Journal of Uncertain Systems
Vol.6, No.3, pp.200-213, 2012
Online at: www.jus.org.uk

Multi-choice Stochastic Transportation Problem with Exponential Distribution

Sankar Kumar Roy[1,*], Deshabrata Roy Mahapatra[1], Mahendra Prasad Biswal[2]

[1] *Department of Applied Mathematics with Oceanology and Computer Programming*
Vidyasagar University, Midnapore-721102, West Bengal, India

[2] *Department of Mathematics, Indian Institute of Technology, Kharagpur-721302, West Bengal, India*

Received 16 October 2011; Revised 21 April 2012

Abstract

A stochastic transportation problem is considered here on which exponential distribution is used to all constraints containing parameters like supply and demand and cost coefficients of objective function have multi-choice. The proposed multi-choice stochastic transportation problem is transferred to an equivalent deterministic model. A new transformation technique is introduced to manipulate cost coefficients of objective function involving multi-choice or goals for binary variables with additional restriction. The additional restriction depends on the number of aspiration levels associated with each cost coefficient of objective function. The specified probabilistic constraints are transformed into an equivalent deterministic constraints using stochastic programming approach. Finally, an example is presented to illustrate the transformation technique and to demonstrates the effectiveness and usefulness of the specified proposed model.

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Keywords: transportation problem, multi-choice programming, stochastic programming, transformation technique, exponential distribution, mixed-integer programming

1 Introduction

The classical transportation problem can be described to a special case of linear programming problem and its models applied to determine an optimal solution of the transportation problem required for deterministic of how many units of commodity to be shipped from each origin to various destinations, satisfying availability (source) and demand (destination), while minimizing the total cost of transportation. In such situation, we should consider the requirement of goods at each demand points, variety of shipping routes and associated cost of distribution or transportation of goods or products from each origin to each destination as the exponential random variables on replacement of the parameters.

Stochastic programming deals with situations where some or all of the parameters of the optimization problem are described by random variables rather than by deterministic quantity. The random variables of the sources and destinations may be several, depending on the nature and the type of problem. Decision making problems of stochastic optimization arises when certain coefficients of the optimization model are not fixed or known but are instead, of some extent, random quantities.

The exponential distribution is an example of a continuous distribution. Gamma distribution becomes an exponential distribution by suitable choice of the parameters. It has lot of applications such as the elapse

*Corresponding author. Email: sankroy2006@gmail.com (S.K. Roy).

of time before an earthquake in a given region, a customer wait for entering in a shop, a piece of machinery work without breaking down, the rate of incoming telephone calls differ according to the time of a day, the time until a radioactive particle decays, the amount of time in months a car battery lasts, the duration of a certain time interval comprised between the time and for any time instant, etc. Here we can also apply this distribution in transportation problem.

The probability density function of gamma distribution with shape and scale parameters δ and λ as follows:

$$f(x;\lambda,\delta) = \frac{\lambda^{\delta}x^{\delta-1}}{\Gamma(\delta)}e^{-\lambda x}, \quad x \geq 0, \quad \lambda > 0, \delta \geq 0. \tag{1.1}$$

If we consider the shape parameter $\delta = 1$ and scale parameter $\lambda = 1/\theta$, where $\theta > 0$, then the above density function of gamma distribution transferred into an exponential distribution as:

$$f(x;\theta) = \frac{1}{\theta}e^{-\frac{x}{\theta}}, \quad x > 0. \tag{1.2}$$

In recent years, methods of multi-choice stochastic optimization have become increasingly important in scientifically based on decision making involved in practical problem arising in economic, industry, health care, transportation, agriculture, military purpose and technology.

Gupta et al. [5] observed to generate the gamma random variate using generalized exponential distribution when shape parameter lies between 0 and 1. Sahoo et al. [10] presented the probabilistic problems with exponential random variables.

In the typical transportation problem, we consider the situation having m origins (sources) S_i $(i = 1, 2, \cdots, m)$ and n destinations (demands) D_j $(j = 1, 2, \cdots, n)$. The sources may be production facilities, warehouses, supply points and the destinations are consumption facilities, warehouse or demand points. Let the supplies $(a_1, a_2, \cdots, a_m)$ be the quantity of homogeneous product, which we want to transport from m origins S_i $(i = 1, 2, \cdots, m)$ to n destinations D_j $(j = 1, 2, \cdots, n)$ which satisfy the demands $(b_1, b_2, \cdots, b_n)$ are respectively. The coefficient C_{ij}^k of the objective function could represent the transportation cost, unfulfilled supply and demand, and others, are provided with transporting a unit of product from source i to destination j.

The mathematical model of the multi-choice transportation problem is presented as follows:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^k x_{ij}, \qquad k = 1, 2, \cdots, K \tag{1.3}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1, 2, \cdots, m \tag{1.4}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, 2, \cdots, n \tag{1.5}$$

$$x_{ij} \geq 0, \qquad \forall\ i \text{ and } j. \tag{1.6}$$

Here z represents the minimum value of the objective function and it is assumed that $a_i > 0$, $b_j > 0$, and $C_{ij}^k > 0$ and $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$ (for unbalanced transportation problem).

