability or stochasticity is random whereas uncertainty due to lack of information, ignorance and/or subjectivity is fuzzy. Probability theory deals with randomness whereas possibility theory or fuzzy theory deals with ambiguity.

In a classical optimization model, the system and element lifetimes are assumed to be random variables and the system performance such as system reliability is evaluated by using the probability theory. Unfortunately, this assumption is not appropriate in a wide range of situations. In many practical cases, the probability distribution function of the system and element lifetimes may be unknown or partially known. In fact, from a practical viewpoint, the fuzziness and randomness of the element life times are often mixed up with each other. In real life, the data cannot be recorded or collected precisely due to human errors or some unexpected situations. So one may consider ambiguous situations like vague parameters, non-exact objectives and constraint functions in the problem. Such problem may be classified as a non-stochastic imprecise model. In such situations the traditional reliability theory, based on probabilistic and binary state assumptions, do not always provide useful information to the practitioners due to the limitation of being able to handle only quantitative information [2, 3, 12, 26]. The subjective information is not captured during analysis. The results obtained are therefore not much practical. This is primarily due to the fact that there is significant impact

*Corresponding author. Email: harishg58iitr@gmail.com (H. Garg); URL: http://sites.google.com/site/harishg58iitr.

of subjective information. It has therefore become inevitable to consider subjective information along with quantitative databases to arrive at useful results in analysis.

The use of fuzzy set theory [28] is another way to handle the subjective information or uncertainties. Due to incomplete and uncertain input information, mathematical models of such problems are developed in fuzzy environment and the optimization problem under consideration becomes a fuzzy programming problem. Also in the development of fuzzy programming, numerous approaches have been proposed in which both kinds of uncertainties; vagueness and ambiguity. In these approaches the parameters (coefficients) involved are treated as fuzzy parameters, since in practice they are often known to experts as fuzzy numerical data rather than crisp data. Bellman and Zadeh [1] inspired the development of fuzzy optimization by providing the aggregation operators, which combine the fuzzy goals and fuzzy decision space. After this pioneering work, a great number of articles dealing with the fuzzy optimization problems have come out. The collection of papers on fuzzy optimization edited by Delgado et al.[5] and Slowinski [25] gives the main stream of this topic. Zimmermann [29] initiated the application of fuzzy theory to optimization by solving theoretical, fuzzy, linear programming problems.

In reliability optimization problems, it is often required to minimize or maximize several objectives subject to several constraints. For instance, a designer is required to minimize the system cost while simultaneously maximizing the system reliability. Therefore, multiobjective functions become an important aspect in the reliability design of the engineering systems. In practical, the problem of series system reliability may be formed as a typical nonlinear programming problem with nonlinear cost-functions in fuzzy environment. Some researchers applied the fuzzy set theory to reliability analysis. Park [19] used fuzzy in the reliability apportionment problem for a two-component series system subject to single constraints and solved it by fuzzy nonlinear programming technique. Sakawa [23] used the surrogate worth trade off method to a multi-objective formulation of a reliability allocation problem to maximize the system reliability and minimize the system cost. Ravi et al.[21, 22] implemented simulated annealing algorithm for several reliability optimization problem. Huang [10] presented a fuzzy multi-objective optimization decision making method of reliability of series system. Mahapatra and Roy [15] proposed a new fuzzy multi-objective optimization method to solve reliability optimization problem having several conflicting objectives. Other important contributions to fuzzy programming with fuzzy parameters have been found in [7, 9, 11, 16, 17, 18].

The present work is an extension of the work, earlier done by Garg and Sharma [8], in which the cost factor was not considered in mathematical modeling. In this study, a conflicting multi-objective nonlinear programming problem is considered in fuzzy environment where we maximize the reliability and minimize the cost of the system. A conflicting nature between the objectives are resolved with the help of the fuzzy numbers. Linear membership function has been used for fuzzification. Based on the choice of the DM/system expert regarding the prioriy amongst the objectives and objectives' membership functions, the problem is converted to a single objective problem and robust global optimization technique, namely particle swarm optimization (PSO) has been used to get the optimal or nearby optimal values of the objectives'.

