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How Accurately Should We Write on the Board? When Marking Comments on Student Papers?

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Abstract

Writing on the board is an important part of a lecture. Lecturers' handwriting is not always perfect. Usually, a lecturer can write slower and more legibly, this will increase understandability but slow down the lecture. In this paper, we analyze an optimal trade-off between speed and legibility.

Keywords: handwriting, optimal writing speed

1 Formulation of the Problem

People's handwriting is usually not perfect. Most people can write in a better (and more legible) handwriting if they make a special effort and write slower. Our handwritten notes to selves are usually less legible to others than notes to others. An extreme example is a difference between cursive – whose purpose is to make writing faster – against block letters which take longer to write but which make the text more legible.

Teachers and professors use a lot of handwriting:

- when writing on the board when lecturing,
- when answering individual student questions during office hours,
- when writing comments on the student papers, etc.

What is the appropriate degree of handwriting accuracy in each of these cases?

A natural objective for selecting the proper degree of accuracy is to minimize the total effort of the writer *and* of the readers, an effort weighted by the importance of the writer's and the readers' time for the society.

2 Analysis of the Problem

What effort do we need to hardwrite with a given accuracy: analysis of the problem. The inaccuracy of handwriting can be naturally described as noise added to the ideal writing result. In other words, at each moment of time, the actual position of the writing hand slightly differs from its ideal position. Let δt be the time quantum, i.e., the smallest period of time for which the the noise at each moment t and the noise at the next moment $t + \delta t$ are statistically independent.

Every small piece of a letter – that appears as a result of this writing – comes from averaging these positions over the time period that it takes to write this piece of the letter. During the time t that it takes to write a piece of the letter, we average $t/\delta t$ independent noise signals.

It is well known in statistics that if we take an average of n independent identically distributed random variables, then the standard deviation decreases by a factor of $\sqrt{n}$. Thus, the standard deviation (inaccuracy) σ of the resulting writing is equal to

$$\sigma = \frac{\sigma_0}{\sqrt{t/\delta t}},$$

where σ_0 is the standard deviation of the original noise.

It should be mentioned that the value σ_0 may be different for different people:

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- some people write fast and still very accurately;
- for others, accurate handwriting is only possible when they write very slowly.

Let us find out how this accuracy depends on the speed with which we write. Let v_w be the speed of writing, i.e., the number of symbols per unit time that results from this writing. Let n be the average number of pieces contained in a symbol. Then, during a unit time, we write v_w symbols and thus, $n \cdot v_w$ pieces of symbols. So, the time t of writing one piece of a symbol is equal to $t = 1/(n \cdot v_w)$. Substituting this expression into the above formula for σ, we conclude that

$$\sigma = \frac{\sigma_0}{\sqrt{(1/n \cdot v_w) \cdot \delta t}} = \sigma_0 \cdot \sqrt{\frac{n}{\delta t}} \cdot \sqrt{v_w}.$$

Thus, we arrive at the following conclusion:

What effort do we need to hardwrite with a given accuracy: result. If we write with a speed v_w, then the accuracy (standard deviation) of a person's handwriting is equal to $\sigma = c \cdot \sqrt{v_w}$, where the constant c depends on the individual writer.

What effort do we need to understand a handwriting: analysis of the problem. To understand the handwriting, we do not need to observe every single part of each symbol, it is sufficient to understand the symbol's general shape. For example, we usually, when we read a text, do not notice the details of the font – unless, of course, we are specifically paying attention to the font.

In other words, instead of reading every single pixel that forms a symbol, we only pay attention to a few pixels from this symbol. The speed with which we can understand the text depends on the number of pixels that we need to read in order to understand the symbol properly.

Within each part of the symbol (e.g., a linear part of a circular part), we read several pixels, and then take an average of their values to get the general impression about this part. The part is well recognized when the standard deviation of this average is smaller than (or equal to) some threshold value σ_t. If we read v_r symbols per unit time, this means that it takes time $1/v_r$ to read each symbol and thus, time $1/(v_r \cdot n)$ to read each part of the symbol. Let t_p be the time for reading one pixel. This means that for each part of the symbol, we read $n_p = 1/(v_r \cdot n \cdot t_p)$ pixels.