Hitchcock [6] first considered the problem of minimizing the cost of distribution of product from several factories to a number of customers. He developed a procedure to solve the transportation problem, which is close resemblance with the primal simplex transportation method developed by Dantzig [4]. Biswal and Acharya [1] presented the transformation of a multi-choice linear programming problem in which constraints are associated with the multi-choice parameters. A method for modeling the multi-choice goal programming problem, using the multiple terms of binary variables was presented by Chang [2]. He has considered a mathematical model where the multiplicative terms of binary variables are replaced by continuous variable

[2]. He has also proposed a revised method for multi-choice goal programming model which does not involve multiplicative terms of binary variables to model the multiple aspiration levels [3]. Mahapatra et al. [7] discussed the solution procedure using fuzzy programming technique in the objective function and stochastic programming approach has been applied for the randomness of source and destination parameters in an inequality type of constraints of the transportation problem. In [8], Mahapatra et al. have also discussed the computational procedure of multi-objective transportation problem with joint constraints involving the constraints in probabilistic nature. They have assumed that C_{ij}^k $(k = 1, 2, \cdots, K)$ are deterministic constants and a_i $(i = 1, 2, \cdots, m)$ and b_j $(j = 1, 2, \cdots, n)$ may be random variables in multi-objective stochastic transportation problem.

Most of the researchers, such as [7], have presented the solution procedure for using the membership function of the fuzzy programming technique in the objective functions and the constraints involving source, destination are followed by the normal distribution.

In this paper, we present a multi-choice stochastic transportation problem with two probabilistic constraints realizing from the real life problem. Both the probabilistic constraints are inequality type and the parameters (supply and demand) are follow exponential distribution. The cost coefficients of the objective function are multi-choice where we use a new transformation technique to solve it. Binary variable and additional restrictions are introduced to formulate a non-linear mixed integer programming model. In order to solve the present problem, we propose a new methodology to solve multi-choice stochastic transportation problem.

2 Mathematical Model

In this paper, we have considered a mathematical model for multi-choice stochastic transportation problem involving exponential random variable in all constraints and cost coefficients of objective function are also satisfied the multi-choices or goals with binary programming framework as follows:

Model 1

$$\min : z = \sum_{i=1}^{m} \sum_{j=1}^{n} \{C_{ij}^1, C_{ij}^2, \cdots, C_{ij}^k\} x_{ij}, \qquad k = 1, 2, \cdots, K \tag{2.7}$$

$$\text{subject to} \quad \Pr\left(\sum_{j=1}^{n} x_{ij} \leq a_i\right) \geq 1 - \alpha_i, \quad i = 1, 2, \cdots, m \tag{2.8}$$

$$\Pr\left(\sum_{i=1}^{m} x_{ij} \geq b_j\right) \geq 1 - \beta_j, \quad j = 1, 2, \cdots, n \tag{2.9}$$

$$x_{ij} \geq 0, \quad \forall \;\; i \text{ and } \; j \tag{2.10}$$

where $0 < \alpha_i < 1$, $\forall$ i and $0 < \beta_j < 1$, $\forall$ j.

We have assumed that a_i $(i = 1, 2, \cdots, m)$ and b_j $(j = 1, 2, \cdots, n)$ are specified with exponential random variables and $\{C_{ij}^1, C_{ij}^2, \cdots, C_{ij}^k\}$, are known as multi-choices or goals for $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$. Now the following cases are to be considered.

1. Only $a_i (i = 1, 2, \cdots, m)$ has exponential distribution.

2. Only $b_j (j = 1, 2, \cdots, n)$ has exponential distribution.

3. Both $a_i (i = 1, 2, \cdots, m)$ and $b_j (j = 1, 2, \cdots, n)$ have exponential distributions.

2.1 Only $a_i (i = 1, 2, \cdots, m)$ Follows Exponential Distribution

The constraints (2.8) can be represented as below when a_i $(i = 1, 2, \cdots, m)$ follows an exponential random variable.

$$\Pr\left(\sum_{j=1}^{n} x_{ij} \leq a_i\right) \geq 1 - \alpha_i, \quad i = 1, 2, \cdots, m.$$

The above inequality can be further expressed as

$$\Pr\left(\sum_{j=1}^{n} x_{ij} \geq a_i\right) \leq \alpha_i, \quad i = 1, 2, \cdots, m. \tag{2.11}$$

It is assumed that a_i $(i = 1, 2, \cdots, m)$ are independent exponential random variables with parameter θ_i define as positive integers, where mean $E(a_i)= \theta_i$ and variance $Var(a_i)={\sigma_{a_i}}^2 =\theta_i^2$, which are known to us. We know that the probability density function of a_i $(i = 1, 2, \cdots, m)$ is given by

$$f(a_i) \quad = \quad \frac{1}{\theta_i} e^{-\frac{a_i}{\theta_i}}, \quad \text{where} \quad \begin{cases} a_i > 0 \\ \theta_i > 0. \end{cases} \tag{2.12}$$

Now the inequality (2.11) can be expressed as the cumulative density function of exponential distribution:

$$\int_0^{\sum_{j=1}^{n} x_{ij}} f(a_i) d(a_i) \quad \leq \quad \alpha_i. \tag{2.13}$$