2 Fuzzy Multi-objective Optimization Problem

A general multiobjective optimization problem consists in finding the design variables set x that optimizes a vector of objective functions

$$f(x) = \{f_1(x), f_2(x), \dots f_n(x)\}$$

over the feasible design space X. A mathematical formulation of the multiobjective optimization problem is:

$$\begin{array}{rll} \text{Maximize} & f(x) = (f_1(x), f_2(x), \dots, f_n(x))^T & \\ \text{s.t.} & x \in X = \{x \in R^n | g_j(x) \leq 0, j = 1, 2, \dots, m\} & \quad (1) \\ & x_k^l \leq x_k \leq x_k^u \qquad k = 1, 2, \dots, K & \end{array}$$

where $n \geq 2$ is the number of distinct objectives in optimization problem; m is the total number of constraints, $x = (x_1, x_2, \dots, x_K)$ is K dimensional decision variable from some universe $X \subseteq R^n$, objective functions $f_i(x), i = 1, 2, \dots, n$ where, $f_i : X \to R$ and the constrained functions $g_j(x)$ where, $g_j : X \to R$ are all real valued functions on X, $f(x)$ is the multi-objective vector or criteria vector of objective functions. When all $f_i's$ and $g_j's$ are linear, the problem is called a multi-objective linear programming problem. If at least one of the $f_i's$ is nonlinear the problem is nonlinear multi-objective optimization problem.

Some basic definitions on Pareto optimal solutions are introduced below.

Definition: (Complete optimal solution): x^* is said to be complete optimal solution to the problem (1) if and only if there exists $x^* \in X$ such that $f_r(x^*) \geq f_r(x)$, for $r = 1, 2, \ldots, n$ and $x \neq x^* \in X$.

However, in general, such a complete optimal solution that simultaneously maximizes all the leader' and follower's objective functions does not always exist. Instead of a complete optimal solution, a new solution concept, called Pareto optimality, arises and it is defined as follows.

Definition: (Pareto-optimal solution): A vector $x^* \in X$ is a pareto optimal if there exists no feasible vector $x \in X$ such that for one objective function, say $f_i(x)$, $f_i(x) > f_i(x^*)$, and for all other $f_r(x) \geq f_r(x^*), r = 1, 2, \ldots, n; r \neq i$.

The set of all Pareto-optimal solution of a given multi–objective optimization problem is called the Pareto–optimal set, i.e. $P^* = \{x \in X : x \text{ is a Pareto–optimal solution}\}$.

Conventional optimization methods assume that all design information (parameters and goals) of an optimization model are precisely known. But in the real world, we often encounter the situation that we have to make a decision under uncertainty due to the presence of incomplete or imprecise or vagueness in information. Thus, in practical sense, the expression of the objective functions and constraints in the optimization problem (1) are not transparent. In order to make the model more flexible and adoptable to human decision process, the optimization model (1) can be represented by fuzzy non-linear programming problems with fuzzy numbers.

Therefore in fuzzy environment the optimization problem (1) becomes

$$\begin{array}{ll} \text{Maximize} & f(\widetilde{x}) = \{f_1(\widetilde{x}), f_2(\widetilde{x}), \ldots, f_n(\widetilde{x})\} \\ \text{s.t.} & \widetilde{x} \in X = \{\widetilde{x} \in R^n | g_j(\widetilde{x}) \lesssim 0, j = 1, 2, \ldots, m\} \end{array} \tag{2}$$

where $\lesssim$ represents the fuzzy version of $\leq$.

In a multi-objective optimization problem, an optimal solution which simultaneously optimizes all the objectives, and that too when the problem is modeled in a fuzzy environment, is rarely possible. In such situations one usually tries to search for the best possible solution in the presence of incomplete, vague or imprecise information which is as close to the DM's expectations as possible. Search of such a satisfying solution requires solving the multi-objective fuzzy optimization problem iteratively in an interactive manner wherein the DM is initially asked to specify his or her preferences and expectations. Based on these preferences, the problem is solved and the DM is provided with a possible solution. If the DM is satisfied with this solution the problem ends there, otherwise he or she is asked to modify his or her preferences in the light of the earlier obtained results. This iterative procedure is continued till a satisfactory solution is achieved which is closed to DM's expectations. The detail of the technique is given in the following steps.

