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When to Let in Late Students?

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Abstract

Some students are late for classes. If we let in these late, this disrupts the class and decreases the amount of effective lecture time for the students who arrived on time. On the other hand, if many students are late and we do not let them in, these students will miss the whole lecture period. It is therefore reasonable to sometimes let students in, but restrict the times when late students can enter the class. In this paper, we show how, depending on the number of late students – and depending on how late they are –we can find the optimal schedule of letting in late students.

Keywords: late students, optimal scheduling

1 Formulation of the Problem

Letting in late students is disruptive. Some students are late for class. Letting them walk in all the time disrupts others. As a result, some teachers in schools and even some professors at the universities do not let late students in at all.

Comment. This is not only about classes. The famous Russian theater reformer Stanislasvky started his reform by not letting late spectators in – and thus, minimizing disruptions for others. This tradition is held in many theaters now.

Not letting in late students is probably too harsh. On the other hand, such a no-late policy may be too harsh, especially if we take into account that lateness is often caused by things beyond a student's control – e.g., on a commuter campus like ours, an accident on a freeway that caused traffic delays make students arrive late to their first class of the day.

Resulting problem: when to let in late students? Based on the above discussion, we conclude that:

- in principle, it is desirable to let late students in, but
- we cannot let them in all the time.

So, we should select specific times when the students will be allowed to enter.

How this problem is solved now? Sometimes, these times are determined by the event. For example, in a symphony concert, late patrons have to wait for the end of the first musical piece to enter. What shall we do in a lecture where there are no such easily determined least-disruption times?

There are many heuristic ways of dealing with such situations. For example, a recent recollection volume by students from the Mathematical Department of St. Petersburg University, Russia, mentions that some professors teaching big Calculus classes would ask late students to wait until it is exactly 10 minutes after the beginning of the lecture, and let in all accumulated late students [1].

Need for an optimal solution. Instead of relying on such heuristic rules, it is desirable to come up with a precise solution to the problem – a solution obtained by optimizing an appropriately chosen objective function.

This is what we do in this paper.

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2 Formalization and Analysis of the Problem

Towards formalization of the problem. Let T be the duration of the lecture, and let Δ denote the disruption time cause by letting late students in. For every real number $t \in [0, T]$, Let $f(t)$ denote the total proportion of students who arrive between the beginning of the lecture and time t after the lecture started.

Our objective is to minimize the total disruption, i.e., minimize the disruption time per student.

When we do not let in late students. If we do not let any late students in, then the only disruption comes from late students missing the class. Each of the late students missed time T, and the proportion of late students is $f(T)$. Thus, the resulting disruption per student is

$$D_0 = T \cdot f(T).$$

When we let in late students at one single moment of time. If we let in late students at a single moment of time t_1, then there are three sources of disruption:

- there is a disruption $\Delta \cdot (1 - f(T))$ (caused by letting students in) for all students who arrived on time;
- there is a disruption $t_1 \cdot f(t_1)$ caused by the fact that students who arrive between times 0 and t_1 miss time t_1;
- finally, there us a disruption $T \cdot (f(T) - f(t_1))$ caused by the fact that students who arrive after moment t_1 miss the whole lecture.

The resulting overall disruption is equal to

$$d_1(t_1) = \Delta \cdot (1 - f(T)) + t_1 \cdot f(t_1) + T \cdot (f(T) - f(t_1)).$$

The time t_1 should be selected from the condition that the resulting overall disruption is the smallest possible. For such an optimal value t_1, the resulting disruption is equal to

$$D_1 = \min\{\Delta \cdot (1 - f(T)) + t_1 \cdot f(t_1) + T \cdot (f(T) - f(t_1)) : 0 \le t_1 \le T\}.$$

The optimal value t_1 can be determined by the condition that the derivative of the minimized function is equal to 0, i.e., that

$$f(t_1) + t_1 \cdot f'(t_1) - T \cdot f'(t_1) = 0,$$

where $f'(t)$ denoted the derivative of the function $f(t)$. This condition can be equivalently reformulated as

$$f(t_1) = (T - t_1) \cdot f'(t_1).$$

Shall we let in students or shall we not? Whether we should let in students at all or not depends on whether $D_1 = d_1(t_1) < D_0$. The corresponding inequality has the form

$$\Delta \cdot (1 - f(T)) + t_1 \cdot f(t_1) + T \cdot (f(T) - f(t_1)) < T \cdot f(T),$$

i.e., equivalently,

$$\Delta \cdot (1 - f(T)) + t_1 \cdot f(t_1) < T \cdot f(t_1),$$

which is, in turn, equivalent to

$$\Delta \cdot (1 - f(T)) < (T - t_1) \cdot f(t_1).$$

General case. Let us now consider the general case, in which we let students in at several ($k \geq 0$) moments of time $0 < t_1 < t_2 < \ldots < t_k < T$. To simplify the description of this inequality, it makes sense to set $t_0 = 0$ and $t_{k+1} = T$, then this inequality has the form

$$0 = t_0 < t_1 < t_2 \ldots < t_k < t_{k+1} = T.$$

For every i from 0 to k, students who arrive between times t_i and t_{i+1} lose time t_{i+1} – the next time late students are let in. The proportion of such students is $f(t_{i+1}) - f(t_i)$, so the disruption for all these students is equal to $t_{i+1} \cdot (f(t_{i+1} - f(t_i))$. To those students who have already been sitting in class by the time of the i-th disruption – their proportion is $1 - f(T) + f(t_{i-1})$ – the disruption is equal to $\Delta \cdot (1 - f(T) + f(t_{i-1}))$. Thus, the overall disruption is equal to

$$d_k(t_1, \ldots, t_k) = \sum_{i=1}^{k} \Delta \cdot (1 - f(T) + f(t_{i-1})) + \sum_{i=0}^{k} t_{i+1} \cdot (f(t_{i+1} - f(t_i)).$$