Using (2.12), the above integral can be expressed as:

$$\int_0^{\sum_{j=1}^{n} x_{ij}} \frac{1}{\theta_i} e^{-\frac{a_i}{\theta_i}} d(a_i) \quad \leq \quad \alpha_i. \tag{2.14}$$

Let $-a_i/\theta_i = z$. The above integral can be expressed as:

$$\int_0^{-\frac{\sum_{j=1}^{n} x_{ij}}{\theta_i}} -e^z d(z) \leq \alpha_i \tag{2.15}$$

which can be integrated as:

$$\left[-e^z\right]_0^{-\frac{\sum_{j=1}^{n} x_{ij}}{\theta_i}} \leq \alpha_i. \tag{2.16}$$

Taking logarithm on both sides, we have

$$-\frac{\sum_{j=1}^{n} x_{ij}}{\theta_i} \quad \geq \quad \ln(1 - \alpha_i), \quad i = 1, 2, \cdots, m. \tag{2.17}$$

Finally, the stochastic constraint (2.8) can be transformed into an equivalent deterministic constraints as follows:

$$\sum_{j=1}^{n} x_{ij} \quad \leq \quad -\theta_i \ln(1 - \alpha_i), \; i = 1, 2, \cdots, m. \tag{2.18}$$

Therefore, we have obtained a multi-choice deterministic transportation problem (see **Model 2**) instead of multi-choice stochastic transportation problem (**Model 1**) as follows:

$$\textbf{Model 2} \quad \min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^{1}, C_{ij}^{2}, \cdots, C_{ij}^{k}\}x_{ij}, \qquad k = 1, 2, \cdots, K \tag{2.19}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq -\theta_i \ln(1-\alpha_i), \quad i = 1, 2, \cdots, m \tag{2.20}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j, \quad j = 1, 2, \cdots, n \tag{2.21}$$

$$x_{ij} \geq 0, \quad \forall \ i \ \text{ and } \ j. \tag{2.22}$$

2.2 Only $b_j (j = 1, 2, \cdots, n)$ Follows Exponential Distribution

Let us assume that b_j $(j = 1, 2, \cdots, n)$ be an exponential random variable. Then the constraints (2.9) can be represented as follows:

$$\Pr\left(\sum_{i=1}^{m} x_{ij} \geq b_j\right) \quad \geq \quad 1 - \beta_j, \quad j = 1, 2, \cdots, n.$$

Here b_j $(j = 1, 2, \cdots, n)$ are independent exponential random variables with parameters $\theta_j^{'}$ define as positive integers, where mean $= E(b_j) = \theta_j^{'}$ and variance $= Var(b_j) = {\sigma_{b_j}}^2 = \theta_j^{'2}$, which are known to us. Then the probability density function of j-th random variable b_j $(j = 1, 2, \cdots, n)$ is

$$f(b_j) \quad = \quad \frac{1}{\theta_j^{'}} e^{-\frac{b_j}{\theta_j^{'}}}, \quad \text{where} \quad \begin{cases} b_j > 0 \\ \theta_j^{'} > 0. \end{cases} \tag{2.23}$$

The above constraint can be expressed as the cumulative density function of exponential distribution:

$$\int_0^{\sum_{i=1}^{m} x_{ij}} f(b_j) d(b_j) \quad \geq \quad 1 - \beta_j. \tag{2.24}$$

Using (2.23), the above integral can be expressed as:

$$\int_0^{\sum_{i=1}^{m} x_{ij}} \frac{1}{\theta_j^{'}} e^{-\frac{b_j}{\theta_j^{'}}} d(b_j) \quad \geq \quad 1 - \beta_j. \tag{2.25}$$

Let $-b_j/\theta_j^{'} = z^{'}$. The above integral can be expressed as:

$$\int_0^{-\frac{\sum_{i=1}^{m} x_{ij}}{\theta_j^{'}}} -e^{z^{'}} d(z^{'}) \geq 1 - \beta_j \tag{2.26}$$

which can be integrated as:

$$\left[-e^{z^{'}}\right]_0^{-\frac{\sum_{i=1}^{m} x_{ij}}{\theta_j^{'}}} \geq 1 - \beta_j. \tag{2.27}$$

Taking logarithm in both sides, we get

$$-\sum_{i=1}^{m} x_{ij} \quad \leq \quad \theta_j^{'}(\ln \beta_j). \tag{2.28}$$

Finally, the stochastic constraint (2.9) can be transformed into deterministic constraints as follows:

$$\sum_{i=1}^{m} x_{ij} \quad \geq \quad -\theta_j^{'}(\ln \beta_j), \quad j = 1, 2, \cdots, n. \tag{2.29}$$