Step 1: Solve the multi–objective optimization problem (2) as a single objective non-linear problem n times for each problem by taking one of the objective at a time and ignoring the others. The solutions obtained after solving are known as ideal solutions. The solution to the above model is the ideal solution x_t^* of each objective function, f_t, and the corresponding objective function at the ideal solution may be given by

$$f_t^* = f_t(x_t^*), \quad t = 1, 2, \ldots, n.$$

The lower bound (m_i) and upper bound (M_i) corresponding to each objective f_t is calculated as

$$\begin{aligned} m_t &= \min_{1 \leq p \leq n} f_t(x_p^*), \\ M_t &= \max_{1 \leq p \leq n} f_t(x_p^*). \end{aligned}$$

Hence the membership function corresponding to the two constraints $f_t(x) \leq m_t$ (i.e. for minimization) and $f_t(x) \geq M_t$ (i.e. for maximization) is defined as follows:

For $f_t(x) \leq m_t$ $(t = 1, 2, \ldots, n)$,

$$\mu_{f_t}(x) = \begin{cases} 1, & f_t(x) \leq m_t \\ \frac{M_t - f_t(x)}{M_t - m_t}, & m_t \leq f_t(x) \leq M_t \\ 0, & f_t(x) \geq M_t. \end{cases} \tag{3}$$

Here, $\mu_{f_t}(x)$ is strictly monotonically decreasing function of $f_t(x)$.

For $f_t(x) \geq M_t$ $(t = 1, 2, \ldots, n)$,

$$\mu_{f_t}(x) = \begin{cases} 1, & f_t(x) \geq M_t \\ \frac{f_t(x) - m_t}{M_t - m_t}, & m_t \leq f_t(x) \leq M_t \\ 0, & f_t(x) \leq m_t. \end{cases} \tag{4}$$

Here, $\mu_{f_t}(x)$ is strictly monotonically increasing function of $f_t(x)$.

Step 2: Using the achieved objectives' membership functions and DM/system expert preferences in the form of weights $W = [w_1 w_2 \ldots w_n]$, problem is formulated as a single objective optimization problem and is given as:

$$\begin{array}{ll} \text{Maximize} & \left(1 \wedge \dfrac{\alpha_1(x)}{w_1}\right) \wedge \left(1 \wedge \dfrac{\alpha_2(x)}{w_2}\right) \wedge \cdots \wedge \left(1 \wedge \dfrac{\alpha_n(x)}{w_n}\right) \\ \text{s.t.} & \alpha_t(x) = \mu_{f_t}(x) \\ & x_k^l \leq x_k \leq x_k^u, \qquad k = 1, 2, \ldots, K \\ & w_t \in [0, 1], \qquad t = 1, 2, \ldots, n \end{array} \tag{5}$$

where $\wedge$ indicates the intersection or min operator, w_t represents the t^{th} objective weight suggested by DM, α_t is the degree of satisfaction of the t^{th} objective, x is the vector of decision variables, x_k^l and x_k^u are the lower and upper bounds of decision vector x_k, respectively.

Since the problem is nonlinear in nature so it requires an efficient technique to solve this problem. Variety of methods and algorithms exists for optimization of such problems and applied in various technological fields during the last decades. Evolutionary algorithms are found to be very promising global optimizers. PSO is one of the most popular evolutionary algorithms [13, 24] and has been applied effectively to many different problems like system reliability/availability/redundancy allocation optimization [6, 7, 27]. More specifically for application purpose, Fang et al. [6] also used PSO to adjust the membership functions of a Mamadani-fuzzy controller in performing tacking control. Permana and Hashim [20] used PSO to determine the membership function in a fuzzy system. Thus in the light of applicability, this paper use PSO [4, 13, 24] as a tool to solve the optimization problem (5) whose pseudo code is described in Algorithm 1.