The times $t_1, \ldots, t_k$ should be selected from the condition that the resulting disruption is the smallest possible. For such optimal values $t_1, \ldots, t_k$, the resulting disruption is equal to

$$D_k = \min \left\{ \sum_{i=1}^{k} \Delta \cdot (1 - f(T) + f(t_{i-1})) + \sum_{i=0}^{k} t_{i+1} \cdot (f(t_{i+1} - f(t_i)) : 0 < t_1 < t_2 < \ldots < T \right\}.$$

The optimal values $t_1, \ldots, t_i, \ldots, t_k$ can be determined by the condition that the partial derivative of the minimized function with respect to each variable t_i, $1 \leq i \leq k$, is equal to 0. For $i < k$, we get

$$\Delta \cdot f'(t_i) + (f(t_i) - f(t_{i-1}) + t_i \cdot f'(t_i) - t_{i+1} \cdot f'(t_i) = 0.$$

This condition can be equivalently reformulated as

$$f(t_i) - f(t_{i-1}) = (t_{i+1} - t_i - \Delta) \cdot f'(t_i).$$

For $i = k$, we similarly get

$$f(t_k) - f(t_{k-1}) = (T - t_k) \cdot f'(t_k).$$

Whether we should let students in at k different moments of time depends on whether $D_k \leq D_\ell$ for all $\ell \neq k$.

As a result, we arrive at the following solution.

3 When to Let Late Students in: Solution

General solution. For each integer k, we find values $0 = t_0 < t_1 < t_2 < \ldots < t_k < t_{k+1} = T$ for which the expression

$$\sum_{i=1}^{k} \Delta \cdot (1 - f(T) + f(t_{i-1})) + \sum_{i=0}^{k} t_{i+1} \cdot (f(t_{i+1} - f(t_i))$$

is the smallest possible. This can be done, e.g., by solving the following system of equations:

$$f(t_i) - f(t_{i-1}) = (t_{i+1} - t_i - \Delta) \cdot f'(t_i), \ i < k;$$

$$f(t_k) - f(t_{k-1}) = (T - t_k) \cdot f'(t_k).$$

Let us denote the corresponding smallest value of the minimized expression by D_k.

Then, we select k for which the value D_k is the smallest possible, and for this k, take the corresponding minimizing values $t_1, \ldots, t_k$. These are the times at which we let late students in.

Comment. When k increases, the second term in the optimization function tends to the Stiltjes integral $\int t \cdot df(t)$ describing the overall disruption in the case when every late student is let in right away.

Example. Let us illustrate the above idea on the example when the students arrive uniformly, i.e., when $f(t) = f_0 \cdot t$ for some f_0. In this case, the above equation for determining t_i for i_k takes the form

$$f_0 \cdot (t_i - t_{i-1}) = (t_{i+1} - t_i - \Delta) \cdot f_0,$$

i.e., equivalently, that $\Delta t_i \stackrel{\text{def}}{=} t_{i+1} - t_i$ satisfies the condition $\Delta t_i = \Delta t_{i-1} + \Delta$.

In other word, in this case, if there are several moments of time when we let students in, the waiting time before each letting-in increases by Δ from the previous waiting time.

For $i = k$, we similarly conclude that $\Delta t_k = \Delta t_{k-1}$.

In this case, $\Delta t_i = \Delta t_0 + i \cdot \Delta$ for $i < k$, in particular, $\Delta t_{k-1} = \Delta t_0 + (k-1) \cdot \Delta$. Thus, $\Delta t_k = \Delta t_{k-1}$ implies that $\Delta t_k = \Delta t_0 + (k-1) \cdot \Delta$.

We can now express the optimal values t_i in terms of the difference Δt_i. Indeed, since $t_0 = 0$, we have

$$t_i = t_i - t_0 = (t_i - t_{i-1}) + (t_{i-1} - t_{i-2}) + \ldots + (t_1 - t_0) = \Delta t_{i-1} + \Delta t_{i-2} + \ldots + \Delta t_0$$

$$= \Delta t_0 + (i-1) \cdot \Delta + \Delta t_0 + (i-2) \cdot \Delta + \ldots + \Delta t_0 = i \cdot \Delta t_0 + \Delta \cdot (1 + 2 + \ldots + (i-1)),$$

hence

$$t_i = i \cdot \Delta t_0 + \Delta \cdot \frac{(i-1) \cdot i}{2}.$$

To complete our description of the optimal schedule corresponding to the given number k of letting students in, we need to determine the value Δt_0. This value can be determined from the fact that

$$\Delta t_k = T - t_k = \Delta_0 + (k-1) \cdot \Delta.$$

From the above formula, we know that

$$t_k = k \cdot \Delta t_0 + \Delta \cdot \frac{(k-1) \cdot k}{2}.$$

Thus, we conclude that

$$T = t_k + \Delta t_k = (k+1) \cdot \Delta_0 + \left(\frac{(k-1) \cdot k}{2} + (k-1) \right) \cdot \Delta$$

$$= (k+1) \cdot \Delta_0 + \frac{(k-1) \cdot (k+2)}{2} \cdot \Delta.$$

So, we have

$$\Delta t_0 = \frac{T - \dfrac{(k-1) \cdot (k+2)}{2} \cdot \Delta}{k+1}.$$

Substituting the resulting optimal values t_i into the corresponding expression for $d_k(t_1, \ldots, t_k)$, we can find the value D_k for each k and thus, find the optimal number of disruptions k.

References

[1] Epstein, D., Y. Shapiro, and S. Ivanov (eds.), *Mathematics Department of St. Petersburg University*, St. Petersburg, Russia, 2011 (in Russian).