Therefore, we have obtained a multi-choice deterministic transportation problem (see **Model 3**) instead of multi-choice stochastic transportation problem (**Model 1**) as follows:

$$\textbf{Model 3} \quad \min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, \cdots, C_{ij}^k\}x_{ij}, \qquad k = 1, 2, \cdots, K \tag{2.30}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \le a_i, \quad i = 1, 2, \cdots, m \tag{2.31}$$

$$\sum_{i=1}^{m} x_{ij} \ge -\theta_j{}'(\ln \beta_j), \quad j = 1, 2, \cdots, n \tag{2.32}$$

$$x_{ij} \ge 0, \quad \forall \ i \ \text{ and } \ j. \tag{2.33}$$

2.3 Both $a_i (i = 1, 2, \cdots, m)$ and $b_j (j = 1, 2, \cdots, n)$ Follow Exponential Distributions

The mean and variance of a_i and b_j are known and previously defined. In this case, the equivalent deterministic model of the multi-choice stochastic transportation problem can be represented as:

$$\textbf{Model 4} \quad \min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, \cdots, C_{ij}^k\}x_{ij}, \qquad k = 1, 2, \cdots, K \tag{2.34}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \le -\theta_i \ln(1-\alpha_i) \tag{2.35}$$

$$\sum_{i=1}^{m} x_{ij} \ge -\theta_j^{'}(\ln \beta_j) \tag{2.36}$$

$$x_{ij} \ge 0, \quad \forall \ i \ \text{ and } \ j. \tag{2.37}$$

3 Transformation of an Equivalent Model with Cost Coefficients of Objective Function

The proposed model is specified for maximum of seven choices on the cost coefficients of objective function. Seven cases are expressed as following form for $k = 2, 3, \cdots, 8$.

Step 1: <u>*When $k = 2$*</u>

We present the objective function (2.7) as follows:

$$\min : z \;=\; \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2\}x_{ij}. \tag{3.38}$$

The cost coefficients have two choices as $\{C_{ij}^1, C_{ij}^2\}$, out of which one is to be selected. Since total number of elements of the set is 2, so only one binary variable is required. Denoting the binary variable z_{ij}^1, the equation (3.38) is formulated as below:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1 z_{ij}^1 + C_{ij}^2(1 - z_{ij}^1)\}x_{ij}$$

$$x_{ij} \ge 0, \ \ \forall \ i, \ \ \forall \ j \text{ and } \ z_{ij}^k = 0/1, \ k = 1, 2.$$

Step 2: <u>*When $k = 3$*</u>

We present the objective function (2.7) as follows:

$$\min : z \quad = \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, C_{ij}^3\}x_{ij}. \tag{3.39}$$

The cost coefficients have three choices as $\{C_{ij}^1, C_{ij}^2, C_{ij}^3\}$, out of which one is to be selected. Since $2^1 < 3 < 2^2$, so the total number of elements of the set is 3. Denoting the binary variables z_{ij}^1, z_{ij}^2 and introducing additional constraints in two models are formulated as follows:

Model 2(a)

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1(1-z_{ij}^1)(1-z_{ij}^2) + C_{ij}^2 z_{ij}^1(1-z_{ij}^2) + C_{ij}^3(1-z_{ij}^1)z_{ij}^2\}x_{ij}$$
$$z_{ij}^1 + z_{ij}^2 \le 1$$
$$x_{ij} \ge 0, \text{ and } z_{ij}^k = 0/1, \ \forall \ i \text{ and } j, \text{and } k = 1, 2.$$

Model 2(b)

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1 z_{ij}^1 z_{ij}^2 + C_{ij}^2 z_{ij}^1(1-z_{ij}^2) + C_{ij}^3(1-z_{ij}^1)z_{ij}^2\}x_{ij}$$
$$z_{ij}^1 + z_{ij}^2 \ge 1$$
$$x_{ij} \ge 0, \text{ and } z_{ij}^k = 0/1, \ \forall \ i \text{ and } j, \text{and } k = 1, 2.$$

Step 3: *When $k = 4$*

We present the objective function (2.7) as follows:

$$\min : z \quad = \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4\}x_{ij}. \tag{3.40}$$

The cost coefficients of the objective function have four choices as $\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4\}$, out of which one is to be selected. Since the total number of choices is $4 = 2^2$. Denoting the binary variables z_{ij}^1, z_{ij}^2, so we construct following problem:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1 z_{ij}^1 z_{ij}^2 + C_{ij}^2 z_{ij}^1(1-z_{ij}^2) + C_{ij}^3(1-z_{ij}^1)z_{ij}^2 + C_{ij}^4(1-z_{ij}^1)(1-z_{ij}^2)\}x_{ij}$$
$$x_{ij} \ge 0, \text{ and } z_{ij}^k = 0/1, \ \forall \ i \text{ and } j, \text{and } k = 1, 2.$$

Step 4: *When $k = 5$*

We present the objective function (2.7) as follows:

$$\min : z \quad = \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5\}x_{ij}. \tag{3.41}$$

The cost coefficients have five choices as $\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5\}$, out of which one is to be selected. Since $2^2 < 5 < 2^3$, so we need three binary variables $z_{ij}^1, z_{ij}^2, z_{ij}^3$. Then we put restriction to remaining three terms $(8-5)$ by introducing additional constraints in three different models which are as follows:

Model 4(a)

$$\begin{aligned}\min: z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{&C_{ij}^1 z_{ij}^1(1-z_{ij}^2)(1-z_{ij}^3) + C_{ij}^2(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3)\\ &+C_{ij}^3(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3 + C_{ij}^4 z_{ij}^1 z_{ij}^2(1-z_{ij}^3) + C_{ij}^5(1-z_{ij}^1)z_{ij}^2 z_{ij}^3\}x_{ij}\\ &1 \le z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \le 2\\ &z_{ij}^1 + z_{ij}^3 \le 1\\ &x_{ij} \ge 0, \text{ and } z_{ij}^k = 0/1, \ \forall \ i \text{ and } j, \text{and } k = 1,2,3.\end{aligned}$$

Model 4(b)

$$\begin{aligned}\min: z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{&C_{ij}^1 z_{ij}^1(1-z_{ij}^2)(1-z_{ij}^3) + C_{ij}^2(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3)\\ &+C_{ij}^3(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3 + C_{ij}^4 z_{ij}^1 z_{ij}^2(1-z_{ij}^3) + C_{ij}^5 z_{ij}^1(1-z_{ij}^2)z_{ij}^3\}x_{ij}\\ &1 \le z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \le 2\\ &z_{ij}^2 + z_{ij}^3 \le 1\\ &x_{ij} \ge 0, \text{ and } z_{ij}^k = 0/1, \ \forall \ i \text{ and } j, \text{and } k = 1,2,3.\end{aligned}$$

Model 4(c)

$$\begin{aligned}\min: z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{&C_{ij}^1 z_{ij}^1(1-z_{ij}^2)(1-z_{ij}^3) + C_{ij}^2(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3)\\ &+C_{ij}^3(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3 + C_{ij}^4(1-z_{ij}^1)z_{ij}^2 z_{ij}^3 + C_{ij}^5 z_{ij}^1(1-z_{ij}^2)z_{ij}^3\}x_{ij}\\ &1 \le z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \le 2\\ &z_{ij}^1 + z_{ij}^2 \le 1\\ &x_{ij} \ge 0, \text{ and } z_{ij}^k = 0/1, \ \forall \ i \text{ and } j, \text{and } k = 1,2,3.\end{aligned}$$

Step 5: *When $k = 6$*

We present the objective function (2.7) as follows:

$$\min: z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5, C_{ij}^6\}x_{ij}. \tag{3.42}$$

The cost coefficients of the objective function have six choices as $\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5, C_{ij}^6\}$, out of which one is to be selected. Since $2^2 < 6 < 2^3$, so we need three binary variables $z_{ij}^1, z_{ij}^2, z_{ij}^3$. Then we put the restriction to remaining two terms $(8-6)$ by introducing auxiliary constraints in the model are expressed as given below:

$$\begin{aligned}\min: z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{&C_{ij}^1 z_{ij}^1(1-z_{ij}^2)(1-z_{ij}^3) + C_{ij}^2(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3)\\ &+C_{ij}^3(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3 + C_{ij}^4 z_{ij}^1 z_{ij}^2(1-z_{ij}^3)\\ &+C_{ij}^5 z_{ij}^1(1-z_{ij}^2)z_{ij}^3 + C_{ij}^6(1-z_{ij}^1)z_{ij}^2 z_{ij}^3\}x_{ij}\\ &1 \le z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \le 2\\ &x_{ij} \ge 0, \text{ and } z_{ij}^k = 0/1, \ \forall \ i \text{ and } j, \text{and } k = 1,2,3.\end{aligned}$$

Step 6: *When $k = 7$*

We present the objective function (2.7) as follows:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5, C_{ij}^6, C_{ij}^7\}x_{ij}. \tag{3.43}$$

The cost coefficients of the objective function have seven choices as $\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5, C_{ij}^6, C_{ij}^7\}$, out of which one is to be selected. Since $2^2 < 7 < 2^3$, so we need three binary variables $z_{ij}^1, z_{ij}^2, z_{ij}^3$. Then we put the restriction to remaining one term $(8-7)$ by introducing additional constraint in the mathematical model. Two different models are formulated as given below:

Model 6(a)

$$\begin{aligned}\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{&C_{ij}^1(1-z_{ij}^1)(1-z_{ij}^2)(1-z_{ij}^3) + C_{ij}^2 z_{ij}^1(1-z_{ij}^2)(1-z_{ij}^3)\\ &+C_{ij}^3(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3) + C_{ij}^4(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3 + C_{ij}^5 z_{ij}^1 z_{ij}^2(1-z_{ij}^3)\\ &+C_{ij}^6 z_{ij}^1(1-z_{ij}^2)z_{ij}^3 + C_{ij}^7(1-z_{ij}^1)z_{ij}^2 z_{ij}^3\}x_{ij}\\ &z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \le 2\\ &x_{ij} \ge 0, \text{ and } z_{ij}^k = 0/1, \ \forall \ i \text{ and } j, \text{and } k = 1,2,3.\end{aligned}$$