Algorithm 1 Pseudo code of particle swarm optimization (PSO)

1: Objective function: $f(\mathbf{x})$, $\quad \mathbf{x} = (x_1, x_2, \ldots, x_K)$;
2: For each particle:
    Initialize particle position and veclocity
3: Do:
4: For each particle:
   (a) Calculate fitness value
   (b) If the fitness value is better than the best fitness value (pbest) in history.
   (c) Set current value as the new pbest.
5: End for
6: For each particle :
   (a) Find in the particle neighborhood, the particle with the best fitness.
   (b) Calculate particle velocity according to the velocity equation (6).
   (c) Update particle position according to the position equation (7).
   (d) Apply the position constriction.
7: End for
8: While maximum iterations or minimum error criteria is not attained

The velocity and position of particle i at iteration $t+1$ are respectively updated as follows

$$\begin{aligned} v_i(t+1) &= \chi(v_i(t) + c_1 * r_1 * (pbest_i(t) - x_i(t)) \\ &\quad + c_2 * r_2 * (gbest(t) - x_i(t))) \qquad (6) \\ x_i(t+1) &= x_i(t) + v_i(t+1) \qquad (7) \end{aligned}$$

where x_i and v_i represents the position and velocity of the each dimension, $pbest_i$ and $gbest$ are the personal best and global best position of the particle at iteration t. The factor χ, where $\chi = 2/|2-\phi-\sqrt{\phi^2-4\phi}|$ with $\phi = c_1 + c_2,\ \phi > 4$, is called the constriction factor.

To stop the optimization process maximum number of generations and change in population fitness value are used. MATLAB 7.4 has been used for coding purpose.

3 Illustrative Example

To illustrate, a fertilizer plant situated in northern part of India and producing approximately 1500-2000 metric tons per day has been considered as a main system [14]. The fertilizer plant are large, complex and repairable engineering unit which is combination of ammonia and urea plant. The urea plant is composed of synthesis, decomposition, crystallization and prilling system, arranged in predetermined configuration.

System Description

The urea plant considered here is a complex engineering system where the units are arranged in a random fashion and they run continuously for a longer period to produce the required quantity of urea [14]. The plant is a combination of two dependent systems namely ammonia production system and the urea production system. For the production of urea, liquid ammonia and Carbon dioxide are used as inputs which are obtained from ammonia plant. Further, processed in a reactor at controlled pressure and temperature the reactants (urea, ammonium carbonate, water and excess ammonia) are sent to decomposer for urea separation. In the crystallizer, the crystals of urea are separated by centrifuge and conveyed pneumatically to the prilling tower where they are melted, sprayed through distributors and finally fell down at the bottom of the tower, from where it is collected. Among the various functional units in the plant such as urea synthesis, urea decomposition, urea crystallization, urea prilling and urea recovery, urea synthesis is one of the most important and vital functional processes which is the subject of our discussion.

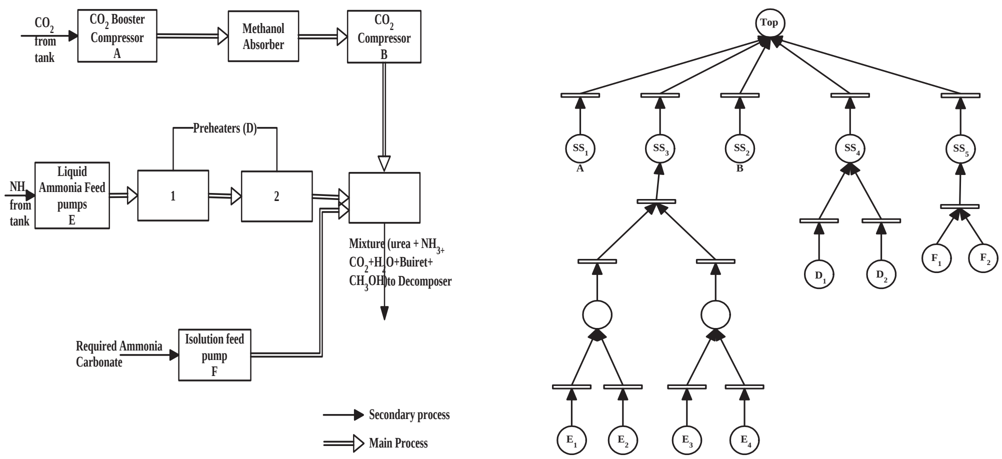
In brief, the various subsystems and the components associated with them are defined as below [8]:

- Subsystem 1 has one unit CO_2 booster compressor (A), a centrifugal type pump, which raise the pressure of CO_2 from 0.1 to 29.5 atm. Its failure causes complete failure of the system.
- Subsystem 2 has one unit CO_2 high-pressure compressor (B), a reciprocating type pump, which raises the pressure of CO_2 from 29.5 to 250atm. Its failure causes complete failure of the system.
- Subsystem 3 contains the liquid ammonia feed pumps (E) which are of reciprocating type that raise the ammonia pressure from 16.5 to 250atm. Two pumps are in operation simultaneously and two remain in cold standby. The system fails when three pumps fail.
- Subsystem 4 has two ammonia pre-heaters (D) arranged in series. The first one raises the temperature of gas upto 53.2^0C and the second heat the gas to 82.3^0C. Failure of either causes complete failure of the system.
- Subsystem 5 has the recycle isolution feed pump (F), a multistage centrifugal pump, to raise the pressure of ammonia carbonate from 17 to 250atm. It has one unit in standby. The system fails only when both units fail.

The schematic diagram and the PetriNet model of this system are shown in Fig. 1, where "Top" in Fig. 1(b) represents the system failure of the synthesis unit.

Mathematical Model of the System

In earlier, Garg and Sharma [8] were found the various reliability parameter such as failure rate, repair time, MTBF, availability, reliability , maintainability and expected number of failures used for predicting the system behavior at different level of uncertainities. The present work is an extension of the earlier work done by [8, 14], in which the cost factor was not considered in mathematical modeling. Since the cost factor plays a key role in reliability analysis and its improvement. Also, in addition to maximization of system



(a) (b)

Figure 1: (a) Systematic diagram and (b)PetriNet model of the synthesis unit

reliability, it is also oftenly required that the cost be minimized simultaneously. But, all these requires the knowledge of precise numerical probabilities and component functional dependencies, the information which is rather difficult to obtain because of incomplete or non-obtainable information. Even, if data is available, it is often inaccurate and thus, subject to uncertainty i.e. historical records/data can only represents the past behavior but may be unable to predict the future behavior of the equipment. If the data are used as such in the calculations, the results will be highly uncertain. To handle the vagueness and uncertainity in data, the crisp data related to failure rates ($\lambda_i's$) and repair times ($\tau_i's$) are converted into fuzzy number/interval with some support (spread) as suggested by decision makers/system analyst etc. Incorporating all these things, the multiobjective reliability optimization problem for the concerned system as follows

$$\begin{aligned}
\text{Maximize } R_s &= \exp(-\lambda_s t)\\
\text{Minimize } C_s &= \sum_{j=1}^{10}\left\{a_j\ log\left(\frac{1}{1-\exp(-\lambda_j t)}\right)+b_j\right\}\\
\text{s.t.} \quad & (1-s)x_k \le x_k \le (1+s)x_k,\\
& x = [\lambda_1, \lambda_2, \ldots, \lambda_{10}, \tau_1, \tau_2, \ldots, \tau_{10}]^T,\\
& \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6;\ \lambda_7 = \lambda_8; \lambda_9 = \lambda_{10}\\
& \tau_3 = \tau_4 = \tau_5 = \tau_6;\ \tau_7 = \tau_8; \tau_9 = \tau_{10}\\
& k = 1, 2, \ldots, 20\\
& s = 0.15 (\text{considered uncertainty level})
\end{aligned}$$

where R_s and C_s denote the total reliability and cost of the system, λ_i and τ_i are the failure rate and repair time of the i^{th} component while λ_s is the system failure rate whose expression is given by

$$\lambda_s = \lambda_1 + \lambda_2 + (\lambda_3\tau_3 + \lambda_4\tau_4)(\lambda_5 + \lambda_6) + (\lambda_3 + \lambda_4)(\lambda_5\tau_5 + \lambda_6\tau_6) + \lambda_7 + \lambda_8 + \lambda_9\lambda_{10}(\tau_9 + \tau_{10}).$$

The different values for the parameters $a_j (j = 1, 2, \ldots, 10)$ are 14, 8, 7.14, 7.14, 7.14, 7.14, 3.33, 3.33, 24, 24 respectively and for $b_j (j = 1, 2, \ldots, 10)$ are 80, 60, 50, 50, 50, 50, 30, 30, 100, 100 respectively are chosen randomly. The input parameters defining the specific instances has the same values as that of [8] and are tabulated in Table 1.