Each pixel is written with standard deviation σ, so when we average over n_p symbols, we get standard deviation

$$\sigma_r = \frac{\sigma}{\sqrt{n_p}} = \sigma \cdot \sqrt{v_r} \cdot \sqrt{n \cdot t_p}.$$

Substituting the above expression for σ in terms of the writing speed v_w, we conclude that

$$\sigma_r = c \cdot \sqrt{v_w} \cdot \sqrt{v_r} \cdot \sqrt{n \cdot t_p}.$$

The maximum reading speed is determine by the condition that this standard deviation is equal to the threshold value σ_t, i.e., that

$$\sigma_r = c \cdot \sqrt{v_w} \cdot \sqrt{v_r} \cdot \sqrt{n \cdot t_p},$$

hence the reading speed v_r can be found as

$$v_r = \frac{c^2 \cdot n \cdot t_p}{\sigma_t^2} \cdot \frac{1}{v_w}.$$

The coefficient at $1/v_w$ depends on the parameter c that describes the accuracy of the writer. Thus, this whole coefficient can be viewed as a parameter describing the writer's accuracy. Thus, we arrive at the following conclusion.

What effort do we need to understand a handwriting: result. The maximal speed v_r with which we can efficiently read a text written with speed v_w is equal to

$$v_r = \frac{C}{v_w},$$

where the parameter C describer the writer's accuracy.

How to describe efforts. Our objective is to find the writing speed v_w for which the overall effort E is the smallest. The overall efforts consists of the writer's effort and the reader's effort.

To write a text consisting of N symbols, the writer need time N/v_w. Let s_w be the societal value of one time unit of the writer – it can be gauged, e.g., by the writer's salary. Then, the total effort used by the writer is $s_w \cdot (N/v_w)$.

Let N_r be the number of intended readers of this text, and let s_r be the average societal value of the reader's time. Each reader needs time N/v_r to read all N symbols, so the total effort of all the readers is $N_r \cdot s_r \cdot (N/v_r)$. Substituting the above formula for v_r in terms of v_w, we conclude that the readers' effort is equal to $N_r \cdot s_r \cdot N \cdot (v_w/C)$. Thus, the overall effort E is equal to

$$E = s_w \cdot \frac{N}{v_w} + N_r \cdot s_r \cdot N \cdot \frac{v_w}{C}.$$

3 Solution to the Problem

Formula describing the optimal writing speed. To minimize the total effort E with respect to v_w, we differentiate E relative to v_w and equate the resulting derivative to 0. As a result, we get

$$-s_w \cdot \frac{1}{v_w^2} + N_r \cdot s_r \cdot N \cdot \frac{1}{C} = 0,$$

hence

$$v_w = \sqrt{\frac{s_w \cdot C}{s_r \cdot N_r}}.$$

Here, s_w and s_r are the societal value of the writer and the reader, respectively, N_r is the number of intended readers, and C is a constant that describes the writer's handwriting ability, the constant that can be determined from the formula $v_r = C/v_w$ that describes the dependence of the reading speed v_r on the speed v_w with which the text was handwritten.

Discussion. The slower the handwriting, the larger the effort. Thus, the formula leads to the following conclusions:

- The more important the writer and the less important the reader, the less he or she need to try to write more accurately. For example, a student handwriting a homework should use more effort in his or her handwriting than an instructor grading this homework.
- The more accurate the writer, the less effort he or she needs to spend writing.
- The larger the reading audience, the more effort the writer needs when writing. For example, the writing on a board when lecturing should be more accurate than a writing comments on an individual homework; when lecturing, the larger the class, the more accurate should be the handwriting.

References

[1] Suddath, C., Morning the death of handwriting, *Time Magazine*, August 03, 2009.