Model 6(b)

$$\begin{aligned}\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{&C_{ij}^1 z_{ij}^1(1-z_{ij}^2)(1-z_{ij}^3) + C_{ij}^2(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3)\\ &+C_{ij}^3(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3 + C_{ij}^4 z_{ij}^1 z_{ij}^2(1-z_{ij}^3) + C_{ij}^5 z_{ij}^1(1-z_{ij}^2)z_{ij}^3\\ &+C_{ij}^6(1-z_{ij}^1)z_{ij}^2 z_{ij}^3 + C_{ij}^7 z_{ij}^1 z_{ij}^2 z_{ij}^3\}x_{ij}\\ &z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \ge 1\\ &x_{ij} \ge 0, \text{ and } z_{ij}^k = 0/1, \ \forall \ i \text{ and } j, \text{and } k = 1,2,3.\end{aligned}$$

Step 7: *When k=8*

We present the objective function (2.7) as follows:

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5, C_{ij}^6, C_{ij}^7, C_{ij}^8\}x_{ij}. \tag{3.44}$$

The cost coefficients of the objective function have eight choices as $\{C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4, C_{ij}^5, C_{ij}^6, C_{ij}^7, C_{ij}^8\}$, out of which one is to be selected. Since the total number of choices is $8 = 2^3$ so we need three binary variables $z_{ij}^1, z_{ij}^2, z_{ij}^3$. So we have formulated only one model as given below:

$$\begin{aligned}\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{&C_{ij}^1 z_{ij}^1 z_{ij}^2 z_{ij}^3 + C_{ij}^2(1-z_{ij}^1)z_{ij}^2 z_{ij}^3 + C_{ij}^3 z_{ij}^1(1-z_{ij}^2)z_{ij}^3\\ &+C_{ij}^4 z_{ij}^1 z_{ij}^2(1-z_{ij}^3) + C_{ij}^5(1-z_{ij}^1)(1-z_{ij}^2)z_{ij}^3 + C_{ij}^6 z_{ij}^1(1-z_{ij}^2)(1-z_{ij}^3)\\ &+C_{ij}^7(1-z_{ij}^1)z_{ij}^2(1-z_{ij}^3) + C_{ij}^8(1-z_{ij}^1)(1-z_{ij}^2)(1-z_{ij}^3)\}x_{ij}\\ &x_{ij} \ge 0, \text{ and } z_{ij}^k = 0/1, \ \forall \ i \text{ and } j, \text{and, } k = 1,2,3.\end{aligned}$$

4 Numerical Example

The numerical example is related to multi-choice stochastic transportation problem in which cost coefficients of the objective function are of multi-choices or goals and the demands and supplies are follow an exponential random variables. In order to show the application of the model, here we present an example of coal transportation problem. Coal is a kind of crucial energy sources in the development of economy and society. Accordingly, how to transport the coal from mines to the different areas economically is also an important issue for coal transportation. A reputed coal supply company transports the coal at the mines area at Asansol in Burdwan district, West Bengal, India from three supply points to the four destination centers at Kolkata, Kharagpur, Haldia and Bankura of West Bengal, India through twelve routes. The main purpose is to minimize the transportation cost and maximize the profit against the market price at different markets. The transportation cost of carrying one unit(100 Kg) of coal from sources to destinations each treated as of multi-choice parameters. Without using the multi-choice programming methodology, the problem can not be solved easily. Due to increasing the fuel price rate and road collection tax, the price rates of transportation costs in each routes are appended below:

x_{11} routes either 10 or 11 or 12 required admissible costs in Rupees.
x_{12} routes either 15 or16 required admissible costs in Rupees.
x_{13} routes either 20 or 21 or 22 or 23 required admissible costs in Rupees.
x_{14} routes either 15 or 16 or 17 required admissible costs in Rupees.
x_{21} routes either 12 or 13 or 14 or 15 or 16 required admissible costs in Rupees.
x_{22} routes either 10 or 11 or 12 or 13 or 14 or 15 required admissible costs in Rupees.
x_{23} routes either 9 or 10 or 11 required admissible costs in Rupees.
x_{24} routes either 18 or 19 required admissible costs in Rupees.
x_{31} routes either 20 or 21 or 22 or 23 or 24 or 25 or 26 required admissible costs in Rupees.
x_{32} routes either 9 or 10 or 11 or 12 or 13 or 14 or 15 or 17 required admissible costs in Rupees.
x_{33} routes either 24 or 25 or 26 required admissible costs in Rupees.
x_{34} routes either 27 or 28 required admissible costs in Rupees.

We formulate a multi-choice stochastic transportation problem where the objective function and the constraints are formulated as:

$$\min : z = \sum_{i=1}^{3}\sum_{j=1}^{4}\{C_{ij}^{1}, C_{ij}^{2}, \cdots, C_{ij}^{k}\}x_{ij}, \qquad k = 1, 2, \cdots, 8 \tag{4.45}$$

$$\text{subject to} \qquad \Pr\left(\sum_{j=1}^{4} x_{ij} \le a_i\right) \ge 1 - \gamma_i, \quad i = 1, 2, 3 \tag{4.46}$$

$$\Pr\left(\sum_{i=1}^{3} x_{ij} \ge b_j\right) \ge 1 - \delta_j, \quad j = 1, 2, 3, 4 \tag{4.47}$$

$$x_{ij} \ge 0,\ i = 1, 2, 3;\ j = 1, 2, 3, 4 \text{ and } 0 < \gamma_i < 1,\ 0 < \delta_j < 1, \forall\ i,\ j. \tag{4.48}$$

Assuming the means and variances of exponential random variables with specified probability levels of supplies i.e, a_i for $i = 1, 2, 3$ are represented in following Table 1.