Table 1: Data for failure rate and repair time [8]

Subsystems→	A	B	E	D	F
	$i = 1$	$i = 2$	$i = 3, \ldots, 6$	$i = 7, 8$	$i = 9, 10$
Failure rate($\lambda_i \times 10^{-3}$hrs^{-1})	3.873	5.395	2.759	4.475	6.691
Repair time(τ_i) (hrs)	4.564	2.838	4.736	3.727	4.384

4 Computational Results

Parameter Setting

The optimization method is implemented in Matlab (MathWorks) and the program has been run on a T6400 @ 2GHz Intel Core(TM) 2 Duo processor with 2GB of Random Access Memory(RAM). In order to eliminate stochastic discrepancy, 25 independent runs are made involving 25 different initial trial solutions with population size 100 and maximum number of generation as 300 along with $c_1 = c_2 = 2.05$. To stop the optimization process maximum number of generations and change in population fitness value are used.

Results and Discussion

A fuzzy region of satisfaction corresponding to the objective functions are to be constructed by using (3) and (4). Using these constructed membership functions and their weight vector as suggested by DM/system experts corresponding to the two objective functions, the equivalent crisp optimization problem is formulated as follow.

$$\begin{aligned}
\text{Maximize} \quad & \left(1 \wedge \frac{\alpha_1(x)}{w_1}\right) \wedge \left(1 \wedge \frac{\alpha_2(x)}{w_2}\right) \\
\text{s.t.} \quad & \alpha_t(x) = \mu_{\tilde{f}_t}(x), \qquad t = 1, 2 \\
& x = [\lambda_1, \lambda_2, \ldots, \lambda_{10}, \tau_1, \tau_2, \ldots, \tau_{10}]^T \\
& \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6;\ \lambda_7 = \lambda_8; \lambda_9 = \lambda_{10} \\
& \tau_3 = \tau_4 = \tau_5 = \tau_6;\ \tau_7 = \tau_8; \tau_9 = \tau_{10} \\
& s = 0.15 (\text{considered uncertainty level}) \\
& w_t \in [0, 1].
\end{aligned} \tag{8}$$

For the solution of the optimization problem (8), PSO is used with the parameter given in section 4. The results corresponding to weight sets $W1 = [1, 1]$, $W2 = [1, 0.5]$, $W3 = [0.8, 0.2]$, $W4 = [0.2, 0.8]$, $W5 = [0.5, 1]$ which are suggested by DMs/system experts (in order of their importance factor) are given in Table 2.

Table 2: Pareto results for different weight sets suggested by decision maker

Sub Components	Decision variables	W1	W2	W3	W4	W5
A	λ_i	0.004032	0.003292	0.003304	0.004355	0.004454
$(i = 1)$	τ_i	5.248600	5.248600	4.134244	4.794343	5.248600
B	λ_i	0.004586	0.004586	0.004804	0.005590	0.006204
$(i = 2)$	τ_i	3.263700	2.412300	2.515186	3.121812	2.412300
D	λ_i	0.003173	0.002703	0.002779	0.003126	0.003173
$(i = 3, \ldots, 6)$	τ_i	4.025600	4.025600	5.116085	4.215035	4.025600
E	λ_i	0.003804	0.003804	0.003890	0.004884	0.003832
$(i = 7, 8)$	τ_i	3.167950	3.167950	3.205497	3.745798	3.167950
F	λ_i	0.007695	0.007202	0.005849	0.007515	0.007695
$(i = 9, 10)$	τ_i	3.726400	3.726400	4.387996	4.203431	3.726400
System R_s		0.843740	0.851231	0.847863	0.814497	0.826239
System C_s		915.909555	926.269636	934.592675	913.187325	912.144580
α_1		0.816717	0.965529	0.898612	0.235780	0.469040
α_2		0.816717	0.482764	0.214475	0.904467	0.938079

In Case I (i.e W1, results shown in 3rd column of Table 2) where DM insists to have no biasness towards any of the objectives (i.e. not to attain maximum reliability or minimum cost but biasness towards the midways between both objectives). In Case II (i.e W2, where the reliability index importance is twice as that of cost index, results shown in 4th column of Table 2), DM wants that the biasness/preference towards maximizing reliability. Also DM does not care about the other objective (i.e. minimizing the cost). So, on solving, the Pareto-optimal will shift towards the maximizing reliability which was preference of the DM in this case. Similarly in Case V (i.e W5, where the cost index importance is twice as that of reliability index, result shown 7th column of Table 2) in which DM wants that the biasness/preference towards minimum cost and also does not care about the other objective (i.e. maximizing the reliability). Hence the Pareto-optimal will shift towards the minimum cost. Moreover, the optimal result of the different preferences can be easily seen through the degrees of membership value i.e. α_1 & α_2. For instance, in Case I the degrees of membership are equal because in that case DM insists no bianess towards any of the objectives while in Case II and Case V, the degrees of membership value are in twice ratio according to the preferences towards the objective funtions. Similarly effects have been observed in all other cases. Thus for different preferences suggested by DMs, optimum values of systems' reliability and cost are achieved. Since each objective function is optimized, system performance under consideration is optimized automatically.

5 Conclusion

Due to growing complexity in the industrial systems, the design of a repairable series-parallel system is insufficient if the analysis is done based on empirical methods. Also, the design cost for the system will increases as it is very difficult to analyze the system behavior based on experience. Applying soft computing techniques such as PSO to analyze and to optimize the design problems of repairable series-parallel systems, appears to be very helpful in decision making of system parameter design. In the present study, an attempt has been made to optimize the system reliability and cost of the synthesis unit of a fertilizer plant with limited, imprecise and vague data. Due to complexity in the system configuration, the data obtained from historical records, is imprecise and inaccurate. The development of fuzzy numbers from available data on components and using fuzzy possibility theory to define the membership function can greatly increase the relevance of the reliability study. The computed failure rates and repair times, may help the maintenance engineer to determine the repair policy by taking into account optimal design and the company maintenance policy. System reliability engineers/analysts may use these results to set the future targets of their interest.

In the nutshell, a structured framework has been developed that may help maintenance engineers to analyze and predict system performance. Attempts have also been made:

- to deal with imprecise, uncertain dependent information related to system performance as the fuzzy methodology provides a better, consistent and mathematically sound method for handling uncertainties in data than conventional methods, such as Bayesian statistics;

- to model and deal with highly complex repairable series-parallel system (synthesis unit of a urea plant) using fuzzy sets, as these sets can deal easily with approximation.

Acknowledgment

The authors would like to thank the referees for providing very helpful comments and suggestions. Also, the corresponding author acknowledges the Ministry of Human Resource and Development (MHRD), Govt. of India, New Delhi for providing a financial help to carry out this work.

References

[1] Bellman, R., and L. Zadeh, Decision making in a fuzzy environment, *Management Science*, vol.17, pp.141–164, 1970.

[2] Cai, K.Y., Fuzzy reliability theories, *Fuzzy Sets and Systems*, vol.40, pp.510–511, 1991.

[3] Cai, K.Y., System failure engineering and fuzzy methodology: an introductory overview, *Fuzzy Sets and Systems*, vol.83, pp.113–133, 1996.

[4] Clerc, M., and J.F. Kennedy, The particle swarm: explosion, stability, and convergence in a multi-dimensional complex space, *IEEE Transactions on Evolutionary Computation*, vol.6, no.1, pp.58–73, 2002.

[5] Delgado, M., J. Kacprzyk, J.L. Verdegay, and M. Vila, *Fuzzy Optimization: Recent Advances*, Physica-Verlag, New York, 1994.

[6] Fang, G., N.M. Kwok, and Q. Ha, Automatic fuzzy membership function tuning using the particle swarm optimisation, *IEEE Pacific-Asia Workshop on Computational Intelligence and Industrial Application*, pp.324–328, 2008.

[7] Garg, H., and S.P. Sharma, Multi-objective optimization of crystallization unit in a fertilizer plant using particle swarm optimization, *International Journal of Applied Science and Engineering*, vol.9, no.4, pp.261–276, 2011.

[8] Garg, H., and S.P. Sharma, Behavior analysis of synthesis unit in fertilizer plant, *International Journal of Quality and Reliability Management*, vol.29, no.2, pp.217–232, 2012.