Mean	Variance	Specified probability level
$E(a_1)=\theta_1=2$	$V(a_1)=4$	α_1=0.03
$E(a_2)=\theta_2=3$	$V(a_2)=9$	α_2=0.04
$E(a_3)=\theta_3=4$	$V(a_3)=16$	α_3=0.05

Table 1:

Again, the means and variances of the exponential random variables with specified probability levels of demands i.e, b_j for $j = 1, 2, 3, 4$ are represented in the following Table 2.

Mean	Variance	Specified probability level
$E(b_1)=\theta_1^{'}=5$	$V(b_1)=25$	β_1=0.06
$E(b_2)=\theta_2^{'}=6$	$V(b_1)=36$	β_1=0.07
$E(b_3)=\theta_3^{'}=7$	$V(b_3)=49$	β_3=0.08
$E(b_4)=\theta_4^{'}=8$	$V(b_4)=64$	β_4=0.09

Table 2:

Using the data provided in Table 1 and Table 2 the following deterministic multi-choice transportation problem is formulated as:

$$\begin{aligned} \min : z = {} & \{10,11,12\}x_{11} + \{15,16\}x_{12} + \{20,21,22,23\}x_{13} \\ & +\{15,16,17\}x_{14} + \{12,13,14,15,16\}x_{21} + \{10,11,12,13,14,15\}x_{22} \\ & +\{9,10,11\}x_{23} + \{18,19\}x_{24} + \{20,21,22,23,24,25,26\}x_{31} \\ & +\{9,10,11,12,13,14,15,17\}x_{32} + \{24,25,26\}x_{33} + \{27,28\}x_{34} \end{aligned} \tag{4.49}$$

$$\text{subject to} \quad \sum_{j=1}^{4} x_{1j} \leq 4.040541464 \tag{4.50}$$

$$\sum_{j=1}^{4} x_{2j} \leq 9.137762245 \tag{4.51}$$

$$\sum_{j=1}^{4} x_{3j} \leq 16.32879781 \tag{4.52}$$

$$\sum_{i=1}^{3} x_{i1} \geq 11.25364287 \tag{4.53}$$

$$\sum_{i=1}^{3} x_{i2} \geq 7.977780111 \tag{4.54}$$

$$\sum_{i=1}^{3} x_{i3} \geq 5.051457289 \tag{4.55}$$

$$\sum_{i=1}^{3} x_{i4} \geq 2.40794509 \tag{4.56}$$

$$x_{ij} \geq 0, \quad i = 1,2,3. \;, \; j = 1,2,3,4.$$

Now using a new transformation technique, we obtain the following multi-choice deterministic transportation problem:

$$
\begin{aligned}
\min: z &= t_{11}x_{11}+t_{12}x_{12}+t_{13}x_{13}+t_{14}x_{14}+t_{21}x_{21}+t_{22}x_{22}\\
&\quad +t_{23}x_{23}+t_{24}x_{24}+t_{31}x_{31}+t_{32}x_{32}+t_{33}x_{33}+t_{34}x_{34}\\
\text{subject to} &\quad (4.50)-(4.56)\\
t_{11} &= 10z_{11}^1z_{11}^2+11z_{11}^1(1-z_{11}^2)+12(1-z_{11}^1)z_{11}^2\\
t_{12} &= 15z_{12}^1+16(1-z_{12}^1)\\
t_{13} &= 20z_{13}^1z_{13}^2+21z_{13}^1(1-z_{13}^2)+22(1-z_{13}^1)z_{13}^2+23(1-z_{13}^1)(1-z_{13}^2)\\
t_{14} &= 15z_{14}^1z_{14}^2+16z_{14}^1(1-z_{14}^2)+17(1-z_{14}^1)z_{14}^2\\
t_{21} &= 12z_{21}^1(1-z_{21}^2)(1-z_{21}^3)+13(1-z_{21}^1)z_{21}^2(1-z_{21}^3)\\
&\quad +14(1-z_{21}^1)(1-z_{21}^2)z_{21}^3+15z_{21}^1z_{21}^2(1-z_{21}^3)+16(1-z_{21}^1)z_{21}^2z_{21}^3\\
t_{22} &= 10z_{22}^1(1-z_{22}^2)(1-z_{22}^3)+11(1-z_{22}^1)z_{22}^2(1-z_{22}^3)+12z_{22}^1z_{22}^2(1-z_{22}^3)\\
&\quad +13(1-z_{22}^1)(1-z_{22}^2)z_{22}^3+14z_{22}^1(1-z_{22}^2)z_{22}^3+15(1-z_{22}^1)z_{22}^2z_{22}^3\\
t_{23} &= 9z_{23}^1z_{23}^2+10z_{23}^1(1-z_{23}^2)+11(1-z_{23}^1)z_{23}^2\\
t_{24} &= 18z_{24}^1+19(1-z_{24}^2)\\
t_{31} &= 20(1-z_{31}^1)(1-z_{31}^2)(1-z_{31}^3)+21z_{31}^1(1-z_{31}^2)(1-z_{31}^3)\\
&\quad +22(1-z_{31}^1)z_{31}^2(1-z_{31}^3)+23(1-z_{31}^1)(1-z_{31}^2)z_{31}^3\\
&\quad +24z_{31}^1z_{31}^2(1-z_{31}^3)+25z_{31}^1(1-z_{31}^2)z_{31}^3+26(1-z_{31}^1)z_{31}^2z_{31}^3\\
t_{32} &= 9z_{32}^1z_{32}^2z_{32}^3+10(1-z_{32}^1)z_{32}^2z_{32}^3+11z_{32}^1(1-z_{32}^2)z_{32}^3\\
&\quad +12z_{32}^1z_{32}^2(1-z_{32}^3)+13(1-z_{32}^1)(1-z_{32}^2)z_{32}^3+14z_{32}^1(1-z_{32}^2)(1-z_{32}^3)\\
&\quad +15(1-z_{32}^1)z_{32}^2(1-z_{32}^3)+17(1-z_{32}^1)(1-z_{32}^2)(1-z_{32}^3)\\
t_{33} &= 24z_{33}^1z_{33}^2+25z_{33}^1(1-z_{33}^2)+26(1-z_{33}^1)z_{33}^2\\
t_{34} &= 27z_{34}^1+28(1-z_{34}^1)\\
&\quad 1\le z_{11}^1+z_{11}^2\le 2\\
&\quad 1\le z_{14}^1+z_{14}^2\le 2\\
&\quad 1\le z_{21}^1+z_{21}^2+z_{21}^3\le 2\\
&\quad z_{21}^1+z_{21}^2\le 1\\
&\quad 1\le z_{22}^1+z_{22}^2+z_{23}^3\le 2\\
&\quad 1\le z_{23}^1+z_{23}^2\le 2\\
&\quad z_{31}^1+z_{31}^2+z_{31}^3\le 2\\
&\quad 1\le z_{33}^1+z_{33}^2\le 2\\
&\quad x_{ij}\ge 0,\quad i=1,2,3 \text{ and } j=1,2,3,4.
\end{aligned}
$$