[9] Gu, Y.K., and H.Z. Huang, Fuzzy mapping between physical domain and function domain in design process, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol.12, no.1, pp.7–20, 2004.

[10] Huang, H.Z., Fuzzy multi-objective optimization decison-making of reliability of series system, *Microelectronics Reliability*, vol.37, no.3, pp.447–449, 1997.

[11] Huang, H.Z., Z.G. Tian, and M.J. Zuo, Intelligent interactive multi-objective optimization method and its application to reliability optimization, *IIE Transactions*, vol.37, no.11, pp.983–993, 2005.

[12] Karwowski, W., and A. Mittal, *Applications of Fuzzy Set Theory in Human Factors*, Elsevier, Amsterdam, 1986.

[13] Kennedy, J., and R.C. Eberhart, Particle swarm optimization, *IEEE International Conference on Neural Networks*, vol.IV, pp.1942–1948, 1995.

[14] Kumar, D., *Analysis and Optimization of Systems Availability in Sugar, Paper and Fertilizer Industries*, Ph.D. Thesis, University of Roorkee (Presently IIT Roorkee), India, 1991.

[15] Mahapatra, G.S., and T.K. Roy, Fuzzy multi-objective mathematical programming on reliability optimization model, *Applied Mathematics and Computation*, vol.174, pp.643–659, 2006.

[16] Nayak, P.K., and M. Pal, Intuitionistic fuzzy optimization technique for nash equilibrium solution of multi-objective bi-matrix games, *Journal of Uncertain Systems*, vol.5, no.4, pp.271–285, 2011.

[17] Nematian, J., A new method for multi-objective linear programming models with fuzzy random variables, *Journal of Uncertain Systems*, vol.6, no.1, pp.38–50, 2012.

[18] Pahlavani, A., and M.S. Mehrabad, A fuzzy multi-objective programming for optimization of hierarchical service centre locations, *Journal of Uncertain Systems*, vol.5, no.3, pp.202–226, 2011.

[19] Park, K.S., Fuzzy apportionment of system reliability, *IEEE Transactions on Reliability*, vol.R-36, pp.129–132, 1987.

[20] Permana, K.E., and S.Z.M. Hashim, Fuzzy membership function generation using particle swarm optimization, *International Journal of Open Problems in Computer Science and Mathematics*, vol.3, no.1, pp.27–41, 2010.

[21] Ravi, V., N.B.S. Murty, and P.J. Reddy, Non equilibrium simulated annealing-algorithm applied to reliability optimization of complex systems, *IEEE Transactions on Reliability*, vol.46, no.2, pp.233–239, 1997.

[22] Ravi, V., P.J. Reddy, and H.J. Zimmermann, Fuzzy global optimization of complex system reliability, *IEEE Transactions on Fuzzy Systems*, vol.8, pp.241–248, 2000.

[23] Sakawa, M., Multi-objective optimization by the surrogate worth trade-off method, *IEEE Transactions on Reliability*, vol.R-27, pp.311–314, 1978.

[24] Shi, Y., and R.C. Eberhart, Parameter selection in particle swarm optimization, *Proceedings of Evolutionary Programming VII*, pp.591–600, 1998.

[25] Slowinski, R., *Fuzzy Sets in Decision Analysis, Operations Research and Statistics*, Kluwer Academic Publishers, Boston, 1998.

[26] Verma, A.K., A. Srividya, and R.S.P. Gaonkar, *Fuzzy Reliability Engineering: Concepts and Applications*, Narosa Publishing House, New Delhi, India, 2007.

[27] Yeh, W.-C., Y.-C. Lin, Y.Y. Chung, and M. Chih, A particle swarm optimization approach based on monte carlo simulation for solving the complex network reliability problem, *IEEE Transaction on Reliability*, vol.59, no.1, pp.212–221, 2010.

[28] Zadeh, L.A., Fuzzy sets, *Information and Control*, vol.8, pp.338–353, 1965.

[29] Zimmermann, H.J., Fuzzy programming and linear programming with several objective functions, *Fuzzy Sets and Systems*, vol.1, no.1, pp.45–55, 1978.