5 Result and Discussion

The above mathematical programming model is treated as a non-linear mixed integer programming problem which solved by Lingo10 package. The optimal solution of the mathematical model is obtained as: x_{11}= 1.632596, x_{14}= 2.407945, x_{21}=4.086305, x_{23}= 5.051457, x_{31}= 5.534742, x_{32}= 7.977780, where rest of the decision variables are zero. The minimum cost of the objective function is 329.4388. For the optimal value of the objective function, the multi-choice cost coefficients are obtained as follows:

x_{ij}	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}
value of C_{ij}^k	10	15	20	15	12	10	9	18	20	9	25	27

If the decision variables $x_{ij},\ i = 1,2,3;\ j = 1,2,3,4$ are considered to be integers, the optimal solution can be obtained as: $x_{11}= 1,\ x_{14}= 3,\ x_{21}= 3,\ x_{23}= 6,\ x_{31}= 8,\ x_{32}= 8$ where rest of the decision variables are zero. The minimum cost of the objective function is 377. For the optimal value of the objective function, the multi-choice cost coefficients are obtained as follows:

x_{ij}	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}
value of C_{ij}^k	10	15	20	15	12	12	9	18	20	9	26	27

Introduction of binary variables is an important concept in multi-choice programming for selection of one choice from the set of multi-choices. In this paper, we have formulated the proposed model by using the auxiliary constraints which contain the binary variables. The number of binary variables for each choices or goals are depend on the relation $\frac{\ln(k_i)}{\ln(2)}$, where k_i is the number of choices or goals. When $k_i = 2,$ or 4, or 8 there is no need of auxiliary constraint to construct of our proposed model. When $k_i = 3,$ or 7 only one auxiliary constraint associated with binary variables is needed for construction of our model. When $k_i = 5,$ or 6, then there is need of 3 and 2 auxiliary constraints respectively. Depending on the number of choices or goals we get different models of our proposed problem and solving all these models, we have obtained the same optimal solution. Hence it is necessary to solve only one model instead of different models.

6 Conclusion

The aim of this paper is to present the solution procedure for multi-choice stochastic unbalanced transportation problem with consideration of exponential random variable. Initially, we have transformed all the probabilistic constraints into an equivalent deterministic constraints by stochastic programming approach and then we have used a transforation technique in such a way that the combination of choices for each coefficients of objective function should provide an optimal solution to our model.

In the real life transportation problem, the cost coefficients of the objective function and supply and demand may not be known previously due to uncountable factors. For this reason, the cost coefficients of the objective function are of multi-choice rather than by single choice and supply and demand are followed random variables. In our paper we have formulated the transportation model by considering both the factors. Finally we have concluded that the formulated model is highly applicable for these types of real life transportation problem and solving this model, the decision maker has provided more information for taking the right decision.

A further study is needed for multi-objective multi-choice interval valued transportation problem under stochastic environment and the number of choices or goals also can be extended.

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